

<b>Name:</b>		<b>Index Number:</b>		<b>Class:</b>	
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## DUNMAN HIGH SCHOOL Preliminary Examination Year 6

MATHEMATICS (Higher 1)

8865/01

Paper 1

**September 2018**

**3 hours**

Additional Materials:      Answer Paper  
    List of Formulae (MF26)

### READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on the work you hand in.  
 Write in dark blue or black pen on both sides of the paper.  
 You may use an HB pencil for any diagrams or graphs.  
 Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

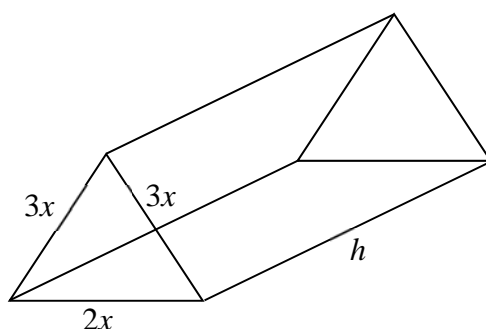
*For teachers' use:*

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Total
<b>Score</b>													
<b>Max Score</b>	<b>4</b>	<b>5</b>	<b>9</b>	<b>9</b>	<b>13</b>	<b>3</b>	<b>6</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>12</b>	<b>12</b>	<b>100</b>

## Section A: Pure Mathematics [40 marks]

- 1 Show algebraically that for all real non-zero  $k$ , the line  $y - kx = 1$  intersects the curve  $y^2 - 3x - 2 = 0$  at two distinct points. [4]
- 2 (i) Differentiate  $\sqrt{(e^{2x} + 1)^3}$  with respect to  $x$ . [2]
- (ii) Hence find the exact value of  $\int_0^{\ln \sqrt{3}} e^x \sqrt{(e^{4x} + e^{2x})} dx$ , simplifying your answer. [3]
- 3 The curve  $C$  has equation  $y = \frac{2}{x^2 - a^2}$ , where  $a > 0$ .
- (i) Sketch the graph of  $C$ , stating the equations of any asymptotes and coordinates of any point(s) where the curve crosses the axes. [4]
- (ii) State the set of values of  $x$ , in terms of  $a$ , for which  $y$  is increasing. [2]
- (iii) The line with equation  $y = -1$  cuts  $C$  at the points  $P$  and  $Q$ . Given that the length of  $PQ$  is 2 units, find the value of  $a$ . [3]

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The edges of a hollow prism are made of aluminium while the rectangular and triangular surfaces are made of acrylic, as shown in the diagram. The triangular surfaces are isosceles with sides  $3x$  cm and base  $2x$  cm. The length of the prism is  $h$  cm and the volume of the prism is fixed at  $20 \text{ cm}^3$ .

- (i) Show that  $h = \frac{5\sqrt{2}}{x^2}$ . [2]
- (ii) The cost of aluminium and acrylic are 80 cents per cm and \$1.20 per  $\text{cm}^2$  respectively. Show that the total cost,  $\$C$ , of constructing the prism is given by
- $$C = 4.8\sqrt{2}x^2 + 12.8x + \frac{48\sqrt{2}}{x} + \frac{12\sqrt{2}}{x^2}. \quad [3]$$
- (iii) Use differentiation to find the value of  $x$  that gives the minimum cost of constructing the prism, leaving your answer correct to 2 decimal places. [4]

- 5 (a) It is given that the curve has equation  $\ln y = ax^2 + bx + c$ , where  $a$ ,  $b$  and  $c$  are constants. The curve passes through the point with coordinates  $\left(\frac{7}{2}, e^{-1}\right)$  and has a turning point at  $\left(\frac{5}{2}, 1\right)$ . Find the values of  $a$ ,  $b$  and  $c$ . [4]
- (b) A company produces  $x$  number of limited edition toys for sale on a weekly basis, where  $0 \leq x \leq 170$ . The manager wants to investigate the profitability of the weekly sale. For producing that number of toys, he found that the weekly cost,  $\$C$ , in hundreds of dollars, is given by the equation  $C = \frac{x^2}{50} + 30\ln(x+1)$  and the weekly revenue,  $\$R$ , in hundreds of dollars, is given by the equation  $R = 5x$ . The company's weekly profit is given by  $\$P$ , in hundreds of dollars.
- (i) Find an expression for  $P$  in terms of  $x$ . [1]
- (ii) Sketch the graph of  $P$  against  $x$ , labelling the coordinates of the end points. [2]
- (iii) Use your calculator to find the value of  $x$  that will enable the company to earn the maximum profit. Hence state the maximum profit, correct to the nearest dollar. [2]
- (iv) Find the range of values of  $x$  such that the company will achieve a weekly profit of more than \$10 000. [2]
- (v) Determine the value of  $x$  when the rate of change of  $C$  is the least as  $x$  varies. [2]

**Section B: Probability and Statistics [60 marks]**

- 6 The length of a string is a random variable with mean 15 cm and standard deviation  $\sqrt{52}$  cm. A random sample of 30 strings is taken. Find the probability that the sample mean length lies between 180 mm and 270 mm. [3]
- 7 Ken has a mobile phone which allows him to set a password consisting of 5 characters. The characters are to be chosen from  $\{1, 2, 3, 4, 5, 6, 7, A, B, C, D, E\}$ .
- (i) Find the number of possible passwords if repetitions are not allowed. [1]
- (ii) Ken has set a password which he could not recall. However, he is certain that he uses 3 distinct digits and 2 distinct letters. He attempts to recall the password. Find the maximum number of failed possible attempts he need to make before he can recall the password correctly. [2]
- (iii) Suppose that repetitions are allowed. Find the probability that a password chosen at random contains the digit 6 exactly twice and only the last 2 characters are letters. [3]
- 8 An interactive simulation ride allows a group of 5 riders to take the ride at a time. The ride time,  $X$  minutes, follows a normal distribution with mean  $\mu$  minutes and standard deviation 2 minutes. The ride starts promptly at 10 am daily with no wait time between any groups of 5 riders. There are only 4 scheduled rides every morning. At 10 am on a particular morning, there are already 20 people queuing for the ride. It is assumed that all the people in the queue will take the ride based on the sequence of the queue and the ride times are independent.
- (i) Show that  $\mu = 14$ , correct to the nearest integer, if  $P(\mu < X < 16) = 0.35$ . [2]
- For the rest of the question, use  $\mu = 14$  for your calculations. A ride is considered long if it has a ride time of at least 15 minutes.
- (ii) Find the probability that the 12<sup>th</sup> person in the queue took the ride before 10.30 am on that morning. [2]
- (iii) Show that the probability of having at least 2 long rides on that morning is 0.363. [2]
- (iv) Given that there are at least 2 long rides on that morning, find the probability that none of these long rides are consecutive. [2]
- 9 The events  $A$  and  $B$  are such that  $P(A' \cap B) = 0.3$  and  $P(A | B) = 0.2$ . It is given that  $A$  and  $B$  are independent.
- (i) Find  $P(A)$  and  $P(B)$ . [4]
- (ii) Find the probability that either  $A$  or  $B$  or both occurs. [2]
- (iii) Find  $P(B' | A)$ . [2]
- (iv) State, with a reason, whether  $A$  and  $B$  are mutually exclusive. [1]

- 10** To study the recent relationship between the property price index,  $p$  (in %) and the stock index,  $s$  (in thousands) of a particular city, Hilton recorded the readings from each of the past 8 quarters in the table below.

Stock Index, $s$ (thousands)	2.12	2.98	2.87	2.70	2.75	2.63	2.83	2.53
Property Price Index, $p$ (%)	105	170	155	140	146	160	150	130

Hilton realised that he has recorded one of the values of  $p$  incorrectly.

- (i) Sketch a scatter diagram for the data and circle the erroneous point  $X$  on your diagram. [2]

[For the remaining parts of this question, you should exclude the point  $X$ .]

- (ii) Find the product moment correlation coefficient and comment on its value in the context of the question. [2]
- (iii) Find the equation of the regression line of  $p$  on  $s$ , in the form  $p = a + bs$ , giving the values of  $a$  and  $b$  correct to 3 significant figures. Sketch this line on your scatter diagram. [2]
- (iv) Assuming that the value of  $s$  at  $X$  is correct, estimate the corresponding value of  $p$ . Give two reasons why you would expect this estimate to be reliable. [3]

Hilton concludes that higher stock index will lead to higher property price index. Comment on his conclusion in the context of the question. [1]

- 11** A test consists of 15 multiple choice questions, where each question has  $n$  possible options, of which only one is correct. A student took the test by randomly choosing the answer to each question. It is known that the probability of answering exactly 3 questions correctly is the same as the probability of answering exactly 4 questions correctly.

- (i) Show that  $n = 4$ . [3]

Each correct answer is awarded 3 marks and each incorrect answer carries a penalty of 1 mark. The score is the total marks awarded based on the number of correct and incorrect answers.

- (ii) Find the expected score,  $s$ , obtained by the student. [3]
- (iii) Find the probability that the score obtained by the student is within 4 marks of  $s$ . [2]

There are now  $k$  students who took the test by randomly choosing the answer to each question, where  $k \geq 20$ . Given the probability that the average number of correct answers obtained is less than 4 is at least 0.9, find the least possible value of  $k$ . [4]

- 12** The speeds of cars along a busy stretch of road follow a normal distribution. Studies show that a mean speed of 50 km/h is needed to ensure a smooth flow of traffic. When the mean speed falls below 50 km/h, it may lead to road congestion. If this happens, Wireless Road Pricing (WRP) will be used to charge motorists to discourage them from using the road, hence improving the traffic condition. A random sample of the speeds,  $x$  km/h, of 120 cars along the stretch of road is recorded and the data are summarised by

$$\sum x = 5415, \quad \sum x^2 = 351500.$$

- (i) Calculate unbiased estimates of the population mean and variance of the speeds of the cars. [2]
- (ii) What do you understand by the term ‘unbiased estimate’? [1]
- (iii) Test at the 3% significance level whether WRP is needed. [4]

The Road Transport Authority decides to implement WRP on the same stretch of road. After the implementation, the speeds of a second sample of 80 cars are recorded with a mean speed of 60 km/h and a variance of 1100 (km/h)<sup>2</sup>.

- (iv) A test at the 8% significance level of the second sample suggests that the mean speed has increased beyond  $\mu_0$ . Use an algebraic method to find the maximum value of  $\mu_0$ . [4]
- (v) State one assumption used in obtaining the sample statistics for the second sample. [1]