General Certificate of Education Advanced Level

Higher 1

JC2 Preliminary Examination

MATHEMATICS **8864/01**

Paper 1 **24 August 2016**

3 hours

Additional Materials: List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, arrange your answers in **NUMERCAL ORDER**.
Place the cover sheet given in front and fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

Name: ____________________________ **Class:** ________________

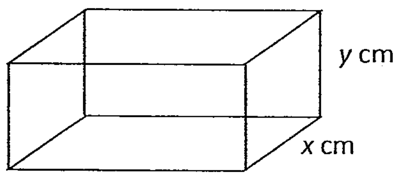
Section A: Pure Mathematics [35 marks]

1. The curve C has equation $y = \frac{2-x}{x-3}$.

   (i) Sketch the graph of C, stating the exact coordinates of the points of intersection with the axes and the equations of the asymptotes. [3]

   (ii) Find the set of values of k for which C and the line $y = kx - 1$, where $k > 0$, have two distinct points of intersection. [3]

2. The diagram shows a rectangular box with breadth and height given by x cm and y cm respectively. A piece of wire, P cm long, is used to make the twelve edges of the rectangular box. Given that the box has a fixed volume of 24000 cm^3 and its length is twice its breadth,



   (i) show that $P = 12x + \frac{48\,000}{x^2}$, [2]

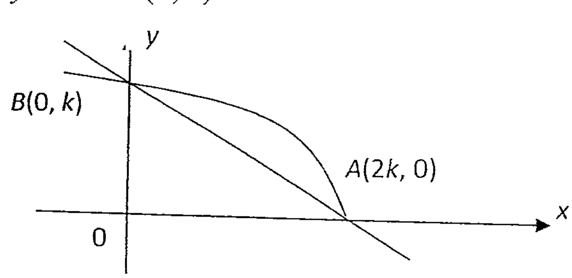
   (ii) find, as x varies, the stationary value of P and determine if this value is a maximum or a minimum. [4]

3. **(a)** Differentiate $\ln\left(\frac{20x}{10-x}\right)$ with respect to x, where $0 < x < 10$. [2]

   (b) Find $\frac{d}{dx}\left(\frac{1}{1+x^2}\right)$ and hence evaluate $\int_0^2 \frac{x}{(1+x^2)^2}\,dx$. [3]

   (c) Find the exact value of constant a, where $a \neq 0$, such that $\int_1^a (e^{1-x} + \frac{1}{x^2})\,dx = -\frac{1}{a}$. [2]

4. The diagram shows the curve C with the equation $y = \sqrt{2k - x}$, where k is a constant, and the straight line L passing through the points A and B. The curve C and the line L both meet the x-axis at $A(2k, 0)$ and the y-axis at $B(0, k)$.



   (i) Show that the equation of line L is given by $y = -\frac{1}{2}x + k$. [1]

   (ii) Find, in terms of k, the area of the region bounded by C and L. [3]

   (iii) Hence, or otherwise, find the value of k if $\frac{\text{Area of } D}{\text{Area of } E} = \frac{4}{3}$ where D is the region bounded by the curve $y = \sqrt{2k - x}$ and the x- and y-axes, and E is the region bounded by L and the x- and y-axes. [3]

5. **(i)** Sketch the graph of curve C with equation $y = e^{3-2x} + 7x - 7$, stating the exact coordinates of the points of intersection with the y-axis. [2]

(ii) Given that the gradient of tangent to C at the point A is 5, show that the x-coordinate of A is $\frac{3}{2}$. [2]

(iii) Find the equation of the normal to C at A. [3]

The normal to C at A meets the y-axis at B.

(iv) Find the length of AB. [2]

Section B: Statistics [60 marks]

6. A fast food company MadRonald has 9000 employees in Singapore. For each employee, the mode of transport to work is either public transport or privately owned vehicle. The number of employees using each mode of transport are recorded according to age in years, and the data is summarised as follows.

	Public Transport	Privately Owned Vehicle
Under 30 years	2000	400
30-65 years	1500	2500
Over 65 years	1000	1600

The Chief Executive Officer would like to investigate the travelling time to work of his employees. He decides to obtain a random sample of 200 employees.

(i) Describe how he might obtain a stratified sample, identifying the strata and finding the size of the sample taken from each strata. [3]

His assistant suggested using systematic sampling to obtain the sample of 200 employees.

(ii) State, with a reason, which sampling method is more suitable in this context. [1]

7. Events A and B are such that $P(A' \cap B') = 0.25$ and $P(A \mid B) = P(B \mid A) = 0.5$.

(i) Find $P(A \cup B)$. [1]

(ii) Show that $P(A) = P(B)$. [1]

(iii) Hence or otherwise, find $P(A)$ and $P(A \cap B)$. [3]

(iv) State, with a reason, whether A and B are independent. [1]

8. A coffee dispenser machine dispenses coffee in three cup sizes: small, regular and large. The volume of coffee, in millilitres (ml), dispensed for the different cup sizes follow independent normal distributions with means and standard deviations as given below:

	Volume Dispensed	
	Mean (ml)	Standard Deviation (ml)
Small	120	15
Regular	250	13
Large	350	k

(i) Calculate the probability that a randomly selected customer who bought a regular cup of coffee from the machine, gets less than twice the volume of coffee in a random small cup. [3]

During a particular hour, it is noted that the number of cups of coffee sold through the dispensing machine for small, regular and large cup sizes are three, four and five respectively.

(ii) If the probability that the total volume dispensed by the machine during that particular hour is not less than three litres is greater than 0.99, find the range of values of k. [4]

9. The average monthly household income, $\$ I$, in thousands of dollars, and the average monthly household expenditure, $\$ E$, in thousands of dollars, were recorded for a random sample of nine households of a certain constituency. The results are given in the following table.

Household	1	2	3	4	5	6	7	8	9
I	0.80	1.25	1.50	2.00	3.00	3.40	4.00	6.30	8.35
E	0.43	1.00	1.69	1.90	1.58	3.00	4.50	5.00	7.90

(i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]

(ii) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iii) Find the equations of the regression line of E on I, and the regression line of I on E. Sketch both regression lines on your scatter diagram, indicating clearly the point of intersection. [4]

(iv) Use the appropriate regression line to estimate the average monthly household expenditure whose average monthly household income is \$5 000. Comment on the reliability of your estimate. [2]

10. At a shooting practice, each shooter is required to fire five shots. The probability that a shooter achieves a successful shot is 0.48.
State, in context, an assumption needed for the number of successful shots achieved by a random shooter to be well modelled by a binomial distribution. [1]

(i) Find the probability that a shooter achieves at least three successful shots. [2]

In a marksmanship contest, each shooter is required to fire 50 shots. To obtain a marksmanship badge, a shooter needs to achieve at least 30 successful shots.

(ii) By using a suitable approximation, find the probability that a shooter obtains a marksmanship badge. [3]

(iii) A random sample of N shooters is taken, where N is a large number. Given that the probability that the average number of successful shots achieved by one random shooter exceeds 23 is less than 0.998, find the largest possible value of N. [4]

11. The National Library Board conducted a national poll on the time spent by teenagers reading magazines. The results showed that the average time spent by a random teenager was 4.5 hours per week. A school principal thinks that his students spend more time reading magazines than this average. To test his hypothesis, he selects a random sample of 65 students and the time spent reading magazines in a week by each student, x hours, are summarised by

$$\sum(x-4.5)=21, \quad \sum(x-4.5)^2=85.$$

(i) Calculate the unbiased estimates of the population mean and variance. [2]

(ii) Test, at 1% significance level, whether the principal's hypothesis is true. [4]
State, giving a reason, whether any assumption about the school population is needed in order for the test to be valid. [1]

Another test is conducted using the same data as above and also at 1% level of significance with the following hypotheses:

Null hypothesis: the population mean time spent on reading magazines by students in a week is equal to μ_0 hours.

Alternative hypothesis: the population mean time spent on reading magazines by students in a week is less than μ_0 hours.

(iii) Given that the null hypothesis is rejected in favour of the alternative hypothesis, find the range of possible values of μ_0. [3]

12. At a carnival, Mr Lim plays a game which involves throwing darts to burst balloons which may contain a winning ticket. The probability that Mr Lim successfully bursts a balloon in a random throw is p. If he successfully bursts a balloon, there is a 10% chance that the balloon contains a winning ticket. If he misses the balloons on his first try, he is given a second try. The game ends when he successfully bursts a balloon or after his second try.

(i) Draw a tree diagram to represent the possible outcomes. [2]

The probability that Mr Lim obtains a winning ticket in a game is 0.05.

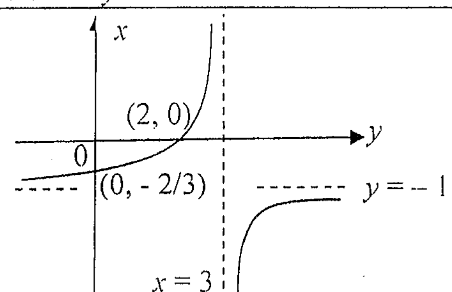
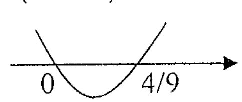
(ii) Show that $p = 0.293$, correct to three significant figures. [2]

(iii) Given that Mr Lim obtains a winning ticket, find the probability that it is from his second try. [3]

(iv) Find the probability that Mr Lim misses all the balloons for two games. [2]

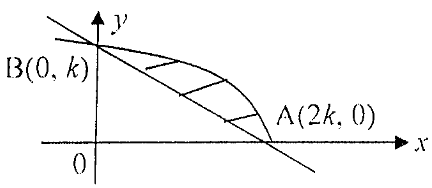
(v) If each game cost \$1, find the optimum amount Mr Lim needs to spend in order to have the highest chance of obtaining a winning ticket once. [4]

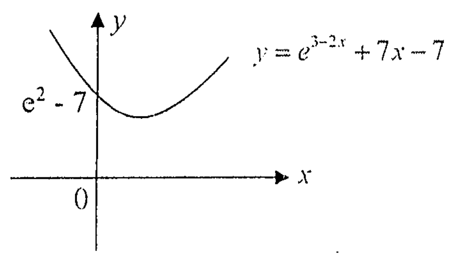
~ END OF PAPER ~

	Suggested Solutions
1i	When $x = 0, y = -\frac{2}{3}$ When $y = 0, x = 2$ V.A. is $x = 3$, H.A. , using long division, $y = \frac{2-x}{x-3} = -\frac{x-2}{x-3} = -\frac{x-3+1}{x-3} = -1 - \frac{1}{x-3}$ So, H.A. is $y = -1$
	
1ii	To find points of intersection, $\frac{2-x}{x-3} = kx - 1$, for $x \neq 3$
	$2 - x = (kx-1)(x-3)$ $2 - x = kx^2 - x - 3kx + 3$ $kx^2 - 3kx + 1 = 0$ For two distinct points of intersection, $b^2 - 4ac > 0$
	$(-3k)^2 - 4(k)(1) > 0$
	$k(9k-4) > 0$  So $k < 0$ or $k > 4/9$ Since $k > 0$, hence $\{k \in \mathbb{R}: k > 4/9\}$
2i	volume of box, $V = (2x)(x)(y)$

	$2x^2y = 24\,000$ $y = \dfrac{12\,000}{x^2}$
	$P = 4x + 4(2x) + 4y$ $= 12x + 4(\dfrac{12\,000}{x^2})$
	$= 12x + \dfrac{48\,000}{x^2}$ (Shown)
2ii	$\dfrac{dP}{dx} = 12 - 2(48\,000\,x^{-3})$
	For stationary values, set $\dfrac{dP}{dx} = 0$ $12 - 2(48\,000\,x^{-3}) = 0$
	$12 - \dfrac{2 \times 48\,000}{x^3} = 0$ $\dfrac{1}{x^3} = \dfrac{12}{2 \times 48\,000} = \dfrac{1}{8000}$ $x^3 = 8000$ $x = 20$ cm
	x: 20^-, 20, 20^+ $\dfrac{dP}{dx}$: - ve, 0, +ve
	So $x = 20$ cm gives P min Minimum value of $P = 12(20) + \dfrac{48\,000}{(20)^2}$ $= 240 + 120 = 360$ cm.
3(a)	$\dfrac{d}{dx} \ln\left(\dfrac{20x}{10 - x}\right)$ $= \dfrac{d}{dx}[\ln 20x - \ln(10 - x)]$

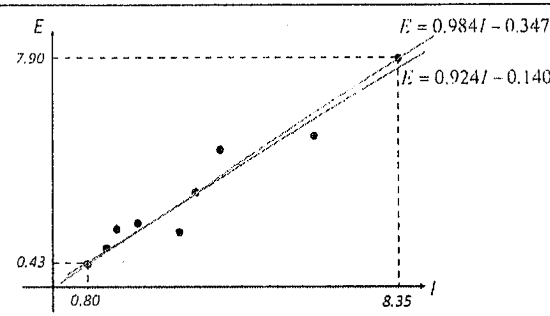
	$=\frac{1}{x}+\frac{1}{10-x}$
3(b)	$\frac{d}{dx}\left(\frac{1}{1+x^2}\right)=-\frac{1}{(1+x^2)^2}(2x)=-\frac{2x}{(1+x^2)^2}$
	$\int_0^2 \frac{x}{(1+x^2)^2}dx$ $=-\frac{1}{2}\int_0^2 \frac{-2x}{(1+x^2)^2}dx$
	$=-\frac{1}{2}\left[\frac{1}{1+x^2}\right]_0^2$ $=-\frac{1}{2}(\frac{1}{5}-1)$ $=-\frac{1}{2}(-\frac{4}{5})$ $=\frac{2}{5}$
3(c)	$\int_1^a (e^{1-x}+\frac{1}{x^2})\,dx=-\frac{1}{a}$ $\left[-e^{1-x}-\frac{1}{x}\right]_1^a=-\frac{1}{a}$
	$-e^{1-a}-\frac{1}{a}-(-e^{1-1}-\frac{1}{1})=-\frac{1}{a}$ $-e^{1-a}-\frac{1}{a}+1+1=-\frac{1}{a}$ $-e^{1-a}=-2$ $1-a=\ln 2$ $a=1-\ln 2$ $-e^{1-a}=-2$ $1-a=\ln 2$ $a=1-\ln 2$
4i	Since the line AB passes through points $A(2k, 0)$ and $B(0, k)$, Gradient of the line AB $=\frac{k-0}{0-2k}=-\frac{1}{2}$

	Hence equation of line AB is $\frac{y-0}{x-2k} = -\frac{1}{2}$
	$\Rightarrow y = -\frac{1}{2}x + k$. (shown)
4ii	
	Area required = area under the curve – area of triangle $= \int_0^{2k} \sqrt{2k-x}\,dx - \frac{1}{2}(2k)(k)$
	$= \int_0^{2k} (2k-x)^{1/2}\,dx - k^2$ $= \left[\frac{(2k-x)^{3/2}}{\left(\frac{3}{2}\right)(-1)}\right]_0^{2k} - k^2$
	$= -\frac{2}{3}\left[0-(2k)^{3/2}\right] - k^2$ $= \frac{2}{3}(2k)^{3/2} - k^2$ units^2
4iii	From (ii), area of $D = \frac{4\sqrt{2}}{3}k^{3/2}$ area of $E = k^2$ So, $\frac{\text{Area of } D}{\text{Area of } E} = \frac{4}{3}$ $\Rightarrow \frac{\frac{2}{3}(2k)^{3/2}}{k^2} = \frac{4}{3}$
	$\Rightarrow \quad (2k)^{3/2} = 2k^2$ Squaring both sides,

	$\Rightarrow \quad (2k)^3 = (2k^2)^2$ $\Rightarrow \quad 8k^3 = 4k^4$ $\Rightarrow \quad k = 2$ (since $k \neq 0$)
5i	when $x = 0,\ y = e^3 - 7$ 
5ii	$y = e^{3-2x} + 7x - 7$ $\frac{dy}{dx} = -2e^{3-2x} + 7$
	At point A, gradient is 5 So $-2e^{3-2x} + 7 = 5$ $-2e^{3-2x} = -2$ $e^{3-2x} = 1$ $3 - 2x = 0$
	$x = \frac{3}{2}$ (Shown)
5iii	When $x = \frac{3}{2}$, then $y = e^{3-2(3/2)} + 7\left(\frac{3}{2}\right) - 7 = 1 + \frac{21}{2} - 7 = \frac{9}{2}$ So, A is $(\frac{3}{2}, \frac{9}{2})$
	Equation of normal at A is $\frac{y - \frac{9}{2}}{x - \frac{3}{2}} = -\frac{1}{5}$

	$y-\frac{9}{2}=-\frac{1}{5}x+\frac{3}{10}$ $y=-\frac{1}{5}x+\frac{48}{10}$ $\therefore y=-\frac{1}{5}x+\frac{24}{5}$
5iv	At B, $x=0$, so $y=\frac{24}{5}$ So, AB $=\sqrt{\left(\frac{3}{2}\right)^2+\left(\frac{24}{5}-\frac{9}{2}\right)^2}$ $=\sqrt{2.34}=1.529705854\approx 1.53$ units
6i	Divide the population into 6 strata, namely Public Transport Under 30 years, Public Transport 30-65 years, Public Transport Over 65 years, Privately Owned Vehicle Under 30 years, Privately Owned Vehicle 30-65 years and Privately Owned Vehicle Over 65 years. Calculate the proportionate number to select from each strata.
	No. of Public Transport Under 30 years $=\frac{2000}{9000}\times 200=44.44\approx 44$ No. of Public Transport 30-65 years $=\frac{1500}{9000}\times 200=33.33\approx 33$ No. of Public Transport Over 65 years $=\frac{1000}{9000}\times 200=22.22\approx 22$
	No. of Privately Owned Vehicle Under 30 years $=\frac{400}{9000}\times 200=8.88\approx 9$ No. of Privately Owned Vehicle 30-65 years $=\frac{2500}{9000}\times 200=55.55\approx 56$ No. of Privately Owned Vehicle Over 65 years $=\frac{1600}{9000}\times 200=35.55\approx 36$
	Select the respective number randomly from each strata
6ii	Stratified sampling is more suitable as it is more representative of the employee population.

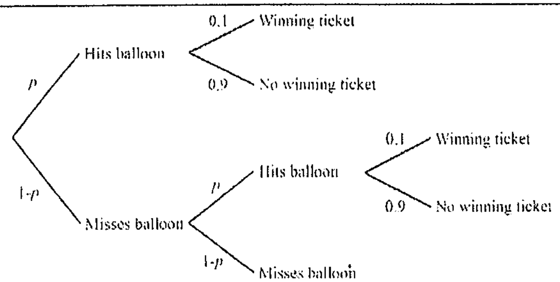
	Or Also systematic sampling may give disproportionately fewer employees from Under 30 years and Private transport as number is very small compared to the other categories.
7i	$\mathrm{P}(A \cup B) = 1 - \mathrm{P}(A' \cap B') = 1 - 0.25 = 0.75$
7ii	$\mathrm{P}(A \mid B) = \mathrm{P}(B \mid A)$ $\frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)} = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(A)}$ $\therefore \mathrm{P}(A) = \mathrm{P}(B)$ (shown)
7iii	$\mathrm{P}(A \cap B) = 0.5\mathrm{P}(A)$ $\mathrm{P}(A \cap B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cup B)$ $0.5\mathrm{P}(A) = \mathrm{P}(A) + \mathrm{P}(A) - 0.75$ $1.5\mathrm{P}(A) = 0.75$ $\therefore \mathrm{P}(A) = 0.5$ $\therefore \mathrm{P}(A \cap B) = 0.5(0.5) = 0.25$
7iv	$\mathrm{P}(A) \times \mathrm{P}(B) = 0.5 \times 0.5 = 0.25 = \mathrm{P}(A \cap B)$ Since $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$, A and B are independent.
8i	Let S, R and L be the random variable denoting volume of coffee dispensed for a small, regular and large cup respectively. $R - 2S \sim N(250 - 2 \times 240,\ 13^2 + 2^2 \times 15^2)$ $R - 2S \sim N(10, 1069)$
	Required probability $= P(R < 2S)$ $= P(R - 2S < 0)$ $= 0.380$ (to 3 s.f.)
8ii	Let A be the random variable denoting the total volume of coffee dispensed in a particular hour. $A \sim N(3 \times 120 + 4 \times 250 + 5 \times 350,$ $3 \times 15^2 + 4 \times 13^2 + 5 \times k^2)$ $A \sim N(3110, 1351 + 5k^2)$
	$P(A \geq 3000) > 0.99$

	$P(Z \geq \frac{3000-3110}{\sqrt{1351+5k^2}}) > 0.99$ $P(Z \geq \frac{-110}{\sqrt{1351+5k^2}}) > 0.99$
	$\frac{-110}{\sqrt{1351+5k^2}} < -2.326347877$ $\frac{110}{\sqrt{1351+5k^2}} > 2.326347877$ $\frac{110}{2.326347977} > \sqrt{1351+5k^2}$ $k^2 < \frac{1}{5}\left[\left(\frac{110}{2.326347977}\right)^2 - 1351\right]$ $k^2 < 176.963193$ $\lvert k \rvert < 13.30275127$
	$0 < k < 13.3$ (since $k > 0$)
9i	$(\overline{x}, \overline{y}) = (3.4, 3)$ 
9ii	From the GC, $r = 0.969$ (to 3sf)
	There is a strong positive linear relationship between the average monthly household income and the average monthly household expenditure. As the average monthly household income increases, the average monthly household expenditure increases proportionately.
9iii	From the GC, equation of the regression line of E on I is, $E = 0.924I - 0.140$ From the GC, equation of the regression line of I on E is,

	$I = 1.02E + 0.352$ $\Rightarrow E = 0.984I - 0.347$
9iv	When $I = 5$, $E = 0.92356(5) - 0.14013 = 4.47767$ The average monthly household expenditure is \$4480 (to 3 s.f.).
	Since $I = 5$ is within the given data range of $0.80 \le I \le 8.35$ (an interpolation), and the value of $r = 0.969$ is close to 1, the predicted value of $E = 4480$ is a reliable estimate.
10	It is assumed that the **outcome of each shot is independent of each other.** **Or** The probability of achieving a successful shot is a constant of 0.48 for all shots.
10i	Let X be the random variable denoting the number of successful shots out of 5. $X \sim B(5, 0.48)$
	$P(X \ge 3) = 1 - P(X \le 2)$ $= 0.46254$ $= 0.463$ (to 3 s.f.)
10ii	Let Y be the random variable denoting the number of successful shots out of 50. $Y \sim B(50, 0.48)$ Since $n = 50$ is large, $np = 24 > 5$ and $nq = 26 > 5$,
	$Y \sim N(24, 12.48)$ approx.
	$P(Y \ge 30) = P(Y > 29.5)$ c.c. $= 0.059749$ $= 0.0597$ (to 3 s.f.)
10iii	$\overline{Y} = \frac{Y_1 + Y_2 + \ldots + Y_N}{N}$. Since N is large enough, by CLT, $\overline{Y} \sim N(24, \frac{12.48}{N})$ approx.
	$P(\overline{Y} \ge 23) < 0.998$
	$P(\overline{Y} \le 23) > 0.002$

	$P(Z \le \frac{23-24}{\sqrt{\frac{12.48}{N}}}) > 0.002$ $\frac{23-24}{\sqrt{\frac{12.48}{N}}} > -2.87816$ $N < 103.4$
	Alternative Method Using GC, Y = normcdf(23, 10^{99},24, $\sqrt{\frac{12.48}{N}}$) N $P(\overline{Y} \ge 23)$ 102 0.99787 103 0.99797 104 0.99805 $N \le 103$
	Largest value of $N = 103$
	Alternative Method Let Y be the random variable denoting the number of successful shots out of 50N. $Y \sim B(50N, 0.48)$ Since require average number of successful shots exceed 23, $P(Y > 23N) < 0.998$ Using GC, Y = 1- binomcdf(50N, 0.48, 23N) N $P(Y > 23)$ 104 0.99798 105 0.99806 106 0.99815 $N \le 104$ Largest value of $N = 104$
11i	Unbiased estimate of the population mean

	$= \frac{\Sigma(x-4.5)}{65} + 4.5$ $= \frac{21}{65} + 4.5$ $= 4.823076923$ ≈ 4.82 (3 s.f.)
	Unbiased estimate of the population variance $= \frac{1}{65-1}(85 - \frac{21^2}{65})$ $= 1.222115385$ ≈ 1.22 (3 s.f)
11ii	$H_0 : \mu = 4.5$ $H_1 : \mu > 4.5$ Level of sig: $1\% = 0.01$ Test statistic: $z = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$, where $\bar{x} = 4.8231$, $s = \sqrt{1.2221}$ and $n = 65$ Using G.C., p-value $= 0.00923 < 0.01$
	Therefore we reject H_0, and conclude that there is sufficient evidence at 1% level of significance that the students from the school spend more time reading magazines as compared to the national average.
	No assumption needed. This is because the sample size is large and thus by Central Limit Theorem, $\overline{X}$ follows a normal distribution.
11iii	$H_0 : \mu = \mu_0$ $H_1 : \mu < \mu_0$ The null hypothesis is rejected at 1% level of significance $\Rightarrow \frac{4.8231 - \mu_0}{\sqrt{\frac{1.2221}{65}}} < -2.32634$ $\mu_0 > 5.1421$

	$\mu_0 > 5.14$ (to 3 s.f.)
12i	
12ii	$p(0.1)+(1-p)p(0.1)=0.05$
	$p(0.1)+(1-p)p(0.1)=0.05$ $0.1p+0.1p-0.1p^2=0.05$ $0.1p^2-0.2p+0.05=0$ $2p^2-4p+1=0$ $p=0.29289$ or $p=1.7071$ (Reject $\because p>1$) $\therefore p=0.293$ (shown)
12iii	P(winning ticket is from second try | obtains a winning ticket) $=\dfrac{\text{P(winning ticket is from second try} \cap \text{obtains a winning ticket)}}{\text{P(obtains a winning ticket)}}$
	$=\dfrac{(1-0.29289)(0.29289)(0.1)}{0.05}$ $=0.41421$ ≈ 0.414 (to 3sf)
12iv	P(misses all the balloons in both games) $=(1-0.29289)^4$ $=0.25000$ ≈ 0.250 (to 3sf)
12v	Let X be the random variable denoting the number of wins out of n games. $X \sim B(n, 0.05)$

	P(X = 1) = highest probability is the mode.
	Using GC n | P(X = 1) 18 | 0.37631 19 | 0.3777353602536 20 | 0.3777353602535 21 | 0.37641
	Since highest probability occurs at X = 19, hence optimal amount is \$19. (\$20 can be also accepted)