ANDERSON JUNIOR COLLEGE
JC2 Preliminary Examination 2017

MATHEMATICS **8865/01**
Higher 1 **11 September 2017**
Paper 1 **3 hours**
Additional Materials:
Graph Paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagram or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions
Give non-exact numerical answers correct to **3 significant figures**, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
When unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in [ ] at the end of each question or part question.

Name:_______________________________ PDG:__________

1	2	3	4	5	6

7	8	9	10	11	TOTAL

This Question paper consists of 6 printed pages

Section A: Pure Mathematics [40 marks]

1	Find the values of k for which $3(k-2)x^2-6x+k>0$ for all values of x. [4] Hence deduce the values of k for which the function $y=(k-2)x^3-3x^2+kx+5$ is strictly increasing for all real values of x. [2]
2	The curve C has equation $y=\frac{1}{2}e^{1-3x^2}$. (i) Without using a calculator, find the equation of the tangent to C at the point P where $x=1$, giving your answer in the form where $y=mx+c$, where m and c are constants in exact terms to be found. [3] The tangent to C at P cuts x-axis at the point A and the y-axis at the point B. (ii) Find the exact coordinates of the midpoint of AB. [2] (iii) Find the length of AB, giving your answer to 3 significant figures. [2]
3	(a) Show that $\frac{d}{dx}\ln\left(\frac{x^3}{1+x^2}\right)=\frac{x^2+3}{x\left(1+x^2\right)}$. [2] Hence deduce the exact value of $\int_1^2 \frac{x^2+3}{2x(x^2+1)}dx$, simplifying your answer to a single term. [3] (b) State the numerical value of $\int_1^2 \ln\left(\frac{x^3}{1+x^2}\right)dx$. [1]
4	Sketch the graph of the curve C with equation $y=2(k-x)x$, where k is a positive constant, showing clearly the coordinates of the points where C cuts the axes. [1] (i) Show that the line $y=\frac{k}{2}x$ always intersects C at two distinct points. [2] The line $y=\frac{k}{2}x$ intersects C at the origin O and another point A where $x=\frac{3k}{4}$. (ii) Find the area of the region between C and the line $y=\frac{k}{2}x$. [3] (iii) State the values of x for which $2kx-2x^2\le\frac{k}{2}x$. [1] Consider the case where $k=2$. (iv) Use your answer in (iii) to deduce the exact values of x for which $4\ln x\ -2\left(\ln x\right)^2\le\ln x$ [2]
5	A new company manufactures souvenirs. The cost, C thousand dollars for producing x hundred souvenirs, is modelled by the equation $C=\frac{169}{2x+1}+2x$, $0\le x\le 20$. (i) Use differentiation to find the number of souvenirs that must be produced to minimise the cost. State the minimum cost, justifying that this cost is a minimum. [5] (ii) Sketch the graph of C against x, showing clearly the coordinates of any

turning points and any intersections with the axes [1]

The daily revenue collected R thousand dollars, varies with the time t days. The CEO believes that the connection between the rate of change of the daily revenue, $\frac{\mathrm{d}R}{\mathrm{d}t}$, and the time t days, can be modelled by the equation $\frac{\mathrm{d}R}{\mathrm{d}t} = 3 - \mathrm{e}^{-2t}$, $t \geq 0$.

(iii) Sketch the graph of $\frac{\mathrm{d}R}{\mathrm{d}t}$ against t, showing clearly the coordinates of the point(s) where the curve cuts the vertical axis and the equation of any asymptote(s).
Give a practical interpretation of the asymptote(s). [2]

(iv) The daily revenue collected when $t = 0$ is \$1000. Find, in terms of t, the daily revenue collected, R thousand dollars, on day t. [3]

(v) Hence state the value of t when the daily revenue collected first reaches \$21500. [1]

(vi) The daily revenue collected when $t = 0$ is \$1000. Find, in terms of t, the daily revenue collected, R thousand dollars, on day t. [3]

(vii) Hence state the value of t when the daily revenue collected first reaches \$21500. [1]

Section B: Probability and Statistics [60 marks]

6

1. Independent events A and B are such that $P(A) = 0.45$ and $P(B) = 0.4$.

(i) Find $P(A \cup B)$. [2]

Event C is such that $P(C) = 0.4$, $P(B \mid C) = 0.4$, $P(A \cap C) = 0.18$ and $P(A \cap B \cap C) = 0.1$.

(ii) Find $P(B \cap C)$ and hence deduce $P(A' \cap B \cap C)$. [2]

(iii) Show that $P(A \cup B \cup C) = 0.83$ and hence find $P(A' \cap B' \cap C')$ [3]

7

A salad bar in a restaurant has 7 types of greens, 3 types of proteins and 6 types of toppings. There are also 2 types of soup and 2 types of yogurt for selection.
A promotional set meal consists of a salad plate, plus either a soup or a yoghurt. For the salad plate, a customer needs to choose 3 different types of greens, 1 type of protein, and 2 different types of toppings.

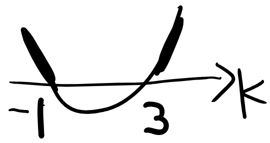
(i) Find the number of ways the customers may customise his set meal. [2]

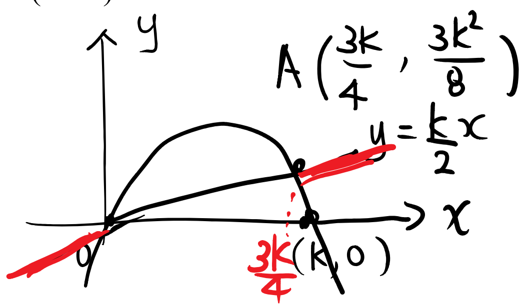
Each morning, the employee has to key a password to access the company accounts. The password consists of 3 digits from 1 to 9, followed by 2 letters

	of the alphabet. Each digit or letter may be used any number of times. Find the number of possible passwords if (ii) there is no other restriction, [1] (iii) the password has exactly one even digit and at least one vowel. [3] One morning, the employee forgot the password. However, he is certain that the digits are all different, but the alphabets are identical. He makes an attempt to type in the password. (iv) Find the probability that the employee gets the password correct in his first attempt. [2]
8	A nursery sells a large number of rose seeds. 25% of the seeds are red rose seeds, and the rest are either yellow or pink rose seeds. The nursery sells the seeds in packs of 12, and each pack contains a random selection of rose seeds. For these packs, the mean number of yellow rose seeds is 3.6. A pack of rose seeds is chosen at random. (i) Show that the probability that the pack contains at most three yellow rose seeds is 0.4925. [2] (ii) Find the probability that more than half of the seeds in the pack are either red or yellow rose seeds. [2] A box contains 200 packs of seeds. (iii) Find the probability that at least 30%, but less than 60% of the packs contain at most three yellow rose seeds. [2] John buys a pack of rose seeds. His pack of seeds contains three red rose seeds, four yellow rose seeds and five pink rose seeds. His child randomly picks three seeds from the pack to plant them in a row. Find the probability that (iv) there are at least two pink rose seeds planted, [3] (v) the third seed planted is a pink rose seed if it is known that at least two pink rose seeds are planted. [3]
9	A college has a large number of students taking mathematics and chemistry. In the block test, the scores of the mathematics test, X marks, is normally distributed with mean 50 marks and standard deviation 8 marks. 3 students are chosen at random. Find the probability that (i) each of the three students score more than 40 marks, [2] (ii) the total marks of the first two students differ from twice the marks of the third student by more than 15 marks. [3] The mathematics marks are moderated to Y marks, using the formula $Y = aX + b$, where a and b are positive constants. 2.04 % of the students have

	a moderated score of less than 42 marks, while 2.04% of students have a moderated score of more than 78 marks. (iii) Find the value of $\mathrm{E}(Y)$ and show that $\mathrm{Var}(Y) = 77.432$. [3] (iv) Find the values of a and b. [3] The chemistry marks of the college block test, C marks, has a mean of 52 marks and standard deviation 10 marks. A group of 40 chemistry students are randomly selected to attend a feedback session. (v) Find the probability that the average chemistry mark of the group is within 1 mark of the college mean chemistry mark. [2]
10	A baker claims that the mean mass of his 'Xtra' loaf of bread is 800 g. The mass of the loaves is known to have a standard deviation of 10.1 g. A random sample of 50 loaves was taken, and found to have a mean mass of 797.7 grams. (i) Test the baker's claim at the 5% level of significance. [4] (ii) Meanwhile, a group of consumers used the same sample to carry out a different test. They conclude that the baker is overstating the mean mass at the k% significance level. Find the smallest value of k to three significant figures. [3] The bakery also claims that the average mass of a certain compound in each loaf of healthy bread is 150 mg. The mass of the compound in the loaves is normally distributed and the standard deviation is σ mg. A random sample of 60 loaves of healthy bread is taken, and the mass of compound in each loaf y mg is observed. The results are summarised as $\sum(y-150) = 60$. A test at 6% shows that the baker is understating the average mass of compound. (iii) Find the possible values that σ can take. [4]
12	A company is selling a particular make of cars. The age of the car x, in months, and the advertised selling price P, in hundreds of dollars, for 8 cars are given below. <table><tr><td>x</td><td>5</td><td>10</td><td>20</td><td>50</td><td>70</td><td>80</td><td>100</td><td>120</td></tr><tr><td>P</td><td>546</td><td>500</td><td>433</td><td>329</td><td>278</td><td>249</td><td>187</td><td>100</td></tr></table> (i) Give a sketch of the scatter diagram for the data, as shown on your GC. [1] (ii) Find the product moment correlation coefficient and comment on its value in the context of the question. [2] (iii) Find the equation of the regression line of P on x, and sketch this line on your diagram. [2] (iv) Estimate the age of a car that can be bought from the company with a budget of

	\$28 000. Give reasons why you expect this estimate to be reliable. The number of remaining months the Certificate of Entitlement COE of a car is valid is denoted by y months. It is known that $y = 120 - x$. (v)Find the equation of the regression line of P on y. [2]

1	$(k-2)x^2-6x+k>0$ for all values of x $3(k-2)>0$ ----(1) and $(-6)^2-4(3)(k-2)(k)<0$ ------(2) From (1): $k>2----(1)$ and From (2): $36-12k(k-2)<0$ $\Rightarrow 36-12k^2+24k<0$ $\Rightarrow -k^2+2k-3<0$ $\Rightarrow k^2-2k+3>0$ $\Rightarrow (k+1)(k-3)>0$ $\Rightarrow k<-1$ or $k>3------(2)$  From (1) and (2) : solution is $k>3$ $y=(k-2)x^3-3x^2+kx+5 \Rightarrow \frac{dy}{dx}=3(k-2)x^2-6x+k$ If function is strictly increasing, $\frac{dy}{dx}>0$ for all values of x So $(k-2)x^2-6x+k>0$ From above, solution is $k>3$
2	(i) $y=\frac{1}{2}e^{1-3x^2} \Rightarrow \frac{dy}{dx}=\frac{1}{2}e^{1-3x^2}\left(-6x\right)=-3xe^{1-3x^2}$ At P, $x=1$, $y=\frac{1}{2}e^{1-3}=\frac{1}{2}e^{-2}$, $\frac{dy}{dx}=-3e^{1-3}=-3e^{-2}$ Equation of tangent is $y-\left(\frac{1}{2}e^{-2}\right)=(-3e^{-2})(x-1)$ $y-\left(\frac{1}{2}e^{-2}\right)=(-3e^{-2})(x-1)$ $y=-3e^{-2}x+3e^{-2}+\frac{1}{2}e^{-2}=-3e^{-2}x+\frac{7}{2}e^{-2}$ (ii) At B, $x=0$, $y=\frac{7}{2}e^{-2}$ At A, $y=0$, $-3e^{-2}x+\frac{7}{2}e^{-2}=0 \Rightarrow x=\frac{\left(-\frac{7}{2}e^{-2}\right)}{-3e^{-2}}=\frac{7}{6}$ $A(\frac{7}{6},0)$ $B(0,\frac{7}{2}e^{-2})$ Midpoint of AB is $\left(\frac{\frac{7}{6}+0}{2},\frac{0+\frac{7}{2}e^{-2}}{2}\right)=\left(\frac{7}{12},\frac{7}{4}e^{-2}\right)$ (iii) $AB=\sqrt{\left(\frac{7}{6}\right)^2+\left(\frac{7}{2}e^{-2}\right)^2}$ =1.26

3	a(i) $\ln\left(\frac{x^3}{1+x^2}\right)=\ln x^3-\ln(1+x^2)=3\ln x-\ln(1+x^2)$ $\frac{d}{dx}\ln\left(\frac{x^3}{1+x^2}\right)=\frac{3}{x}-\frac{2x}{1+x^2}$ $=\frac{3(1+x^2)-2x(x)}{x(1+x^2)}$ $=\frac{3+3x^2-2x^2}{x(1+x^2)}=\frac{x^2+3}{x(1+x^2)}$ a(ii) $\int_1^2\frac{x^2+3}{2x(1+x^2)}dx=\frac{1}{2}\int_1^2\frac{x^2+3}{x(1+x^2)}dx$ $=\frac{1}{2}\left[\ln\frac{x^3}{1+x^2}\right]_1^2=\frac{1}{2}\left[\ln\frac{2^3}{1+2^2}-\ln\frac{1}{2}\right]$ $=\frac{1}{2}(\ln\frac{8}{5}-\ln\frac{1}{2})=\frac{1}{2}(\ln\frac{\left(\frac{8}{5}\right)}{\left(\frac{1}{2}\right)})$ $=\frac{1}{2}\ln\frac{16}{5}$ 3b) $\int_1^2\ln\left(\frac{x^3}{1+x^2}\right)dx=-0.0103$
4	$y=2(k-x)x$  (i) At point of intersection of $y=\frac{k}{2}x$ and $y=2(k-x)x$ $\frac{k}{2}x=2(k-x)x\Rightarrow\frac{k}{2}x=2kx-2x^2\Rightarrow 2x^2-2kx+\frac{k}{2}x=0$ $\Rightarrow 2x^2-\frac{3k}{2}x=0$---------(1) Method 1: Observe that Discriminant is $D=\left(-\frac{3k}{2}\right)^2-4(2)(0)=\frac{9k^2}{4}>0$ (since k>0 $\Rightarrow k^2>0\Rightarrow\frac{9}{4}k^2>0$ for all positive values of k.

Hence, the quadratic equation (1) will have 2 distinct roots.
So the line intersects the curve at two distinct points.

<u>Alternative Method:</u>

From (1) $x\left(2x-\frac{3k}{2}\right)=0 \Rightarrow x=0 \quad \text{or} \quad x=\frac{3k}{4}\neq 0$

Hence, the quadratic equation (1) will have 2 distinct roots.
So the line intersects the curve at two distinct points.

At A, When $x=\frac{3k}{4}, y=\frac{k}{2}\left(\frac{3k}{4}\right)=\frac{3k^2}{8}$ & A $\left(\frac{3k}{4},\frac{3k^2}{8}\right)$

(ii) Area $=\int_0^{\frac{3k}{4}} \left(2\left(k-x\right)x-\frac{k}{2}x\right)\, dx =$

$$\int_0^{\frac{3k}{4}} -2x^2+\frac{3k}{2}x\ dx=\left[-\frac{2x^3}{3}+\frac{3kx^2}{4}\right]_0^{\frac{3k}{4}}$$

$$=\left(-\frac{2}{3}\left(\frac{3k}{4}\right)^3+\frac{3k}{4}\left(\frac{3k}{4}\right)^2\right)-0=-\frac{2}{3}\left(\frac{27k^3}{64}\right)+\frac{3k}{4}\left(\frac{9k^2}{16}\right)$$

$$=-\frac{9k^3}{32}+\frac{27k^3}{64}=\left(-\frac{9}{32}+\frac{27}{64}\right)k^3=\frac{9}{64}k^3$$

<u>Alternative Method</u>:

Area $=\int_0^{\frac{3k}{4}} 2\left(k-x\right)x\ dx-\frac{1}{2}\left(\frac{3k}{4}\right)\left(\frac{3k^2}{8}\right)$

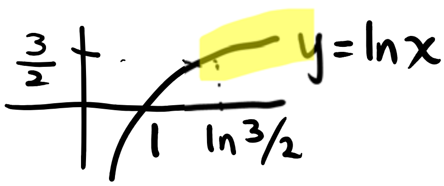
(iii) $2kx-2x^2\leq\frac{k}{2}x$ means $2(k-x)x\leq\frac{k}{2}x\Rightarrow x\leq 0 \quad \text{or} \quad x\geq\frac{3k}{4}$

(iv) Replace x by $\ln x$ and k by 2 in the solution above:

$4\ln x-2\left(\ln x\right)^2\leq \ln x$

$\Rightarrow \ln x\leq 0 \quad \text{or} \quad \ln x\geq\frac{3}{2}$

$\Rightarrow 0<x\leq 1 \quad \text{or}\ x\geq e^{\frac{3}{2}}$



5

(i) $C=\frac{169}{2x+1}+2x=169(2x+1)^{-1}+2x$

$\frac{dC}{dx}=169(-1)(2x+1)^{-2}\left(2\right)+2=\frac{-338}{(2x+1)^2}+2$

Min C: $\frac{dC}{dx}=0\Rightarrow\frac{-338}{(2x+1)^2}+2=0$

$$2=\frac{338}{(2x+1)^2}\Rightarrow(2x+1)^2=\frac{338}{2}=169$$

$$2x+1=13 \quad \text{or} \quad 2x+1=-13$$

$$x=6 \text{ or } x=\text{-}2 \ \ (\text{rejected}, x\geq 0)$$

Method 1: $\dfrac{d^2C}{dx^2} = \dfrac{676}{(2x+1)^3}$

At $x = 6, \dfrac{d^2C}{dx^2} > 0$; so C is minimum when x=6.

Method 2:

x	6^-	6	6^+
$\dfrac{dC}{dx}$	-	0	+
Outline	\	—	/

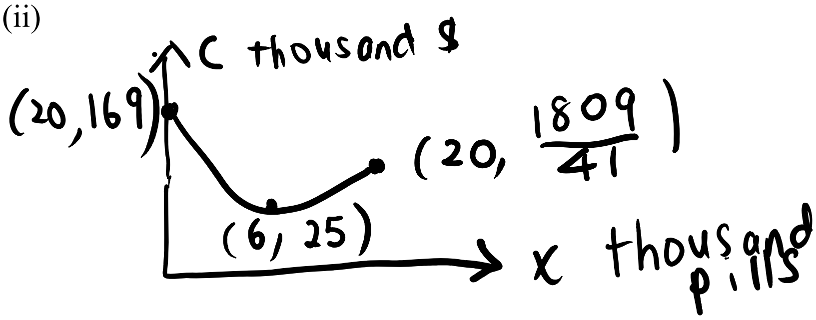
C is minimum.

$C = \dfrac{169}{(2\times 6+1)^2} + 2(6) = 25$

6000 pills must be produced.

Minimum production cost is $25000.

(ii)



(iii)

$\dfrac{dR}{dt} = 3 - e^{-2t}$

$\dfrac{dR}{dt}$ increases and approaches 3 when t is very large.

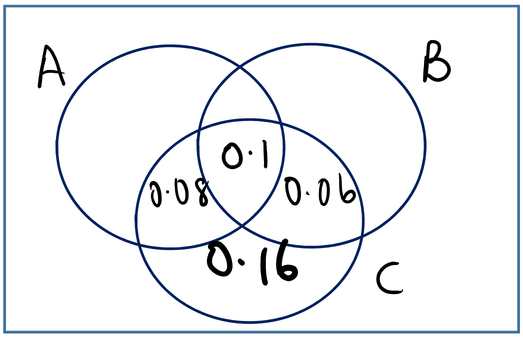
The daily revenue collected increases at a rate of approximately **3 thousand dollars per day** in the **long run**

(iv) $R = \int 3 - e^{-2t} dt = 3t - \dfrac{e^{-2t}}{-2} + C = 3t + \dfrac{e^{-2t}}{2} + C$

t = 0, R=1: $3(0) + \dfrac{e^0}{2} + C = 1 \Rightarrow \dfrac{1}{2} + C = 1 \Rightarrow C = \dfrac{1}{2}$

$R = 3t + \dfrac{e^{-2t}}{2} + \dfrac{1}{2}$

(v)The revenue first reaches $21500 when $t = 7$

6	(i) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ $=P(A)+P(B)-P(A)\times P(B)$ $\because$ A & B independent $=0.45+0.4-(0.45)(0.4)=0.67$ (ii) $P(B\mid C)=0.4\Rightarrow\dfrac{P(B\cap C)}{P(C)}=0.4$ $P(B\cap C)=0.4P(C)=0.4(0.4)=0.16$ $P(A'\cap B\cap C)=P(B\cap C)-P(A\cap B\cap C)=0.16-0.1=0.06$ (iii) <u>Method 1</u>(Formula) $P(A\cup B\cup C)$ $=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$ =0.45+0.4+0.4 -0.18-0.16-0.18+0.1=0.83 <u>Alternative method (From Venn diagram)</u> $P(A\cap B'\cap C)=P(A\cap C)-P(A\cap B\cap C)=0.18-0.1=0.08$ $P(A'\cap B'\cap C)=P(C)-0.1-0.08-0.06=0.16$  $P(A\cup B\cup C)=P(A\cup B)+0.16=0.67+0.16=0.83$ $P(A'\cap B'\cap C')=1-P(A\cup B\cup C)=1-0.83=0.17$
7	(i) No of ways= ${}^{7}C_{3}\times{}^{3}C_{1}\times{}^{6}C_{2}\times{}^{4}C_{1}$ =6300 (ii) No of codes that can be formed = $9\times9\times9\times26\times26=492804$ (iii) Case 1: one even digit & 2 odd digits, one vowel & one consonant Case 2: one even digit & 2 odd digits, 2 vowels. No of codewords= $3(4\times5\times5)\times2(5\times21)+3(4\times5\times5)\times(5\times5)$ =63000+7500=70500 (iv) No of passwords with all different digits, & identical letters $=9\times8\times7\times26\times1=13104$ Probability = $\dfrac{1}{13104}$
8	Let X be the number of yellow rose seeds out of 12. $X \sim B(12, 0.3)$

Since $E(X)=12p=3.6 \Rightarrow p=\frac{3.6}{12}=0.3$

$P(X \le 3)=0.4925158 \approx 0.4925$

(ii) Let Y be the number of seeds that are either red or yellow rose seeds

$Y \sim B(12,0.55)$ Since P(yellow or red)=0.3+0.25=0.55

$P(Y>6)=P(Y\ge 7)=1-P(Y\le 6)=0.527$

(iii) Let W be the number of packs that contain at most three yellow rose seeds, out of 200 packs . $W \sim B(200,0.4925)$

$P(30\% \text{ of } 200 \le W < 60\% \text{ of } 200)=P(60\le W<120)$

$=P(60\le W\le 119)=P(W\le 119)-P(W\le 59)$

$=0.998545\approx 0.999$

(iv)P(at least 2 pink) $=\left(\frac{5}{12}\times\frac{4}{11}\times\frac{3}{10}\right)+3\left(\frac{5}{12}\times\frac{4}{11}\times\frac{7}{10}\right)=\frac{4}{11}$

(v)P(third seed is pink|at least 2 pink)= $\frac{P(PPP \text{ or } P\bar{P}P \text{ or } \bar{P}PP)}{P(\text{at least 2 pink})}$

$$=\frac{\left(\frac{5}{12}\times\frac{4}{11}\times\frac{3}{10}\right)+2\left(\frac{5}{12}\times\frac{4}{11}\times\frac{7}{10}\right)}{\frac{4}{11}}=\frac{\left(\frac{17}{66}\right)}{\left(\frac{4}{11}\right)}=\frac{17}{24}$$

9

$X \sim N(50,8^2)$

(i)Prob$=\left(P(X>40)\right)^3$ $=(0.894351)^3=0.715$(ii)

$X_1+X_2-2X_3 \sim N(50+50-2(50),8^2+8^2+4(8^2))$ i.e $N(0,384)$

$P(X_1+X_2-2X_3<-15 \text{ or } X_1+X_2-2X_3>15)$

$=P(X_1+X_2-2X_3<-15)+P(X_1+X_2-2X_3>15)$

$=0.221997+0.221997=0.444$

Alternative: $1-P(-15<X_1+X_2-2X_3<15)=1-0.556006=0.444$

(iii) Let $Y \sim N(\mu,\sigma^2)$

$P(Y<42)=P(Y>78)\Rightarrow E(Y)=\frac{42+78}{2}=60$ (by symmetry)

$P(Y<42)=0.0204\Rightarrow P(Z<\frac{42-60}{\sigma})=0.0204\Rightarrow P(Z<\frac{-18}{\sigma})=0.0204$

$\frac{-18}{\sigma}=-2.0455567\Rightarrow \sigma=\frac{-18}{-2.0455567}=8.79956$

Var(Y)=8.79956^2=77.4322655=77.432

$Y=aX+b$

$E(Y)=aE(X)+b=a(50)+b=50a+b$

$50a+b=60$ --------(1)

$Var(Y)=a^2Var(X)=64a^2$

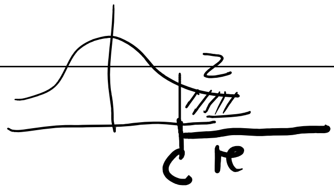
$64a^2$ =77.4333---------(2)

	$a^2 = \frac{77.4322655}{64} = 1.209879149$ $a = 1.099995 \approx 1.10$ 50(1.099995)+b=60 b=5.0025≈5 (v) $\overline{C} = \frac{C_1 + C_2 + ...C_{40}}{40}$ Since sample size=40>30 is large, By **CLT**, $\overline{C} \sim N(52, \frac{10^2}{40})$ $P(52-1 < \overline{X} < 52+1) = P(51 < \overline{C} < 53) = 0.473$
10	(i) Let X be the mass of a randomly chosen 'Xtra' loaf of bread, and μ the population mean. *X* has a unknown distribution Test $H_0 : \mu = 800$ (baker's claim) vs $H_1 : \mu \neq 800$ Test statistic: Under Ho and since sample size n=50≥30is large**, by** **Central Limit Theorem,** $\overline{X} \sim N\left(800, \frac{10.1^2}{50}\right)$ approximately, $Z = \frac{\overline{X} - 800}{\sqrt{\frac{10.1^2}{50}}} \sim N(0,1)$ Two **tailed** test at the 5% level of significance. From sample, $\overline{x} = 797.7$,z = - 1.61, p=0 .107 Since *p* = 0.107>0.05 , do **not reject H_0**. There is **insufficient evidence** at the **5%** level to conclude that the average mass is not 800 g. We do **not** reject the baker's claim. OR: There is **insufficient evidence** at the **5%** level to conclude that the baker's claim is not valid. (ii) If Test $H_0 : \mu = 800$ (baker's claim) vs $H_1 : \mu < 800$ **(baker is overstating)** Then p=0.05367 If bakery is overstating , reject Ho at k%, p=0.05367< $\frac{k}{100}$ ⇒ k>5.367 smallest k is 5.37 (iii) Let Y be the mass of compound in a randomly chosen healthy loaf and μ the population mean. *Y* has a normal distribution Test $H_0 : \mu = 150$ (bakery's claim) vs $H_1 : \mu > 150$ (understating **)** Test statistic: Under Ho $\overline{Y} \sim N\left(150, \frac{\sigma^2}{60}\right)$ and $Z = \frac{\overline{Y} - 150}{\frac{\sigma}{\sqrt{60}}} \sim N(0,1)$ One-**tailed** test at the 6% level of significance.

Critical Value:

$P(Z \le C) = 0.94 \Rightarrow C = 1.554774$

Reject Ho if $z > 1.554774$



Since our sample mean $\bar{y} = \dfrac{\sum (y-150)}{60} + 150 = \dfrac{60}{60} + 150 = 151$ Bakery is understating (Reject Ho)

$$\frac{151-150}{\frac{\sigma}{\sqrt{60}}} > 1.554774 \quad \Rightarrow \frac{1}{\left(\frac{\sigma}{\sqrt{60}}\right)} > 1.554774 \quad \Rightarrow \frac{\sqrt{60}}{\sigma} > 1.554774$$

$$\Rightarrow \sqrt{60} > 1.554774\sigma \Rightarrow \sigma < \frac{\sqrt{60}}{1.554774} = 4.98205$$

$$\Rightarrow \sigma < \frac{\sqrt{60}}{1.554774} = 4.98205$$

$0 \le \sigma \le 4.98$

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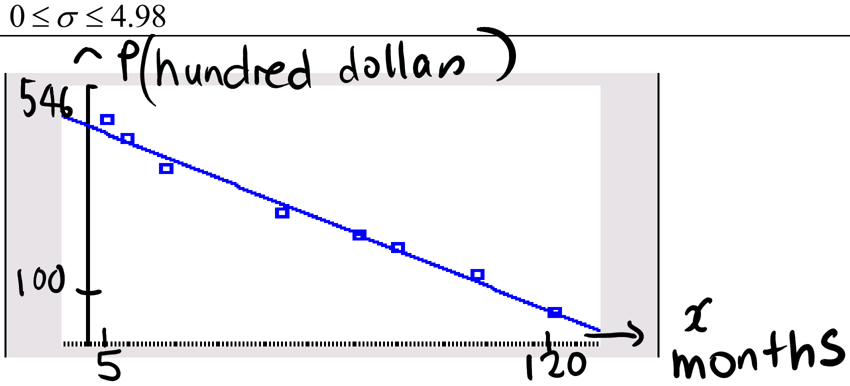
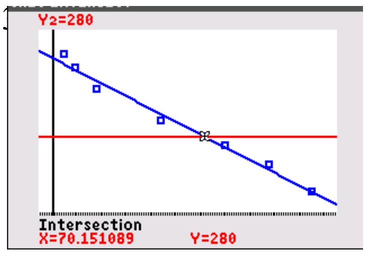


(ii) r= -0.992

Since r is close to -1, there is a strong negative linear correlation between the age of the car (x) and the advertised selling price (P). As the age of the car increases, the advertised selling price tends to decrease.

(iii)Regression line is $P = -3.60x + 5$



(iv)Using P on x:

$280 = -3.59669x + 532.3118$

$x = 70.2$

The estimated age of the car is 64.4 months.

The estimate is reliable because r= -0.992 is close to -1, and $P = 280$ is within the sample data range of $130 < P < 546$. Interpolation for2 strongly linearly correlated variables is reliable.

(v)

$y = 120 - x \Rightarrow x = 120 - y$. Replace x by $120 - y$:

$P = -3.5966918(120 - y) + 532.3118$

$P = 3.60y + 101$