Name: ________________________________________ Class: ________________________

JURONG JUNIOR COLLEGE

Preliminary Examination 2017

# MATHEMATICS
# Higher 1

**8865/01**

**28 August 2017**

Paper 1

**3 hours**

Additional materials: Answer Paper
Cover Page
List of Formulae (MF 26)

**READ THESE INSTRUCTIONS FIRST**

Write your name and class on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work together securely, with the cover page in front.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages.

**[Turn over**

## Section A: Pure Mathematics [40 marks]

**1** John has a total of \$100 000 to be invested in stocks, bonds and gold. The rate of return for stocks, bonds and gold are 12%, 8% and 4% per year respectively. The income generated from stocks is the same as the income generated from bonds and gold combined. John has stipulated that the amount invested in stock should exceed twice the amount invested in bonds by \$4000. Find the total income from his investments at the end of the first year. [4]

**2** The curve $C$ has equation $x^2+(y-1)^2=4$.

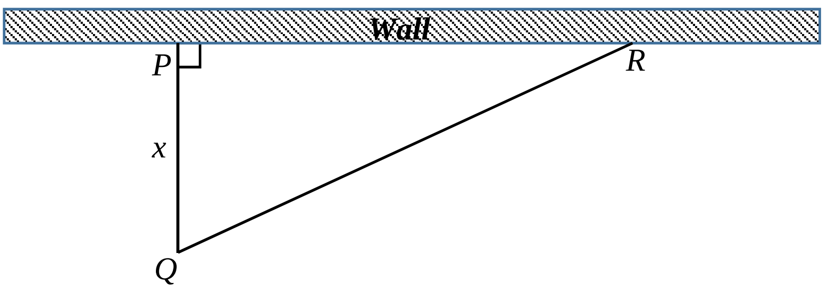
**(i)** Sketch $C$ and write down the range of values of $k$ such that $x^2+(k-1)^2=4$ has no real roots. [2]

**(ii)** Find the range of values of $k$ such that the line $y=x+k+1$ intersects $C$ exactly twice. [3]

**3** A farmer wishes to make an animal enclosure using the wall as one side of the enclosure. The farmer intends to use 100 m of fencing. If $PQ$ is $x$,

**(i)** show that area of the enclosure is $A=5\sqrt{100x^2-2x^3}$, [3]

**(ii)** find the value of $x$ that gives the maximum possible area of the enclosure. [3]



**4** **(i)** Sketch on the same diagram, the graphs of $C_1$: $y=\ln(4x+2)$ and $C_2$: $y=2+\dfrac{3}{2x-7}$, stating the equation of any asymptotes and the exact coordinates of the points of intersection with the axes. [4]

**(ii)** Solve $\ln(4x+2)>2+\dfrac{3}{2x-7}$. [2]

**(iii)** Using differentiation, find the equation of the tangent to $C_1$ at $x=2$. [3]

**(iv)** Find $\int 2+\dfrac{3}{2x-7}\,\mathrm{d}x$. [1]

**(v)** Using your answer to part **(iv)**, find the exact area of the region bounded by $C_2$, the lines $y=x$, $x=6$ and $x=8$. [3]

**5** **(a)** The number of Type *A* bacteria (in millions) *N* after *t* days is modelled by

$$N=\frac{K}{1+0.5\mathrm{e}^{-0.6t}}.$$

**(i)** Find the initial number of Type *A* bacteria (in millions) in terms of *K*. [1]

**(ii)** Find the long term population size of Type *A* bacteria in terms of *K*. [1]

**(iii)** After 2 days, the number of Type *A* bacteria is 320 millions. Find *K*. [2]

**(iv)** Sketch the graph of $N=\dfrac{K}{1+0.5\mathrm{e}^{-0.6t}}$. [2]

**(v)** Find $\dfrac{\mathrm{d}N}{\mathrm{d}t}$. [2]

**(vi)** Find the rate of increase of the number of Type *A* bacteria after the third day. [2]

**(b)** The growth rate of Type *B* bacteria is given by $\dfrac{\mathrm{d}P}{\mathrm{d}t}=\dfrac{60\mathrm{e}^{-1.1t}}{\left(1+\mathrm{e}^{-1.1t}\right)^2}$ millions per day.

**(i)** Evaluate $P=\int_0^5 \dfrac{60\mathrm{e}^{-1.1t}}{\left(1+\mathrm{e}^{-1.1t}\right)^2}\,\mathrm{d}t$. [1]

**(ii)** What does *P* represent in the context of this question? [1]

**[Turn Over**

## Section B: Statistics [60 marks]

**6** A fair six-sided die is tossed once. If the score on the die is 1 or 2, a ball is picked from bag $A$. If the score on the die is 3, 4, 5 or 6, a ball is picked from bag $B$. Bag $A$ contains 6 red and 4 blue balls. Bag $B$ contains 5 red, 3 blue and 2 green balls. Events $A$ and $R$ are defined as follows:

Event $A$ = {A ball is picked from bag $A$}

Event $R$ = {A red ball is picked}

Find

**(i)** $\mathrm{P}(R)$, [2]

**(ii)** $\mathrm{P}(A'|R)$. [3]

State with a reason whether events $A$ and $R$ are independent. [1]

**7** On average, every 3 out of 8 appointments of a hospital consultant will start late. The number of these appointments which start late is the random variable $L$.

**(i)** State, in context, two assumptions needed for $L$ to be well modelled by a binomial distribution. [2]

The consultant has seven appointments daily. Assume now that $L$ follows a binomial distribution.

**(ii)** Find the most likely numbers of appointments that start late. [2]

**(iii)** Find the probability that at least half the appointments start late. [2]

**(iv)** For a particular week, the consultant works 5 days a week. Find the probability that, for no more than 2 of the days, at least half of the appointments start late. [2]

**8** Members of the choir are in one these four vocal ranges: soprano, alto, tenor and bass. The sopranos and altos are women while the tenors and basses are men. A choir has four sopranos, three altos, three tenors and two basses.

**(a)** Find the number of ways to arrange the members of the choir in a row

**(i)** without restrictions, [1]

**(ii)** such that those of the same vocal range are together. [2]

**(b)** Five people are randomly selected from the choir. Find the probability that

**(i)** all the tenors are chosen, [2]

**(ii)** at least one women is chosen given that all the tenors are chosen. [3]

**9** The sleep pattern of 250 babies were tracked over 24 months. The age, $x$ months, and the average daily sleep time, $t$ hours, are given in the following table.

| $X$ | 1 | 3 | 6 | 9 | 12 | 18 | 24 |
|---|---|---|---|---|---|---|---|
| $T$ | 15.1 | 14.5 | 14.2 | 13.9 | 13.5 | 13.5 | 13 |

**(i)** Give a sketch of the scatter diagram for the data, ass shown on your calculator. [2]

**(ii)** Find $\bar{x}$ and $\bar{t}$ and mark the point ($\bar{x}$, $\bar{t}$) on your scatter diagram. [2]

**(iii)** Find the product moment correlation coefficient. [1]

**(iv)** Find the equation of the regression line of $t$ on $x$ and draw this line on your scatter diagram. [2]

**(v)** Calculate an estimate of the average daily sleep time of a 4–month old baby. Comment on the reliability of the estimate. [2]

**(vi)** Explain why it is not appropriate to estimate the average daily sleep time of a 32–month baby using the equation found in part **(iv)**. [1]

**10** A company claims that the mean weight of bags of cashew nuts packed, in grams, is 500. A random sample of 60 bags is selected and the weight, $x$ grams, of each bag is taken. The results obtained are summarised as follows:

$$\sum(x-500)=318 \text{ and } \sum(x-500)^2=25548.4$$

**(i)** Find unbiased estimates of the population mean and the variance. [3]

**(ii)** Test at 3% level of significance whether the company's claim is valid. [4]

**(iii)** State, giving a reason, whether it is necessary to assume a normal distribution in order for the test to be valid. [1]

**(iv)** Find an inequality satisfied by the level of significance in order for the null hypothesis to be rejected. What conclusion can be drawn if the test is conducted at the 5% level of significance? [2]

**[Turn Over**

The company now decides to test the claim that the mean weight of bags of cashew nuts packed, in grams, is at least 500. A second random sample of 70 bags is selected and the sample variance is $23.4^2\ \mathrm{g}^2$ . Using a 5% level of significance, the company finds that the mean weight of bags of cashew nuts is not at least 500.

**(v)** Find the set of values within which the mean weight of bags of cashew nuts of this sample must lie to 2 decimal places. You may assume that the weight of bags of cashew nuts follows a normal distribution. [4]

**11** Cheese tarts of a certain brand are sold in boxes containing 6 tarts. The masses, in grams, of the cheese tarts and of the empty boxes have independent normal distributions with means and standard deviations as shown in the following table.

| | Mean (g) | Standard deviation (g) |
|---|---|---|
| Cheese tart | 60 | 3.5 |
| Empty box | 52 | 0.8 |

**(i)** Find the probability that the mass of a cheese tart is less than 58 grams. [1]

**(ii)** Find the probability that a randomly chosen box of cheese tarts contains exactly 2 cheese tarts with mass less than 58 grams. [2]

**(iii)** Find the probability that the total mass of a box containing 6 cheese tarts is more than 415 grams. [3]

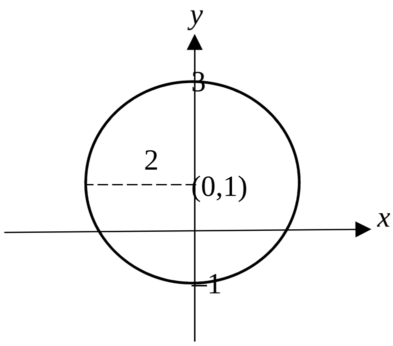
The cost of producing cheese tarts is 2.1 cents per gram and the cost of producing empty boxes is 0.3 cents per gram.

**(iv)** Find the probability that the total cost of producing a box containing 6 cheese tarts is between 747 cents and 774 cents. State the mean and variance of the distribution that you use. [5]

A rival brand of mini cheese tarts have masses with mean 35 grams and standard deviation 3.5 grams. A random sample of 30 mini cheese tarts is taken.

**(v)** Find the mean mass exceeded by 20% of these mini cheese tarts. [3]

# Jurong Junior College 2017 H1 Mathematics (8865) Preliminary Examination Solutions

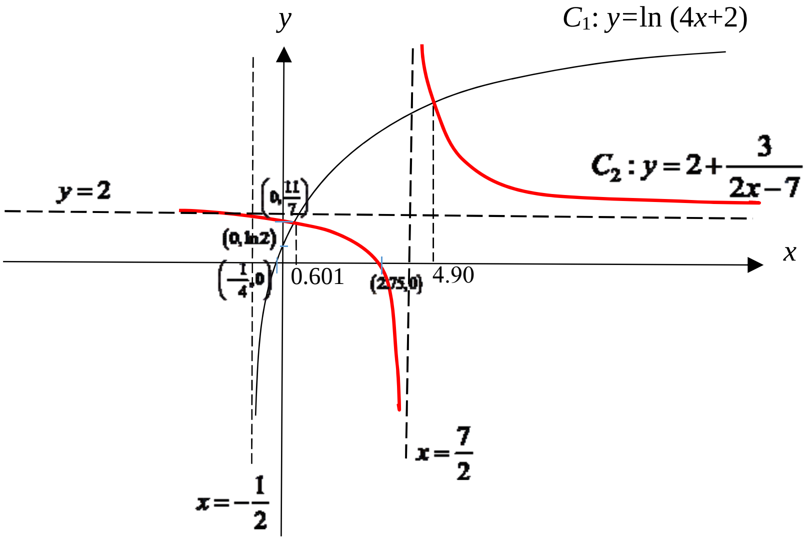
| Qn | Soln |
|---|---|
| 1 | Let \$$x$, \$$y$, \$$z$ be the amount invested in stocks, bonds and gold respectively.<br>$x+y+z=100\ 000$<br>$0.12x=0.08y+0.04z \Rightarrow \quad 0.12x-0.08y-0.04z=0$<br>$x-2y=4000$<br>Using GC, $x=28\ 000,\ y=12\ 000,\ z=60\ 000$<br>Total income $=0.12(28000)+0.08(12000)+0.04(60000)=6720$ |
| 2(a) | $x^2+(y-1)^2=4$<br>Curve $C$ is a circle with centre (0,1) and radius 2 units.<br><br><br>$x^2+(k-1)^2=4$ has no real roots<br>$\Rightarrow$ $y=k$ does not intersect the circle<br>$\Rightarrow$ $k<-1$ or $k>3$ |
| (b) | Sub $y=x+k+1$ into $x^2+(y-1)^2=4$,<br>$x^2+(x+k)^2=4$<br>$x^2+x^2+2kx+k^2=4$<br>$2x^2+2kx+k^2-4=0$<br>Since line cuts $C$ twice, Discriminant > 0<br>$(2k)^2-4(2)(k^2-4)>0$<br>$4k^2-8k^2+32>0$<br>$-4k^2+32>0$<br>$k^2-8<0$<br>$(k+\sqrt{8})(k-\sqrt{8})<0$<br>$-\sqrt{8}<k<\sqrt{8}$ |
| 3(i) | $QR=100-x$<br>$PR=\sqrt{(100-x)^2-x^2}$<br>$=\sqrt{10000-200x+x^2-x^2}$<br>$=\sqrt{10000-200x}$ |

| | |
|---|---|
| | $A=\frac{1}{2}x\sqrt{10000-200x}$<br>$=\frac{1}{2}\sqrt{x^2(10000-200x)}$<br>$=\frac{1}{2}\sqrt{10000x^2-200x^3}$<br>$=\frac{10}{2}\sqrt{100x^2-2x^3}$<br>$=5\sqrt{100x^2-2x^3}\quad(\text{shown})$ |
| (ii) | $\frac{\mathrm{d}A}{\mathrm{d}x}=5\left(\frac{1}{2}\right)\left(100x^2-2x^3\right)^{-\frac{1}{2}}\left(200x-6x^2\right)=\frac{5\left(100x-3x^2\right)}{\sqrt{100x^2-2x^3}}$<br>$\frac{5\left(100x-3x^2\right)}{\sqrt{100x^2-2x^3}}=0$<br>$100x-3x^2=0$<br>$x(100-3x)=0$<br>$x=0$ (N.A.) *or* $x=\frac{100}{3}$ |

| $x$ | 33.2 | $\frac{100}{3}=33.3$ | 33.4 |
|---|---|---|---|
| $\frac{\mathrm{d}A}{\mathrm{d}x}$ | 0.345 | 0 | $-0.174$ |
| Slope | / | - | \ |

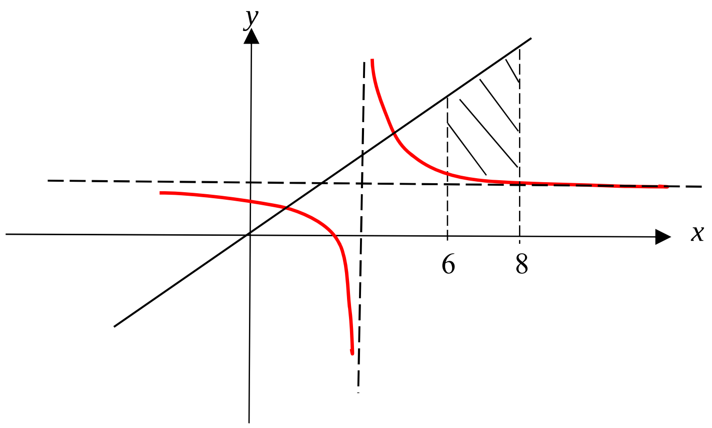
A is maximum when $x=\frac{100}{3}$

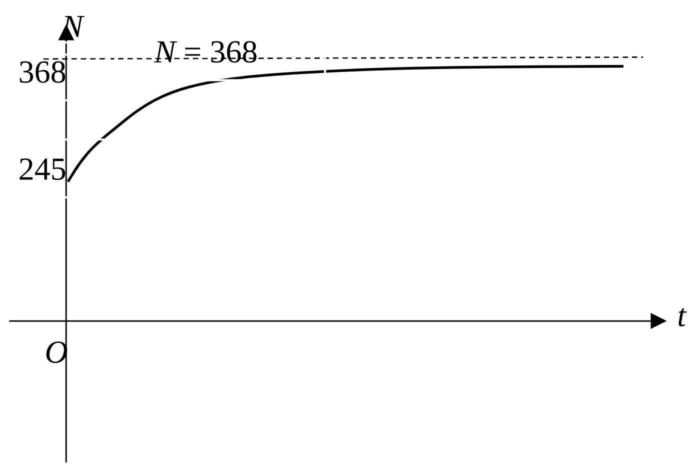
4 (i)




$C_1$: $y=\ln(4x+2)$: $(0,\ln 2)$ and $\left(-\frac{1}{4},0\right)$. Asymptote: $x=-\frac{1}{2}$

$C_2$: $y=2+\frac{3}{2x-7}$: $\left(0,\frac{11}{7}\right)$ and $(2.75,0)$ . Asymptotes: $x=\frac{7}{2}$ and $y=2$.

| | |
|---|---|
| (ii) | For $\ln(4x+2) > 2+\dfrac{3}{2x-7}$, <br> $0.601 < x < \dfrac{7}{2}$ or $x > 4.90$ |
| (iii) | $y=\ln(4x+2)$ <br> $\dfrac{dy}{dx}=\dfrac{4}{4x+2}=\dfrac{2}{2x+1}$ <br> When $x=2$, $y=\ln 10$, $\dfrac{dy}{dx}=\dfrac{2}{5}$ <br> Equation of the tangent to the curve: <br> $y-\ln 10=\dfrac{2}{5}(x-2)$ <br> $y=\dfrac{2}{5}x-\dfrac{4}{5}+\ln 10$ |
| (iv) | $\int 2+\dfrac{3}{2x-7}\ dx = 2x+\dfrac{3}{2}\ln(2x-7)+c$ |
| (v) |   <br> Area <br> $=\int_6^8 x\ dx-\int_6^8 2+\dfrac{3}{2x-7}\ dx$ <br> $=\left[\dfrac{x^2}{2}\right]_6^8-\left[2x+\dfrac{3}{2}\ln(2x-7)\right]_6^8$ <br> $=\left[\dfrac{8^2}{2}-\dfrac{6^2}{2}\right]-\left[\left[2(8)+\dfrac{3}{2}\ln(2(8)-7)\right]-\left[2(6)+\dfrac{3}{2}\ln(2(6)-7)\right]\right]$ <br> $=14-\left[4+\dfrac{3}{2}\ln 9-\dfrac{3}{2}\ln 5\right]$ <br> $=10-\dfrac{3}{2}\ln\dfrac{9}{5}$ |
| | <u>Alternative</u> <br> Area = Area of trapezium $-\int_6^8 2+\dfrac{3}{2x-7}\ dx$ |

| | |
|---|---|
| | $=\frac{1}{2}(6+8)(2)-\int_6^8 2+\frac{3}{2x-7}\,dx$<br>$=14-\left[2x+\frac{3}{2}\ln(2x-7)\right]_6^8$<br>$=14-\left[\left[2(8)+\frac{3}{2}\ln(2(8)-7)\right]-\left[2(6)+\frac{3}{2}\ln(2(6)-7)\right]\right]$<br>$=14-\left[4+\frac{3}{2}\ln 9-\frac{3}{2}\ln 5\right]$<br>$=10-\frac{3}{2}\ln\frac{9}{5}$ |
| 5 (a) (i) | $N=\frac{K}{1+0.5e^{-0.6t}}$<br>When $t$ = 0, $N=\frac{K}{1+0.5e^{0}}=\frac{2}{3}K$ |
| (ii) | As $t\to\infty, e^{-0.6t}\to 0, N\to K$<br>The long term population size is $K$ millions. |
| (iii) | $t$ = 2, $N$=320<br>$320=\frac{K}{1+0.5e^{-0.6(2)}}$<br>$K=320\left(1+0.5e^{-1.2}\right)=368.191\approx 368$ |
| (iv) |  |
| (v) | $N=\frac{368.19}{1+0.5e^{-0.6t}}=368.19(1+0.5e^{-0.6t})^{-1}$<br>$\frac{dN}{dt}=-368.29(1+0.5e^{-0.6t})^{-2}\left[(-0.6)0.5e^{-0.6t}\right]$<br>$=\frac{368.19\times 0.5\times 0.6e^{-0.6t}}{\left(1+0.5e^{-0.6t}\right)^2}$<br>$\approx\frac{110e^{-0.6t}}{\left(1+0.5e^{-0.6t}\right)^2}$ |

**(vi)** After 3 days, the rate of increase is:

$$\frac{\mathrm{d}N}{\mathrm{d}t}=\frac{110\mathrm{e}^{-0.6(3)}}{\left(1+0.5\mathrm{e}^{-0.6(3)}\right)^2}=15.5127\approx 15.5 \text{ millions per day.}$$

**(b) (i)**

$$P=\int_0^5 \frac{60\mathrm{e}^{-1.1t}}{\left(1+\mathrm{e}^{-1.1t}\right)^2}\,\mathrm{d}t=27.051\approx 27.1$$

**(ii)** $P$ represents the growth of number of Type B bacteria in millions during the first 5 days.

**6(i)**

$$\mathrm{P}(R)=\left(\frac{2}{6}\times\frac{6}{10}\right)+\left(\frac{4}{6}\times\frac{5}{10}\right)=\frac{8}{15}$$

**(ii)**

$$\mathrm{P}(A'\,|\,R)=\frac{\mathrm{P}(A'\cap R)}{\mathrm{P}(R)}=\frac{\frac{4}{6}\times\frac{5}{10}}{\frac{8}{15}}=\frac{5}{8}$$

$A$ and $R$ are not independent

Any of the following reasons:

(1) $\mathrm{P}(R\,|\,A)=\dfrac{6}{10}\neq \mathrm{P}(R)=\dfrac{8}{15}$

OR

(2) $\mathrm{P}(A)\times\mathrm{P}(R)=\dfrac{2}{6}\times\dfrac{8}{15}=\dfrac{8}{45}$ & $\mathrm{P}(A\cap R)=\dfrac{2}{6}\times\dfrac{6}{10}=\dfrac{1}{5}$

$\mathrm{P}(A\cap R)\neq\mathrm{P}(A)\times\mathrm{P}(R)$

OR

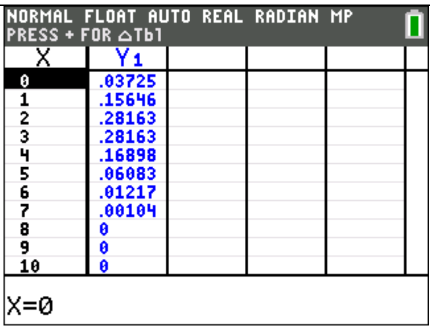
(3) $\mathrm{P}(A'\,|\,R)=\dfrac{5}{8}\neq\mathrm{P}(A')=\dfrac{4}{6}$ , $A'$ and $R$ are not independent, thus $A$ and $R$ are not independent.

**7(i)** (1) The probability that any one appointment will start late remains constant throughout the sample.

(2) The punctuality of each appointment is independent of the punctuality of any other appointments.

OR Whether an appointment start late is independent of any other appointments that start late.

**(ii)**




Using GC, [GC keystrokes: $\mathrm{Y}_1$= binompdf (7, $\frac{3}{8}$ ,$x$)],

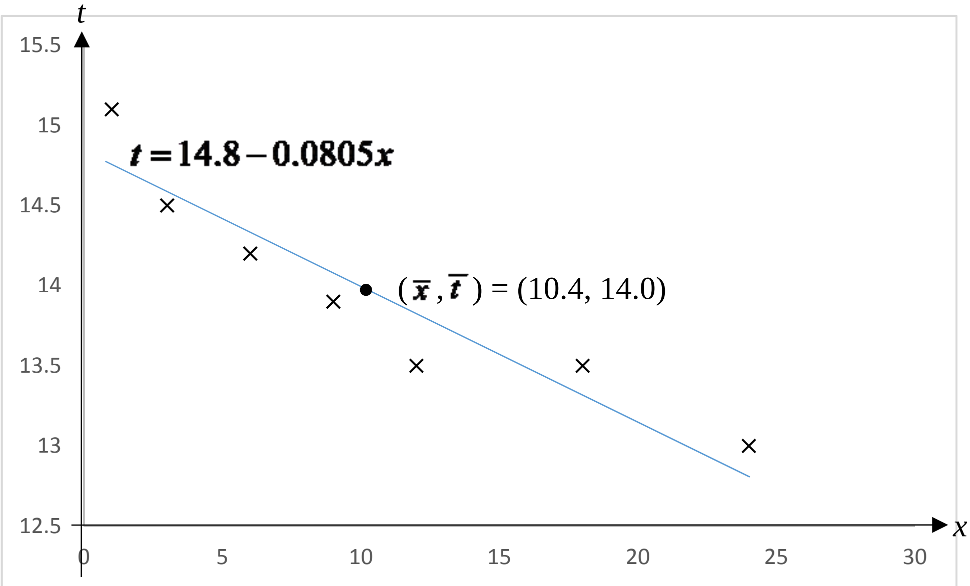
the most likely numbers of appointments that start late = 2 and 3

[Note: most likely number is referring to the **<u>MODE</u>**, not E($X$)]

**(iii)**

$$L\sim B\left(7,\frac{3}{8}\right)$$

$$\begin{aligned}\mathrm{P}(L\geq 3.5)&=\mathrm{P}(L\geq 4)\\&=1-\mathrm{P}(L\leq 3)\\&=0.24302\\&\approx 0.243\ \ (3\text{sf})\end{aligned}$$

| | |
|---|---|
| (iv) | Let $X$ denote the number of days out of 5, with at least half of the appointments starting late.<br>$X \sim B(5, 0.24302)$<br>$\mathrm{P}(X \le 2) = 0.90371 \approx 0.904$ (3sf) |
| 8(a)(i) | No. of ways $= 12! = 479\,001\,600$ |
| (a)(ii) | No. of ways $= (4! \times 3! \times 3! \times 2!) \times 4! = 41472$ |
| (b)(i) | P(all the tenors are chosen)$= \dfrac{{}^{3}C_3 \times {}^{9}C_2}{{}^{12}C_5} = \dfrac{36}{792} = \dfrac{1}{22}$ (or 0.0455 3sf) |
| (b)(ii) | P(at least one woman is chosen \| all the tenors are chosen)<br>$= \dfrac{\text{n(1 woman, 3 tenors, 1 bass) + n (2 woman, 3 tenors)}}{\text{n(all tenors are chosen)}}$<br>$= \dfrac{\left({}^{7}C_1 \times {}^{3}C_3 \times {}^{2}C_1\right) + \left({}^{7}C_2 \times {}^{3}C_3\right)}{{}^{3}C_3\,{}^{9}C_2}$<br>$= \dfrac{35}{36}$ or $0.972(3\text{sf})$ |
| 9(i) |  |
| (ii) | $(\bar{x}, \bar{t}) = (10.4, 14.0)$ |
| (iii) | $r = -0.941$ |
| (iv) | $t = 14.796 - 0.080474x$<br>$t = 14.8 - 0.0805x$ (3 s.f.) |
| (v) | When $x = 4$<br>$t = 14.796 - 0.080474(4) = 14.474 \approx 14.5$ hours (3 s.f.)<br>Since $r = -0.941$ is <u>close to – 1</u>, indicating a strong negative linear correlation between the age and average total sleep time of babies and <u>$x$ = 4 is within the data range</u>, this is an interpolation. Hence, the estimate is reliable. |
| (vi) | $x$ = 32 is outside the data range, we are doing extrapolation. The estimate will not be reliable. |

| | |
|---|---|
| 10(i) | $\bar{x}=\frac{\sum(x-500)}{60}+500=\frac{318}{60}+500=505.3$<br>$s^2=\frac{1}{n-1}\left[\sum(x-500)^2-\frac{\left(\sum(x-500)\right)^2}{n}\right]$<br>$=\frac{1}{59}\left[25548.4-\frac{(318)^2}{60}\right]$<br>$=\frac{23863}{59}$ or $404.46\approx 404$ (3sf) |
| (ii) | $H_0: \mu=500$<br>$H_1: \mu\neq 500$<br>Under $H_0$, $\bar{X}\sim N\left(500,\frac{23863/59}{60}\right)$ approximately by CLT since $n$ = 60 is large.<br><br>Test statistic $Z=\frac{\bar{X}-500}{\sqrt{\frac{23863/59}{60}}}\sim N(0,\ 1)$ approximately.<br>$\alpha$ = 0.03<br>From GC, $p$-value = 0.0412<br><br>Since $p$-value = 0.0412 > $\alpha$ = 0.03, we do not reject $H_0$ at the 3% level of significance and conclude there is insufficient evidence that the population mean weight of bags of cashew nuts is not 500g. ie, there is insufficient evidence at the 3% level of significance that the claim is invalid. |
| (iii) | It is not necessary to assume a normal distribution as the sample size, $n$ = 60, is large, by Central Limit Theorem, $\bar{X}$ has a normal distribution approximately. |
| (iv) | For $H_0$ to be rejected, $\alpha\geq 0.0412$ .<br>If the test is conducted at the 5% level of significance, since $\alpha=0.05>0.0412=p$-value, we reject $H_0$ at the 5% level of significance and conclude there is sufficient evidence that the population mean weight of bags of cashew nuts is not 500g. ie, there is sufficient evidence at the 5% level of significance that the claim is invalid. |
| (v) | $s^2=\frac{70}{69}(23.4^2)=\frac{63882}{115}=555.50\approx 556$<br>$H_0: \mu=500$<br>$H_1: \mu<500$<br><br>Under $H_0$, $\bar{X}\sim N\left(500,\frac{555.50}{70}\right)$<br>Test statistic $Z=\frac{\bar{X}-500}{\sqrt{\frac{555.50}{70}}}\sim N(0,\ 1)$ |

| | |
|---|---|
| | $\alpha = 0.05$<br>mean weight of bags of cashew nuts is not at least 500 g<br>$\Rightarrow$ $H_0$ is rejected at 5% level of significance<br>$\Rightarrow \dfrac{\bar{x}-500}{\sqrt{\frac{555.50}{70}}} \le -1.6449$<br>$\Rightarrow \bar{x} \le 500 - 1.6449\sqrt{\frac{555.50}{70}}$<br>$\Rightarrow \bar{x} \le 495.37(2\text{dp})$ |
| 11 | Let $X$ and $Y$ denote the mass of a cheese tart and an empty box respectively.<br>$X \sim \mathrm{N}(60, 3.5^2)$ and $Y \sim \mathrm{N}(52, 0.8^2)$ |
| (i) | $\mathrm{P}(X<58) = 0.28385 \approx 0.284$ (3sf) |
| (ii) | Let $N$ be the number of tarts in a box with mass less than 58g.<br>$N \sim \mathrm{B}(6, 0.28385)$<br>$\mathrm{P}(N=2) = 0.31790 \approx 0.318$ (3sf) |
| | Alternatively,<br>Probability<br>$= {}^6C_2(0.28385)^2(1-0.28385)^4$<br>$= 0.31790 \approx 0.318$ (3sf) |
| (iii) | Let $T = X_1 + X_2 + \ldots + X_6 + Y$<br>$\mathrm{E}(T) = 6\mathrm{E}(X) + \mathrm{E}(Y) = 6(60) + 52 = 412$<br>$\mathrm{Var}(T) = 6\mathrm{Var}(X) + \mathrm{Var}(Y) = 6(3.5^2) + 0.8^2 = 74.14$<br>$T \sim \mathrm{N}(412, 74.14)$ |
| | $\mathrm{P}(T > 415) = 0.36376 \approx 0.364$ (3sf) |
| (iv) | $C = 2.1(X_1 + X_2 + \ldots + X_6) + 0.3Y$<br>$\mathrm{E}(C) = 2.1 \times 6\mathrm{E}(X) + 0.3\mathrm{E}(Y)$<br>$= (2.1)(6)(60) + 0.3(52)$<br>$= 771.6$<br>$\mathrm{Var}(C) = 2.1^2 \times 6\mathrm{Var}(X) + 0.3^2\mathrm{Var}(Y)$<br>$= 2.1^2(6)(3.5^2) + 0.3^2(0.8^2)$<br>$= 324.1926$<br>$C \sim \mathrm{N}(771.6\ ,\ 324.1926)$ |
| | $\mathrm{P}(747 < C < 774) = 0.467$ (3sf) |
| (v) | Let $M$ denote the mass of a mini cheese tart.<br>$\mathrm{E}(M) = 35,\quad \mathrm{Var}(M) = 3.5^2$<br>$\bar{M} \sim \mathrm{N}(35, \frac{3.5^2}{30})$ approximately by CLT since $n = 30$ is large.<br>$\mathrm{P}(\bar{M} > a) = 0.2$<br>$\mathrm{P}(\bar{M} \le a) = 1 - 0.2 = 0.8$<br>$a = 35.5$ (3sf) |