Candidate Name: ________________________ Class: _____________

**JC2 PRELIMINARY EXAMINATION**
Higher 1

## MATHEMATICS 8865/01

Paper 1

**13 September 2017**
**3 hours**

Additional Materials: Cover page
Answer papers
List of Formulae (MF26)

**READ THESE INSTRUCTIONS FIRST**

Write your full name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

## Section A: Pure Mathematics [40 marks]

**1.** A researcher prepares three types of food samples $X$, $Y$ and $Z$ for his experiment. Each food sample weighs 20 grams and contains three types of ingredients, namely, fibre, wheat and sweetener. The amount of fibre, wheat and sweetener in each sample is given below.

| Sample Type | Amount in grams for each 20 grams of sample | | |
|---|---|---|---|
| | Fibre | Wheat | Sweetener |
| $X$ | 12 | 6 | 2 |
| $Y$ | 8 | 6 | 6 |
| $Z$ | 6 | 14 | 0 |

The researcher wants to prepare a new food sample type $T$ by mixing different amounts of sample types $X$, $Y$ and $Z$ such that in 20 grams of sample type $T$, there are 8.8 grams of fibre, 7.6 grams of wheat and 3.6 grams of sweetener.

Determine the amount, in grams, of samples types $X$, $Y$ and $Z$ in 20 grams of sample type $T$. [4]

**2.** Mr Woo purchased $x$ kg of cherries from fruit stall $A$ for \$$a$. He bought 1 kg less cherries from fruit stall $B$ for \$$a$. He realised that fruit stall $B$ charged him more by \$5 per kg. By considering the difference in the unit price of the cherries bought from the two fruit stalls or otherwise, show that $5x^2 - 5x - a = 0$. [2]

Find the maximum weight of cherries that Mr Woo can buy from fruit stall $A$ if he does not want to spend more than \$50 on cherries. Give your answer to the nearest integer. [3]

**3.** **(a)** Differentiate $e^{x^2} + \dfrac{1}{px^2+1}$ with respect to $x$, where $p$ is a constant. [2]

**(b)** Find $\displaystyle\int \frac{2x-1}{x+3}\,dx$. [3]

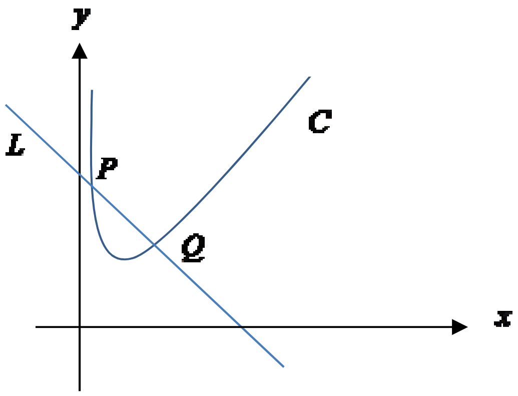
**4.** The curve $C$ has equation $y = qx - \ln(2x^2+1)$, where $q$ is a positive constant.

**(i)** Find, in terms of $q$, the equation of the tangent to $C$ at the point where $x = 1$. Give your answer in the form $y = ax + b$, with $a$ and $b$ in exact forms. [3]

**(ii)** Find the exact value of $q$ such that $C$ has 1 stationary point. [3]

**(iii)** Using the value of $q$ found in **(ii)**, find the equation of the tangent which is parallel to the $x$-axis. [2]

**5.**




The diagram shows the curve $C$ with equation $y=\left(\sqrt{x}+\dfrac{2k}{\sqrt{x}}\right)^2$ and the line $L$ with equation $y=13k-x$, where $k$ is a positive constant. The graphs intersect at $P$ and $Q$ as shown. Show that the $x$-coordinates of $P$ and $Q$ are $\dfrac{1}{2}k$ and $4k$ respectively. [2]

Hence find, in terms of $k$, the area of the region bounded by $C$, $L$, the $x$-axis and the line $x=k$. [4]

**6.** The number of customers (in thousands), $C$, of a new company is believed to be modelled by the equation

$$C=8(1-\mathrm{e}^{-kt}),$$

where $t$ is the number of years from the time the company starts its operation and $k$ is a positive constant.

**(i)** Given that the company has 7 thousand customers at the end of the 3rd year of operation, determine the exact value of $k$, giving your answer in the simplest form. [3]

Using the value of $k$ found in **(i)**,

**(ii)** sketch the graph of $C$ against $t$, stating the equations of any asymptotes, [2]

**(iii)** find the exact value of $\dfrac{\mathrm{d}C}{\mathrm{d}t}$ when $t=2$, simplifying your answer. Give an interpretation of the value you have found, in the context of the question. [3]

At the end of the 6th year of operation, the number of customers of the company is now believed to be modelled by the equation

$$C=-0.05t^2+0.7t+5.475,$$

where $t\geq 6$.

**(iv)** Use differentiation to find the value of $t$, where $t\geq 6$, which gives the maximum value of $C$. Hence, find the maximum value of $C$. [4]

**[Turn over**

## Section B: Statistics [60 marks]

**7.** A group of 10 students consisting of 6 females and 4 males bought tickets to attend a concert. If the tickets were for a particular row of 10 adjacent seats, find the number of possible seating arrangements when

**(i)** the first and last seats were occupied by students of the same gender, [3]

**(ii)** one particular student did not turn up for the concert. [1]

**8.** At a stall in a fun-fair, games of chance are played, where at most 1 prize is won per game. The probability that a prize is won in each game is 0.1. For each day, 80 games are played. The random variable $X$ is the number of prizes being won on a particular day.

**(i)** Find $\mathrm{P}(X > 5)$. [1]

The stall is opened for $n$ days and on each day, 80 games are played.

**(ii)** If $n = 10$, find the probability that there are 4 days with at most 5 prizes being won each day. [2]

**(iii)** If $n$ is large, using a suitable approximation, find the minimum value of $n$ such that the probability that the average number of prizes being won each day exceeds 8.5 is less than 0.1. [4]

**9.** In a box containing a large number of apples, 15% of the apples are rotten. A random sample of 20 apples is drawn to inspect.

**(i)** Explain the significance of the phrase 'large number' in the first sentence of the question. [1]

**(ii)** Find the probability that there is at least 1 but less than 4 rotten apples in the random sample. [3]

A box containing large number of apples is chosen for export if there is no rotten apple from the random sample. If there is at least 1 but less than 4 rotten apples in the random sample, another random sample of 10 apples is drawn from the box to inspect. If there is no rotten apple in the second random sample, the box will be chosen for export. Otherwise, the box will not be chosen for export.

**(iii)** Find the probability that a randomly chosen box is chosen for export. [2]

**(iv)** Four boxes each containing a large number of apples are chosen for inspection. If it is known that the first box is chosen for export, find the probability that exactly two out of the four boxes are chosen for export. [2]

**10.** A magazine claims that the average time a child spends outdoors is no longer than 14 hours a week. To verify this, Henry randomly selects 50 children and the amount of time that each child spends outdoors in a particular week, $x$ hours, is recorded. The results are summarised as follows.

$$\sum(x-14)=3.9 \qquad \sum(x-14)^2=2.7$$

**(i)** Find unbiased estimates of the population mean and variance. [2]

**(ii)** Suggest a reason why, in this context, the given data is summarised in terms of $(x-14)$ rather than $x$. [1]

**(iii)** Test at the 1% significance level whether the claim made in the magazine is valid. [5]

**11.** **(a)** Seven pairs of values of variables $x$ and $y$ are measured where $x$ and $y$ are positive values. Draw a sketch of a possible scatter diagram for each of the following cases:

**(i)** the product moment correlation coefficient is approximately zero, [1]

**(ii)** the product moment correlation coefficient is approximately 0.8. [1]

**(b)** A study on how the trade-in value $p$, in thousand dollars, of a particular make of used car depreciates with the age of the car $t$, in years, is conducted. The data for 7 cars is collected and shown in the following table.

| Age, $t$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| Trade-in value, $p$ | 54.0 | 50.1 | 45.3 | 38.6 | 35.1 | 33.5 | 30.4 |

**(i)** Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]

**(ii)** Find the product moment correlation coefficient and comment on its value in the context of this question. [2]

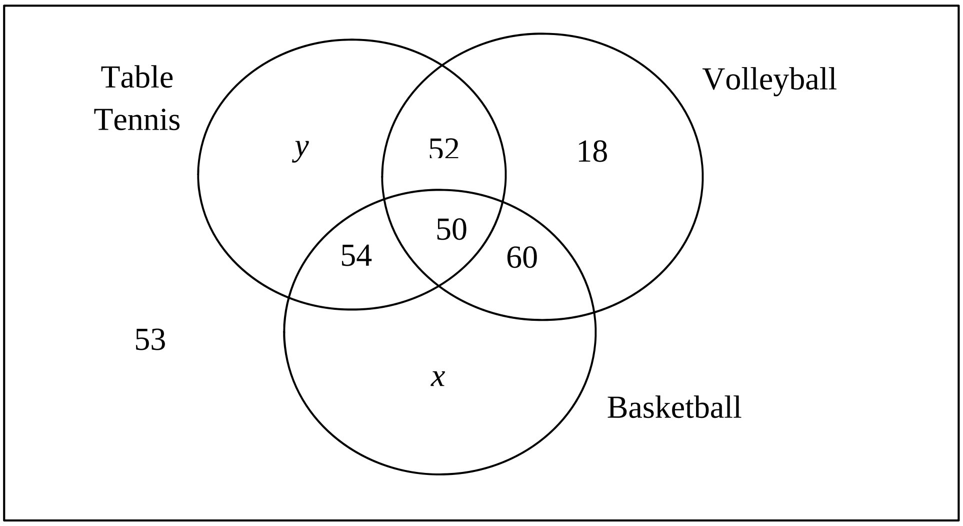
**(iii)** Find the equation of the regression line of $p$ on $t$ in the form $p=mt+c$, giving the values of $m$ and $c$ correct to 2 decimal places. [1]

**(iv)** The data for a second sample of another 6 cars is obtained. The regression lines of $p$ on $t$ and of $t$ on $p$ for the second sample are given respectively as:

$$p=61.45-4.19t \quad \text{and} \quad t=14.39-0.23p\,.$$

Calculate the mean trade-in value and mean age for the combined sample of 13 cars. [3]

**12.**




A group of students are surveyed on the types of sport(s) they can play out of the three sports namely table tennis, volleyball and basketball. The numbers of students who can play the different sport(s) are shown in the Venn diagram. The number of students who can play table tennis only is $y$ and the number of students who can play basketball only is $x$. One of the students is chosen at random.

$T$ is the event that the student can play table tennis.
$B$ is the event that the student can play basketball.
$V$ is the event that the student can play volleyball.

**(i)** Write down the expressions for $P(T)$ and $P(V)$ in terms of $x$ and $y$. Given that $T$ and $V$ are independent, show that $13y - 17x = 199$. [3]

**(ii)** Given that $P(T \cup B) = \dfrac{379}{450}$, find the values of $x$ and $y$. [3]

Using the values of $x$ and $y$ found in **(ii)**, find

**(iii)** $P(B \cap (T \cup V))$, [1]

**(iv)** $P(T \mid V)$. [1]

Three students from the whole group are chosen at random.

**(v)** Find the probability that among the three students, one can play exactly two sports out of the three sports (table tennis, volleyball and basketball), the other one can play table tennis only and the remaining one cannot play any of the sports. [3]

**13.** The masses, in kilograms, of cod fish and salmon fish sold by a fishmonger are normally distributed. The means and standard deviations of these distributions, and the selling prices, in \$ per kilogram, of cod fish and salmon fish are shown in the following table.

| | Mean (kg) | Standard deviation (kg) | Selling price (\$ per kg) |
|---|---|---|---|
| Cod Fish | $a$ | 0.1 | 68 |
| Salmon Fish | 0.6 | 0.15 | 30 |

**(i)** Find the probability that a randomly chosen cod fish has mass less than $(0.2 + a)$ kg. [2]

**(ii)** It is known that 20% of the cod fish sold by the fishmonger have a mass of at least 0.5 kg. Find the value of $a$. [3]

Use $a = 0.4$ for the rest of the question.

**(iii)** Find the probability that the total mass of 4 randomly chosen cod fish is within $\pm\, 0.1$ kg of twice the mass of a randomly chosen salmon fish. [4]

**(iv)** Find the probability that a randomly chosen cod fish has a selling price exceeding \$25 and a randomly chosen salmon fish has a selling price exceeding \$15. [2]

**(v)** State an assumption needed for your calculations in **(iii)** and **(iv)**. [1]

**- End of paper -**

**J2 H1 Math Prelim Exam (Solutions)**

**1.** Let the amount of samples types $X$, $Y$ and $Z$ in 20 grams of sample type $T$ be $x$, $y$ and $z$.

Fibre:

$$\left(\frac{12}{20}\right)x+\left(\frac{8}{20}\right)y+\left(\frac{6}{20}\right)z=8.8$$

$$0.6x+0.4y+0.3z=8.8 \qquad ---(1)$$

Wheat :

$$\left(\frac{6}{20}\right)x+\left(\frac{6}{20}\right)y+\left(\frac{14}{20}\right)z=7.6$$

$$0.3x+0.3y+0.7z=7.6 \qquad ---(2)$$

Sweetener :

$$\left(\frac{2}{20}\right)x+\left(\frac{6}{20}\right)y+\left(\frac{0}{20}\right)z=3.6$$

$$0.1x+0.3y+0z=3.6 \quad ---(3)$$

From GC

$x=6,\quad y=10,\ z=4$

There are 6 g of sample type $X$, 10 g of sample type $Y$ and 4 g of sample type $Z$ in 20 g of sample type $T$.

**2.**

$$\frac{a}{x-1}=\frac{a}{x}+5$$

$$\frac{a}{x-1}=\frac{a+5x}{x}$$

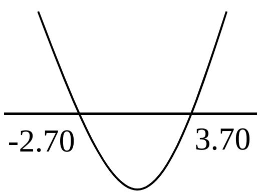
$$ax=ax-a+5x^2-5x$$

$5x^2-5x-a=0$ (shown)

To spend not more than \$50 for $x$ kg of cherries $\Rightarrow a\le 50$

$$5x^2-5x\le 50$$

$$x^2-x-10\le 0$$




Note:
The graphical method to solve the inequality must be shown.

$-2.70\le x\le 3.70$

The maximum amount of cherries Mr Woo can buy is 3 kg.

**3(a)** $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{x^2}+\dfrac{1}{px^2+1}\right)=2x\mathrm{e}^{x^2}-\dfrac{2px}{\left(px^2+1\right)^2}$

**(b)** $\displaystyle\int\frac{2x-1}{x+3}\,\mathrm{d}x=\int 2-\frac{7}{x+3}\,\mathrm{d}x$

$=2x-7\ln(x+3)+C$

**4(i)** $\dfrac{\mathrm{d}y}{\mathrm{d}x}=q-\dfrac{4x}{2x^2+1}$

When $x=1$,

$y=q-\ln 3$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x}=q-\dfrac{4}{3}$

Equation of tangent:

$y=\left(q-\dfrac{4}{3}\right)x+c$

$q-\ln 3=\left(q-\dfrac{4}{3}\right)(1)+c\Rightarrow c=\dfrac{4}{3}-\ln 3$

$\therefore y=\left(q-\dfrac{4}{3}\right)x+\dfrac{4}{3}-\ln 3$

Note:

The bracket in $y=\left(q-\dfrac{4}{3}\right)x+\dfrac{4}{3}-\ln 3$

must be clearly shown.

**(ii)** $\dfrac{\mathrm{d}y}{\mathrm{d}x}=q-\dfrac{4x}{2x^2+1}$

For stationary points,

$\dfrac{\mathrm{d}y}{\mathrm{d}x}=q-\dfrac{4x}{2x^2+1}=0$

$2qx^2+q-4x=0$

Since $C$ has 1 stationary point, $b^2-4ac=0$

$(-4)^2-4(2q)(q)=0$

$16-8q^2=0$

$q^2=2$

$q=-\sqrt{2}$ (NA since $q>0$) or $q=\sqrt{2}$

**(iii)** $y=\sqrt{2}x-\ln(2x^2+1)$

$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\sqrt{2}-\dfrac{4x}{2x^2+1}$

Given that the tangent is parallel to the $x$-axis,

$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\sqrt{2}-\dfrac{4x}{2x^2+1}=0$

$2\sqrt{2}x^2+\sqrt{2}-4x=0$

Using GC, $x=0.70711$ or $\dfrac{1}{\sqrt{2}}$.

Hence, the equation of the tangent is $y=1-\ln 2$ or $y=0.307$.

**5.**

$$\left(\sqrt{x}+\frac{2k}{\sqrt{x}}\right)^2=13k-x$$

$$x+\frac{4k^2}{x}+4k=13k-x$$

$$x^2+4k^2+4kx=13kx-x^2$$

$$2x^2-9kx+4k^2=0$$

$$(2x-k)(x-4k)=0$$

$$x=\frac{1}{2}k \quad \text{or} \quad x=4k \ \text{(shown)}$$

Note:
Factorisation
must be shown

Alternative:

$$x=\frac{-(-9k)\pm\sqrt{(-9k)^2-4(2)(4k^2)}}{2(2)}$$

$$=\frac{9k\pm\sqrt{49k^2}}{4}$$

$$=\frac{9k\pm 7k}{4}$$

$$=\frac{16k}{4} \quad \text{or} \quad \frac{2k}{4}$$

$$=4k \quad \text{or} \quad \frac{k}{2} \quad \text{(shown)}$$

$$\text{Required area }=\int_k^{4k}\left(\sqrt{x}+\frac{2k}{\sqrt{x}}\right)^2 \mathrm{d}x+\int_{4k}^{13k}(13k-x)\,\mathrm{d}x$$

$$=\int_k^{4k}\left(x+\frac{4k^2}{x}+4k\right)\mathrm{d}x+\text{Area of triangle}$$

$$=\left[\frac{x^2}{2}+4k^2\ln x+4kx\right]_k^{4k}+\frac{1}{2}(9k)(9k)$$

$$=\left(8k^2+4k^2\ln 4k+16k^2\right)-\left(\frac{1}{2}k^2+4k^2\ln k+4k^2\right)+\frac{81}{2}k^2$$

$$=60k^2+4k^2\ln 4$$

$$=60k^2+8k^2\ln 2$$

**6(i)**

$$7=8(1-\mathrm{e}^{-3k})$$

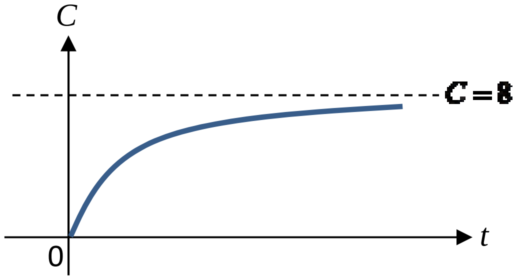
$$\frac{7}{8}=1-\mathrm{e}^{-3k}$$

$$\mathrm{e}^{-3k}=\frac{1}{8}$$

$$-3k=\ln\frac{1}{8}=-3\ln 2$$

$$k=\ln 2$$

**(ii)**




Note:
Curve is in the first
quadrant only

**(iii)** $C = 8(1-e^{-kt}) = 8 - 8e^{-t\ln 2}$

$\dfrac{dC}{dt} = 8\ln 2e^{-t\ln 2}$

$\left.\dfrac{dC}{dt}\right|_{t=2} = 8\ln 2e^{-2\ln 2}$

$= 8\ln 2e^{\ln\frac{1}{4}}$

$= 8\ln 2\left(\dfrac{1}{4}\right)$

$= 2\ln 2$

This value indicates that the number of customers is **increasing** at a rate of 2ln2 **thousands per year at the end of the second year of operation.**

**(iv)** $\dfrac{dC}{dt} = -0.1t + 0.7$

$-0.1t + 0.7 = 0$

$t = 7$

$C = -0.05(7)^2 + 0.7(7) + 5.475 = 7.925$

Hence, the maximum number of customers is 7.925 **thousands** customers or 7925 when $t = 7$.

**7(i)** Case 1: First and last seats occupied by males

Number of ways = $4 \times 3 \times 8!$ = 483 840 ways

OR Number of ways = ${}^4C_2 \times 2! \times 8!$ = 483 840 ways

Case 2: First and last seats occupied by females

Number of ways = $6 \times 5 \times 8!$ = 1209 600 ways

OR Number of ways = ${}^6C_2 \times 2! \times 8!$ = 1209 600 ways

$\therefore$ Total number of ways $= 483\ 840 + 1209\ 600 = 1693\ 440$ ways

**(ii)** Number of ways = $10! =$ 36 288 00 ways

**8(i)** Given $X$ denotes the number of prizes being won out of 80 games on a particular day.

Then $X \sim B(80, 0.1)$

$P(X > 5) = 1 - P(X \le 5) \approx 0.82308 \approx 0.823$

**(ii)** $P(X \le 5) \approx 0.17692$

Let $Y$ denotes the number of days with at most 5 prizes being won each day out of 10 days. Then $Y \sim B(10,\ 0.17692)$

Note: For $p$ to be in 5 sf

$P(Y = 4) \approx 0.063970 \approx 0.0640$

**(iii)** E($X$) = $80 \times 0.1 = 8$
Var($X$) = $80 \times 0.1 \times 0.9$ = 7.2

Since $n$ is large, by Central Limit Theorem, $\overline{X} \sim \text{N}(8, \frac{7.2}{n})$ approximately.

$\text{P}(\overline{X} > 8.5) < 0.1$

$$\Rightarrow \text{P}\left(Z > \frac{8.5-8}{\sqrt{\frac{7.2}{n}}}\right) < 0.1$$

$$\Rightarrow 1 - \text{P}\left(Z \le \frac{0.5}{\sqrt{\frac{7.2}{n}}}\right) < 0.1$$

$$\Rightarrow \text{P}\left(Z \le \frac{0.5}{\sqrt{\frac{7.2}{n}}}\right) > 0.9$$

Take note on the change in the inequality

$$\Rightarrow \frac{0.5}{\sqrt{\frac{7.2}{n}}} > 1.2816$$

Solving, $n > 47.300$ (5 s.f.)

The minimum value of $n$ is 48.

<u>Alternative (Using table)</u>
Using GC,

When $n$ = 47, $\text{P}(\overline{X} > 8.5) \approx 0.10072 > 0.1$

When $n$ = 48, $\text{P}(\overline{X} > 8.5) \approx 0.09835 < 0.1$

When $n$ = 49, $\text{P}(\overline{X} > 8.5) \approx 0.09605 < 0.1$

The minimum value of $n$ is 48.

**9(i)** The phrase 'large number' in the first sentence is required in order to assume that the probability of a rotten apple is approximately constant at 0.15.

**(ii)** Let $X$ denote the number of rotten apples in a random sample of 20 apples.
Then $X \sim \text{B}(20, 0.15)$
$\text{P}(1 \le X \le 3) = \text{P}(X \le 3) - \text{P}(X = 0) \approx 0.60897 \approx 0.609$
Or
$\text{P}(1 \le X \le 3) = \text{P}(X = 1) + \text{P}(X = 2) + \text{P}(X = 3) \approx 0.60897 \approx 0.609$

**(iii)** Let $Y$ denote the number of rotten apples in a random sample of 10 apples.
Then $Y \sim \text{B}(10,\ 0.15)$
P(A randomly chosen box is chosen for export)
$= \text{P}(X = 0) + \text{P}(1 \le X \le 3) \times \text{P}(Y = 0)$
$\approx 0.15865 \approx 0.159$

**(iv)** Required probability =

$$\frac{\text{P(First box is chosen for export)} \times \text{P(One out of the 3 remaining boxes chosen for export)} \times 3}{\text{P(First box chosen for export)}}$$

$\approx\ 0.15865\ \times\ (1 - 0.15865)^2 \times 3$
$\approx 0.33691 \approx 0.337$

Alternative solution:
Let $W$ denote the number of boxes chosen for export out of 3.
Then $W \sim \text{B}(3,\ 0.15865)$
$\text{P}(W = 1) \approx 0.337$

**10(i)** Unbiased estimate of the population mean, $\bar{x} = \dfrac{3.9}{50} + 14 = 14.078$

Unbiased estimate of the population variance,

$$s^2 = \frac{1}{49}\left[2.7 - \frac{(3.9)^2}{50}\right] \approx 0.048894 \approx 0.0489$$

**(ii)** Possible reasons:
- Keep the recorded values small since they are around 14 hours.
- Give an indication of the variations around the hypothesised mean of 14 hours.

**(iii)** $\text{H}_0: \mu = 14$ vs $\text{H}_1: \mu > 14$

Since $n$ = 50 is large, by Central Limit Theorem, $\overline{X} \sim \text{N}\left(14,\ \dfrac{0.048894}{50}\right)$ approximately.

Level of significance: 1%
Critical region: $z \ge\ 2.3263$

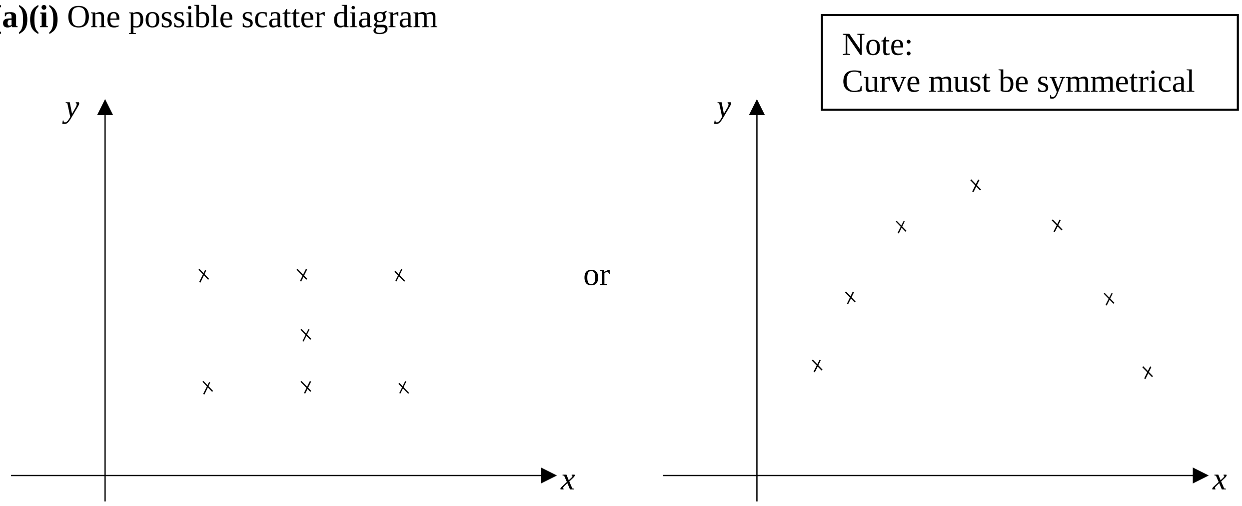
Note: State the distribution of $\overline{X}$

Standardised test statistic $z = \dfrac{14.078 - 14}{\sqrt{\dfrac{0.048894}{50}}} \approx 2.4943\ > 2.3263$

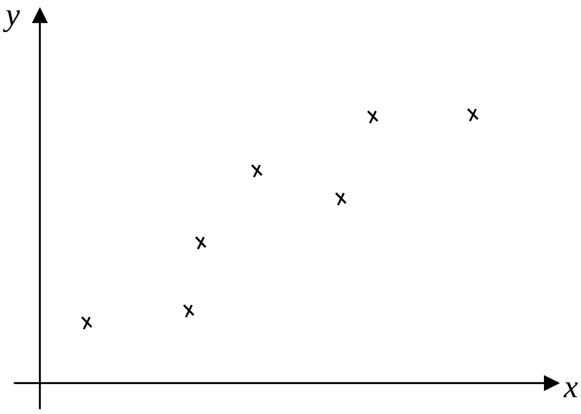
Using GC, $p$-value $\approx 0.0063100\ < 0.01$
Since the $p$-value is smaller than the level of significance, we reject $\text{H}_0$. There is sufficient evidence, at 1% level of significance, to conclude that Henry's claim is invalid.

Alternative conclusion:
Since the standardised test statistic falls inside the critical region, we reject $\text{H}_0$. There is sufficient evidence, at 1% level of significance, to conclude that Henry's claim is invalid.

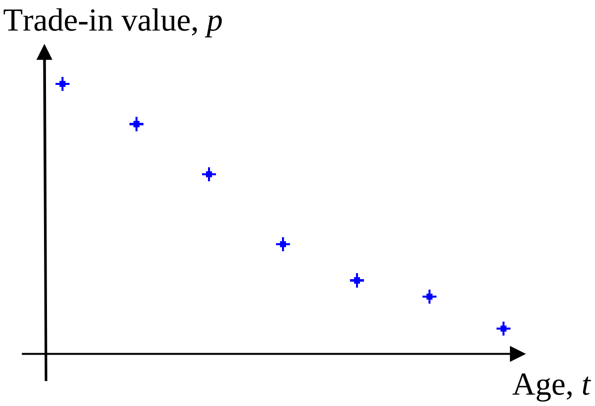
**11(a)(i)** One possible scatter diagram




**(ii)** One possible diagram




**(b)(i)**




Note:
Label the axes correctly

**(ii)** $r = -0.986$

There is a strong negative **linear** correlation between the age of the car and the average trade-in value of the car.

**(iii)** $p = -4.0786t + 61.3929$

$p \approx -4.08t + 61.39$ (to 2 d.p.)

**(iv)** Let the mean trade-in value and mean age of the 6 cars be $a$ and $b$ respectively.

$p = 61.45 - 4.19t \Rightarrow p + 4.19t = 61.45$
$t = 14.39 - 0.23p \Rightarrow 0.23p + t = 14.39$
Solving using GC, $a = 31.843$ and $b = 7.0661$

Mean age for 13 cars
$= \dfrac{2+3+4+5+6+7+8+6(7.0661)}{13}$
$\approx 5.95$ (3 s.f)

Mean age for 13 cars is 5.95

Mean trade-in value for 13 cars
$= \dfrac{54+50.1+45.3+38.6+35.1+33.5+30.4+6(31.843)}{13}$
$= 36.8$
Mean trade-in value for 13 cars is 36.8 thousand dollars (or $36800).

**12(i)** P($T$) = $\dfrac{156+y}{287+x+y}$ , P($V$) = $\dfrac{180}{287+x+y}$ , P($T \cap V$) = $\dfrac{102}{287+x+y}$
Given that $T$ and $V$ are independent, this means that
P($T \cap V$) = P($T$) × P($V$)
$\Rightarrow \dfrac{102}{287+x+y} = \dfrac{156+y}{287+x+y} \times \dfrac{180}{287+x+y}$
$\Rightarrow$ 102 (287 + $x$ + $y$) = 180 (156 + $y$)
$\Rightarrow 29\ 274 + 102x + 102y = 28\ 080 + 180y$
$\Rightarrow 1194 + 102x - 78y = 0$
$\Rightarrow -39y + 51x + 597 = 0$
$\Rightarrow 39y - 51x = 597$
$\therefore 13y - 17x = 199$ (Shown) ----- (1)

**(ii)** P($T \cup B$) = $\dfrac{379}{450}$
$\Rightarrow \dfrac{216+x+y}{287+x+y} = \dfrac{379}{450}$
$\Rightarrow 450(216+x+y) = 379(287+x+y)$
$\Rightarrow 97\ 200 + 450x + 450y = 108\ 773 + 379x + 379y$
$\Rightarrow 71x + 71y = 11\ 573$
$\therefore x + y = 163$ -----(2)

Using GC, solving (1) and (2) simultaneously, $x = 64$ and $y = 99$

**(iii)** P($B \cap (T \cup V)$) = $\dfrac{164}{450} = \dfrac{82}{225}$

**(iv)** $\mathrm{P}(T \mid V) = \dfrac{\mathrm{n}(T \cap V)}{\mathrm{n}(V)} = \dfrac{102}{180} = \dfrac{17}{30}$

**(v)** Required probability = $\dfrac{166}{450} \times \dfrac{99}{449} \times \dfrac{53}{448} \times 3! \approx 0.0577$

**13(i)** Let $C$ denote the mass of cod fish. Then $C \sim \mathrm{N}(a,\ 0.1^2)$.

$\mathrm{P}(C < 0.2 + a) = \mathrm{P}\left(Z < \dfrac{(0.2+a)-a}{0.1}\right) = \mathrm{P}(Z < 2) \approx 0.97725 \approx 0.977$

**(ii)** Given that $\mathrm{P}(C \geq 0.5) = 0.2$

$\Rightarrow \mathrm{P}(C < 0.5) = 0.8$

$\Rightarrow \mathrm{P}\left(Z < \dfrac{0.5-a}{0.1}\right) = 0.8$

$\Rightarrow \dfrac{0.5-a}{0.1} \approx 0.84162$

$\therefore\ a \approx 0.416$

**(iii)** Using $a = 0.4$,

Consider $T = C_1 + C_2 + C_3 + C_4 - 2S \sim \mathrm{N}(0.4,\ 0.13)$

$\mathrm{P}(-0.1 < T < 0.1) \approx 0.11993 \approx 0.120$

**(iv)** Using $a = 0.4$,

Let $68C$ be the selling price of cod fish. Then $68C \sim \mathrm{N}(27.2,\ 46.24)$

Let $30S$ be the selling price of salmon fish. Then $30S \sim \mathrm{N}(18,\ 20.25)$

$\mathrm{P}(68C > 25) \times \mathrm{P}(30S > 15) \approx 0.46858 \approx 0.469$

<u>Alternative</u>

$\mathrm{P}\left(C > \dfrac{25}{68}\right) \times \mathrm{P}\left(S > \dfrac{15}{30}\right) \approx 0.46858 \approx 0.469$

**(v)** The mass of all the fish are independent of one another.

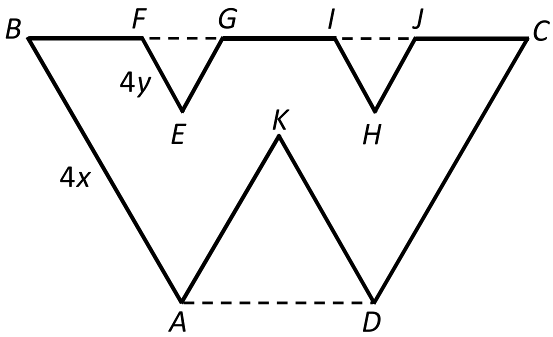
**RI H1 Mathematics 2017 Prelim Exam Paper Question**

**1** Differentiate

**(a)** $\ln\{7\sqrt{(1+6x^2)}\}$, [2]

**(b)** $\dfrac{1}{6(1-7x)^2}$. [2]

**2** A designer wishes to make a logo for her client. Her design for the logo is shown in the diagram below, which is formed by removing three triangles *EFG*, *HIJ* and *AKD* from a trapezium *ABCD*.




For aesthetic purpose, her design has the following features:

(1) Angles *ABC* and *BCD* are both 60°.

(2) Triangles *EFG* and *HIJ* are identical and equilateral.

(3) The points *F*, *G*, *I* and *J* are on *BC* such that $BF = GI = JC$.

(4) Triangle *AKD* is also equilateral, and the point *K* lies on the intersection of the line segments *AI* and *GD*.

It is given that $AB = CD = 4x$ mm and $EF = 4y$ mm.

**(i)** Show that $BF = 2(x-y)$ mm. [1]

**(ii)** Show that $AD = 2(x+y)$ mm. [1]

The perimeter and area of *ABFEGIHJCDKA* is 566 mm and $4655\sqrt{3}$ mm$^2$ respectively.

**(iii)** Find the values of $x$ and $y$. [4]

**3** **Do not use a calculator in answering this question.**

**(a)** Solve the inequality $\dfrac{(x-4)(x^2-4x+4)}{x+4} \geq 0$. [4]

**(b)** Find the range of values of $k$ for which $(k-1)x^2 - 2x + k + 2 < 0$ for all real values

| | |
|---|---|
| | of $x$. [4] |
| **4** | A jug containing liquid is taken from a refrigerator and placed in a room with a constant temperature of 25 °C. The temperature of the liquid $\theta$ °C after time $t$ minutes is given by $$\theta = 25 - A\mathrm{e}^{kt},$$ where $A$ and $k$ are real constants.<br><br>Initially the temperature of the liquid is 9 °C. After 20 minutes, the temperature of the liquid increases to 17 °C.<br><br>**(i)** Find the value of $A$ and show that $k = \frac{1}{20}\ln\frac{1}{2}$. [4]<br><br>**(ii)** Find the temperature of the liquid after 25 minutes. [1]<br><br>**(iii)** Find the exact duration it takes for the temperature of the liquid to increase from 17 °C to 23 °C. [2]<br><br>**(iv)** State what happens to $\theta$ for large values of $t$. [1]<br><br>**(v)** Sketch a graph of $\theta$ against $t$. [2] |
| **5** | The curve $C_1$ has equation $y = \ln(1 + x)$ and the curve $C_2$ has equation $y = \ln 2 + 1 - x$.<br><br>**(i)** Sketch the graphs of $C_1$ and $C_2$ on the same diagram, stating the equations of any asymptotes and the exact coordinates of any points where the curves crosses the axes. [3]<br><br>**(ii)** Verify that $C_1$ and $C_2$ intersect at $x = 1$. [1]<br><br>**(iii)** Find, correct to 2 decimal places, the area of the finite region bounded by $C_1$, $C_2$ and the $x$-axis. [3]<br><br>**(iv)** Use integration to find the exact area of the finite region bounded by $C_1$, $C_2$ and the $y$-axis. Leave your answer in the form $A + B\ln 2$, where $A$ and $B$ are constants to be determined. [5] |
| **6** | It is given that $X \sim \mathrm{N}(\mu, 7)$ and $\mathrm{P}(X < 7) = 0.7$.<br><br>**(i)** Find the value of $\mu$ correct to 3 decimal places. [2]<br><br>**(ii)** Find $\mathrm{P}(X_1 < X_2 + 1)$, where $X_1$ and $X_2$ are independent observations of $X$. [2] |
| **7** | A traditional bakery produces two types of biscuits – one with sweet fillings and one |

| | |
|---|---|
| | with salted fillings. The biscuits are sold in packs of 8, and each pack has a random selection of the two types of biscuits.<br><br>The mean number of biscuits with sweet fillings in each pack is 3.2.<br><br>**(i)** Find the probability that a randomly chosen pack contains no biscuits with sweet fillings. [2]<br><br>**(ii)** Show that the probability that a randomly chosen pack contains at least four biscuits with sweet fillings is 0.406 correct to 3 significant figures. [2]<br><br>A customer buys 18 packs of biscuits for a wedding.<br><br>**(iii)** Find the probability that at most 9 of these packs contains at least four biscuits with sweet fillings. [2] |
| **8** | Box *A* contains 2 green marbles and 6 red marbles. Box *B* contains 4 green marbles and 2 red marbles. Two fair dice are tossed at the same time. Box *A* is selected if at least one '6' is shown. Otherwise, box *B* is selected. One marble is then chosen from the selected box and its colour noted.<br><br>Draw a tree diagram to represent this situation. [4]<br><br>Find the probability that<br><br>**(i)** the marble chosen is green, [2]<br><br>**(ii)** the marble chosen is from box *A*, given that its colour is red. [2]<br><br>Give your answers as a fraction in its lowest term. |
| **9** | Caffeine is said to affect our sleep at night. In a student research study, different amounts of caffeine, $x$ grams, were given to a test subject for 8 consecutive evenings and the times, $t$ minutes, for the test subject to fall asleep at night were recorded.<br><br>The results are given in the table below.<br><br>(table below)<br><br>**(i)** Draw a scatter diagram to illustrate the data. [2]<br><br>**(ii)** Calculate the equation of the regression line of $t$ on $x$, and draw this line on your scatter diagram. [1]<br><br>**(iii)** Find $\bar{x}$ and $\bar{t}$, and mark the point $(\bar{x}, \bar{t})$ on your scatter diagram. [1] |

| Day | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $x$ | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 |
| $t$ | 15 | 16 | 20 | 23 | 25 | 24 | 30 | 35 |

**(iv)** Find the product moment correlation coefficient and comment on its value in the context of the data. [2]

**(v)** Use the regression line in part **(ii)** to predict the time taken for the test subject to fall asleep when 1.00 grams of caffeine was given. Comment on the validity of this prediction. [2]

**10** A farmer grows watermelons. He claims that the average mass of watermelons in his farm is at least 10 kg. To test this claim, a random sample of 70 watermelons is checked and the masses of watermelons, $x$ kg, are summarised as follows:

$$\sum(x-10) = -28, \quad \sum(x-10)^2 = 267.$$

**(i)** Find unbiased estimates of the population mean and variance. [3]

**(ii)** Test at the 5 % significance level whether the farmer's claim is valid. [4]

The farming process is improved and the new population variance is known to be $3.31\text{ kg}^2$. A new random sample of 70 watermelons is checked and the total mass of this sample is $m$ kg. A test at the 5 % significance level shows that there is sufficient evidence to suggest that the population mean mass of watermelons is more than 10 kg.

**(iii)** Find the range of possible values of $m$. [3]

**11** A pet shop owner carries African bullfrogs. The masses, in kg, of the male and female frogs are independent and are normally distributed with means and standard deviations as shown in the following table:

| | Mean mass | Standard deviation |
|---|---|---|
| Male | 1.4 | 0.28 |
| Female | 0.7 | 0.14 |

**(i)** A male and a female bullfrog are chosen at random. Find the probability that the mass of the female frog is greater than the mass of the male frog, stating clearly the mean and variance of any distribution that you use. [3]

The owner wishes to build a tank for up to four frogs. If he uses material $X$, the total mass of the frogs must not exceed the maximum safety limit of 4.5 kg.

**(ii)** Two male and two female bullfrogs are chosen at random. Find the probability that their total mass do not exceed the maximum safety limit of 4.5 kg, stating clearly the mean and variance of any distribution that you use.

If the owner uses material $Y$, the maximum safety limit of the tank is improved to $L$ kg.

| | |
|---|---|
| | **(iii)** It is 95 % certain that four male bullfrogs chosen at random have a total mass not exceeding the maximum safety limit of $L$ kg. Find, correct to 1 decimal place, the least value of $L$. [4] |
| **12** | Find the number of ways in which the letters of the word SECTION can be arranged if<br><br>**(i)** the letters are not in alphabetical order, [1]<br><br>**(ii)** the consonants (S, C, T, N) and vowels (E, I, O) must alternate, [2]<br><br>**(iii)** all the vowels are together, [2]<br><br>**(iv)** all the vowels are separated, [2]<br><br>**(v)** there must be exactly two letters between the two letters E and O. [2]<br><br>Find, as a fraction in its lowest term, the probability that after arranging the letters of the word SECTION, there is at least one consonant and at least one vowel between the two letters E and O. [4] |