H2 Mathematics 2017 Prelim Exam Paper 1 Question

1

Mr Subash returned to Singapore after his tour in Europe and wishes to convert his foreign currencies back to Singapore Dollars (S$). Three money changers offer the following exchange
rates:

Money Changer	1 Swiss Franc	1 British Pound	1 Euro	Total amount of S$ Mr Subash would receive after currency conversion
A	S$1.35	S$1.80	S$1.55	S$1151.50
B	S$1.40	S$1.85	S$1.65	S$1208.25
C	S$1.45	S$1.75	S$1.60	S$1189.25

How much of each currency has Mr Subash left after his tour? [4]

2

(a) Find $\int \sin(2\theta)\cos(3\theta)\,\mathrm{d}\theta$. [2]

(b) Use the substitution $\theta = \sqrt{x}$ to find the exact value of $\int_{\sqrt{\frac{\pi}{2}}}^{\sqrt{\pi}} \theta^3 \cos\left(\theta^2\right) \mathrm{d}\theta$. [5]

3

(i) Using the formula for $\sin P - \sin Q$, show that

$$\sin\left[(2r+1)\theta\right] - \sin\left[(2r-1)\theta\right] \equiv 2\cos(2r\theta)\sin\theta.$$ [1]

(ii) Given that $\sin\theta \neq 0$, using the method of differences, show that

$$\sum_{r=1}^{n} \cos(2r\theta) = \frac{\sin\left[(2n+1)\theta\right] - \sin\theta}{2\sin\theta}.$$ [2]

(iii) Hence find $\sum_{r=1}^{n} \cos^2\left(\frac{r\pi}{5}\right)$ in terms of *n*.

Explain why the infinite series

$$\cos^2\left(\frac{\pi}{5}\right) + \cos^2\left(\frac{2\pi}{5}\right) + \cos^2\left(\frac{3\pi}{5}\right) + \ldots$$

is divergent. [3]

4

A fund is started at $6000 and compound interest of 3% is added to the fund at the end of each year. If withdrawals of $*k* are made at the beginning of each of the subsequent years, show that the amount in the fund at the beginning of the $(n+1)$th year is

$$\$\frac{100}{3}\left[(180-k)(1.03)^n + k\right].$$ [5]

(i) It is given that $k = 400$. At the beginning of which year, for the first time, will the amount in the fund be less than \$1000? [2]

(ii) If the fund is fully withdrawn at the beginning of sixteenth year, find the least value of k to the nearest integer. [2]

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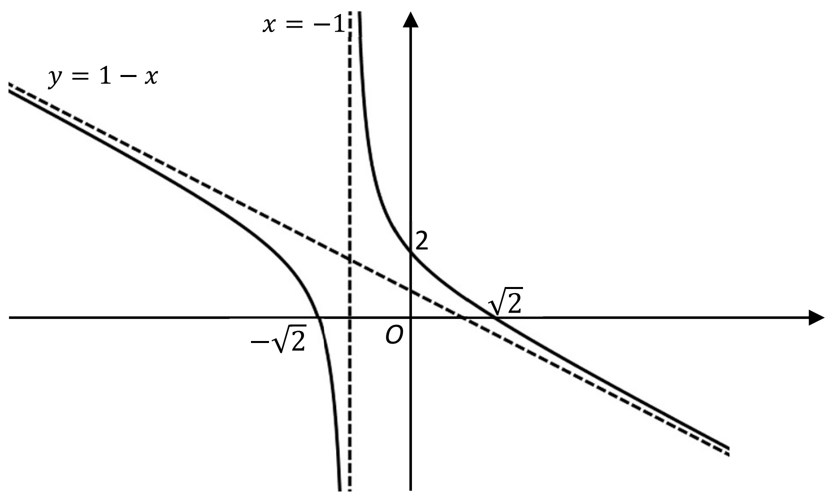
5 **(a)** The curve C has the equation

$$(x-2)^2 = a^2\left(1-y^2\right), \qquad 1 < a < 2.$$

Sketch C, showing clearly any intercepts and key features. [2]

(b) The diagram shows the graph of $y = \mathrm{f}(x)$, which has an oblique asymptote $y = 1 - x$, a vertical asymptote $x = -1$, x-intercepts at $\left(\sqrt{2}, 0\right)$ and $\left(-\sqrt{2}, 0\right)$, and y-intercept at $(0, 2)$.



Sketch, on separate diagrams, the graphs of

(i) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

(ii) $y = \mathrm{f}'(x)$, [3]

showing clearly all relevant asymptotes and intercepts, where possible.

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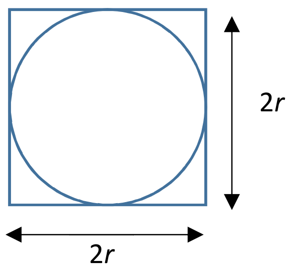
6 With respect to the origin O, the position vectors of the points U, V and W are $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ respectively. The mid-points of the sides VW, WU and UV of the triangle UVW are M, N and P respectively.

(i) Show that $\overrightarrow{UM} = \dfrac{1}{2}\left(\mathbf{v} + \mathbf{w} - 2\mathbf{u}\right)$. [2]

(ii) Find the vector equations of the lines UM and VN. Hence show that the position vector of the point of intersection, G, of UM and VN is $\dfrac{1}{3}\left(\mathbf{u} + \mathbf{v} + \mathbf{w}\right)$. [5]

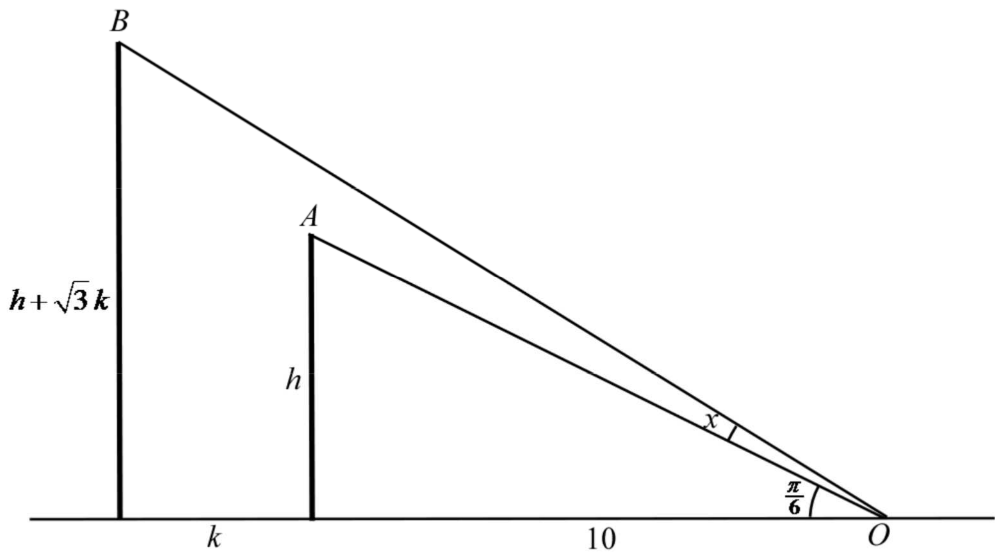
	(iii) It is now given that $\mathbf{u}=\begin{pmatrix}1\\0\\0\end{pmatrix}$, $\mathbf{v}=\begin{pmatrix}0\\1\\0\end{pmatrix}$, $\mathbf{w}=\begin{pmatrix}0\\0\\1\end{pmatrix}$. Find the direction cosines of $\overrightarrow{OG}$. [2]
7	**(a)** If $u=2-\mathrm{i}\sin^2\theta$ and $v=2\cos^2\theta+\mathrm{i}\sin^2\theta$ where $-\pi<\theta\le\pi$, find $u-v$ in terms of $\sin^2\theta$, and hence determine the exact expression for $\lvert u-v\rvert$ and the exact value of $\arg(u-v)$. [6] **(b)** The roots of the equation $x^2+(\mathrm{i}-3)x+2(1-\mathrm{i})=0$ are α and β, where α is a real number and β is not a real number. Find α and β. [4]
8	**(a)** When a liquid is poured onto a flat surface, a circular patch is formed. The area of the circular patch is expanding at a constant rate of $6\pi\ \text{cm}^2/\text{s}$. **(i)** Find the rate of change of the radius 24 seconds after the liquid is being poured. [3] **(ii)** Explain whether the rate of change of the radius will increase or decrease as time passes. [1] **(b)** A cylindrical can of volume $355\ \text{cm}^3$ with height h cm and base radius r cm is made from 3 pieces of metal. The curved surface of the can is formed by bending a rectangular sheet of metal, assuming that no metal is wasted in creating this surface. The top and bottom surfaces of the can are cut from square sheets of metal with length $2r$ cm, as shown below. The cost of the metal sheets is \$$K$ per cm^2.

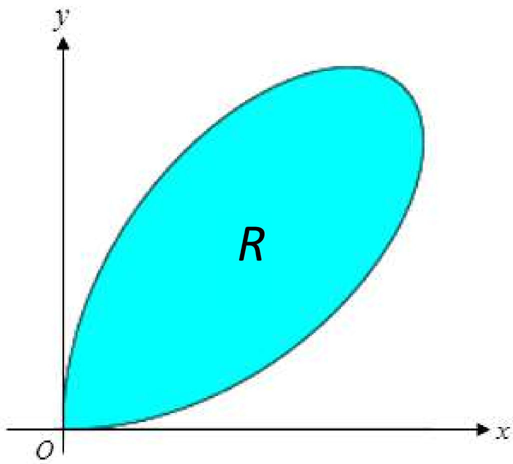


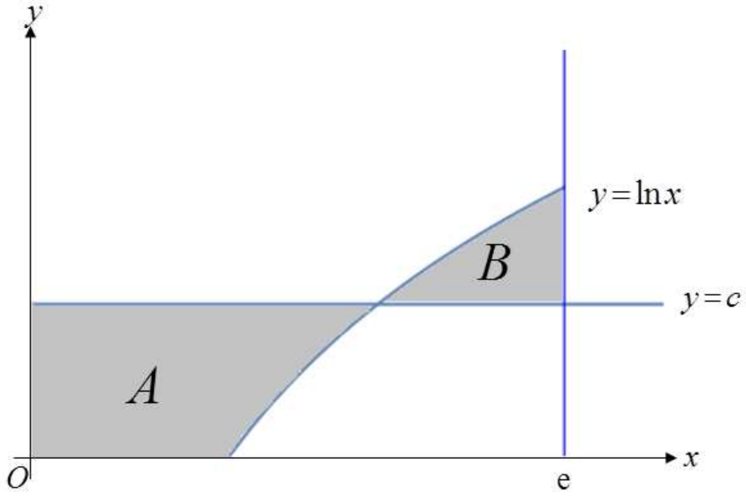
(i) Show that the total cost of metal used, denoted by \$$C$, is given by

$$C=K\left(\frac{710}{r}+8r^2\right).$$ [3]

(ii) Use differentiation to show that, when the cost of metal used is a minimum, then $\frac{h}{r}=\frac{8}{\pi}$. [5]

9	**(i)** Express $\sqrt{3}\cos x - \sin x$ in the form $R\cos(x+\alpha)$ where R and α are exact positive constants to be found. [2] **(ii)** State a sequence of transformations which transform the graph of $y=\cos x$ to the graph of $y=\sqrt{3}\cos x-\sin x$. [2] The function f is defined by $\mathrm{f}: x \mapsto \sqrt{3}\cos x - \sin x,\ 0 \le x \le 2\pi$. **(iii)** Sketch the graph of $y=\mathrm{f}(x)$ and state the range of f. [3] The function g is defined by $\mathrm{g}: x \mapsto \mathrm{f}(x),\ 0 \le x \le k$. **(iv)** Given that g^{-1} exists, state the largest exact value of k and find $\mathrm{g}^{-1}(x)$. [3] The function h is defined by $\mathrm{h}: x \mapsto x-2,\ x \ge 0$. **(v)** Explain why the composite function fh does not exist. [1]
10	 A laser from a fixed point O on a flat ground projects light beams to the top of two vertical structures A and B as shown above. To project the beam to the top of A, the laser makes an angle of elevation of $\frac{\pi}{6}$ radians. To project the beam to the top of B, the laser makes an angle of elevation of $\left(\frac{\pi}{6}+x\right)$ radians. The two structures A and B are of heights h m and $\left(h+\sqrt{3}\,k\right)$ m respectively and are 10 m and (10 + k) m away from O respectively. **(i)** Show that the length of the straight beam from O to A is $\frac{20}{\sqrt{3}}$ m. [1] **(ii)** Show that the length of AB is $2k$ m and that the angle of elevation of B from A is $\frac{\pi}{3}$ radians. [3]

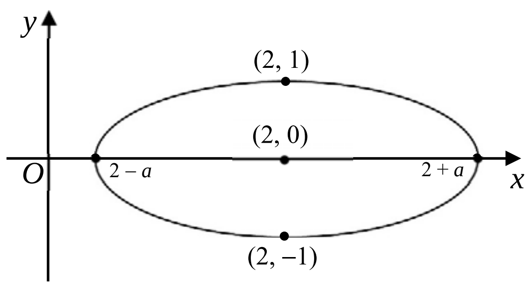
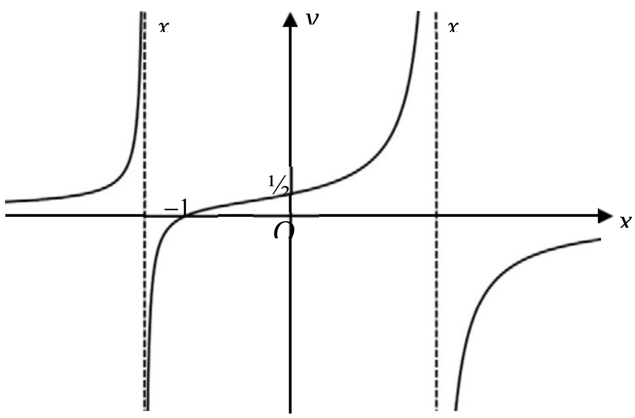
	(iii) Hence, using the sine rule, show that $k = \dfrac{10\sin x}{\sqrt{3}\sin\left(\dfrac{\pi}{6} - x\right)}$. [2] **(iv)** If x is sufficiently small, show that $k \approx \dfrac{20}{\sqrt{3}}\left(x + ax^2\right)$, where a is a constant to be determined. [6]
11	**(a)** The diagram below shows a section of *Folium of Descartes* curve which is defined parametrically by $x = \dfrac{3m}{1+m^3}, \quad y = \dfrac{3m^2}{1+m^3}, \quad m \geq 0.$  **(i)** It is known that the curve is symmetrical about the line $y = x$. Find the values of m where the curve meets the line $y = x$. [1] **(ii)** Region R is the region enclosed by the curve in the first quadrant. Show that the area of R is given by $2\left(\int_0^{\frac{3}{2}} x \, \mathrm{d}y - \dfrac{9}{8}\right)$, and evaluate this integral. [5] **(b)** The diagram below shows a horizontal line $y = c$ intersecting the curve $y = \ln x$ at a point where the x-coordinate is such that $1 < x < e$.

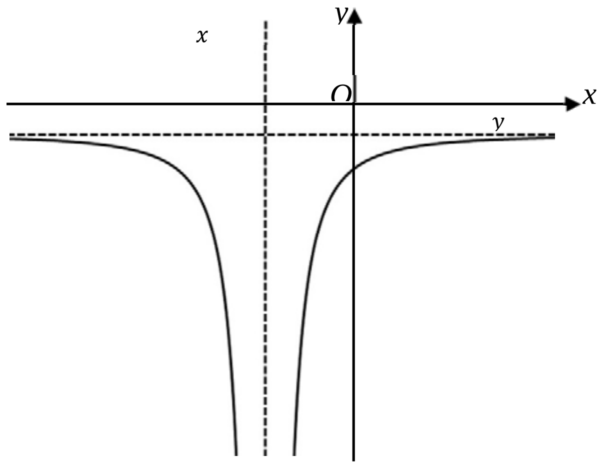


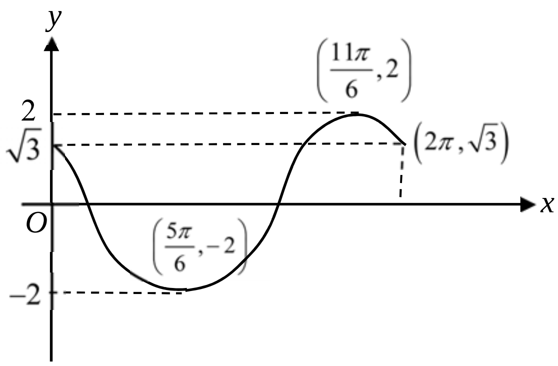
The region A is bounded by the curve, the line $y = c$, the x-axis and the y-axis while the region B is bounded by the curve and the lines $x = e$ and $y = c$. Given that the volumes of revolution when A and B are rotated completely about the y-axis are equal, show that $c = \dfrac{e^2 + 1}{2e^2}$. [6]

ANNEX B

XXJC H2 Math JC2 Preliminary Examination Paper 1

QN	Topic Set	Answers
1	Equations and Inequalities	He has 250 francs, 125 pounds and 380 euros left.
2	Integration techniques	2(a) $\frac{1}{2}\cos\theta-\frac{1}{10}\cos(5\theta)+c$ 2(b) $-\frac{1}{2}-\frac{\pi}{4}$
3	Sigma Notation and Method of Difference	3(iii) $\dfrac{\sin\frac{(2n+1)\pi}{5}}{4\sin\frac{\pi}{5}}-\frac{1}{4}+\frac{1}{2}n$
4	AP and GP	4(i) at the beginning of 19th year 4(ii) Least $k=$ 503
5	Graphs and Transformation	5(a)  5(b) (i) 

		5(b) (ii) 
6	Vectors	6(ii) Line *UM*: $\mathbf{r} = \mathbf{u} + \lambda\left(\mathbf{w} + \mathbf{v} - 2\mathbf{u}\right),\ \lambda \in \mathbb{R}$ Line *VN*: $\mathbf{r} = \mathbf{v} + \mu\left(\mathbf{w} + \mathbf{u} - 2\mathbf{v}\right),\ \mu \in \mathbb{R}$ 6(iii) Direction cosines of $\overrightarrow{OG}$ are $\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}$
7	Complex numbers	7(a) $u - v = 2\sin^2\theta - 2\mathrm{i}\sin^2\theta$ $\lvert u - v\rvert = 2\sqrt{2}\sin^2\theta,\ \arg\left(u - v\right) = -\frac{\pi}{4}$ 7(b) $\alpha = 2,\ \beta = 1 - \mathrm{i}$
8	Differentiation & Applications	8(a) (i) $\frac{1}{4}$ cm/ s , (ii) $\frac{\mathrm{d}r}{\mathrm{d}t}$ will decrease as time passes
9	Functions	9(i) $R = 2,\ \alpha = \frac{\pi}{6}$ 9(ii) $y = \cos x \xrightarrow{A} y = \cos(x + \alpha) \xrightarrow{B} y = R\cos(x + \alpha)$ *A*: Translation by α radians in the negative *x*-direction, followed by *B*: Scaling parallel to the *y*-axis by a scale factor *R*.

		9(iii)  Range of f, $R_f = [-2, 2]$ 9(iv) Largest $k = \frac{5\pi}{6}$ $g^{-1}(x) = \cos^{-1}\frac{x}{2} - \frac{\pi}{6}$ 9(v) Since $R_h = [-2, +\infty)$ and $D_f = [0, 2\pi]$, $R_h \not\subset D_f$, fh does not exist.
10	Binomial Expansion	10(iv) $a = \sqrt{3}$
11	Application of Integration	11(a) (i) $m = 0$ or 1 11(a) (ii) $\frac{3}{2}$ units^2

H2 Mathematics 2017 Prelim Exam Paper 1 Solution

1	Let x, y and z be the amount of Francs, Pounds & Euro Mr Subash has left respectively. $1.35x+1.80y+1.55z=1151.50$ $1.40x+1.85y+1.65z=1208.25$ $1.45x+1.75y+1.60z=1189.25$ Using GC, x = 250, y = 125, z = 380. He has 250 francs, 125 pounds and 380 euros left.
2	**(a)** By Factor Formula, $\sin(2\theta)\cos(3\theta) = \frac{1}{2}\left[\sin(5\theta)+\sin(-\theta)\right]$ $= \frac{1}{2}\left[\sin(5\theta)-\sin(\theta)\right]$ $\int \sin(2\theta)\cos(3\theta)\,\mathrm{d}\theta = \int \frac{1}{2}\left[\sin(5\theta)-\sin(\theta)\right]\mathrm{d}\theta$ $= \frac{1}{2}\cos\theta - \frac{1}{10}\cos(5\theta)+c$ **(b)** $\theta=\sqrt{\pi} \Rightarrow \sqrt{x}=\sqrt{\pi} \Rightarrow x=\pi$ $\theta=\sqrt{\frac{\pi}{2}} \Rightarrow \sqrt{x}=\sqrt{\frac{\pi}{2}} \Rightarrow x=\frac{\pi}{2}$ $\theta=\sqrt{x} \Rightarrow \frac{\mathrm{d}\theta}{\mathrm{d}x}=\frac{1}{2\sqrt{x}}$. $\int_{\sqrt{\frac{\pi}{2}}}^{\sqrt{\pi}} \theta^3\cos\left(\theta^2\right)\mathrm{d}\theta$ $=\int_{\frac{\pi}{2}}^{\pi} x\sqrt{x}(\cos x)\left(\frac{1}{2\sqrt{x}}\right)\mathrm{d}x$ $=\frac{1}{2}\int_{\frac{\pi}{2}}^{\pi} x\cos x\,\mathrm{d}x$ $u = x$, $\frac{\mathrm{d}u}{\mathrm{d}x} = 1$; $\frac{\mathrm{d}v}{\mathrm{d}x} = \cos x$, $v = \sin x$ $=\frac{1}{2}\left[\left[x\sin x\right]_{\frac{\pi}{2}}^{\pi} - \int_{\frac{\pi}{2}}^{\pi} 1(\sin x)\,\mathrm{d}x\right]$ $=\frac{1}{2}\left(0-\frac{\pi}{2}+\left[\cos x\right]_{\frac{\pi}{2}}^{\pi}\right)$ $=\frac{1}{2}\left[-\frac{\pi}{2}+(-1-0)\right]$ $=-\frac{1}{2}-\frac{\pi}{4}$

3	**(i)** $\sin\left[(2r+1)\theta\right]-\sin\left[(2r-1)\theta\right]$ $\equiv 2\cos\frac{(2r+1)\theta+(2r-1)\theta}{2}\sin\frac{(2r+1)\theta-(2r-1)\theta}{2}$ $\equiv 2\cos(2r\theta)\sin\theta$ [Shown] **(ii)** From (i), $\sin\left[(2r+1)\theta\right]-\sin\left[(2r-1)\theta\right]\equiv 2\cos(2r\theta)\sin\theta$ $\Rightarrow \cos(2r\theta)=\frac{\sin\left[(2r+1)\theta\right]-\sin\left[(2r-1)\theta\right]}{2\sin\theta}$ $\therefore \sum_{r=1}^{n}\cos(2r\theta) = \sum_{r=1}^{n}\frac{\sin\left[(2r+1)\theta\right]-\sin\left[(2r-1)\theta\right]}{2\sin\theta}$ $=\frac{1}{2\sin\theta}\left[\begin{array}{l}\sin3\theta-\sin\theta\\+\sin5\theta-\sin3\theta\\+\sin7\theta-\sin5\theta\\+\cdots\\+\sin(2n-1)\theta-\sin(2n-3)\theta\\+\sin(2n+1)\theta-\sin(2n-1)\theta\end{array}\right]$ $=\frac{\sin\left[(2n+1)\theta\right]-\sin\theta}{2\sin\theta}$ [Shown] **(iii)** $\sum_{r=1}^{n}\cos^2\left(\frac{r\pi}{5}\right) = \sum_{r=1}^{n}\frac{\cos\left(\frac{2r\pi}{5}\right)+1}{2}$ $=\frac{1}{2}\sum_{r=1}^{n}\cos\left(\frac{2r\pi}{5}\right)+\sum_{r=1}^{n}\frac{1}{2}$ $\left(\text{Let } \theta=\frac{\pi}{5}\right)$ $=\frac{1}{2}\left[\frac{\sin\frac{(2n+1)\pi}{5}-\sin\frac{\pi}{5}}{2\sin\frac{\pi}{5}}\right]+\frac{1}{2}n$ $=\frac{\sin\frac{(2n+1)\pi}{5}}{4\sin\frac{\pi}{5}}-\frac{1}{4}+\frac{1}{2}n$ As $n\to\infty$, $-\frac{1}{4}+\frac{1}{2}n\to\infty$ and $\left\|\sin\frac{(2n+1)\pi}{5}\right\|\le 1$, $\therefore \sum_{r=1}^{n}\cos^2\left(\frac{r\pi}{5}\right)\to\infty.$ $\therefore$ the series $\cos^2\left(\frac{\pi}{5}\right)+\cos^2\left(\frac{2\pi}{5}\right)+\cos^2\left(\frac{3\pi}{5}\right)+\ldots$ is divergent.

4

Yr	Amount at the beginning of yr	Amount at the end of yr
1	6000	6000(1.03)
2	$6000(1.03)-k$	$[6000(1.03)-k](1.03)$ $=6000(1.03)^2-k(1.03)$
3	$6000(1.03)^2-k(1.03)-k$ $=6000(1.03)^2-k(1.03)-k$	$\left[6000(1.03)^2-k(1.03)-k\right](1.03)$ $=6000(1.03)^3-k(1.03)^2-k(1.03)$

By inspection, amount in the fund at the end of nth year

$$= 6000(1.03)^n - k(1.03)^{n-1} - k(1.03)^{n-2} - \ldots - k(1.03)$$

Amount in the fund at the beginning of $(n + 1)$th year

$$= 6000(1.03)^n - k(1.03)^{n-1} - k(1.03)^{n-2} - \ldots - k(1.03) - k$$

$$= 6000(1.03)^n - k\left[1+1.03+(1.03)^2+\cdots+(1.03)^{n-1}\right]$$

$$= 6000(1.03)^n - k\left\{\frac{1\left[1-(1.03)^n\right]}{1-1.03}\right\}$$

$$= 6000(1.03)^n + \frac{100}{3}k\left[1-(1.03)^n\right]$$

$$= \frac{100}{3}\left[180(1.03)^n + k - k(1.03)^n\right]$$

$$= \frac{100}{3}\left[(180-k)(1.03)^n + k\right] \quad \text{[Shown]}$$

(i) Given $k = 400$,

$$\frac{100}{3}\left[(180-400)(1.03)^n + 400\right] < 1000$$

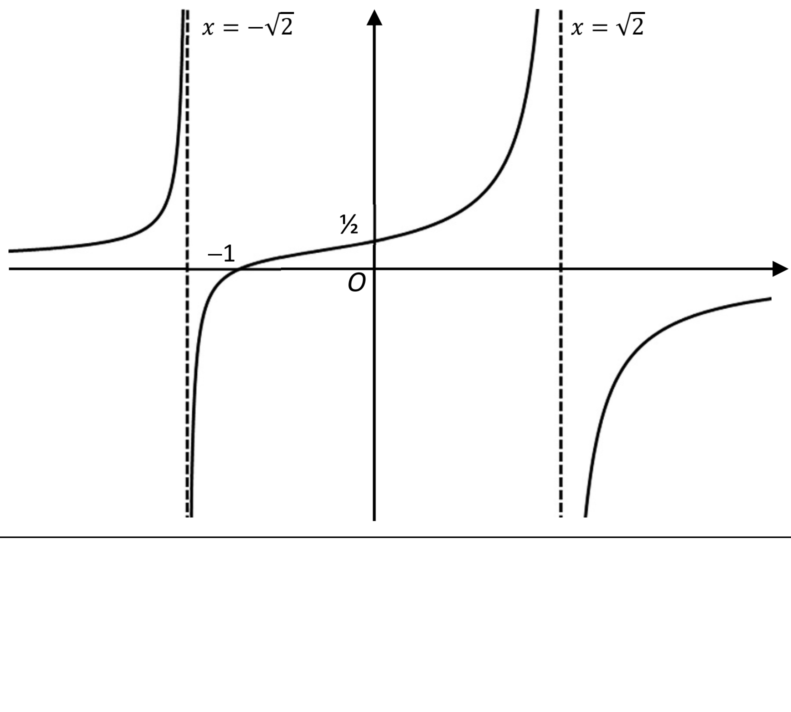
$$-220(1.03)^n + 400 < 30$$

$$(1.03)^n > \frac{37}{22} \text{ (or 1.6818)}$$

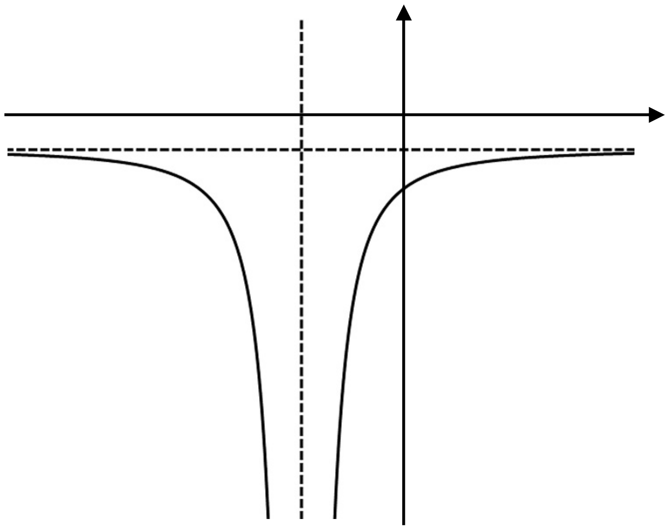
$$n\ln 1.03 > \ln\frac{37}{22}$$

$$n > \frac{\ln\frac{37}{22}}{\ln 1.03} = 17.6 \text{ (3 sf)}$$

$$\text{Least } n = 18$$

	Or: use GC, table of values gives least n = 18 $n+1$ = 19 Therefore, at the beginning of 19th year, the amount in the fund will be less than \$1000 for the first time **(ii)** When $n+1=16 \Rightarrow n=15$, $\frac{100}{3}\left[(180-k)(1.03)^{15}+k\right] \le 0$ $(180-k)(1.03)^{15}+k \le 0$ $180(1.03)^{15}+k\left[1-(1.03)^{15}\right] \le 0$ $k\left[1-(1.03)^{15}\right] \le -180(1.03)^{15}$ $k\left[(1.03)^{15}-1\right] \ge 180(1.03)^{15}$ $k \ge \frac{180(1.03)^{15}}{(1.03)^{15}-1}$ $k \ge 502.6$ Least k = 503 (nearest integer) Or: from GC (plot graph or table of values), least k = 503 (nearest integer)
5	**(a)** $(x-2)^2=a^2(1-y^2)$ $\Rightarrow \frac{(x-2)^2}{a^2}+y^2=1$ $\Rightarrow \frac{(x-2)^2}{a^2}+\frac{(y-0)^2}{1^2}=1$, $1<a<2$ **(b)(i)** $y=\frac{1}{\mathrm{f}(x)}$ 

(b)(ii) $y = \mathrm{f}'(x)$



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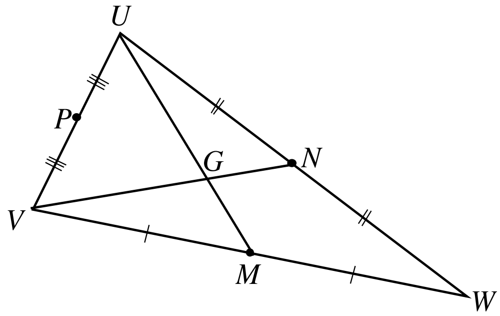


(i)

By Ratio Theorem, $\overrightarrow{UM} = \dfrac{\overrightarrow{UW} + \overrightarrow{UV}}{2}$

$= \dfrac{\mathbf{w} - \mathbf{u} + \mathbf{v} - \mathbf{u}}{2}$

$= \dfrac{1}{2}(\mathbf{v} + \mathbf{w} - 2\mathbf{u})$ (Shown)

(ii) Vector equation of line *UM* is $\mathbf{r} = \mathbf{u} + \lambda(\mathbf{w} + \mathbf{v} - 2\mathbf{u}),\ \lambda \in \mathbb{R}$

$\overrightarrow{VN} = \dfrac{\overrightarrow{VW} + \overrightarrow{VU}}{2}$

$= \dfrac{\mathbf{w} - \mathbf{v} + \mathbf{u} - \mathbf{v}}{2} = \dfrac{1}{2}(\mathbf{w} + \mathbf{u} - 2\mathbf{v})$

Vector equation of line *VN* is $\mathbf{r} = \mathbf{v} + \mu(\mathbf{w} + \mathbf{u} - 2\mathbf{v}),\ \mu \in \mathbb{R}$

At point of intersection *G*,

$\mathbf{u} + \lambda(\mathbf{w} + \mathbf{v} - 2\mathbf{u}) = \mathbf{v} + \mu(\mathbf{w} + \mathbf{u} - 2\mathbf{v})$

For $\mathbf{u}$: $1 - 2\lambda = \mu$

For $\mathbf{w}$: $\lambda = \mu$

Solving, $\lambda = \dfrac{1}{3} = \mu$

	$\overrightarrow{OG} = \mathbf{u}+\frac{1}{3}(\mathbf{w}+\mathbf{v}-2\mathbf{u})$ $= \frac{1}{3}(\mathbf{u}+\mathbf{v}+\mathbf{w})$ (Shown) **(iii)** $\mathbf{u}=\begin{pmatrix}1\\0\\0\end{pmatrix}, \mathbf{v}=\begin{pmatrix}0\\1\\0\end{pmatrix}, \mathbf{w}=\begin{pmatrix}0\\0\\1\end{pmatrix}$ $\overrightarrow{OG} = \frac{1}{3}\left[\begin{pmatrix}1\\0\\0\end{pmatrix}+\begin{pmatrix}0\\1\\0\end{pmatrix}+\begin{pmatrix}0\\0\\1\end{pmatrix}\right]=\begin{pmatrix}\frac{1}{3}\\\frac{1}{3}\\\frac{1}{3}\end{pmatrix}$ $\left\|\overrightarrow{OG}\right\| = \sqrt{3\left(\frac{1}{3^2}\right)}=\sqrt{\frac{1}{3}}$ Direction cosines of $\overrightarrow{OG}$ are $\frac{\frac{1}{3}}{\sqrt{\frac{1}{3}}}, \frac{\frac{1}{3}}{\sqrt{\frac{1}{3}}}, \frac{\frac{1}{3}}{\sqrt{\frac{1}{3}}}$, i.e., $\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}$
7	**(a)** $u=2-\mathrm{i}\sin^2\theta$, $v=2\cos^2\theta+\mathrm{i}\sin^2\theta$ $u-v = 2-\mathrm{i}\sin^2\theta-2\cos^2\theta-\mathrm{i}\sin^2\theta$ $= 2-2\cos^2\theta-2\mathrm{i}\sin^2\theta$ $= 2(1-\cos^2\theta)-2\mathrm{i}\sin^2\theta$ $= 2\sin^2\theta-2\mathrm{i}\sin^2\theta$ or $2(\sin^2\theta)(1-\mathrm{i})$ $\|u-v\| = 2\|\sin^2\theta-\mathrm{i}\sin^2\theta\|$ $= 2\sqrt{\sin^4\theta+\sin^4\theta}$ $= 2\sqrt{2\sin^4\theta}$ $= 2\sqrt{2}\sin^2\theta$ or $2\|\sin^2\theta\|\|1-\mathrm{i}\|$ $= 2(\sin^2\theta)\sqrt{1+1}$ $= 2\sqrt{2}\sin^2\theta$ Note that $u-v$ lies in the 4th quadrant. $\arg(u-v) = -\tan^{-1}\frac{2\sin^2\theta}{2\sin^2\theta}$ $= -\tan^{-1}1 = -\frac{\pi}{4}$ Or: $\arg(u-v) = \arg(2\sin^2\theta-2\mathrm{i}\sin^2\theta)=\arg\left[2(\sin^2\theta)(1-\mathrm{i})\right]$ $= \arg(2\sin^2\theta)+\arg(1-\mathrm{i})$

$$= 0+\left(-\frac{\pi}{4}\right) = -\frac{\pi}{4}$$

(b) Method 1 Solve α first then factorise quadratic expression or use sum of roots

$$x^2+(\mathrm{i}-3)x+2(1-\mathrm{i}) = 0$$

Sub. $x=\alpha\in\mathbb{R}$,

$$\alpha^2+(\mathrm{i}-3)\alpha+2(1-\mathrm{i}) = 0$$

$$(\alpha^2-3\alpha+2)+\mathrm{i}(\alpha-2) = 0$$

Comparing imaginary parts,

$$\alpha-2 = 0$$

$$\alpha = 2$$

$$x^2+(\mathrm{i}-3)x+2(1-\mathrm{i}) = (x-2)(x-\beta)$$

Comparing constants,

$$2(1-\mathrm{i}) = 2\beta$$

$$\therefore \beta = 1-\mathrm{i}$$

Or: Sum of roots, $\alpha+\beta = -(\mathrm{i}-3)$

$$2+\beta = 3-\mathrm{i}$$

$$\therefore \beta = 1-\mathrm{i}$$

Method 2 Factorise the quadratic expression first

$$x^2+(\mathrm{i}-3)x+2(1-\mathrm{i}) = (x-\alpha)(x-\beta)$$

Comparing coefficients of x,

$$\mathrm{i}-3 = -(\alpha+\beta)$$

$$\alpha+\beta = 3-\mathrm{i} \quad (1)$$

Comparing constants,

$$\alpha\beta = 2-2\mathrm{i} \quad (2)$$

From (1), $\beta = 3-\mathrm{i}-\alpha$ (3)

Sub. (3) into (2), $\alpha(3-\mathrm{i}-\alpha) = 2-2\mathrm{i}$

$$3\alpha-\alpha^2-\alpha\mathrm{i} = 2-2\mathrm{i}$$

Comparing imaginary parts, $\alpha = 2$

Sub. into (3), $\beta = 3-\mathrm{i}-2$

$$\therefore \beta = 1-\mathrm{i}$$

Or:

Let $\beta=a+b\mathrm{i}$, where $a\in\mathbb{R}$, $b\in\mathbb{R}$ and $b\neq 0$

$$x^2+(\mathrm{i}-3)x+2(1-\mathrm{i}) = (x-\alpha)[x-(a+b\mathrm{i})]$$

Comparing coefficients of x,

$$\mathrm{i}-3 = -a-b\mathrm{i}-\alpha$$

$b = -1$ (Comparing imaginary parts)

$a+\alpha = 3$ (1) (Comparing real parts)

Comparing constants,

$$2-2\mathrm{i} = \alpha(a+b\mathrm{i})$$

$$= \alpha(a-\mathrm{i}) = \alpha a - \alpha\mathrm{i}$$

$$\underline{\underline{\alpha = 2}} \qquad \text{(Comparing imaginary parts)}$$

Sub. into (1), $a = 3-\alpha = 3-2 = 1$

$$\therefore \underline{\underline{\beta = 1-\mathrm{i}}}$$

<u>Method 3 Solve *x* first using quadratic formula</u>

$$x^2+(\mathrm{i}-3)x+2(1-\mathrm{i}) = 0$$

$$x = \frac{-(\mathrm{i}-3)\pm\sqrt{(\mathrm{i}-3)^2-4(1)\left[2(1-\mathrm{i})\right]}}{2}$$

$$= \frac{3-\mathrm{i}\pm\sqrt{\mathrm{i}^2-6\mathrm{i}+9-8+8\mathrm{i}}}{2} = \frac{3-\mathrm{i}\pm\sqrt{2\mathrm{i}}}{2}$$

$$= \frac{3-\mathrm{i}\pm(1+\mathrm{i})}{2} \qquad \text{(use GC to find } \sqrt{2\mathrm{i}}\text{)}$$

$$= 2 \text{ or } 1-\mathrm{i}$$

$$\therefore \underline{\underline{\alpha = 2}} \text{ and } \underline{\underline{\beta = 1-\mathrm{i}}}$$

For comparison purpose:
If GC is **not** used to find $\sqrt{2\mathrm{i}}$, then the algebraic works will look as follows:

Let $\sqrt{2\mathrm{i}} = a+b\mathrm{i}$, where $a \in \mathbb{R}$, $b \in \mathbb{R}$

$$2\mathrm{i} = a^2-b^2+2ab\mathrm{i}$$

Compring real parts, $a^2-b^2 = 0$

$$a^2 = b^2$$

$$a = \pm b \quad (1)$$

Compring imaginary parts, $ab = 1 \quad (2)$

When $a = b$,

Sub. into (2), $a^2 = 1$

$$a = \pm 1$$

When $a = 1$, $b = 1$. When $a = -1$, $b = -1$

$$\pm\sqrt{2\mathrm{i}} = \pm(1+\mathrm{i})$$

When $a = -b$

Sub. into (2), $-b^2 = 1 \quad (\text{NA } \because b \in \mathbb{R})$

$$\therefore x = \frac{3-\mathrm{i}\pm(1+\mathrm{i})}{2} = 2 \text{ or } 1-\mathrm{i}$$

$$\therefore \underline{\underline{\alpha = 2}} \text{ and } \underline{\underline{\beta = 1-\mathrm{i}}}$$

---

8 **(a)(i)** Let A cm^2 be area of the circular patch.

$$A = \pi r^2$$

$$\frac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r$$

Given $\dfrac{\mathrm{d}A}{\mathrm{d}t} = 6\pi \text{ cm}^2/\text{s}$, a **constant**

This means that, in 1 s, A increases by 6π cm^2 **constantly**.

When $t = 0$, $A = 0$

When $t = 24$,

$$\begin{aligned} A &= 24\times 6\pi = 144\pi \\ \pi r^2 &= 144\pi \\ r &= 12 \quad \text{(reject } r=-12 \text{ since } r>0\text{)} \\ \frac{dA}{dr} &= 2\pi(12) = 24\pi \\ \frac{dA}{dt} &= \frac{dA}{dr}\times\frac{dr}{dt} \\ 6\pi &= 24\pi\frac{dr}{dt} \\ \frac{dr}{dt} &= \frac{1}{4} \end{aligned}$$

$\therefore$ rate of change of the radius is $\underline{\underline{\frac{1}{4}\text{ cm/ s}}}$.

(a)(ii)

$$\begin{aligned} \frac{dA}{dt} &= \frac{dA}{dr}\times\frac{dr}{dt} \\ 6\pi &= 2\pi r\frac{dr}{dt} \\ \frac{dr}{dt} &= \frac{6\pi}{2\pi r} = \frac{3}{r} \end{aligned}$$

Method 1

As r increases, $\frac{dr}{dt} = \frac{3}{r}$ decreases, $\therefore \frac{dr}{dt}$ will $\underline{\underline{\text{decrease}}}$ as time passes.

Method 2

$$\begin{aligned} \frac{d\left(\frac{dr}{dt}\right)}{dt} &= \frac{d\left(\frac{3}{r}\right)}{dr}\times\frac{dr}{dt} \\ &= \frac{-3}{r^2}\left(\frac{3}{r}\right) = \frac{-9}{r^3} < 0 \end{aligned}$$

$\therefore \frac{dr}{dt}$ will $\underline{\underline{\text{decrease}}}$ as time passes.

(b)(i)

$$\begin{aligned} V &= \pi r^2 h \\ 355 &= \pi r^2 h \\ \pi r h &= \frac{355}{r} \\ C &= K(2\pi r h) + 2K(4r^2) \\ &= K\left[2\left(\frac{355}{r}\right) + 8r^2\right] \end{aligned}$$

	$= K\left(\frac{710}{r}+8r^2\right)$ (Shown) **(b)(ii)** $\frac{dC}{dr} = \left(-\frac{710}{r^2}+16r\right)K$ For C to be a minimum, $\frac{dC}{dr}=0$. $-\frac{710}{r^2}+16r = 0$ $-710+16r^3 = 0$ $r^3 = \frac{355}{8}$ $r = \sqrt[3]{\frac{355}{8}} = 3.54$ (3 sf) $\frac{d^2C}{dr^2} = \left(\frac{1420}{r^3}+16\right)K = \left(\frac{1420}{\frac{355}{8}}+16\right)K = 48K > 0$ Or

r	3.5	$\sqrt[3]{\frac{355}{8}} \approx 3.54$	3.6
$\frac{dC}{dr}$	$-1.96K<0$	0	$2.82K>0$

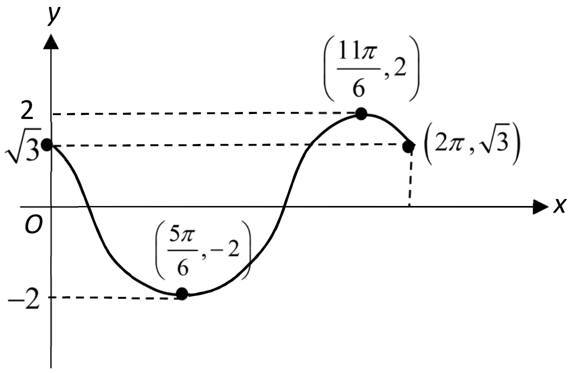
	So, $r=\sqrt[3]{\frac{355}{8}}$ does give the minimum cost. Recall $355 = \pi r^2 h$ $h = \frac{355}{\pi r^2}$ $\therefore \frac{h}{r} = \frac{355}{\pi r^3} = \frac{355}{\pi\left(\frac{355}{8}\right)}$ $= \frac{8}{\pi}$ (Shown)
9	**(i)** $\sqrt{3}\cos x - \sin x = R\cos(x+\alpha)$ $R = \sqrt{\left(\sqrt{3}\right)^2+1^2} = \sqrt{4} = \underline{\underline{2}}$ $\alpha = \tan^{-1}\frac{1}{\sqrt{3}} = \underline{\underline{\frac{\pi}{6}}}$ **(ii)** $y=\sqrt{3}\cos x-\sin x = 2\cos\left(x+\frac{\pi}{6}\right)$ $y=\cos x \xrightarrow{A} y=\cos(x+\alpha) \xrightarrow{B} y=R\cos(x+\alpha)$

A: Translation by α radians in the negative *x*-direction, followed by

B: Scaling parallel to the *y*-axis by a scale factor *R*.

[can be *B* followed by *A*]

(iii) $\text{f} : x \mapsto \sqrt{3}\cos x - \sin x,\ 0 \le x \le 2\pi$



Range of f, $R_f = [-2, 2]$.

(iv) $\text{g} : x \mapsto \text{f}(x),\ 0 \le x \le k$.

Largest $k = \dfrac{5\pi}{6}$.

Let

$$y = \text{g}(x).$$

$$y = 2\cos\left(x + \frac{\pi}{6}\right)$$

$$\cos\left(x + \frac{\pi}{6}\right) = \frac{y}{2}$$

$$\Rightarrow x = \cos^{-1}\frac{y}{2} - \frac{\pi}{6}$$

$$\therefore \text{g}^{-1}(x) = \cos^{-1}\frac{x}{2} - \frac{\pi}{6}$$

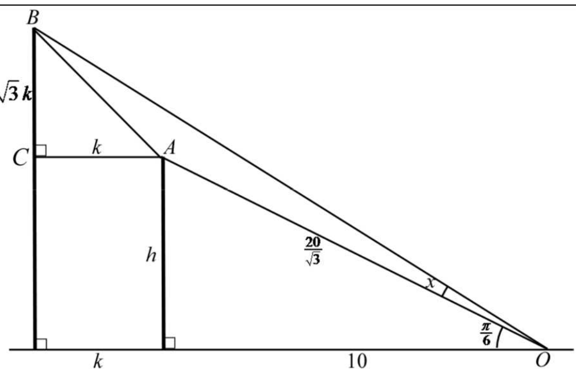
(v) $\text{h} : x \mapsto x - 2,\ x \ge 0$

Since $R_h = [-2, +\infty)$ and $D_f = [0, 2\pi]$,

$R_h \not\subset D_f$, fh does not exist.

10

(i)



$$\cos\frac{\pi}{6} = \frac{10}{OA}$$
$$\frac{\sqrt{3}}{2} = \frac{10}{OA}$$
$$OA = \frac{20}{\sqrt{3}} \text{ m}$$

(Shown)

(ii)

$$AB = \sqrt{k^2 + 3k^2} = \sqrt{4k^2} = 2k \qquad \text{(Shown)}$$
$$\angle BAC = \tan^{-1}\frac{\sqrt{3}\,k}{k} = \tan^{-1}\sqrt{3}$$
$$= \frac{\pi}{3} \qquad \text{(Shown)}$$

(iii)

$$\angle CBO = \tfrac{\pi}{2} - \left(\tfrac{\pi}{6} + x\right) = \tfrac{\pi}{3} - x$$
$$\angle CBA = \tfrac{\pi}{2} - \tfrac{\pi}{3} = \tfrac{\pi}{6}$$
$$\angle ABO = \tfrac{\pi}{3} - x - \tfrac{\pi}{6} = \tfrac{\pi}{6} - x$$

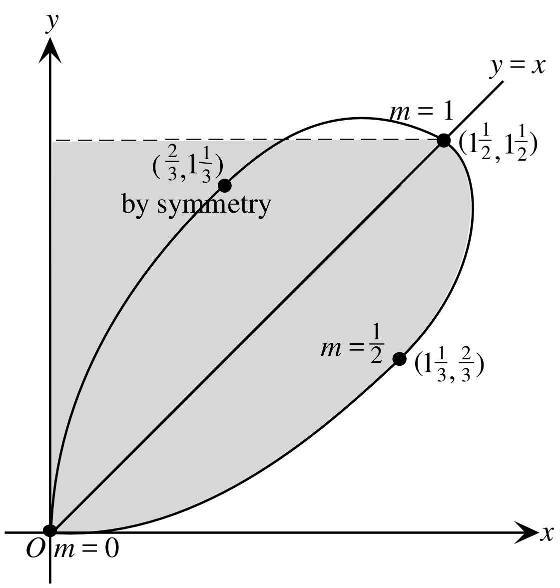
Or:

$$\angle BAO = 2\pi - \tfrac{\pi}{2} - \tfrac{\pi}{3} - \tfrac{\pi}{3} \quad (\angle \text{ at a pt})$$
$$= \tfrac{5\pi}{6}$$
$$\angle ABO = \pi - x - \tfrac{5\pi}{6} = \tfrac{\pi}{6} - x$$

In ΔABO,

$$\frac{2k}{\sin x} = \frac{\frac{20}{\sqrt{3}}}{\sin\left(\frac{\pi}{6} - x\right)}$$
$$k = \frac{10\sin x}{\sqrt{3}\sin\left(\frac{\pi}{6} - x\right)}$$

(iv)

	$k=\dfrac{10\sin x}{\sqrt{3}\sin\left(\frac{\pi}{6}-x\right)}$ $=\dfrac{10\sin x}{\sqrt{3}\left(\sin\frac{\pi}{6}\cos x-\cos\frac{\pi}{6}\sin x\right)}$ $\approx\dfrac{10x}{\sqrt{3}\left[\frac{1}{2}\left(1-\frac{x^2}{2}\right)-\frac{\sqrt{3}}{2}x\right]}$ $=\dfrac{10x}{\frac{\sqrt{3}}{2}\left[\left(1-\frac{x^2}{2}\right)-\sqrt{3}x\right]}$ $=\dfrac{20x}{\sqrt{3}}\left[1-\left(\sqrt{3}x+\frac{x^2}{2}\right)\right]^{-1}$ $\approx\dfrac{20x}{\sqrt{3}}\left(1+\sqrt{3}x\right)$ $=\underline{\underline{\dfrac{20}{\sqrt{3}}\left(x+\sqrt{3}x^2\right)}}$
11	**(a)(i)** $x=\dfrac{3m}{1+m^3},\quad y=\dfrac{3m^2}{1+m^3},\quad m\geq 0$ $y = x$ $\dfrac{3m^2}{1+m^3}=\dfrac{3m}{1+m^3}$ $m(m-1)=0$ $m=\underline{\underline{0 \text{ or } 1}}$ **(a)(ii)** When $m = 0$, $y = 0$. When $m = 1$, $y=\dfrac{3}{1+1}=\dfrac{3}{2}$.  **<u>Notes</u>**: Use GC to trace the path to see how m varies when the point moves along the path.

Area of (lower) half of the "leaf" is

$$\frac{1}{2}A = \int_0^{\frac{3}{2}} x\,\mathrm{d}y - \text{area of }\Delta \qquad \left(\text{Note: } \int_0^{\frac{3}{2}} x\,\mathrm{d}y = \text{shaded area}\right)$$

$$A = 2\left[\int_0^{\frac{3}{2}} x\,\mathrm{d}y - \frac{1}{2}\left(\frac{3}{2}\right)\left(\frac{3}{2}\right)\right]$$

$$= 2\left(\int_0^{\frac{3}{2}} x\,\mathrm{d}y - \frac{9}{8}\right) \qquad \text{(Shown)}$$

$$2\left(\int_0^{\frac{3}{2}} x\,\mathrm{d}y - \frac{9}{8}\right) = 2\int_0^1 \frac{3m}{1+m^3}\left[\frac{6m\left(1+m^3\right)-3m^2\left(3m^2\right)}{\left(1+m^3\right)^2}\right]\mathrm{d}m - \frac{9}{4}$$

$$= 2\int_0^1 \frac{3m\left(6m-3m^4\right)}{\left(1+m^3\right)^3}\mathrm{d}m - \frac{9}{4}$$

$$= \frac{15}{4} - \frac{9}{4} \qquad \text{(by GC)}$$

$$= \underline{\underline{\frac{3}{2}}}$$

(b)

$$y = \ln x$$
$$x = \mathrm{e}^y$$

$$V_A = \pi\int_0^c \left(\mathrm{e}^y\right)^2 \mathrm{d}y$$
$$= \pi\int_0^c \mathrm{e}^{2y}\,\mathrm{d}y$$
$$= \pi\left[\frac{1}{2}\mathrm{e}^{2y}\right]_0^c$$
$$= \frac{\pi}{2}\left(\mathrm{e}^{2c}-1\right)$$

$$V_B = (1-c)\pi\mathrm{e}^2 - \pi\int_c^1 \left(\mathrm{e}^y\right)^2 \mathrm{d}y \quad \text{or} \quad \pi\int_c^1 \left[\mathrm{e}^2 - \left(\mathrm{e}^y\right)^2\right]\mathrm{d}y$$
$$= \pi(1-c)\mathrm{e}^2 - \pi\left[\frac{1}{2}\mathrm{e}^{2y}\right]_c^1$$
$$= \pi(1-c)\mathrm{e}^2 - \frac{\pi}{2}\left(\mathrm{e}^2 - \mathrm{e}^{2c}\right)$$

$$V_A = V_B$$
$$\frac{\pi}{2}\left(\mathrm{e}^{2c}-1\right) = \pi(1-c)\mathrm{e}^2 - \frac{\pi}{2}\left(\mathrm{e}^2-\mathrm{e}^{2c}\right)$$
$$\mathrm{e}^{2c}-1 = 2\mathrm{e}^2(1-c) - \mathrm{e}^2 + \mathrm{e}^{2c}$$
$$= 2\mathrm{e}^2 - 2c\mathrm{e}^2 - \mathrm{e}^2 + \mathrm{e}^{2c}$$
$$2c\mathrm{e}^2 = \mathrm{e}^2 + 1$$

	$c = \dfrac{e^2+1}{2e^2}$ (Shown)

H2 Mathematics 2017 Prelim Exam Paper 2 Question

1 It is given that $y=\ln(1+\sin x)$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. Show that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\mathrm{e}^{-y}$. [4]

(ii) Express $\frac{\mathrm{d}^4y}{\mathrm{d}x^4}$ in terms of $\frac{\mathrm{d}y}{\mathrm{d}x}$ and e^{-y}. [3]

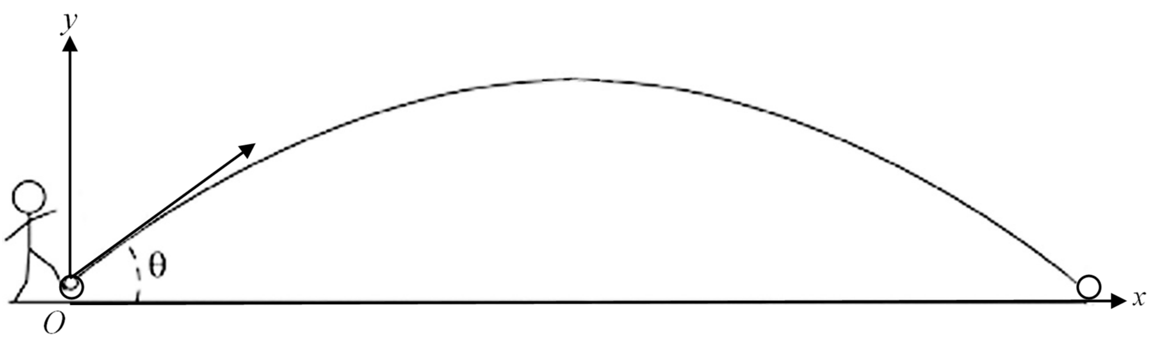
(iii) Hence, find the first four non-zero terms in the Maclaurin series for $\ln(1+\sin x)$.[3]

2 John kicked a ball at an acute angle θ made with the horizontal, and it moved in a projectile motion, as shown in the diagram. The initial velocity of the ball is u m s^{-1}. Taking John's position where he kicked the ball as the origin O, the ball's displacement curve is given by the parametric equations:

horizontal displacement, $x=ut\cos\theta$,

vertical displacement, $y=ut\sin\theta-5t^2$,

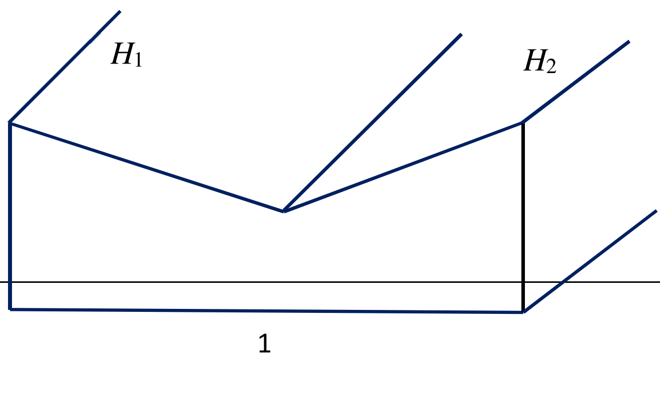
where u and θ are constants and t is the time in seconds after the ball is kicked.



(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x}=\tan\theta-\frac{10}{u}t\sec\theta$. [2]

(ii) If the initial velocity of the ball is 30 m s^{-1}, find the equation of the tangent to the displacement curve at the point where $t=\frac{1}{2}$, giving your answer in the form $y=(a\tan\theta+b\sec\theta)x+c$, where a, b and c are constants to be determined. [3]

3 Peter is using equations of planes to model two hillsides that meet along a river. The river is modelled by the line where the two planes meet.



One of the hillsides, H_1, contains the points *A*, *B* and *C* with coordinates (3, 0, 2), (1, 0, 3) and $(2, -3, 5)$ respectively. The point *A* is on the river. The other hillside H_2 has equation $2x - y + kz = 14$, where *k* is a constant.

(i) Find a vector equation of H_1 in scalar product form. [4]

(ii) Show that $k = 4$ and deduce that point *B* is also on the river. [3]

(iii) Write down a cartesian equation of the river. [1]

(iv) Show that *B* is the point on the river that is nearest to *C*. Hence find the exact distance from *C* to the river. [3]

(v) Find the acute angle between *BC* and H_2. [2]

4 To determine whether the amount of preservatives in a particular brand of bread meets the safety limit of preservatives present, the Food Regulatory Authority (FRA) conducted a test to examine the growth of fungus on a piece of bread over time after its expiry date. The piece of bread has a surface area of $100\ \text{cm}^2$. The staff from FRA estimate the amount of fungus grown and the rate at which it is growing by finding the area of the piece of bread the fungus covers over time. They believe that the area, $A\ \text{cm}^2$, of fungus present *t* days after the expiry date is such that the rate at which the area is increasing is proportional to the product of the area of the piece of bread covered by the fungus and the area of the bread not covered by the fungus. It is known that the initial area of fungus is $20\ \text{cm}^2$ and that the area of fungus is $40\ \text{cm}^2$ five days after the expiry date.

(i) Write down a differential equation expressing the relation between *A* and *t*. [1]

(ii) Find the value of *t* at which 50% of the piece of bread is covered by fungus, giving your answer correct to 2 decimal places. [6]

(iii) Given that this particular brand of bread just meets the safety limit of the amount of preservatives present when the test is concluded 2 weeks after the expiry date, find the range of values of *A* for any piece of bread of this brand to be deemed safe for human consumption in terms of the amount of preservatives present, giving your answer correct to 2 decimal places. [2]

(iv) Write the solution of the differential equation in the form $A = \text{f}(t)$ and sketch this curve. [3]

5 The probability distribution of a discrete random variable, *X*, is shown below.

x	1	2
$\text{P}(X = x)$	a	b

Find $\text{E}(X)$ and $\text{Var}(X)$ in terms of *a*. [5]

6 **(i)** Find the number of 3-digit numbers that can be formed using the digits 1, 2 and 3 when

(a) no repetitions are allowed, [1]

(b) any repetitions are allowed, [1]

	(c) each digit may be used at most twice. [2] **(ii)** Find the number of 4-digit numbers that can be formed using the digits 1, 2 and 3 when each digit may be used at most twice. [5]
7	At a canning factory, cans are filled with potato puree. The machine which fills the cans is set so that the volume of potato puree in a can has mean 420 millilitres. After the machine is recalibrated, a quality control officer wishes to check whether the mean volume has changed. A random sample of 30 cans of potato puree is selected and the volume of the puree in each can is recorded. The sample mean volume is $\bar{x}$ millilitres and the sample variance is 12 millilitres^2. **(i)** Given that $\bar{x} = 418.55$, carry out a test at the 1% level of significance to investigate whether the mean volume has changed. State, giving a reason, whether it is necessary for the volumes to have a normal distribution for the test to be valid. [6] **(ii)** Use an algebraic method to calculate the range of values of $\bar{x}$, giving your answer correct to 2 decimal places, for which the result of the test at the 1% level of significance would be to reject the null hypothesis. [3]
8	In this question you should state clearly the values of the parameters of any normal distribution you use. The mass of a tomato of variety *A* has normal distribution with mean 80 g and standard deviation 11 g. **(i)** Two tomatoes of variety *A* are randomly chosen. Find the probability that one of the tomatoes has mass more than 90 g and the other has mass less than 90 g. [3] The mass of a tomato of variety *B* has normal distribution with mean 70 g. These tomatoes are packed in sixes using packaging that weighs 15 g. **(ii)** The probability that a randomly chosen pack of 6 tomatoes of variety *B* including packaging, weighs less than 450 g is 0.8463. Show that the standard deviation of the mass of a tomato of variety *B* is 6 g, correct to the nearest gram. [4] **(iii)** Tomatoes of variety *A* are packed in fives using packaging that weighs 25 g. Find the probability that the total mass of a randomly chosen pack of variety *A* is greater than the total mass of a randomly chosen pack of variety *B*, using 6 g as the standard deviation of the mass of a tomato of variety *B*. [5]
9	A jar contains 5 identical balls numbered 1 to 5. A fixed number, *n*, of balls are selected and the number of balls with an even score is denoted by *X*. **(i)** Explain how the balls should be selected in order for *X* to be well modelled by a binomial distribution. [2] Assume now that *X* has the distribution $\mathrm{B}\left(n, \frac{2}{5}\right)$. **(ii)** Given that $n = 10$, find $\mathrm{P}(X \geq 4)$. [2] **(iii)** Given that the mean of *X* is 4.8, find *n*. [2] **(iv)** Given that P(*X* = 0 or 1) < 0.01, write down an inequality for *n* and find the least value of *n*. [3]

Shawn and Arvind take turns to draw one ball from the jar at random. The first person who draws a ball with an even score wins the game. Shawn draws first.

(v) Show that the probability that Shawn wins the game is $\frac{3}{5}$ if the selection of balls is done without replacement. [2]

(vi) Find the probability that Shawn wins the game if the selection of balls is done with replacement. [2]

10 **(a)** Traffic engineers are studying the correlation between traffic flow on a busy main road and air pollution at a nearby air quality monitoring station. Traffic flow, x, is recorded automatically by sensors and reported each hour as the average flow in vehicles per hour for the preceding hour. The air quality monitoring station provides, each hour, an overall pollution reading, y, in a suitable unit (higher readings indicate more pollution). Data for a random sample of 8 hours are as follows.

Traffic flow, x	1796	1918	2120	2315	2368	2420	2588
Pollution reading, y	1.0	2.2	3.5	4.2	4.3	4.5	4.9

(i) Draw the scatter diagram for these values, labelling the axes. [2]

It is thought that the pollution y can be modelled by one of the formulae

$$y = a + bx \qquad\qquad y^2 = c + dx$$

where a, b, c and d are constants.

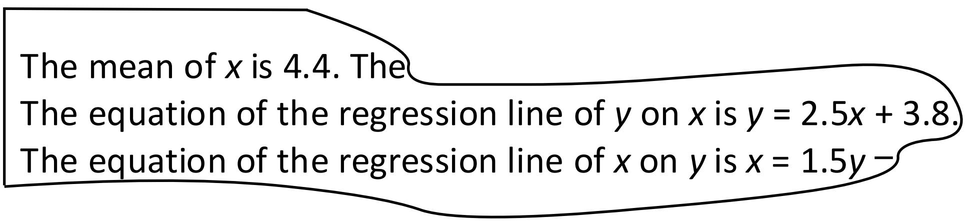
(ii) Find the value of the product moment correlation coefficient between
(a) x and y,
(b) x and y^2. [2]

(iii) Use your answers to parts **(i)** and **(ii)** to explain which of $y = a + bx$ or $y^2 = c + dx$ is the better model. [2]

(iv) It is required to estimate the value of y for which $x = 2000$. Find the equation of a suitable regression line, and use it to find the required estimate. [2]

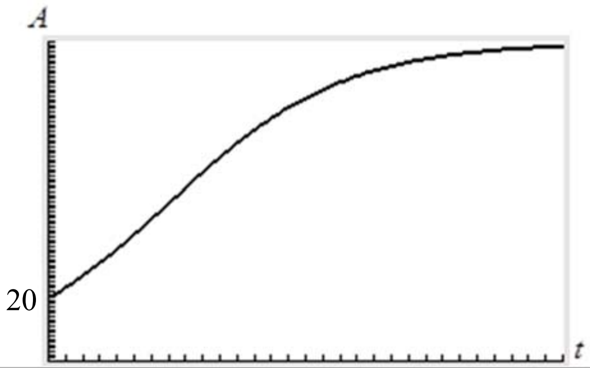
(v) The local newspaper carries a headline "Heavy traffic causes air pollution". Comment briefly on the validity of this headline in the light of your results. [1]

(b) The diagram below shows an old research paper that has been partially destroyed. The surviving part of the paper contains incomplete information about some bivariate data from an experiment. Calculate the missing constant at the end of the equation of the second regression line. [3]



ANNEX B

JJC H2 Math JC2 Preliminary Examination Paper 2

QN	Topic Set	Answers
1	Maclaurin series	(i) $\dfrac{dy}{dx} = \dfrac{\cos x}{1+\sin x}$ (ii) $\dfrac{d^4y}{dx^4} = -\left(e^{-y}\right)^2 - e^{-y}\left(\dfrac{dy}{dx}\right)^2$ or $-e^{-y}\left[e^{-y} + \left(\dfrac{dy}{dx}\right)^2\right]$ (iii) $\ln\left(1+\sin x\right) = x - \dfrac{1}{2}x^2 + \dfrac{1}{6}x^3 - \dfrac{1}{12}x^4 + \ldots$
2	Differentiation & Applications	(ii) $y = \left(\tan\theta - \dfrac{1}{6}\sec\theta\right)x + \dfrac{5}{4}$
3	Vectors	(i) $\mathbf{r}\bullet\begin{pmatrix}3\\5\\6\end{pmatrix} = 21$ (iii) $\dfrac{x-3}{-2} = z-2,\ y=0$ or $\dfrac{x-1}{-2} = z-3,\ y=0$ (iv) $\sqrt{14}$ (v) $49.3°$ or 0.861 rad
4	Differential Equations	(i) $\dfrac{dA}{dt} = kA\left(100-A\right)$ (ii) 7.07 days (iii) $79.58 \le A \le 100$ (iv) $A = \dfrac{100e^{\left(\frac{1}{5}\ln\frac{8}{3}\right)t}}{4+e^{\left(\frac{1}{5}\ln\frac{8}{3}\right)t}}$ or $\dfrac{100e^{0.196t}}{4+e^{0.196t}}$ 
5	DRV	$E\left(X\right) = 2-a$ and $\text{Var}\left(X\right) = a-a^2$
6	P&C, Probability	(i) (a) 6, (b) 27, (c) 24 (ii) 54

7	Hypothesis Testing	(i) Since p-value = 0.0242 > $\alpha = 0.01$, we do not reject H_0 at 1% level of significance and conclude that there is insufficient evidence that the population mean volume has changed. It is not necessary for the volumes to have a normal distribution for the test to be valid as $n = 30$ is large. (ii) $\bar{x} \le 418.34$ or $\bar{x} \ge 421.66$
8	Normal Distribution	(i) 0.297 (iii) 0.364
9	Binomial Distribution	(i) (1) Selection of balls is done with replacement. (2) The balls are thoroughly mixed before each selection. (ii) 0.618 (iii) 12 (iv) $\left(\frac{3}{5}\right)^n + n\left(\frac{2}{5}\right)\left(\frac{3}{5}\right)^{n-1} < 0.01$, least n = 14 (vi) $\frac{5}{8}$ or 0.625
10	Correlation & Linear Regression	(a) (ii) (a) 0.959, (b) 0.995 (iii) $y^2 = c + dx$ is the better model since • From **(i)**, the points on the scatter diagram seem to lie on a concave downward curve. • From **(ii)**, the product moment correlation coefficient between x and y^2 is closer to 1, as compared to that between x and y. (iv) $y^2 = 0.0279x - 48.0$, y = 2.79 when x 2000. (v) May not be valid as coorelation does not necessarily imply causation. (b) 17.8

H2 Mathematics 2017 Prelim Exam Paper 2 Solution

1	**(i)** $y = \ln(1+\sin x) \Rightarrow e^{y} = 1+\sin x$ $\frac{dy}{dx} = \frac{\cos x}{1+\sin x}$ [B1] $\frac{d^2y}{dx^2} = \frac{(1+\sin x)(-\sin x)-(\cos x)(\cos x)}{(1+\sin x)^2}$ $= \frac{-\sin x-\sin^2 x-\cos^2 x}{(1+\sin x)^2}$ $= \frac{-\sin x-1}{(1+\sin x)^2}$ [A1] $= \frac{-(\sin x+1)}{(1+\sin x)^2}$ $= \frac{-1}{1+\sin x}$ $= \frac{-1}{e^{y}}$ $= -e^{-y}$ (Shown) **(ii)** $\frac{d^3y}{dx^3} = -e^{-y}\left(-\frac{dy}{dx}\right)$ $= e^{-y}\frac{dy}{dx}$ $\frac{d^4y}{dx^4} = e^{-y}\frac{d^2y}{dx^2} + e^{-y}\left(-\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right)$ $= e^{-y}\left(-e^{-y}\right) - e^{-y}\left(\frac{dy}{dx}\right)^2$ (from **(i)**) $= -\left(e^{-y}\right)^2 - e^{-y}\left(\frac{dy}{dx}\right)^2$ or $-e^{-y}\left[e^{-y}+\left(\frac{dy}{dx}\right)^2\right]$ **(iii)** When $x=0$, $y = \ln 1 = 0$ $\frac{dy}{dx} = \frac{\cos 0}{1+\sin 0} = 1$ $\frac{d^2y}{dx^2} = -e^{0} = -1$ $\frac{d^3y}{dx^3} = 1$ $\frac{d^4y}{dx^4} = -1-1=-2$

	$\therefore \ln(1+\sin x) = 0 + x + \frac{(-1)}{2}x^2 + \frac{1}{3!}x^3 + \frac{(-2)}{4!}x^4 + \ldots$ $= x - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{12}x^4 + \ldots$
2	**(i)** $\frac{dx}{dt} = u\cos\theta, \quad \frac{dy}{dt} = u\sin\theta - 10t,$ $\frac{dy}{dx} = \frac{u\sin\theta - 10t}{u\cos\theta}$ $= \tan\theta - \frac{10t}{u\cos\theta}$ $= \tan\theta - \frac{10}{u}t\sec\theta$ (Shown) **(ii)** When $u = 30$ and $t = \frac{1}{2}$, $x = 15\cos\theta, \; y = 15\sin\theta - \frac{5}{4}, \; \frac{dy}{dx} = \tan\theta - \frac{1}{6}\sec\theta$ Equation of tangent is $y - 15\sin\theta + \frac{5}{4} = \left(\tan\theta - \frac{1}{6}\sec\theta\right)(x - 15\cos\theta)$ $= \left(\tan\theta - \frac{1}{6}\sec\theta\right)x - 15\sin\theta + \frac{5}{2}$ $\therefore y = \left(\tan\theta - \frac{1}{6}\sec\theta\right)x + \frac{5}{4}$
3	**(i)** $A(3, 0, 2)$, $B(1, 0, 3)$, $C(2, -3, 5)$ $\overrightarrow{AB} = \begin{pmatrix}1\\0\\3\end{pmatrix} - \begin{pmatrix}3\\0\\2\end{pmatrix} = \begin{pmatrix}-2\\0\\1\end{pmatrix} \qquad \overrightarrow{AC} = \begin{pmatrix}2\\-3\\5\end{pmatrix} - \begin{pmatrix}3\\0\\2\end{pmatrix} = \begin{pmatrix}-1\\-3\\3\end{pmatrix}$ $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix}-2\\0\\1\end{pmatrix} \times \begin{pmatrix}-1\\-3\\3\end{pmatrix} = \begin{pmatrix}3\\5\\6\end{pmatrix}$ Take $\mathbf{n}_1 = \begin{pmatrix}3\\5\\6\end{pmatrix}$, $\mathbf{a} \bullet \mathbf{n}_1 = \begin{pmatrix}1\\0\\3\end{pmatrix} \bullet \begin{pmatrix}3\\5\\6\end{pmatrix} = 3 + 0 + 18 = 21$ A vector equation of H_1 is $\mathbf{r} \cdot \begin{pmatrix}3\\5\\6\end{pmatrix} = 21$

	(ii)	Equation of H_2 is $2x - y + kz = 14$. Sub. $A(3, 0, 2)$ into equation of H_2, $2(3) - 0 + k(2) = 14$ $\therefore\ k = 4$ (Shown) Sub. $B(1, 0, 3)$ into LHS of equation of H_2, LHS = $2x - y + 4z = 2(1) - 0 + 4(3) = 14 =$ RHS $\therefore$ B is also in H_2. Since B is in both H_1 and H_2, $\therefore$ B is on the river. (Deduced)
	(iii)	Recall $\overrightarrow{AB} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$, using $A(3, 0, 2)$ or $B(1, 0, 3)$, a cartesian equation of the river (line AB) is $\underline{\underline{\dfrac{x-3}{-2} = z - 2,\ y = 0}}$ or $\underline{\underline{\dfrac{x-1}{-2} = z - 3,\ y = 0}}$
	(iv)	Since $\overrightarrow{BC} \cdot \overrightarrow{AB} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} = 1(-2) + (-3)(0) + 2(1) = 0$, BC is perpendicular to AB. $\therefore$ B is the point on the river that is nearest to C. Exact distance from C to the river $= \left\lvert \overrightarrow{BC} \right\rvert = \left\lvert \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \right\rvert = \sqrt{1 + 9 + 4} = \underline{\underline{\sqrt{14}}}$
	(v)	Acute angle between BC and H_2 $\theta = \sin^{-1} \dfrac{\left\lvert \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} \right\rvert}{\sqrt{14}\sqrt{21}} = \sin^{-1} \dfrac{13}{\sqrt{14}\sqrt{21}}$ $= \underline{\underline{49.3^\circ}}$ or $\underline{\underline{0.861 \text{ rad}}}$
4	**(i)**	$\dfrac{\mathrm{d}A}{\mathrm{d}t} = kA(100 - A)$
	(ii)	$\displaystyle\int \frac{1}{A(100 - A)}\,\mathrm{d}A = \int k\ dt$ By partial fractions, $\dfrac{1}{A(100 - A)} = \dfrac{1}{100A} + \dfrac{1}{100(100 - A)}$

$$\therefore \frac{1}{100}\int\left(\frac{1}{A}+\frac{1}{100-A}\right)\,\mathrm{d}A = kt+c$$

$$\frac{1}{100}\left(\ln|A|-\ln|100-A|\right) = kt+c \quad (\because A>0 \text{ and } 100-A>0)$$

$$\frac{1}{100}\left[\ln A-\ln(100-A)\right] = kt+c$$

$$\ln\frac{A}{100-A} = 100(kt+c)$$

$$\frac{A}{100-A} = \mathrm{e}^{100(kt+c)} = \mathrm{e}^{100kt}\mathrm{e}^{100c} = D\mathrm{e}^{k_1 t}$$

where $k_1 = 100k$ and $D = \mathrm{e}^{100c}$.

When $t=0, A=20$,

$$\frac{20}{100-20} = D$$

$$D = \frac{1}{4}$$

When $t=5, A=40$,

$$\frac{40}{100-40} = \frac{1}{4}\mathrm{e}^{5k_1}$$

$$\frac{1}{4}\mathrm{e}^{5k_1} = \frac{2}{3}$$

$$\mathrm{e}^{5k_1} = \frac{8}{3}$$

$$5k_1 = \ln\frac{8}{3}$$

$$k_1 = \frac{1}{5}\ln\frac{8}{3}$$

$$\therefore \frac{A}{100-A} = \frac{1}{4}\mathrm{e}^{\left(\frac{1}{5}\ln\frac{8}{3}\right)t}$$

When $A = 0.5\times100 = 50$,

$$\frac{50}{100-50} = \frac{1}{4}\mathrm{e}^{\left(\frac{1}{5}\ln\frac{8}{3}\right)t}$$

$$1 = \frac{1}{4}\mathrm{e}^{\left(\frac{1}{5}\ln\frac{8}{3}\right)t}$$

$$\mathrm{e}^{\left(\frac{1}{5}\ln\frac{8}{3}\right)t} = 4$$

$$\left(\frac{1}{5}\ln\frac{8}{3}\right)t = \ln 4$$

$$t = \frac{\ln 4}{\frac{1}{5}\ln\frac{8}{3}} = 7.07 \text{ (2 dp)}$$

The required time is <u>7.07 days</u>.

(iii) When $t = 14$ (days),

$$\frac{A}{100-A} = \frac{1}{4}e^{\left(\frac{1}{5}\ln\frac{8}{3}\right)(14)}$$

Method 1 Solve algebraically

$$\frac{A}{100-A} = 3.8963 \text{ (5 sf)}$$

$$A = (100-A)(3.8963)$$

$$= 389.63 - 3.8963A$$

$$4.8963A = 389.63$$

$$A = 79.58 \text{ (2 dp)}$$

For the bread to be deemed safe for human consumption in terms of the amount of preservatives present, $\underline{\underline{79.58 \le A \le 100}}$

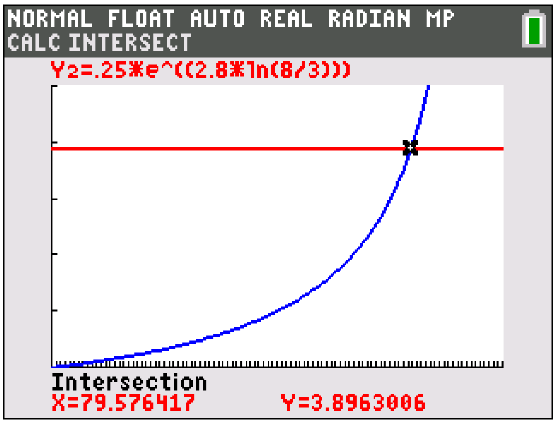
Method 2 Use GC to plot graphs

Use GC to plot $y = \frac{A}{100-A}$ and $y = \frac{1}{4}e^{\left(\frac{1}{5}\ln\frac{8}{3}\right)(14)}$ (≈ 3.8963)

Look for the point of intersection (adjust window).

$$A = 79.58 \text{ (2 dp)}$$

For the bread to be deemed safe for human consumption in terms of the amount of preservatives present, $\underline{\underline{79.58 \le A \le 100}}$



(iv)

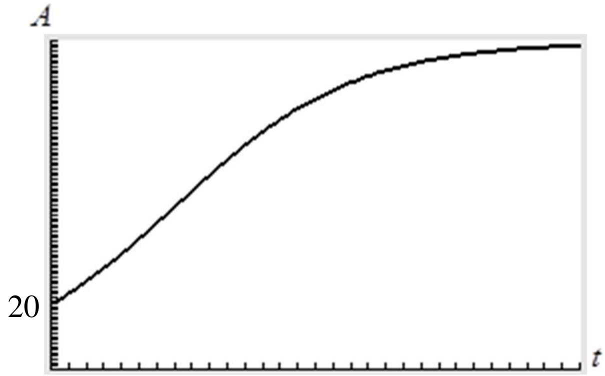
$$\frac{A}{100-A} = \frac{1}{4}e^{\left(\frac{1}{5}\ln\frac{8}{3}\right)t}$$

$$A = \frac{1}{4}e^{\left(\frac{1}{5}\ln\frac{8}{3}\right)t}(100-A)$$

$$4A = e^{\left(\frac{1}{5}\ln\frac{8}{3}\right)t}(100-A) = 100e^{\left(\frac{1}{5}\ln\frac{8}{3}\right)t} - Ae^{\left(\frac{1}{5}\ln\frac{8}{3}\right)t}$$

$$\left[4 + e^{\left(\frac{1}{5}\ln\frac{8}{3}\right)t}\right]A = 100e^{\left(\frac{1}{5}\ln\frac{8}{3}\right)t}$$

$$A = \underline{\underline{\frac{100e^{\left(\frac{1}{5}\ln\frac{8}{3}\right)t}}{4 + e^{\left(\frac{1}{5}\ln\frac{8}{3}\right)t}}}} \text{ or } \underline{\underline{\frac{100e^{0.196t}}{4 + e^{0.196t}}}}$$



5 $b = 1 - a$

x	1	2
$\mathrm{P}(X = x)$	a	$1-a$

$\mathrm{E}(X) = 1(a) + 2(1-a)$

$= \underline{\underline{2-a}}$

$\mathrm{E}(X^2) = 1^2(a) + 2^2(1-a)$

$= 4 - 3a$

$\mathrm{Var}(X) = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2$

$= 4 - 3a - (2-a)^2$

$= 4 - 3a - (4 - 4a + a^2)$

$= \underline{\underline{a - a^2}}$

6 **(i)** Use 1, 2 and 3 to form 3-digit numbers

(a) $3! = \underline{\underline{6}}$

(b) $3 \times 3 \times 3 = \underline{\underline{27}}$

(c) <u>Method 1 Consider the complement</u>

Number of 3-digit numbers with all 3 digits the same (AAA) = 3

Required number = $27 - 3 = \underline{\underline{24}}$

<u>Method 2 Consider cases</u>

<u>Case 1 Each digit is used exactly once</u>

Number of 3-digit numbers = 6 (from (i)(a))

<u>Case 2 One digit is used twice (AAB)</u>

Number of 3-digit numbers = ${}^3\mathrm{P}_2 \times \frac{3!}{2!} = 18$

(${}^3\mathrm{P}_2 = 3 \times 2$: 3 ways to select a digit to be used twice; 2 ways to select another digit)

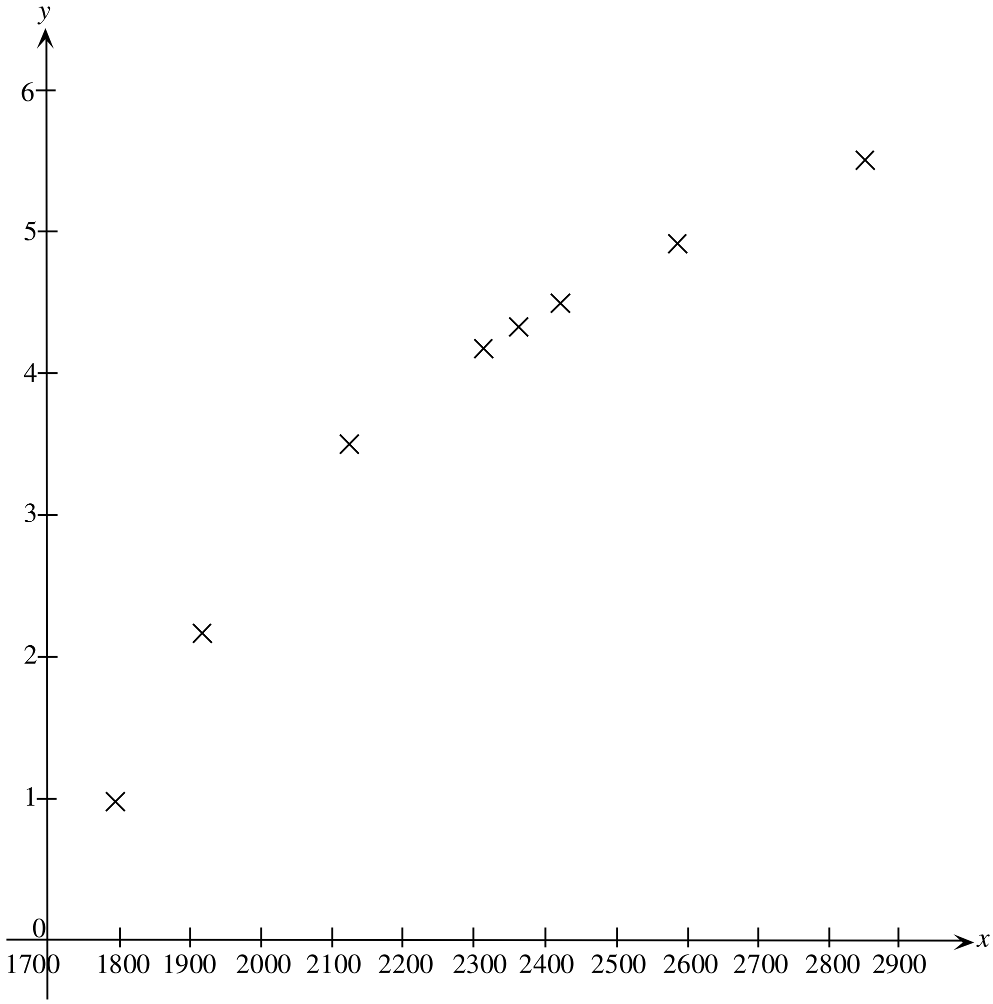
Total number of 3-digit numbers = 6 + 18 = $\underline{\underline{24}}$

(ii) Use 1, 2 and 3 to form 4-digit numbers

		<u>Method 1 Consider the complement</u> Total number of 4-digit numbers = $3^4 = 81$ <u>Case 1 AAAB</u> Number of 4-digit numbers = ${}^3P_2 \times \frac{4!}{3!}$ = 24 (${}^3P_2 = 3\times 2$: 3 ways to select a digit to be used thrice; 2 ways to select another digit) <u>Case 2 AAAA</u> Number of 4-digit numbers = 3 Total number of 4-digit numbers = $81-(24+3) =$ <u>54</u> <u>Method 2 Consider cases</u> <u>Case 1 AABC</u> Number of 4-digit numbers = $3\times\frac{4!}{2!}$ = 36 (3 ways to select the digit to be used twice) <u>Case 2 AABB</u> Number of 4-digit numbers = ${}^3C_2 \times \frac{4!}{2!\times 2!}$ = 18 (${}^3C_2$ ways to select the 2 digits each to be used twice) Total number of 4-digit numbers = 36 + 18 = <u>54</u>
7	**(i)**	$H_0 : \mu = 420$ $H_1 : \mu \neq 420$ $s^2 = \frac{30}{29}(12)$ = 12.414 Under H_0, since $n = 30$ is large, by Central Limit Theorem, $\bar{X} \sim N\left(420, \frac{12.414}{30}\right)$ approximately. Hence it is <u>not necessary</u> for the volumes to have a normal distribution for the test to be valid. Test statistic $Z = \frac{\bar{X}-420}{\sqrt{\frac{12.414}{30}}} \sim N(0,1)$ approximately $\alpha = 0.01$ From GC, $z = \frac{418.55-420}{\sqrt{\frac{12.414}{30}}} = -2.2541$ p-value = 0.0242 (3 sf) Since p-value = 0.0242 > $\alpha = 0.01$, we <u>do not reject H_0</u> at 1% level of

	significance and conclude that there is insufficient evidence that the population mean volume has changed. **(ii)** $\alpha = 0.01 \Rightarrow \frac{\alpha}{2} = 0.005$ Reject H_0 if $z \le -2.5758$ or $z \ge 2.5758$ $\frac{\bar{x}-420}{\sqrt{\frac{12.414}{30}}} \le -2.5758$ or $\frac{\bar{x}-420}{\sqrt{\frac{12.414}{30}}} \ge 2.5758$ $\bar{x} \le 420 - 2.5758\sqrt{\frac{12.414}{30}}$ or $\bar{x} \ge 420 + 2.5758\sqrt{\frac{12.414}{30}}$ $\bar{x} \le 418.34$ or $\bar{x} \ge 421.66$
8	Let A g be the mass of a tomato of variety A and B g be the mass of a tomato of variety B. $A \sim \mathrm{N}(80, 11^2)$ **(i)** $\mathrm{P}(A > 90) = 0.18165$ P(one greater than 90 g and one less than 90 g) $= 2 \times \mathrm{P}(A > 90) \times \mathrm{P}(A < 90)$ $= 2(0.18165)(1 - 0.18165)$ $= 0.297$ (3 sf) Let $B \sim \mathrm{N}(70, \sigma^2)$. **(ii)** Let $S_B = B_1 + B_2 + ... + B_6 + 15$ $S_B \sim \mathrm{N}(6 \times 70 + 15, 6\sigma^2)$ i.e., $\mathrm{N}(435, 6\sigma^2)$ $\mathrm{P}(S_B < 450) = 0.8463$ $\mathrm{P}\left(Z < \frac{450-435}{\sqrt{6}\,\sigma}\right) = 0.8463$ $\frac{15}{\sqrt{6}\,\sigma} = 1.0207$ $\sigma = \frac{15}{1.0207\sqrt{6}} = 6$ (nearest g) (Shown) **(iii)** $S_B \sim \mathrm{N}(435, 216)$ Let $S_A = A_1 + A_2 + ... + A_5 + 25$ $S_A \sim \mathrm{N}(5 \times 80 + 25, 5 \times 11^2)$ i.e., $\mathrm{N}(425, 605)$ $S_A - S_B \sim \mathrm{N}(425 - 435, 605 + 216) = \mathrm{N}(-10, 821)$ $\mathrm{P}(S_A > S_B) = \mathrm{P}(S_A - S_B > 0)$ $= 0.364$ (3 sf)
9	**(i)** (1) Selection of balls is done with replacement.

	(2) The balls are thoroughly mixed before each selection. **(ii)** Given $X \sim \text{B}\left(10, \frac{2}{5}\right)$ $\text{P}(X \geq 4) = 1 - \text{P}(X \leq 3)$ $= \underline{\underline{0.618}}$ (3 sf) **(iii)** Given $\text{E}(X) = 4.8$ $\Rightarrow \frac{2}{5}n = 4.8$ $n = \underline{\underline{12}}$ **(iv)** Given $X \sim \text{B}\left(n, \frac{2}{5}\right)$ $\text{P}(X = 0 \text{ or } 1) < 0.01$ $\Rightarrow \text{P}(X = 0) + \text{P}(X = 1) < 0.01$ $\Rightarrow \left(\frac{3}{5}\right)^n + n\left(\frac{2}{5}\right)\left(\frac{3}{5}\right)^{n-1} < 0.01$ From GC, least $n = \underline{\underline{14}}$ **(v)** Without replacement, P(Shawn wins the game) $= \frac{2}{5} + \frac{3}{5}\left(\frac{2}{4}\right)\left(\frac{2}{3}\right)$ $= \frac{3}{5}$ (Shown) **(vi)** With replacement, P(Shawn wins the game) $= \frac{2}{5} + \frac{3}{5}\left(\frac{3}{5}\right)\left(\frac{2}{5}\right) + \frac{3}{5}\left(\frac{3}{5}\right)\left(\frac{3}{5}\right)\left(\frac{3}{5}\right)\left(\frac{2}{5}\right) + \ldots$ $= \frac{2}{5} + \frac{2}{5}\left(\frac{3}{5}\right)^2 + \left(\frac{2}{5}\right)\left(\frac{3}{5}\right)^4 + \ldots$ $= \dfrac{\frac{2}{5}}{1 - \left(\frac{3}{5}\right)^2}$ $= \underline{\underline{\frac{5}{8}}}$ or $\underline{\underline{0.625}}$
10	**(i)**



(ii) **(a)** Between x and y: $r = \underline{\underline{0.959}}$

(b) Between x and y^2: $r = \underline{\underline{0.995}}$

(iii) From **(i)**, since as x increases, y increases at a decreasing rate, the points on the scatter diagram take the shape of the graph of $y^2 = c + dx$.

Or: From **(i)**, the points on the scatter diagram seem to lie on a concave downward curve.

From **(ii)**, the product moment correlation coefficient between x and y^2 is closer to 1, as compared to that between x and y,

$\therefore$ the model $\underline{\underline{y^2 = c + dx}}$ is the better model.

(iv) From GC, the regression line of y^2 on x is

$y^2 = 0.027897x - 47.985$

$\underline{\underline{y^2 = 0.0279x - 48.0}}$ (3 sf)

When $x = 2000$,

	$y^2 = 0.027897(2000) - 47.985$ $= 7.809$ $\therefore\ y = \underline{\underline{2.79}}$ (3 sf) or $\underline{\underline{2.8}}$ (1 dp, as shown in the table of values) **(v)** May not be valid as correlation does not necessarily imply causation. Or: May not be valid as there could be other factors relating traffic flow and air pollution. **(b)** $y = 2.5x + 3.8$ $\overline{y} = 2.5\overline{x} + 3.8$ $= 2.5(4.4) + 3.8$ $= 14.8$ Let $x = 1.5y - k$ $\overline{x} = 1.5\overline{y} - k$ $4.4 = 1.5(14.8) - k$ $k = 22.2 - 4.4$ $= \underline{\underline{17.8}}$