RI H2 Mathematics 2017 Prelim Exam Paper 1 Question

1 A local wholesaler sells Pikachi plushies in two sizes, small and large. The number of Pikachi plushies bought by three particular retailers and the total amount they paid are shown in the following table.

Retailer	Small	Large	Total Amount paid
A	30	50	\$1375
B	k	$2k$	\$2704
C	$2k$	k	\$2522

Find the price of each small and each large Pikachi plushy and determine the value of k. [4]

2 A right circular cone has base radius r cm and height h cm. As r and h vary, its curved surface area, $\pi r\sqrt{\left(r^2+h^2\right)}$ cm^2, remains constant.

It is given that when $r=\sqrt{2}$ cm, the magnitude of the rate of change of h is 10 times the magnitude of the rate of change of r. Given also that $h>r$, find the height of the cone at this instant. [4]

3

(a) Find $\displaystyle\int \frac{x+2}{\sqrt{\left(1-8x-4x^2\right)}}\,\mathrm{d}x$. [4]

(b) Use the substitution $x=2\sec\theta$ to find the exact value of $\displaystyle\int_2^4 \frac{1}{x}\sqrt{\left(x^2-4\right)}\,\mathrm{d}x$. [4]

4 A curve C has equation $y=\mathrm{f}(x)$, where

$$\mathrm{f}(x)=\frac{a}{(x+b)^2}+cx,$$

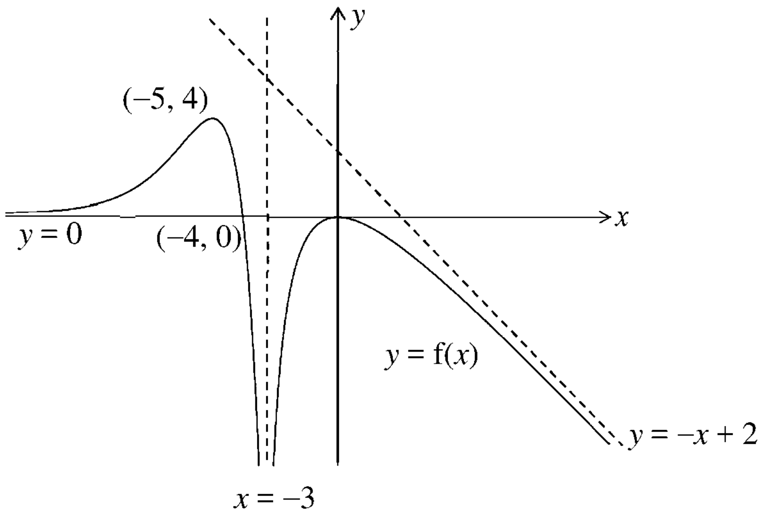
and a, b and c are constants. It is given that C has a vertical asymptote $x=-1$ and a minimum point at $(0,\ 1)$.

(i) Find the values of a, b and c. [4]

(ii) Sketch the graph of $y=\mathrm{f}\left(|x|\right)$, stating the coordinates of any point(s) of intersection with the axes and the equation(s) of any asymptote(s). [3]

(iii) Hence, solve the inequality $f(|x|)-4>0$. [2]

5



The diagram shows the curve $y=f(x)$. The curve has maximum points at $(-5,\ 4)$ and the origin, and crosses the x-axis at $(-4, 0)$. The lines $y = 0$, $x = -3$ and $y = -x + 2$ are the horizontal, vertical and oblique asymptotes to the curve respectively.

On separate diagrams, draw sketches of the graphs of

(a) $y=\dfrac{1}{f(x)}$, [3]

(b) $y=f'(x)$, [3]

(c) $y=f\left(\dfrac{x+1}{2}\right)$, [3]

labelling clearly the equation(s) of any asymptote(s), coordinates of any axial intercept(s) and turning point(s) where applicable.

6

(i) Given that $y=\ln(1+\sin 2x)$, show that $e^{y}\dfrac{d^2y}{dx^2}+e^{y}\left(\dfrac{dy}{dx}\right)^2=-4\sin 2x$.

Find the first three non-zero terms in the Maclaurin's series for y. [5]

(ii) It is given that the three terms found in part **(i)** are equal to the first three terms in the series expansion of $ax(1+bx)^n$ for small x. Find the exact values of the constants a, b and n and use these values to find the coefficient of x^4 in the expansion of $ax(1+bx)^n$, giving your answer as a simplified rational number. [5]

7

Mr Tan is planning to set up a home theatre in his spacious rectangular living room. A projector screen with height a metres is to be positioned against one of the walls b metres above the eye level (see Fig. 1). He is trying to decide on the horizontal distance between the sofa and the screen so that the viewing angle α of the projection screen is as large as possible.

(i) Show that $\alpha = \tan^{-1}\frac{a+b}{x} - \tan^{-1}\frac{b}{x}$, where x is the horizontal distance between the sofa and the screen in metres. [1]

(ii) Use differentiation to show that the value of x which gives the maximum value of α satisfies the equation

$$\frac{a+b}{x^2+(a+b)^2} = \frac{b}{x^2+b^2}.$$

Solve for x and leave your answer in terms of a and b. [4]
[It is not necessary to verify the nature of the maximum point in this part.]

Mrs Tan proposed an alternative way of arrangement. She proposed to place the sofa against the wall opposite the screen, which is c metres away, and to vary the vertical position of the screen placed y metres above the eye level in order to maximise the angle α (see Fig. 2).

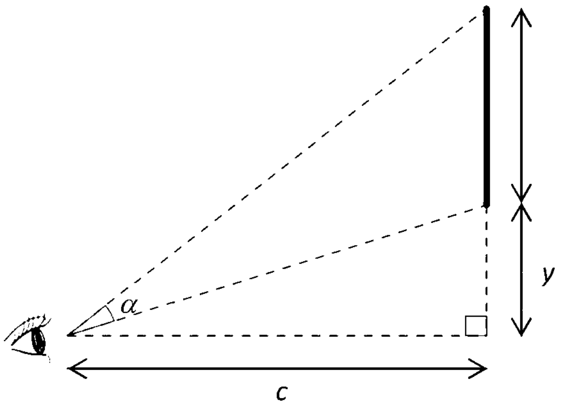


Fig. 2

(iii) Use differentiation to find the value of y which gives the maximum value of α, leaving your answer in terms of a. Interpret the answer in this context. [5]

8 A curve C has parametric equations

$$x = \sin^2 t, \quad y = 2\cos t, \quad \text{for } 0 \le t \le \frac{\pi}{2}.$$

(i) Find a cartesian equation of C. [2]

The tangent to the curve at the point P where $t = \frac{\pi}{3}$ is denoted by l.

(ii) Find an equation of l. [3]

(iii) On the same diagram, sketch C and l, stating the coordinates of the axial intercepts and the point of intersection. [3]

The region R is bounded by the curve C, the line l and the y-axis.

(iv) Find the exact value of the volume of revolution formed when R is rotated completely about the x-axis. [3]

9 **Do not use a calculator in answering this question.**

(a) One root of the equation $z^4 + 2z^3 + az^2 + bz + 50 = 0$, where a and b are real, is $z = 1 +$

(i) Show that $a = 7$ and $b = 30$ and find the other roots of the equation. [5]

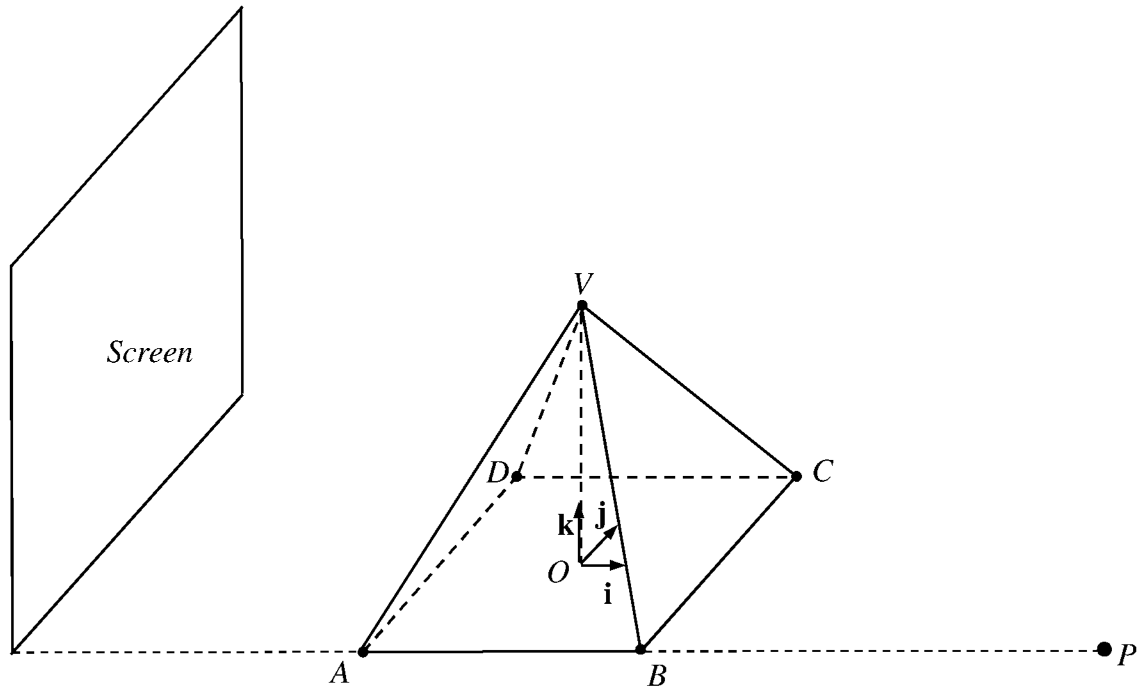
(ii) Deduce the roots of the equation $w^4 - 2\mathrm{i}w^3 - 7w^2 + 30\mathrm{i}w + 50 = 0$. [2]

(b) Given that $p^* = \dfrac{\left(-\dfrac{1}{\sqrt{3}} + \mathrm{i}\right)^5}{(1-\mathrm{i})^4}$, by considering the modulus and argument of p^*, find the exact expression for p, in cartesian form $x + \mathrm{i}y$. [4]

10 In a model of forest fire investigation, the proportion of the total area of the forest which has been destroyed is denoted by x. The destruction rate of the fire is defined to be the rate of change of x with respect to the time t, in hours, measured from the instant the fire is first noticed. A particular forest fire is initially noticed when 20% of the total area of the forest is destroyed.

(a) One model of forest fire investigation shows that the destruction rate is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{10}x(1-x).$$

	(i) Express the solution of the differential equation in the form $x = \mathrm{f}(t)$ and sketch the part of the curve for $t \geq 0$. [6] **(ii)** Find the exact time when the destruction rate is at its maximum. [2] **(iii)** Explain briefly why this model cannot be used to estimate how long the forest has been burning when it is first noticed. [1] **(b)** A second model for the investigation of forest fire is suggested and given by the differential equation $$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{5\pi\left[1+\left(\frac{t}{10}+\tan\frac{\pi}{10}\right)^2\right]}.$$ Determine how long the forest has been burning when the fire is first noticed. [3]
11	 A right opaque pyramid with square base *ABCD* and vertex *V* is placed at ground level for a shadow display, as shown in the diagram. *O* is the centre of the square base *ABCD*, and perpendicular unit vectors **i**, **j**, **k** are in the directions of $\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{OV}$ respectively. The length of *AB* is 8 units and the length of *OV* is 2*h* units. A point light source for this shadow display is placed at the point *P*(20, −4, 0) and a screen of height 35 units is placed with its base on the ground such that the screen lies on a plane with

vector equation $\mathbf{r}\cdot\begin{pmatrix}1\\0\\0\end{pmatrix}=\alpha$ where $\alpha<-4$ (see diagram).

(i) Find a vector equation of the line depicting the path of the light ray from P to V in terms of h. [2]

(ii) Find an inequality between α and h so that the shadow of the pyramid cast on the screen will not exceed the height of the screen. [3]

The point light source is now replaced by a parallel light source whose light rays are perpendicular to the screen and it is also given that $h = 10$.

(iii) Find the exact length of the shadow cast by the edge VB on the screen. [3]

A mirror is placed on the plane VBC to create a special effect during the display.

(iv) Find a vector equation of the plane VBC and hence find the angle of inclination made by the mirror with the ground. [4]

RAFFLES INSTITUTION
H2 Mathematics (9758)
2017 Year 6

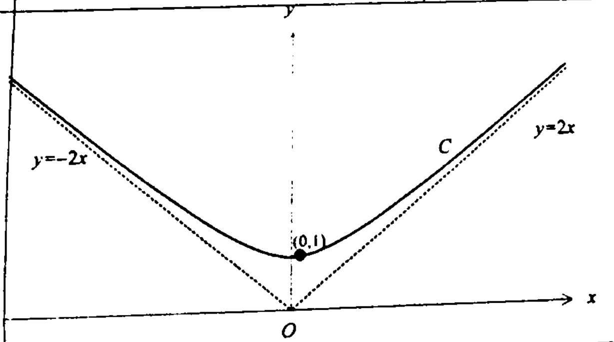
2017 H2 Math 9758 Preliminary Examination Paper 1 : Suggested Solutions

Qn 1		Comments
[4]	Let \$$x$ and \$$y$ be the price of each small and each large Pikachi plushy respectively. Retailer A : $30x + 50y = 1375$ … (1) Retailer B : $kx + 2ky = 2704$ ÷ by k $x + 2y - 2704\left(\frac{1}{k}\right) = 0$ … (2) Retailer C : $2kx + ky = 2522$ ÷ by k $2x + y - 2522\left(\frac{1}{k}\right) = 0$ … (3) From GC : $x = 15$, $y = 18.5$, $\frac{1}{k} = \frac{1}{52}$ Hence, $k = 52$, each small Pikachi plushy costs \$15, and each large Pikachi plushy costs \$18.50.	Most students were able to form the three equations. Only a handful realized the need to linearise the equations before using GC to solve for the unknowns.

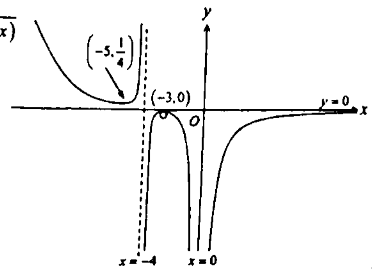
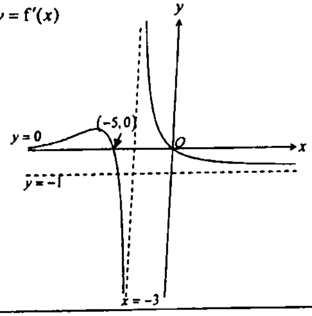
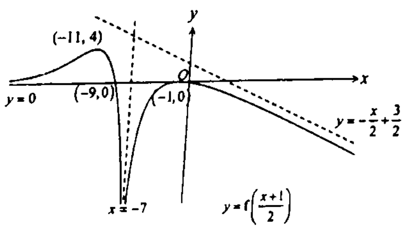
Qn 2		Comments
[4]	Let $A = \pi r\sqrt{r^2 + h^2}$. square both sides $\Rightarrow \quad A^2 = \pi^2 r^2 (r^2 + h^2)$ Differentiate w.r.t. t : $r^2\left(2r\frac{dr}{dt} + 2h\frac{dh}{dt}\right) + (r^2 + h^2)\left(2r\frac{dr}{dt}\right) = 0$ (Note: $\frac{dA}{dt} = 0$ since A is a constant) Since $r \neq 0$, $\quad (2r^2 + h^2)\frac{dr}{dt} + hr\frac{dh}{dt} = 0$ $\Rightarrow \quad \left(\frac{dh}{dt}\right) \div \left(\frac{dr}{dt}\right) = \frac{2r^2 + h^2}{-hr}$ When $r = \sqrt{2}$, $\frac{dh}{dt} = -10\frac{dr}{dt} \Rightarrow \frac{4 + h^2}{-\sqrt{2}h} = -10$ $\Rightarrow h^2 - 10\sqrt{2}h + 4 = 0$ Solving: $h = 13.9$ (3sf) or $h = 0.289$ (3sf) Since $h > r$, the height of the cone required is 13.9 cm (to 3 sf).	Students must remember that r and h are variables and not constants. When performing implicit differentiation on the variables with respect to r , h or t , you must have your $\frac{dh}{dr}$ or $\frac{dr}{dt}$ etc. Do remember to substitute the given conditions **after** differentiation and not ***before*** differentiation!

Qn 3		Comments
(a) [4]	$\int \frac{x+2}{\sqrt{1-8x-4x^2}}\,dx$ $= -\frac{1}{8}\int \frac{-8x-8}{\sqrt{1-8x-4x^2}}\,dx + \int \frac{1}{\sqrt{1-8x-4x^2}}\,dx$ complete the square $= -\frac{1}{8}\int \frac{-8x-8}{\sqrt{1-8x-4x^2}}\,dx + \int \frac{1}{\sqrt{5-4(x+1)^2}}\,dx$ $= -\frac{1}{4}\sqrt{1-8x-4x^2} + \frac{1}{2}\sin^{-1}\frac{2\sqrt{5}(x+1)}{5} + C$	This question is not well done. Most split the integrand to $\int \frac{x}{\sqrt{1-8x-4x^2}}\,dx + \int \frac{2}{\sqrt{1-8x-4x^2}}\,dx$ and have difficulty integrating the first term. There are quite a few who wrote $-\frac{1}{8}\int \frac{-8x-8}{\sqrt{1-8x-4x^2}}\,dx = -\frac{1}{8}\frac{(1-8x-4x^2)^{\frac{3}{2}}}{\frac{3}{2}} + C$
(b) [4]	$x = 2\sec\theta \Rightarrow \frac{dx}{d\theta} = 2\sec\theta\tan\theta$ When $x = 2$, $\sec\theta = 1 \Rightarrow \cos\theta = 1 \Rightarrow \theta = 0$ When $x = 4$, $\sec\theta = 2 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$ $\int_2^4 \frac{1}{x}\sqrt{(x^2-4)}\,dx$	There is a significant number of students who do not know how to integrate $\tan^2\theta$ and $\sec^2\theta$. Make sure you know how to integrate all the trigonometric function ($\sin x$, $\cos x$, $\tan x$, $\sec x$, $\csc x$, $\cot x$) and $(\text{trigonometric})^2$ function, such as $\sin^2 x, \cos^2 x, \tan^2 x, \sec^2 x, \csc^2 x, \cot^2 x$, and be familiar with the formulae/identities given in MF26.

	$= \int_0^{\frac{\pi}{3}} \frac{\sqrt{4\sec^2\theta - 4}}{2\sec\theta}(2\sec\theta\tan\theta)\,d\theta$ $= \int_0^{\frac{\pi}{3}} \frac{2\tan\theta}{2\sec\theta}(2\sec\theta\tan\theta)\,d\theta$ since $\sqrt{4\sec^2\theta - 4} = 2\sqrt{\tan^2\theta} = 2\tan\theta$ for $0 \le \theta \le \frac{\pi}{3}$ $= \int_0^{\frac{\pi}{3}} 2\tan^2\theta\,d\theta$ $= 2\int_0^{\frac{\pi}{3}} \sec^2\theta - 1\,d\theta$ $= 2[\tan\theta - \theta]_0^{\pi/3}$ $= 2\left[\sqrt{3} - \frac{\pi}{3}\right]$	The other common mistake is not changing the limits of the integration when substituting x by 2 sec θ.

Qn 4		Comments
(i) [4]	$y = \frac{a}{(x+b)^2} + cx$ C has a vertical asymptote $x = -1 \Rightarrow b = 1$ C passes through $(0,1) \Rightarrow a = 1$ $\frac{dy}{dx} = -\frac{2}{(x+1)^3} + c$ At $(0,1)$, $\frac{dy}{dx} = 0 \Rightarrow c = \frac{2}{1^3} = 2$	When finding the derivative of $\frac{a}{(x+b)^2}$ with respect to x, there is no need to use the quotient rule. If the unknown constants bother you, ask yourself how you would proceed to find its derivative if you assume some real values for the unknown constants. Eg $\frac{1}{(x+1)^2}$ If you can identify the values of some of the unknowns immediately, substituting these values into the original expression will help to simplify your calculations.
(ii) [3]	 	Whenever you are sketching a graph, you should always remember SIA (shape, intercepts, asymptotes). Question also states that (0, 1) is a minimum point, so if it is not featured in the graph,

		something is wrong! Your graph should also be symmetrical about the y-axis.
(iii) [2]	$f(\lvert x\rvert) - 4 > 0 \Leftrightarrow f(\lvert x\rvert) > 4$ The line $y = 4$ cuts the graph of $y = f(\lvert x\rvert)$ at $x = \pm 1.94$ (3sf). $\therefore f(\lvert x\rvert) - 4 > 0 \Leftrightarrow x < -1.94$ or $x > 1.94$	Question did not ask for exact answers, so it is not necessary to solve for the intersection points algebraically. You just need to plot a graph and find its intersection with the x-axis using a GC. Your final answer should be symmetrical about the y axis, and remember to give the final non exact answers to 3 sf.

Qn 5		Comments
(a) [3]	$y = \frac{1}{f(x)}$ 	**General** Co-ordinates are required for axial intercept(s) and turning point(s). We want to see $(-3,0)$ marked in part (a), and so on. Equations of asymptotes must be labelled. Many students left out "$y = 0$" and/or "$x = 0$". Curve sketched near asymptote must exhibit the characteristic of approaching nearer and nearer to it. Use ruler to draw (dotted) straight lines to represent asymptotes! **For (a)**: An empty circle ought to be marked at the point $(-3,0)$ to indicate the exclusion of the point.
(b) [3]	$y = f'(x)$ 	**For (c)**: We can't leave the equation of the oblique asymptote as $y = -\frac{x+1}{2} + 2$. Simplify it. Many students had the wrong translation unit and/or scaling factor. There are 2 options for your consideration : (1) $f(x) \to f\left(\frac{x}{2}\right) \to f\left(\frac{x+1}{2}\right)$ (2) $f(x) \to f\left(x + \frac{1}{2}\right) \to f\left(\frac{x}{2} + \frac{1}{2}\right)$
(c) [3]		

Qn 6		Comments
(i) [5]	$y = \ln(1 + \sin 2x)$ so $e^y = 1 + \sin 2x$ Differentiating with respect to x: $e^y \frac{dy}{dx} = 2\cos 2x$ $e^y \frac{d^2y}{dx^2} + e^y \left(\frac{dy}{dx}\right)^2 = -4\sin 2x$ (shown) $e^y \frac{d^3y}{dx^3} + e^y \left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right) + 2e^y \left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right) + e^y \left(\frac{dy}{dx}\right)^3 = -8\cos 2x$ $e^y \frac{d^3y}{dx^3} + 3e^y \left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right) + e^y \left(\frac{dy}{dx}\right)^3 = -8\cos 2x$ When $x = 0$, $e^y = 1 \Rightarrow y = 0$, $\frac{dy}{dx} = 2$, $\frac{d^2y}{dx^2} = -4$, $\frac{d^3y}{dx^3} = 8$. By Maclaurin's Theorem, $y = 2x - 4\left(\frac{x^2}{2!}\right) + 8\left(\frac{x^3}{3!}\right) + \cdots$ $= 2x - 2x^2 + \frac{4}{3}x^3 + \cdots$	Since we need to derive the 2[nd] order DE which involves e^y, we should express $y = \ln(1 + \sin 2x)$ as $e^y = 1 + \sin 2x$ and apply implicit differentiation. Direct differentiation can be complicated at times. Note that $\frac{d}{dx}\left(\frac{dy}{dx}\right)^2 = 2\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)$ An alternative solution involves applying standard Maclaurin expansion although this is not the intended method.
(ii) [5]	$ax(1+bx)^n$ $= ax\left[1 + n(bx) + \frac{n(n-1)}{2!}(bx)^2 + \frac{n(n-1)(n-2)}{3!}(bx)^3 + \ldots\right]$ $= ax + nabx^2 + \frac{n(n-1)}{2}ab^2x^3 + \ldots$ By comparing coefficients, $a = 2$ $nab = -2 \Rightarrow nb = -1$	A common mistake involves omitting the term, ax, in the expansion of $ax(1+bx)^n$. Also, since we do not know whether n is a positive integer or not, we should not use $\binom{n}{r}$ or $n!$.

$$\frac{n(n-1)}{2}ab^2 = \frac{4}{3}$$

$$\Rightarrow n^2b^2 - nb^2 = \frac{4}{3}$$

$$\Rightarrow (-1)^2 - (-1)b = \frac{4}{3}$$

$$\Rightarrow b = \frac{1}{3} \quad \text{and} \quad n = -3$$

x^4 term in the expansion of $2x\left(1+\frac{1}{3}x\right)^{-3}$

$$= 2x\left[\frac{-3(-4)(-5)}{3!}\left(\frac{1}{3}x\right)^3\right] = -\frac{20}{27}x^4$$

$\therefore$ coefficient of $x^4 = -\frac{20}{27}$

Qn 7

(i) [1]

Let β be the angle of elevation of the bottom of the screen from eye-level.

$$\tan(\alpha+\beta) = \frac{a+b}{x} \Rightarrow \alpha+\beta = \tan^{-1}\frac{a+b}{x}$$

$$\tan(\beta) = \frac{b}{x} \Rightarrow \beta = \tan^{-1}\frac{b}{x}$$

$$\alpha = (\alpha+\beta)-\beta = \tan^{-1}\frac{a+b}{x} - \tan^{-1}\frac{b}{x}$$

Comments: Students should define the angles properly. They can also draw a diagram to indicate the angles α and β.

(ii) [4]

$$\frac{d\alpha}{dx} = \frac{1}{1+\left(\frac{a+b}{x}\right)^2}\left(-\frac{a+b}{x^2}\right) - \frac{1}{1+\left(\frac{b}{x}\right)^2}\left(-\frac{b}{x^2}\right)$$

$$= -\frac{a+b}{x^2+(a+b)^2} + \frac{b}{x^2+b^2}$$

For maximum α: $\frac{d\alpha}{dx} = -\frac{(a+b)}{x^2+(a+b)^2} + \frac{b}{x^2+b^2} = 0$

$$\frac{(a+b)}{x^2+(a+b)^2} = \frac{b}{x^2+b^2} \quad \text{(Shown)}$$

$$(a+b)(x^2+b^2) = b[x^2+(a+b)^2]$$

$$(a+b)x^2 + (a+b)b^2 = bx^2 + b(a+b)^2$$

$$ax^2 = b(a+b)[a+b-b] = ab(a+b)$$

$x = \sqrt{b(a+b)}$ or $-\sqrt{b(a+b)}$ (NA since $x>0$)

Comments: Students should note the application of chain rule in the showing of the differentiation in the first line.

(iii) [5]

Let the screen to be positioned y metres above the eye level.

$$\alpha = \tan^{-1}\frac{a+y}{c} - \tan^{-1}\frac{y}{c}$$

$$\frac{d\alpha}{dy} = \frac{1}{1+\left(\frac{a+y}{c}\right)^2}\left(\frac{1}{c}\right) - \frac{1}{1+\left(\frac{y}{c}\right)^2}\left(\frac{1}{c}\right)$$

$$= \frac{c}{c^2+(a+y)^2} - \frac{c}{c^2+y^2}$$

$$= \frac{c\left[c^2+y^2-c^2-(a+y)^2\right]}{\left[c^2+(a+y)^2\right](c^2+y^2)}$$

$$= \frac{c\left[y+(a+y)\right]\left[y-(a+y)\right]}{\left[c^2+(a+y)^2\right](c^2+y^2)}$$

$$= \frac{-ac[a+2y]}{\left[c^2+(a+y)^2\right](c^2+y^2)} = \frac{-2ac\left[y+\frac{a}{2}\right]}{\left[c^2+(a+y)^2\right](c^2+y^2)}$$

For maximum α: $\frac{d\alpha}{dy} = \frac{-ac[a+2y]}{\left[c^2+(a+y)^2\right](c^2+y^2)} = 0$

$$\Rightarrow y = -\frac{a}{2} \text{ (since } a \neq 0 \text{ and } c \neq 0)$$

y	$\left(-\frac{a}{2}\right)^-$	$\left(-\frac{a}{2}\right)$	$\left(-\frac{a}{2}\right)^+$
$\left[y+\frac{a}{2}\right]$	<0	0	>0
$-2ac\left[y+\frac{a}{2}\right]$, where $-2ac<0$	>0	0	<0
$\frac{d\alpha}{dy}$	>0	0	<0

Therefore $y = -\frac{a}{2}$ gives the maximum viewing angle α.

Interpretation of the answer:
In order to maximise the viewing angle α, the centre of the screen need to be placed at eye level regardless of the position of the sofa.

Comments: Students should note that it is eye level and not ground level given in the question, for those who concluded that it is impossible to get y being negative.

Students should show how they deduced the signs of the derivative for $y = \left(-\frac{a}{2}\right)^-$ and $y = \left(-\frac{a}{2}\right)^+$

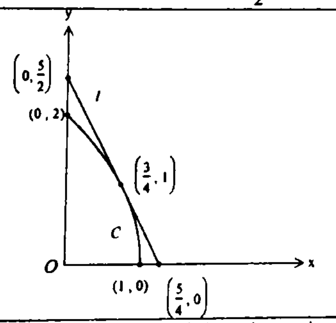
Some were also successful in using the second derivative test, although it is more demanding to obtain.

Qn 8

(i) [2]

Using $\sin^2 t + \cos^2 t = 1$, a cartesian equation of C is

Comments: Important to remember the Trigo identities. Answers such as $y = 2\cos\left(\sin^{-1}\sqrt{x}\right)$

	$x+\left(\frac{y}{2}\right)^2=1$ $\Rightarrow y^2=4-4x,\ 0\le x\le 1,\ 0\le y\le 2$ or $\Rightarrow y=2\sqrt{1-x},\ 0\le x\le 1$	are **not** accepted as they are **not** simplified. Essential to state $0\le x\le 1$ and/or $0\le y\le 2$ as C is defined for $0\le t\le\frac{\pi}{2}$.
(ii) [3]	Differentiate with respect to x: $1+\frac{y}{2}\frac{dy}{dx}=0 \quad\Rightarrow\quad \frac{dy}{dx}=-\frac{2}{y}$ When $t=\frac{\pi}{3}$, $x=\sin^2\left(\frac{\pi}{3}\right)=\frac{3}{4}$, $y=2\cos\left(\frac{\pi}{3}\right)=1$, $\frac{dy}{dx}=-2$ Hence, an equation of l is $y-1=-2\left(x-\frac{3}{4}\right)$ $y=-2x+\frac{5}{2}$	$\frac{dy}{dx}$ can also be found by parametric or explicit differentiation.
(iii) [2]		Sketch C for $0\le x\le 1$ only. State the coordinates of the point of intersection and axial intercepts. Sketch should illustrate that line l is a tangent to curve C at $(\frac{3}{4},1)$.
(iv) [3]	Volume of revolution of R rotated about the x-axis $=\pi\int_0^{\frac{3}{4}}\left(-2x+\frac{5}{2}\right)^2 dx-\pi\int_0^{\frac{3}{4}}(4-4x)\,dx$ $=\pi\left[\frac{\left(-2x+\frac{5}{2}\right)^3}{3(-2)}\right]_0^{\frac{3}{4}}-\pi\left[4x-2x^2\right]_0^{\frac{3}{4}}$ $=-\frac{1}{6}\pi\left[1^3-\left(\frac{5}{2}\right)^3\right]-\pi\left[3-2\left(\frac{3}{4}\right)^2\right]$	As "exact value" is required, you are to **show clear working** instead of using the GC to obtain the values of the integrals. (You may, of course, use the GC to check your answer).

	$=\frac{39}{16}\pi-\frac{15}{8}\pi$ $=\frac{9}{16}\pi$ units^3 OR Use volume of cone $=\frac{1}{3}\pi r^2h$, i.e. $\left[\frac{1}{3}\pi\left(\frac{5}{2}\right)^2\left(\frac{5}{4}\right)-\frac{1}{3}\pi(1)^2\left(\frac{1}{2}\right)\right]-\pi\int_0^{\frac{3}{4}}(4-4x)\,dx$	

Qn 9		Comments
(a) (i) [5]	Since $z=1+3i$ is a root and the polynomial has real coefficients, $z=1-3i$ is also a root to the polynomial. Hence a quadratic factor of the polynomial is $(z-(1+3i))(z-(1-3i))=$ $(z^2-z(1+3i+1-3i)+(1+3i)(1-3i))=(z^2-2z+10)$ $z^4+2z^3+az^2+bz+50$ $=(z^2-2z+10)(Az^2+Bz+C)$ for some constants A, B and C. By comparing coefficient of z^4 and z^3, $A=1$ and $B-2A=2\Rightarrow B=4$ By comparing the constant term, $C=5$ Hence $z^4+2z^3+az^2+bz+50=(z^2-2z+10)(z^2+4z+5)$ Comparing coefficient of z^2 and z, we have $a=-8+10+5=7$ and $b=40-10=30$ (shown). Solving $z^2+4z+5=0$, $z=\frac{-4\pm\sqrt{4^2-4(5)}}{2}=\frac{-4\pm\sqrt{-4}}{2}=\frac{-4\pm\sqrt{4}\sqrt{-1}}{2}=-2\pm i.$ Hence the other roots are $z=1-3i$, $z=-2+i$ and $z=-2-i$. Alternative Solution (more tedious): Since $1+3i$ is a root, $(1+3i)^4+2(1+3i)^3+a(1+3i)^2+b(1+3i)+50=0$ --(1) $(1+3i)^2=1^2+2(3i)+(3i)^2=(1-9)+6i=-8+6i$ $(1+3i)^3=(1+3i)(-8+6i)=(-8-18)+i(6-24)=-26-18i$	Note the following instruction is given at the start of the question - **"Do not use a calculator in answering this question"**. Obviously you can use GC to check your answer, but you are required to show clear working.

$(1+3i)^4 = (-8+6i)^2 = 64-96i-36 = 28-96i$

Applying above results on (1),

$(28-96i)+2(-26-18i)+a(-8+6i)+b(1+3i)+50=0$

$(26-8a+b)+(-132+6a+3b)i=0$

Comparing real and imaginary parts,

$26-8a+b=0$ and $-132+6a+3b=0$

equivalent to $-44+2a+b=0$

Solving, $-44-26+10a=0 \Rightarrow a=7$ and $b=8(7)-26=30$

$\therefore a=7,\ b=30$ (shown)

Since $z=1+3i$ is a root and the polynomial has real coefficients, $z=1-3i$ is also a root to the polynomial.

$z^4+2z^3+7z^2+30z+50$

$=(z-(1+3i))(z-(1-3i))(z^2+Az+B)$

$=(z^2-2z+10)(z^2+Az+B)$

By comparing coefficients, we have $A=4,\ B=5$.

Solving $z^2+4z+5=0$, $z=\dfrac{-4\pm\sqrt{-4}}{2}=\dfrac{-4\pm\sqrt{4}\sqrt{-1}}{2}=-2\pm i$.

Hence the other roots are $z=1-3i$, $z=-2+i$ and $z=-2-i$.

(ii) [2]

Let $z=iw$, then we get $(iw)^4+2(iw)^3+7(iw)^2+30(iw)+50=0$

$\Rightarrow w^4-2iw^3-7w^2+30iw+50=0.$

$z=iw \Rightarrow w=-iz.$

Hence the roots are $w=-i-3$, $w=-i+3$, $w=2i+1$ and $w=2i-1$.

Note that $\dfrac{1}{i}=-i$

(b) [4]

$|p|=|p^*|=\dfrac{\left|\left(-\frac{1}{\sqrt{3}}+i\right)^5\right|}{|(1-i)|^4}=\dfrac{\left(\frac{2}{\sqrt{3}}\right)^5}{\left(\sqrt{2}\right)^4}=\dfrac{32}{4}\left(\dfrac{1}{\sqrt{3}}\right)^5=\dfrac{8}{9\sqrt{3}}$ or $\dfrac{8\sqrt{3}}{27}$

$\arg(p)=-\arg(p^*)=-\left(5\arg\left(-\dfrac{1}{\sqrt{3}}+i\right)-4\arg(1-i)\right)+2\pi+2\pi$

$=-\left(5\left(\dfrac{2\pi}{3}\right)-4\left(-\dfrac{\pi}{4}\right)\right)+2\pi+2\pi$

$=-\dfrac{\pi}{3}$

$p=\dfrac{8}{9\sqrt{3}}\left(\cos\left(-\dfrac{\pi}{3}\right)+i\sin\left(-\dfrac{\pi}{3}\right)\right)=\dfrac{8}{9\sqrt{3}}\left(\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i\right)=\dfrac{4}{9\sqrt{3}}-\dfrac{4}{9}i$ or $\dfrac{4\sqrt{3}}{27}-\dfrac{4}{9}i$

Note that this question requires you to consider $|p^*|$ and $\arg(p^*)$.

Remember to simplify your surds.

Note that $\arg(1-i)=-\dfrac{\pi}{4}$

Qn 10	Comments

(a)(i) [6]

$\dfrac{dx}{dt}=\dfrac{1}{10}x(1-x)$

$\Rightarrow \displaystyle\int\left(\frac{1}{x}+\frac{1}{1-x}\right)dx=\int\frac{1}{10}dt$

$\Rightarrow \ln\left|\dfrac{x}{1-x}\right|=\dfrac{1}{10}t+C$,

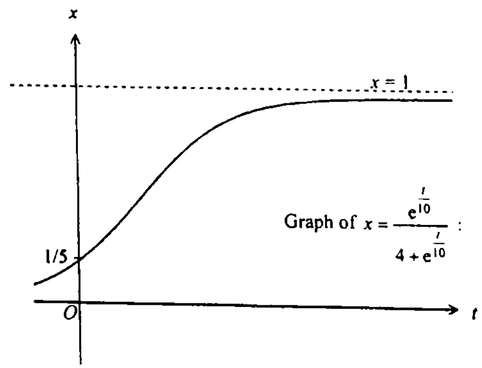
where C is an arbitrary constant

$\Rightarrow \dfrac{x}{1-x}=Ae^{\frac{t}{10}}$, where $A=\pm e^C$

$\Rightarrow x=\dfrac{Ae^{\frac{t}{10}}}{1+Ae^{\frac{t}{10}}}$

When $t=0$, $x=\dfrac{1}{5}$. That is, $\dfrac{1}{5}=\dfrac{A}{1+A}\Rightarrow A=\dfrac{1}{4}$.

Hence, $x=\dfrac{\frac{1}{4}e^{\frac{t}{10}}}{1+\frac{1}{4}e^{\frac{t}{10}}}=\dfrac{e^{\frac{t}{10}}}{4+e^{\frac{t}{10}}}=1-\dfrac{4}{4+e^{\frac{t}{10}}}$.



Graph of $x=\dfrac{e^{\frac{t}{10}}}{4+e^{\frac{t}{10}}}$:

If $\ln\dfrac{x}{1-x}$ is given instead of $\ln\left|\dfrac{x}{1-x}\right|$, then $0<x<1$ should be stated to justify the removal of the modulus

Final answer should be in simplified form, it should not contain a fraction within a fraction like $\dfrac{\frac{1}{4}e^{\frac{t}{10}}}{1+\frac{1}{4}e^{\frac{t}{10}}}$.

(ii) [2]

Since $\dfrac{dx}{dt}=\dfrac{1}{10}x(1-x)$ is a quadratic expression, destruction rate is at its maximum when $x=\dfrac{0+1}{2}=\dfrac{1}{2}$

Or when $\dfrac{d}{dx}\left(\dfrac{dx}{dt}\right)=\dfrac{1}{10}-\dfrac{1}{5}x=0$ i.e., $x=\dfrac{1}{2}$.

The "**destruction rate** is at its maximum" refers to maximum

	Therefore, $\dfrac{\frac{1}{2}}{1-\frac{1}{2}}=\frac{1}{4}e^{\frac{t}{10}}$. $\Rightarrow \quad e^{\frac{t}{10}}=4 \Rightarrow \quad t=10\ln 4$	$\frac{dx}{dt}$, not maximum x.
(iii) [1]	From $x=\dfrac{e^{\frac{t}{10}}}{4+e^{\frac{t}{10}}}$, $\quad x=0 \Rightarrow e^{\frac{t}{10}}=0$. Since $e^{\frac{t}{10}}>0$ for all real t, there is no value of t for $x=0$. **OR** Note that $\quad x=1-\dfrac{4}{4+e^{\frac{t}{10}}}$. Since $0<\dfrac{4}{4+e^{\frac{t}{10}}}<1$, we will have $0<x<1$. Hence, $x>0$ for all real values of t, and there is no value of t for $x=0$. **OR** As $x\to-\infty$, $e^{\frac{t}{10}}\to 0^+$, $x=1-\dfrac{4}{4+e^{\frac{t}{10}}}\to 0^+$. Hence, $x=0$ is a horizontal asymptote and there are no values of t giving $x=0$.	The question asks to explain why the model **cannot** be used to estimate (i.e. why we are unable to estimate using the model), it does not ask for why the model may not give a good estimate. So answers like "extrapolation is not reliable" or "the model is not valid for $t<0$" are not accepted.
(b) [3]	$\dfrac{dx}{dt}=\dfrac{1}{5\pi\left[1+\left(\frac{t}{10}+\tan\frac{\pi}{10}\right)^2\right]}$ $x=\dfrac{1}{5\pi}\displaystyle\int \dfrac{1}{1+\left(\frac{t}{10}+\tan\frac{\pi}{10}\right)^2}\,dt$ $=\dfrac{10}{5\pi}\displaystyle\int \dfrac{\frac{1}{10}}{1+\left(\frac{t}{10}+\tan\frac{\pi}{10}\right)^2}\,dt$	

	$=\dfrac{2}{\pi}\tan^{-1}\left(\dfrac{t}{10}+\tan\dfrac{\pi}{10}\right)+C$ When $t=0$, $x=\frac{1}{5}$. Hence $\frac{1}{5}=\frac{2}{\pi}\tan^{-1}\left(\tan\frac{\pi}{10}\right)+C \Rightarrow C=0$ That is, $x=\dfrac{2}{\pi}\tan^{-1}\left(\dfrac{t}{10}+\tan\dfrac{\pi}{10}\right)$. From G.C., when $x=0$, $t=-3.25$ (3 s.f.) Hence, the forest have been burning for 3.25 hours when it is first noticed.	It is important to have "$+C$" then show that $C=0$. Without this step, no mark can be awarded for the final answer.

Qn 11		Comments
(i) [2]	$\overrightarrow{OP}=\begin{pmatrix}20\\-4\\0\end{pmatrix}$, $\overrightarrow{OV}=\begin{pmatrix}0\\0\\2h\end{pmatrix}$ $\overrightarrow{PV}=\begin{pmatrix}-20\\4\\2h\end{pmatrix}=2\begin{pmatrix}-10\\2\\h\end{pmatrix}$ Vector equation of the line depicting the path of the light ray from P to V is $\mathbf{r}=\begin{pmatrix}20\\-4\\0\end{pmatrix}+\lambda\begin{pmatrix}-10\\2\\h\end{pmatrix}$, $\lambda\in\mathbb{R}$	Note that the vector equation of a line is of the form: $\mathbf{r}=\mathbf{a}+\lambda\mathbf{d}$, $\lambda\in\mathbb{R}$ It is not written as $l=\mathbf{a}+\lambda\mathbf{d}$, $\lambda\in\mathbb{R}$ or equation of line $=\mathbf{a}+\lambda\mathbf{d}$, $\lambda\in\mathbb{R}$
(ii) [3]	$\mathbf{r}.\begin{pmatrix}1\\0\\0\end{pmatrix}=\alpha$ $\mathbf{r}=\begin{pmatrix}20\\-4\\0\end{pmatrix}+\lambda\begin{pmatrix}-10\\2\\h\end{pmatrix}$, $\lambda\in\mathbb{R}$ $\begin{pmatrix}20-10\lambda\\-4+2\lambda\\\lambda h\end{pmatrix}.\begin{pmatrix}1\\0\\0\end{pmatrix}=\alpha$ $20-10\lambda=\alpha\Rightarrow\lambda=\dfrac{20-\alpha}{10}$	"Does not exceed" means that the height is ≤ 35 units. Students who attempted this question by similar triangles must take note that α is a negative value.

	For shadow of the pyramid cast on the screen to not exceed the height of the screen, length of shadow, $\lambda h = \left(\frac{20-\alpha}{10}\right)h \le 35$ $\Rightarrow h \le \frac{350}{20-\alpha}$ since $\alpha < -4$ implies $20-\alpha > 0$	
(iii) [3]	Given that $h = 10$ $\overrightarrow{OB} = \begin{pmatrix}4\\-4\\0\end{pmatrix}$, $\overrightarrow{OV} = \begin{pmatrix}0\\0\\20\end{pmatrix} \Rightarrow \overrightarrow{BV} = \begin{pmatrix}-4\\4\\20\end{pmatrix}$ Length of the shadow cast by edge VB $= \left\lvert \begin{pmatrix}-4\\4\\20\end{pmatrix} \times \begin{pmatrix}1\\0\\0\end{pmatrix} \right\rvert = \left\lvert \begin{pmatrix}0\\20\\-4\end{pmatrix} \right\rvert = \sqrt{416} = 4\sqrt{26}$	Please note that the shadow cast by the edge VB on the screen is <u>not</u> $\lvert\overrightarrow{VB}\rvert$.
(iv) [4]	$\overrightarrow{OC} = \begin{pmatrix}4\\4\\0\end{pmatrix}$, $\overrightarrow{OV} = \begin{pmatrix}0\\0\\20\end{pmatrix}$. $\Rightarrow \overrightarrow{CV} = \begin{pmatrix}-4\\-4\\20\end{pmatrix} = -4\begin{pmatrix}1\\1\\-5\end{pmatrix}$ $\overrightarrow{BV} = \begin{pmatrix}-4\\4\\20\end{pmatrix} = 4\begin{pmatrix}-1\\1\\5\end{pmatrix}$ ~~$\overrightarrow{BV} \times \overrightarrow{CV}$~~ A vector normal to plane VBC is $= \begin{pmatrix}1\\1\\-5\end{pmatrix} \times \begin{pmatrix}-1\\1\\5\end{pmatrix} = \begin{pmatrix}10\\0\\2\end{pmatrix} = 2\begin{pmatrix}5\\0\\1\end{pmatrix}$ Vector equation of the plane VBC is $\mathbf{r}.\begin{pmatrix}5\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\20\end{pmatrix}.\begin{pmatrix}5\\0\\1\end{pmatrix} \Rightarrow \mathbf{r}.\begin{pmatrix}5\\0\\1\end{pmatrix} = 20$ Angle of inclination made by the mirror with the ground	Students must be more careful when computing vectors. There is a lot of computation error for $\overrightarrow{CV}$ and $\overrightarrow{BV}$. This is just a direct application of angle between 2 planes with the normal of the ground (x-y plane) taken to be $\begin{pmatrix}0\\0\\1\end{pmatrix}$. Note that this angle of inclination is **not** the same

	is $\cos^{-1}\left(\frac{\left\lvert\begin{pmatrix}0\\0\\1\end{pmatrix}.\begin{pmatrix}5\\0\\1\end{pmatrix}\right\rvert}{\sqrt{1}\sqrt{25+1}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{26}}\right) = 78.7°$ (correct to 1 d.p.)	as the angle between $\overrightarrow{BV}$ and $\overrightarrow{BA}$ which is equal to 78.9°

RI H2 Mathematics 2017 Prelim Exam Paper 2 Question

1 Referred to the origin O, the points A, B and C have position vectors **a**, **b** and **c** respectively such that

$$\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}, \quad \mathbf{b} = 5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \mathbf{c} = 4\mathbf{i} + \mathbf{j} - 2\mathbf{k}.$$

(i) Given that M is the mid-point of AC, use a vector product to find the exact area of triangle ABM. [4]

(ii) Find the position vector of the point N on the line AB such that $\overrightarrow{MN}$ is perpendicular to $\overrightarrow{AB}$. [4]

2 **(a)** **(i)** Show that $\frac{1}{r-1} - \frac{2}{r} + \frac{1}{r+1} = \frac{2}{r(r-1)(r+1)}$. [1]

(ii) Hence find $\sum_{r=3}^{n} \frac{4}{r(r-1)(r+1)}$.

(There is no need to express your answer as a single algebraic fraction). [4]

(b) Amy and her brother Ben are saving money together for their family trip. In the first week of 2017, Amy saves \$25 and Ben saves \$2. In each subsequent week, Amy saves \$4 more than the amount she saved in the previous week, and Ben saves 22% more than the amount he saved in the previous week.

(i) Which is the first week in which Ben saves more than Amy in that week? [2]

(ii) They need a combined total of \$2400 for the trip. How many complete weeks do Amy and Ben need to save before they can achieve their targeted amount? [2]

3 The function f is defined as follows.

$$\mathrm{f} : x \mapsto \sqrt{3} \sin x + \cos x, \quad x \in \mathbb{R}, \quad 0 < x < \pi.$$

(i) Write $\mathrm{f}(x)$ as $R\sin(x+\alpha)$, where R and α are constants with exact values to be found. [2]

(ii) Sketch the graph of $y = \mathrm{f}(x)$, stating the axial intercepts, and find the range of f. [3]

(iii) Hence, solve $\mathrm{f}(x) \leq 1$ exactly. [2]

The function g is defined as follows:

$$\mathrm{g} : x \mapsto 2\cos\left(x + \frac{\pi}{6}\right), \quad x \in \mathbb{R}, \quad -\frac{\pi}{6} \leq x \leq b.$$

(iv) Write down the largest exact value of b, for g^{-1} to exist. [1]

(v) Taking the value of b found in part **(iv)**, show that the composite function $\mathrm{g}^{-1}\mathrm{f}$ exists and solve $\mathrm{g}^{-1}\mathrm{f}(x) = x$ exactly. [3]

4	The line l_1 has equation $\dfrac{x}{-3}=\dfrac{y}{12}=\dfrac{z-1}{4}$ and the line l_2 has equation $\dfrac{x-1}{-3}=y-4=\dfrac{z-1}{4}$. **(i)** Show that l_1 and l_2 are skew lines. [3] **(ii)** Find a cartesian equation of the plane p which is parallel to l_1 and contains l_2. [3] **(iii)** The point $A(0, a, 1)$ is equidistant from p and l_1. Calculate the possible values of a exactly. [6]
5	For events X and Y, it is given that $\text{P}(X \mid Y)=\dfrac{1}{2}$, $\text{P}(Y \mid X)=\dfrac{2}{3}$ and $\text{P}(X \cup Y)=\dfrac{5}{6}$. Find **(i)** $\text{P}(X)$, [3] **(ii)** $\text{P}(X \cup Y')$. [2]
6	The power consumption of a randomly chosen Effixion laptop has a normal distribution. The salesman at Elf Superstore claims that the average power consumption of an Effixion laptop is 100 watts. The power consumption, w watts, is measured for a random sample of 50 Effixion laptops. The results are summarised as follows. $\sum(w-100)=26 \qquad \sum(w-100)^2=273$ Test whether this data provides evidence at the 3% level of significance, that the salesman has made an understatement. [6] The power consumption of another random sample of 50 Effixion laptops is measured. It is found that the sample variance is 6.25. Using this sample only, find the set of values of $\overline{w}$, correct to 2 decimal places, for which the test would result in the rejection of the null hypothesis in favour of the alternative hypothesis at the 1% level of significance. [4]
7	An unbiased cubical die has the number 1 on one face, the number 2 on two faces and the number 3 on three faces. Adrian invites Benny to play a game. In each round, Benny rolls the die twice. Adrian pays Benny $\$a$ if the total score is 2 and \$3 if the total score is 3. However, if the total score is 4, Benny pays Adrian \$2. No payment is made otherwise. **(i)** Find the probability that Adrian pays Benny at least 5 times in 20 rounds. [4] The random variable X represents Benny's winnings in each round. **(ii)** Given that $a=6$, find the probability distribution of X. Hence, help Benny decide if he should accept Adrian's invitation to play the game. Justify your answer. [5] **(iii)** Determine the value of a for the game to be fair. [1]
8	**(a)** In Country S, each household's monthly income per capita is calculated by taking the gross household income divided by the total number of members in the household. It is assumed

	that this amount for a randomly chosen household consisting of 3 members follows a normal distribution with mean \$2601 and standard deviation \$768. **(i)** The Ministry of Education offers financial aid to students from households consisting of 3 members each and with a household monthly income per capita lower than \$1800. Find the probability that a randomly chosen household with 3 members does not qualify for financial aid. [1] **(ii)** It is found that there is a 50% chance that a randomly chosen household with 3 members has a gross household income between \$5000 and \$$a$, where $a > 5000$. Find the value of a, correct to the nearest dollar. [3] **(b)** Mr Tan is self-employed and his monthly income follows a normal distribution with mean \$6000 and standard deviation \$1000 whereas Mrs Tan works part-time and earns a fixed amount of \$1500 a month. Their family's monthly expenditure follows a normal distribution with mean μ dollars and standard deviation 650 dollars. **(i)** It was found that 10% of the time they spend more than \$5900 in a month. Find the value of μ, correct to the nearest dollar. [2] **(ii)** Mr and Mrs Tan save the remaining amount of their income after deducting their expenditure every month. Find the probability that their monthly savings in August and in September differ by more than \$1000. [4] **(iii)** State an assumption needed for your calculation in part **(b)(ii)**. [1]
9	**(i)** Sketch a scatter diagram that might be expected when x and y are related approximately as given in each of the cases (A) and (B) below. In each case, your diagram should include 6 points, approximately equally spaced with respect to x, and with all x- and y-values positive. The letters $a, b,$ c and d represent constants. (A) $y = a + bx^2$, where a is positive and b is negative, (B) $y = c + d\ln x$, where c is positive and d is negative. [2] The following table shows the Gross Domestic Product (GDP) per capita, \$$x$, and infant mortality rate, y, for a sample of 9 countries. x (\$): 1375, 2502, 10569, 2966, 11539, 2036, 4260, 1433, 7427 y: 115, 69, 18, 65, 17, 83, 44, 112, 27 **(ii)** Draw a scatter diagram for these values, labelling the axes clearly. [2] **(iii)** Calculate the product moment correlation coefficient, and explain why its value does not necessarily mean that a linear model is the best model for the relationship between x and y. [2]

x (\$)	1375	2502	10569	2966	11539	2036	4260	1433	7427
y	115	69	18	65	17	83	44	112	27

	(iv)	State which of the two cases in part **(i)** is more appropriate for modelling the relationship between x and y. Calculate the product moment correlation coefficient and the equation of the appropriate regression line for this case. [3]
	(v)	Use the regression line in part **(iv)** to find an estimate of the infant mortality rate for a country with GDP per capita of $723. Comment on the reliability of your estimate. [3]

10 **(a)** It is given that the probability that 21 randomly chosen people were all born on different days of the year is 0.55631, correct to 5 decimal places.

Find the probability that in a random sample of 22 people, there are at least 2 people with the same date of birth. [3]

[You may assume there are 365 days in a year and the probability that a person is born on any of the 365 days is the same.]

(b) A soccer team consists of 1 goalkeeper, 4 defenders, 4 midfielders and 2 forwards.

Country N has a squad of 3 goalkeepers, 6 defenders, 9 midfielders and 4 forwards.

(i) How many different soccer teams can be formed by country N? [2]

One of the defenders and one of the midfielders in the squad are twin brothers.

(ii) How many different teams can be formed which include at most one of the twin brothers? [3]

The following table shows the dates of birth of the 22 players in the squad of country N:

Jersey Number	Position	Date of birth
1	Goalkeeper	29 October
2	Defender	3 May
3	Defender	17 July
4	Defender	15 May
5	Defender	14 December
6	Midfielder	12 October
7	Midfielder	15 May

Jersey Number	Position	Date of birth
12	Midfielder	15 August
13	Defender	11 July
14	Midfielder	29 March
15	Defender	22 October
16	Midfielder	13 March
17	Forward	29 November
18	Goalkeeper	20 December

8	Forward	10 May	19	Midfielder	5 February
9	Forward	1 July	20	Midfielder	1 March
10	Midfielder	1 April	21	Forward	27 March
11	Midfielder	29 October	22	Goalkeeper	31 October

(iii) Find the probability that the team formed by country N contains no players with the same date of birth. [4]

RAFFLES INSTITUTION
H2 Mathematics (9758)
2017 Year 6

2017 H2 Math 9758 Preliminary Examination Paper 2 : Suggested Solutions

Qn 1	Solutions	Comments
(i) [4]	$\overrightarrow{OA}=\begin{pmatrix}2\\3\\-1\end{pmatrix}, \overrightarrow{OB}=\begin{pmatrix}5\\-2\\3\end{pmatrix}, \overrightarrow{OC}=\begin{pmatrix}4\\1\\-2\end{pmatrix}$ Since M is the mid-point of AC, $\overrightarrow{OM}=\dfrac{\begin{pmatrix}2\\3\\-1\end{pmatrix}+\begin{pmatrix}4\\1\\-2\end{pmatrix}}{2}=\frac{1}{2}\begin{pmatrix}6\\4\\-3\end{pmatrix}$ $\overrightarrow{AB}=\begin{pmatrix}5\\-2\\3\end{pmatrix}-\begin{pmatrix}2\\3\\-1\end{pmatrix}=\begin{pmatrix}3\\-5\\4\end{pmatrix}$ $\overrightarrow{AM}=\frac{1}{2}\begin{pmatrix}6\\4\\-3\end{pmatrix}-\begin{pmatrix}2\\3\\-1\end{pmatrix}=\begin{pmatrix}1\\-1\\-\frac{1}{2}\end{pmatrix}$ Area of ΔABM $=\frac{1}{2}\left\|\overrightarrow{AB}\times\overrightarrow{AM}\right\|$ $=\frac{1}{2}\left\|\begin{pmatrix}3\\-5\\4\end{pmatrix}\times\begin{pmatrix}1\\-1\\-\frac{1}{2}\end{pmatrix}\right\|$ $=\frac{1}{4}\left\|\begin{pmatrix}3\\-5\\4\end{pmatrix}\times\begin{pmatrix}2\\-2\\-1\end{pmatrix}\right\|=\frac{1}{4}\left\|\begin{pmatrix}13\\11\\4\end{pmatrix}\right\|=\frac{\sqrt{306}}{4}=\frac{3\sqrt{34}}{4}$	Many candidates did this question correctly. Some candidates wrongly expressed $\overrightarrow{OM}$ as $\frac{1}{2}\overrightarrow{AC}$. A small minority did not realize that the question asked for "exact area", and left their answer as 4.37, rounded to 3 sf, which is not acceptable. For those who attempted to express the final answer as exact, do note that values like $\frac{\sqrt{76.5}}{2}$ can be simplified to $\frac{3\sqrt{34}}{4}$ by writing 76.5 as a fraction and rationalizing the denominator.

	Or: $\overrightarrow{AB}=\begin{pmatrix}5\\-2\\3\end{pmatrix}-\begin{pmatrix}2\\3\\-1\end{pmatrix}=\begin{pmatrix}3\\-5\\4\end{pmatrix}, \overrightarrow{AC}=\begin{pmatrix}4\\1\\-2\end{pmatrix}-\begin{pmatrix}2\\3\\-1\end{pmatrix}=\begin{pmatrix}2\\-2\\-1\end{pmatrix}$ Area of ΔABC $=\frac{1}{2}\left\|\overrightarrow{AB}\times\overrightarrow{AC}\right\|$ $=\frac{1}{2}\left\|\begin{pmatrix}3\\-5\\4\end{pmatrix}\times\begin{pmatrix}2\\-2\\-1\end{pmatrix}\right\|$ $=\frac{1}{2}\left\|\begin{pmatrix}13\\11\\4\end{pmatrix}\right\|=\frac{\sqrt{306}}{2}$ Area of $\Delta ABM=\frac{1}{2}$Area of $\Delta ABC=\frac{\sqrt{306}}{4}=\frac{3\sqrt{34}}{4}$	
(ii) [4]	l_{AB}: $\mathbf{r}=\begin{pmatrix}2\\3\\-1\end{pmatrix}+\lambda\begin{pmatrix}3\\-5\\4\end{pmatrix}, \lambda\in\mathbb{R}$ Since point N is on the line AB, $\overrightarrow{ON}=\begin{pmatrix}2\\3\\-1\end{pmatrix}+\lambda\begin{pmatrix}3\\-5\\4\end{pmatrix}$ for some λ $\overrightarrow{MN}=\begin{pmatrix}3\lambda-1\\1-5\lambda\\\frac{1}{2}+4\lambda\end{pmatrix}$ For $\overrightarrow{MN}$ to be perpendicular to $\overrightarrow{AB}$, $\overrightarrow{MN}.\overrightarrow{AB}=0$ $\begin{pmatrix}3\lambda-1\\1-5\lambda\\\frac{1}{2}+4\lambda\end{pmatrix}.\begin{pmatrix}3\\-5\\4\end{pmatrix}=0$ $\lambda=\frac{3}{25}$ $\overrightarrow{ON}=\begin{pmatrix}2\\3\\-1\end{pmatrix}+\frac{3}{25}\begin{pmatrix}3\\-5\\4\end{pmatrix}=\frac{1}{25}\begin{pmatrix}59\\60\\-13\end{pmatrix}$	Most candidates demonstrated understanding of the concept of finding $\overrightarrow{MN}$ and using it to find λ. A significant number of the candidates who got the wrong answer made careless mistakes in the following arithmetic operations i.e. $\overrightarrow{MN}=\overrightarrow{ON}-\overrightarrow{OM}$ $=\begin{pmatrix}2+3\lambda\\3-5\lambda\\-1+4\lambda\end{pmatrix}-\begin{pmatrix}3\\2\\-1.5\end{pmatrix}$ $=\begin{pmatrix}\vdots\\ \\2.5+4\lambda\end{pmatrix}$ and $\begin{pmatrix}3\lambda-1\\1-5\lambda\\\frac{1}{2}+4\lambda\end{pmatrix}.\begin{pmatrix}3\\-5\\4\end{pmatrix}=0$ $\cdots\cdots\cdots+2+8\lambda=0$

Qn 2	Solution	Comments
(a)(i) [1]	$\frac{1}{r-1}-\frac{2}{r}+\frac{1}{r+1}=\frac{r(r+1)-2(r-1)(r+1)+r(r-1)}{r(r-1)(r+1)}$ $=\frac{r^2+r-2(r^2-1)+r^2-r}{r(r-1)(r+1)}$ $=\frac{2}{r(r-1)(r+1)}$	Almost all students managed to show this. Just a note that it might be easier to start from LHS and combine the fraction to arrive at RHS instead of trying to break up RHS into partial fractions.
(a)(ii) [4]	$\sum_{r=3}^{n}\frac{4}{r(r-1)(r+1)}=2\sum_{r=3}^{n}\frac{2}{r(r-1)(r+1)}$ $=2\sum_{r=3}^{n}\left(\frac{1}{r-1}-\frac{2}{r}+\frac{1}{r+1}\right)$ $=2\Big[\frac{1}{2}-\frac{2}{3}+\frac{1}{4}$ $+\frac{1}{3}-\frac{2}{4}+\frac{1}{5}$ $+\frac{1}{4}-\frac{2}{5}+\frac{1}{6}$ $+\frac{1}{5}-\frac{2}{6}+\frac{1}{7}$ $+\ldots$ $+\frac{1}{n-3}-\frac{2}{n-2}+\frac{1}{n-1}$ $+\frac{1}{n-2}-\frac{2}{n-1}+\frac{1}{n}$ $+\frac{1}{n-1}-\frac{2}{n}+\frac{1}{n+1}\Big]$ $=2\left(\frac{1}{6}-\frac{1}{n}+\frac{1}{n+1}\right)=\frac{1}{3}-\frac{2}{n}+\frac{2}{n+1}$	Most students were able to recognize that this involved MOD. However, some common mistakes were still prevalent: 1. Forgot about the factor 2, i.e. $\sum_{r=3}^{n}\frac{4}{r(r-1)(r+1)}$ $=\sum_{r=3}^{n}\left(\frac{1}{r-1}-\frac{2}{r}+\frac{1}{r+1}\right)$ 2. Did not write down the correct leftover terms after the cancellations, i.e either missed out $\frac{1}{3}$ or $\frac{1}{n}$ in the final expression. 3. Question mentioned that "There is no need to express answer as a single algebraic fraction" but that does not mean that liked terms need not be simplified. 4. Some students tried to split the sum up without realizing that r **cannot** start

		from 1, e.g. $\sum_{r=3}^{n}\left(\frac{1}{r-1}-\frac{2}{r}+\frac{1}{r+1}\right)$ $=\sum_{r=1}^{n}\left(\frac{1}{r-1}-\frac{2}{r}+\frac{1}{r+1}\right)$ $-\sum_{r=1}^{2}\left(\frac{1}{r-1}-\frac{2}{r}+\frac{1}{r+1}\right)$
(b)(i) [2]	Amount Amy saves in nth week $=25+(n-1)(4)=21+4n$ Amount Ben saves in nth week $=ar^{n-1}=2(1.22)^{n-1}$ When Ben saves more than Amy, $2(1.22)^{n-1}>21+4n$ From GC, in the 20th week, Amy saves \$101, Ben saves \$87.47 in the 21st week, Amy saves \$105, Ben saves \$106.72 Hence, Ben first saves more than Amy in the 21st week. Or: $2(1.22)^{n-1}>21+4n \Rightarrow 2(1.22)^{n-1}-21-4n>0$ When $n=20$, $2(1.22)^{n-1}-21-4n=-13.5<0$ When $n=21$, $2(1.22)^{n-1}-21-4n=1.72>0$	Students need to recognize the use of GC for these 2 parts based on the number of marks and also the nature of the inequality which is not easy to solve algebraically. There were a number of students who could not progress after forming the inequality. Some common mistakes made for both (i) and (ii): 1. Use the sum of AP and GP formulae for both parts. 2. Use only the nth term formulae for both parts. 3. Forgot either the sum of AP/GP formula or the nth term formula for AP/GP. 4. Take $r = 0.22$.
(b)(ii) [2] use GC!	Total amount in Amy's account after nth week, $=\frac{n}{2}(2a+(n-1)d)=\frac{n}{2}(50+(n-1)(4))=\frac{n}{2}(46+4n)$ Total amount in Ben's account after nth week, $=\frac{a(r^n-1)}{r-1}=\frac{2(1.22^n-1)}{1.22-1}$ For their total saving to exceed \$2400, $\frac{n}{2}(46+4n)+\frac{2(1.22^n-1)}{1.22-1}>2400$ From GC, in the 22nd week, total savings= \$2186.89 < \$2400	

in the 23rd week, total savings= \$2458.72 > \$2400

$\therefore$ least $n = 23$.

Hence, their total saving will exceed \$2400 after 23 complete weeks.

Qn 3 | **Comments**

(i) [2]

$f(x) = R\sin(x+\alpha) = R\sin x\cos\alpha + R\cos x\sin\alpha$

compare with $f(x) = \sqrt{3}\sin x + \cos x$

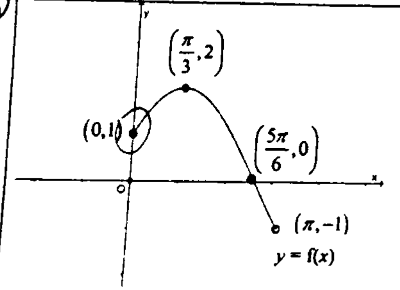
$\Rightarrow R\cos\alpha = \sqrt{3},\ R\sin\alpha = 1$

$\Rightarrow R = \sqrt{1^2 + \sqrt{3}^2} = 2,\quad \alpha = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$

$f(x) = 2\sin\left(x + \frac{\pi}{6}\right)$

Comments: Most students are able to do this part correctly.

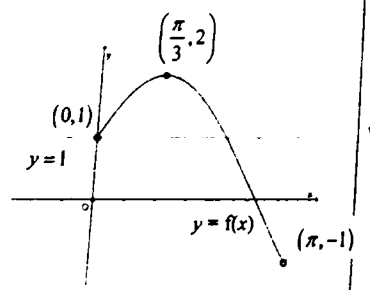
(ii) [3]



The range of f is $(-1, 2]$.

Comments: Students have to learn to read the question carefully. Many did not label the intercepts. Students also need to consider the domain of f and indicate the coordinates of the end points on the sketch. Quite a handful miss out the open circle on both the end points.
A few did not write down the range although they got the graph correct, which is very wasteful.

(iii) [2]



Comments: The key word here is **"exactly"**. This means that students have to show algebraic working to get the correct value of $\frac{2\pi}{3}$. Answers without working will not get any credit.

$f(x) = 2\sin(x + \frac{\pi}{6}) = 1$

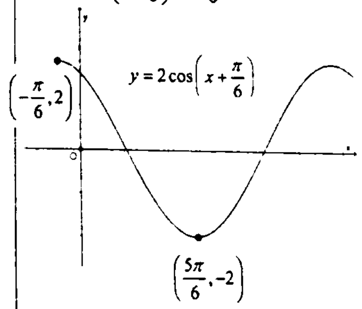
$\Rightarrow x + \frac{\pi}{6} = \frac{\pi}{6}$ or $\pi - \frac{\pi}{6}$

$\Rightarrow x = 0$ or $\frac{2\pi}{3}$

The set of values of x is $\left[\frac{2\pi}{3}, \pi\right)$

(iv) [1]

$g(x) = 2\cos\left(x + \frac{\pi}{6}\right),\quad -\frac{\pi}{6} \le x \le b.$



For g^{-1} to exist, g has to be a 1-1 function.

The largest exact value of b, is $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

(v) [3]

The domain of g^{-1} = the range of g = $[-2, 2]$.

Range of f = $(-1, 2] \subseteq$ Domain of $g^{-1} = [-2, 2]$, therefore $g^{-1}f$ exists.

$g^{-1}f(x) = x,\quad 0 < x < \pi$

$g(x) = f(x)$

$2\cos(x + \frac{\pi}{6}) = 2\sin(x + \frac{\pi}{6})$

$\tan(x + \frac{\pi}{6}) = 1$

$x + \frac{\pi}{6} = \frac{\pi}{4}$

$x = \frac{\pi}{12}$

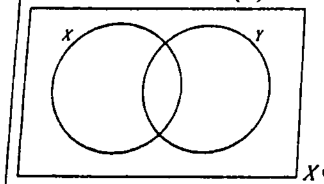
Comments: Many students failed to see that the easiest way to solve this is to solve $g(x) = f(x)$. Many proceeded to find the function $g^{-1}f(x)$.

	(Note: $0 < x \le \frac{5\pi}{6}$ considering domain of f and g)	Again the key word is "exactly". Marks will not be awarded for answers without algebraic working.

Qn 4		Comments
(i) [3]	$l_1: \mathbf{r} = \begin{pmatrix}0\\0\\1\end{pmatrix} + \lambda\begin{pmatrix}-3\\12\\4\end{pmatrix}, \lambda \in \mathbb{R}$ $l_2: \mathbf{r} = \begin{pmatrix}1\\4\\1\end{pmatrix} + \mu\begin{pmatrix}-3\\1\\4\end{pmatrix}, \mu \in \mathbb{R}$ Since $\begin{pmatrix}-3\\12\\4\end{pmatrix}$ and $\begin{pmatrix}-3\\1\\4\end{pmatrix}$ are not parallel, l_1 and l_2 are not parallel. If the two lines intersect, there will be a unique value of λ and μ for the system of equations $\begin{pmatrix}0\\0\\1\end{pmatrix} + \lambda\begin{pmatrix}-3\\12\\4\end{pmatrix} = \begin{pmatrix}1\\4\\1\end{pmatrix} + \mu\begin{pmatrix}-3\\1\\4\end{pmatrix}$ $-3\lambda + 3\mu = 1 \quad (1)$ $12\lambda - \mu = 4 \quad (2)$ $4\lambda - 4\mu = 0 \quad (3)$ Using GC, no solution of λ and μ exist. Hence, the lines do not intersect. Hence, l_1 and l_2 are skew lines.	Common mistakes: - Obtained $\begin{pmatrix}-3\\0\\4\end{pmatrix}$ as the direction vector for l_2 instead of $\begin{pmatrix}-3\\1\\4\end{pmatrix}$. - Concluded that l_1 and l_2 are skew lines without showing that they are not parallel.
(ii) [3]	A normal to $p = \begin{pmatrix}-3\\12\\4\end{pmatrix} \times \begin{pmatrix}-3\\1\\4\end{pmatrix} = \begin{pmatrix}44\\0\\33\end{pmatrix} = 11\begin{pmatrix}4\\0\\3\end{pmatrix}$	Most students did well for this part, although there were a few who did not give the final equation in cartesian form.

	Since (1, 4, 1) lies on l_2 which is on p, $\begin{pmatrix}1\\4\\1\end{pmatrix} \cdot \begin{pmatrix}4\\0\\3\end{pmatrix} = 7$ Hence a cartesian equation for p is $4x + 3z = 7$.	
(iii) [6]	(0, 0, 1) is a point on l_1. $\begin{pmatrix}0\\a\\1\end{pmatrix} - \begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\a\\0\end{pmatrix}$ Distance from A to l $= \dfrac{\left\| \begin{pmatrix}0\\a\\0\end{pmatrix} \times \begin{pmatrix}-3\\12\\4\end{pmatrix} \right\|}{\left\| \begin{pmatrix}-3\\12\\4\end{pmatrix} \right\|} = \frac{1}{13}\left\| \begin{pmatrix}4a\\0\\3a\end{pmatrix} \right\| = \frac{1}{13}\|a\| \left\| \begin{pmatrix}4\\0\\3\end{pmatrix} \right\| = \frac{5}{13}\|a\|$ (1, 4, 1) is a point on p. $\begin{pmatrix}0\\a\\1\end{pmatrix} - \begin{pmatrix}1\\4\\1\end{pmatrix} = \begin{pmatrix}-1\\a-4\\0\end{pmatrix}$ Distance from A to p $= \dfrac{\left\| \begin{pmatrix}-1\\a-4\\0\end{pmatrix} \cdot \begin{pmatrix}4\\0\\3\end{pmatrix} \right\|}{\left\| \begin{pmatrix}4\\0\\3\end{pmatrix} \right\|} = \frac{1}{5}\|-4\| = \frac{4}{5}$	Note that this method is much faster than finding foot of perpendicular, F from A to l and then taking the length of $\overline{AF}$. Instead of factorising a out from $\begin{pmatrix}4a\\0\\3a\end{pmatrix}$, most students simplify the expression using definition, i.e. $\left\| \begin{pmatrix}4a\\0\\3a\end{pmatrix} \right\| = \sqrt{(4a)^2 + 0^2 + (3a)^2} = \sqrt{25a^2}$, which many mistakenly simplify as $5a$ instead of $5\|a\|$, hence obtaining only one value of a as final answer in the last step. Similarly, this method is much faster than finding foot of perpendicular, N from A to p and then taking the length of $\overline{AN}$ Many of those who attempted to find $\overline{OF}$ and $\overline{ON}$ committed careless mistakes and lost marks for not getting the correct position vectors and distances.

	Since the point A is equidistant to p and l_1, $\frac{5}{13}\lvert a\rvert=\frac{4}{5}$ $a=\pm\frac{52}{25}$	

Qn 5		Comments
[3]	$P(X\mid Y)=\frac{1}{2}$ $\Rightarrow \frac{P(X\cap Y)}{P(Y)}=\frac{1}{2}$ $\Rightarrow P(X\cap Y)=\frac{1}{2}P(Y)$ $P(Y\mid X)=\frac{2}{3}$ $\Rightarrow \frac{P(X\cap Y)}{P(X)}=\frac{2}{3}$ $\Rightarrow P(X\cap Y)=\frac{2}{3}P(X)$ $P(X\cap Y)=\frac{1}{2}P(Y)=\frac{2}{3}P(X)$ $P(X\cup Y)=\frac{5}{6}$ $\Rightarrow P(X)+P(Y)-P(X\cap Y)=\frac{5}{6}$ $\Rightarrow P(X)+\frac{4}{3}P(X)-\frac{2}{3}P(X)=\frac{5}{6}$ $\Rightarrow P(X)=\frac{1}{2}$	Most students were able to do this part correctly. The most common method used was to apply standard results to obtain equations connecting $P(X)$, $P(Y)$ and $P(X\cap Y)$. Unfortunately, some wrong formula were seen, for example: $P(X\mid Y)=\frac{P(X\cap Y)}{P(X)}$ instead of $\frac{P(X\cap Y)}{P(Y)}$, $P(Y\mid X)=\frac{P(Y\cap X)}{P(Y)}$ instead of $\frac{P(Y\cap X)}{P(X)}$, and most frequently, $P(X\cup Y)=P(X)+P(Y)+P(X\cap Y)$ instead of $P(X)+P(Y)-P(X\cap Y)$. Interestingly, there were a few students who used their GC to solve the three equations, obtaining $P(X)=\frac{1}{2}$, $P(Y)=\frac{2}{3}$ and $P(X\cap Y)=\frac{1}{3}$ all at one go.
(ii) [2]	$P(X\cap Y)=\frac{2}{3}P(X)=\frac{2}{3}\left(\frac{1}{2}\right)=\frac{1}{3}$  $X\cup Y'$ $P(X\cup Y')=1-P(Y)+P(X\cap Y)$ $=1-\frac{4}{3}\left(\frac{1}{2}\right)+\frac{1}{3}$ $=\frac{2}{3}$	Significant fewer number of students were able to handle this part well. The most common problem was the failure to understand $P(X\cup Y')$. A large number of students tried to simplify it through the result $P(X\cup Y')=P(X)+P(Y')-P(X\cap Y')$, but often end up with a complicated sum difference which eventually worked out to be $P(Y')$ instead. Those students who drew a Venn diagram were generally quick and successful in identifying the correct region and thus were able to obtain the correct answer rather effortlessly.

Qn 6		Comments
[6]	Let $X=W-100$. Then we have $\sum x=26$, $\sum x^2=273$ $\bar{w}=\frac{1}{50}\sum x+100=\frac{26}{50}+100=100.52$ $s_w^2=s_x^2=\frac{1}{n-1}\left[\sum x^2-\frac{\left(\sum x\right)^2}{n}\right]$ $=\frac{1}{49}\left(273-\frac{26^2}{50}\right)$ $=5.29551$ To test $H_0:\mu=100$ vs $H_1:\mu>100$ Perform a 1-tail test at 3% level of significance. Under H_0, $\overline{W}\sim N\left(\mu_0,\frac{s^2}{n}\right)$ **approximately** where $\mu_0=100$ and $n=50$ Using a z-test, p-value $=0.0550$ (3 s.f.) Since p-value $=0.0550>0.03$, we **do not reject** H_0 and conclude that there is **insufficient** evidence, at 3% significance level, that the salesman made an understatement on the average power consumption of the Effixion laptops.	Define your random variable if you are not using W. Do not confuse μ with $\bar{w}$. If we know the population mean, there would be no need to perform hypothesis testing. W is normally distributed, so do not quote CLT. However, there is still an approximation due to use of the unbiased **estimate** of population variance.
[4]	To test $H_0:\mu=100$ vs $H_1:\mu>100$ Perform a 1-tail test at 1% level of significance. Sample variance $=6.25$ $s^2=\frac{n}{n-1}[\text{sample variance}]$	Sample variance is **neither** population variance, **nor** its unbiased estimate.

$= \frac{50}{49}(6.25) = 6.37755$

In order to **reject** H_0 at 1% level of significance, we need

$P(\bar{W} > \bar{w}) \le 0.01$ where $\bar{W} \sim N\left(100, \frac{6.37755}{50}\right)$ approximately

$\Rightarrow P(\bar{W} < \bar{w}) \ge 0.99$

$\Rightarrow \bar{w} \ge 100.8308$

Or $z - value \ge z_{0.99}$

$\frac{\bar{w} - 100}{\sqrt{\frac{6.37755}{50}}} \ge 2.3263$

$\bar{w} \ge 100.8308$ (4 d.p.)

Set of values of $\bar{w}$ is $\{\bar{w} \in \mathbb{R} : \bar{w} > 100.83\}$ or $\{\bar{w} \in \mathbb{R} : \bar{w} \ge 100.84\}$

Note the alternative hypothesis is right-tailed.

Standardisation is not necessary, as parameters of $\bar{W}$ are assumed to be known in the test.

Due to rounding off, 100.83 is not within the range, while 100.84 is.

Qn 7

Comments

(i) [4]

Die shows	1	2	3
Probability	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$

P(Adrian pays Benny in a round)
= P(total score is 2) + P(total score is 3)

$= \frac{1}{6}\times\frac{1}{6} + \left(\frac{1}{6}\times\frac{1}{3}\right)2$

$= \frac{5}{36}$

Let Y be the number of rounds, out of 20, that Adrian pays Benny.

$Y \sim B\left(20, \frac{5}{36}\right)$

$P(Y \ge 5)$

$= 1 - P(Y \le 4)$

$= 0.134$ (3 s.f.)

Comments:

This is NOT the usual 6 sided die. Using a table of this form will help to determine the total score of 2 rolls easily.

For P(total score is 3), there are 2 cases : 1st roll 1 & 2nd roll 2 or 1st roll 2 & 2nd roll 1.

Define the random variable Y <u>properly</u> and state its distribution.

Be careful when taking the complement as the GC can only compute $P(Y \le k)$ and not $P(Y < k)$ for Binomial Distributions.

(ii) [5]

P(total score is 4) $= \left(\frac{1}{6}\times\frac{1}{2}\right)2 + \frac{1}{3}\times\frac{1}{3} = \frac{5}{18}$

<u>Method 1 :</u>

P(total score is 5 or 6) $= 1 - \left(\frac{1}{36} + \frac{1}{9} + \frac{5}{18}\right) = \frac{7}{12}$

<u>Method 2 :</u>

P(total score is 5 or 6) $= \left(\frac{1}{3}\times\frac{1}{2}\right)2 + \frac{1}{2}\times\frac{1}{2} = \frac{7}{12}$

Given : X represents Benny's winnings in each round

x	6	3	-2	0
$P(X = x)$	$\frac{1}{36}$	$\frac{1}{9}$	$\frac{5}{18}$	$\frac{7}{12}$

$E(X) = -\frac{1}{18}$

Since $E(X) < 0$, Benny is expected to lose in the long run. Thus, Benny should not accept Adrian's invitation to play the game.

Comments:

Method 1 is faster, but it does not "help to check" that you are on the right track.
Use Method 2 (unless finding this probability is way too long/tedious) to help you check that $\sum P(X = x) = 1$.

Many did not realize that x can be zero. Do check that $\sum P(X = x) = 1$. If the probabilities do not add up, check the individual probabilities and also re-evaluate if there were missing x values.

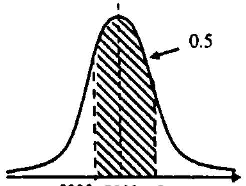
$E(X)$ is the long-run average winnings of Benny in one round. Thus, $E(X) < 0$ does not imply that Benny is expected to lose every round.

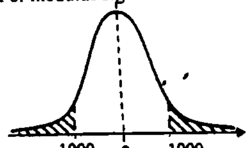
(iii) [1]

For the game to be fair, $E(X) = 0$.

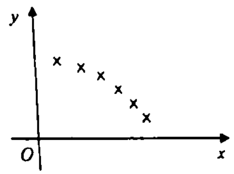
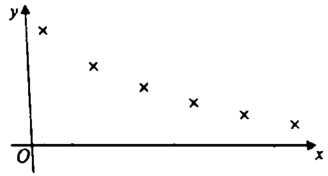
$\frac{a}{36} + \frac{1}{3} - \frac{10}{18} = 0$

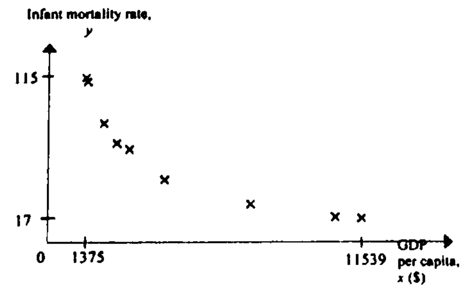
$a = 8$

Qn 8		Comments
(ai) [1]	Let X be the random variable denoting the household income per capita in dollars of a randomly chosen family in Country S. Then $X \sim N(2601,\ 768^2)$. $P(X>1800) = 0.852$ (3s.f.)	A significant number of students misread the question and found the probability to be less than 1800 dollars instead.
(aii) [3]	Let Y be the random variable denoting the gross income in dollars of a randomly chosen family with 3 family members. $Y = 3X \sim N(3\times2601, 9\times768^2)$ $\Rightarrow Y \sim N(7803, 9\times768^2)$ $P(5000<Y<a)=0.5$ $\Rightarrow P(Y<a)-P(Y<5000)=0.5$ $\Rightarrow P(Y<a)=0.5+P(Y<5000)=0.61188$ $\Rightarrow P(Y<a)=0.61188$ $\Rightarrow a=8458$ (to nearest dollars) Alternative: $P(5000<Y<a)=0.5$ $P\left(\frac{5000}{3}<X<\frac{a}{3}\right)=0.5$ since $Y=3X$ $\Rightarrow P\left(X<\frac{a}{3}\right)-P\left(X<\frac{5000}{3}\right)=0.5$ $\Rightarrow P\left(X<\frac{a}{3}\right)=0.5+P\left(X<\frac{5000}{3}\right)=0.61188$ $\Rightarrow \frac{a}{3}=2819.28$ (2dp) $\Rightarrow a=8458$ (to nearest dollar)	Many made the mistake of $Y=X_1+X_2+X_3$, which is the sum of the income per capita of three randomly chosen households. The correct relationship can be derived from the first line in the question as $X=\frac{Y}{3}$. A number of students assumed symmetry which is not true. Many did not get the correct relationship of the probabilities. A sketch of normal distribution curve may help. 
(bi) [2]	Let V be the random variable denoting the family's monthly expenditure in dollars. Then $V \sim N(\mu,\ 650^2)$ $P(V>5900)=0.1$ $\Rightarrow P(V<5900)=0.9$ $\Rightarrow P\left(Z<\frac{5900-\mu}{650}\right)=0.9$ From GC: $P(Z<1.28155)=0.9$ $\Rightarrow \frac{5900-\mu}{650}=1.28155$ $\Rightarrow \mu=5067$ (to the nearest dollar)	Common mistakes: - Some did not find the probability on the left tail. Note: for invNorm on TI 84 Plus C SE, the area is shaded from the left, i.e. lower tail. - Some did the standardisation wrongly as $\frac{\mu-5900}{650}$ Those who arrived at the answer 5067 with both of the above mistakes were not awarded full credits.

(bii) [4]	Let W be the random variable denoting the family's monthly saving in dollars. $W=T+1500-V$, where T denotes Mr Tan's monthly income. Then $W \sim N(7500-5067, 1000^2+0+650^2)$ i.e. $W \sim N(2433, 1422500)$ and $W_1-W_2 \sim N(0, 2845000)$ $\Rightarrow P(\lvert W_1-W_2\rvert>1000)$ $=2P(W_1-W_2<-1000)$ $=0.553$ (3sf)	Some did not find the distribution of the DIFFERENCE between 2 months. For modulus sign: 
(biii) [1]	It is assumed that Mr Tan's income and the family's expenditure in a particular month are independent. Alternatively, Mr Tan's income and the family's expenditure in a month are independent of what he earned and how much the family spent in another month.	We need to assume INDEPENDENCE to be able to add and subtract normal distributions to obtain new normal distributions.

Qn 9		Comments

(i)
[2]
	For (A): $y=a+bx^2$, with $a>0$ and $b<0$.
	
	
	For (B): $y=c+d\ln x$, with $c>0$ and $d<0$.
	
	
		A handful of students plotted points on the axes though the questions asked for positive x and y values.
		
		A number of students also plotted more than 6 data points.
		
		Students should take note that the data points should not lie close to a straight line to clearly illustrate the relationship between x and y.

(ii) [2]		Students should ensure that the scatter plot is sketched to scale. Many scatter plots were out of proportion.
(iii) [2]	From GC, the product moment correlation coefficient is −0.898 (3s.f.). Since −0.898 is close to −1, it suggests a strong negative linear correlation between X and Y. However, it can be observed from the scatter plot that the values of y are decreasing at a decreasing rate with increasing values of x, which will not be the case if the data follows a linear model (the decrease in y should be approximately constant for a linear model).	Many students fail to elaborate on the implication of the value of r in relation to the linear relationship between the variables. They also did not mention the behavior of y as x increases. Many simply mentioned that the scatter plot does not exhibit a linear relationship.
(iv) [3]	(B) is the appropriate model. Using model (B), the product moment correlation coefficient is −0.978 (3s.f.). From GC, $y = 430.30 - 45.010(\ln x)$. $\therefore y = 430 - 45.0\ln x$ (to 3s.f.)	Most students did well in this part of the question, except for those who had made errors in their data entry. Students are reminded to leave their final answers in 3 sf (unless specified otherwise). Thus, intermediate working should be at least 5 sf.
(v) [3]	At a GDP per capita of $723, the infant mortality rate is estimated to be $y = 430.30 - 45.010(\ln 723) = 133.98\ (5\text{s.f.}) = 134\ (3\text{s.f.})$ Since $x = 723$ is outside the range of the data values, the estimation is not reliable.	Many students used 3 sf answer found in (iv) to obtain the estimated value of y. This affects their accuracy of answers.

Qn 10		Comments
(a) [3]	Complement Method: Probability = 1 − P(22 people were all born on different days) $= 1 - \text{P(21 people were all born on different days)} \times \frac{365-21}{365}$ $\approx 1 - 0.55631 \times \frac{344}{365}$ = 0.476 (3sf) Alternative method: P(at least 2 with same date of birth in 21 people) + P(21 people all born on different days and 22nd person shares same date of birth with someone else) $= (1 - 0.55631) + (0.55631) \times \frac{21}{365}$	It is inappropriate to describe the distribution of the number of people who shares the same date of birth as someone else in the group as following the Binomial distribution. There may be more than one common date of birth!
(bi) [2]	Number of ways = $^3C_1 \times {}^6C_4 \times {}^9C_4 \times {}^4C_2 = 34020$	
(ii) [3]	Complement Method: Number of ways = n(teams formed without restriction) − n(teams which include both twins) $= 34020 - {}^3C_1 \times {}^5C_3 \times {}^8C_3 \times {}^4C_2$ = 34020 − 10080 = 23940 Alternative method: Number of ways = n(teams which include the twin defender and not the twin midfielder) + n(teams which include the twin midfielder and not the twin defender) + n(teams which do not include both twins) $= {}^3C_1 \times {}^5C_4 \times {}^8C_4 \times {}^4C_2 + {}^3C_1 \times {}^5C_3 \times {}^8C_4 \times {}^4C_2 + {}^3C_1 \times {}^5C_4 \times {}^8C_3 \times {}^4C_2$ = 6300 + 12600 + 5040	1) *Read the qn carefully!* Qn asks for number of teams with **at most** one twin brother, *not both twin brothers* and *not exactly one twin brother*. 2) Complement method is more efficient since it only involves one case.
(iii) [4]	Let A denote the event that player 1 and player 11 are both in the team. Let B denote the event that player 4 and player 7 are both in the team. $n(A) = {}^6C_4 \times {}^8C_3 \times {}^4C_2 = 5040$ $n(B) = {}^3C_1 \times {}^5C_3 \times {}^8C_3 \times {}^4C_2 = 10080$ $n(A \cap B) = {}^5C_3 \times {}^7C_2 \times {}^4C_2 = 1260$ $\therefore n(A \cup B) = 5040 + 10080 - 1260 = 13860$ Hence required probability $= 1 - \frac{13860}{34020} = \frac{16}{27}$	1) Qn asks for probability, not number of ways. 2) There are many different methods available for this part, but no matter which one you apply, do always check whether your cases are mutually exclusive (ie whether there is "double" counting). There is a need to account for the duplication.