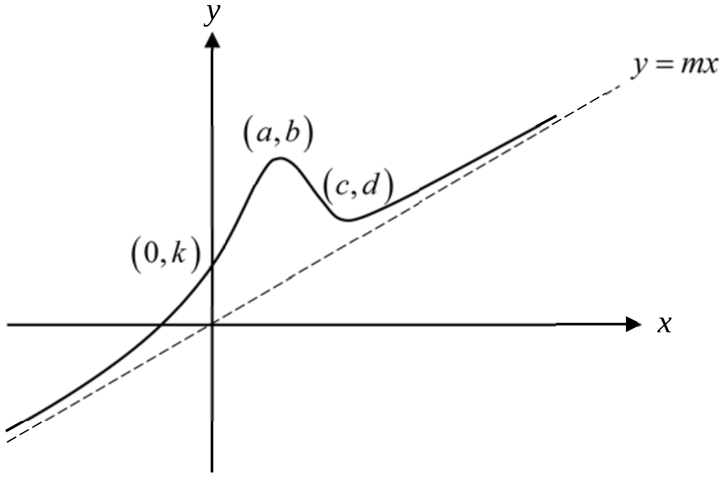
1 The diagram below shows the graph of $y = \mathrm{f}(x)$. The curve has a maximum point at (a, b), a minimum point at (c, d) and cuts the y-axis at $(0, k)$. The equation of the asymptote is $y = mx$, where m is a positive constant.



Sketch the graph of $y = \mathrm{f}'(x)$, giving equation(s) of any asymptotes and coordinate(s) of any intercepts, if it is possible to do so. [2]

2 A drone is programmed to fly from a point *O* at ground level eastwards to a point *C* at the top of a building. The point *C* is 0.1 km vertically above ground level and 0.6 km horizontally from *O*. The drone passes through two checkpoints *A* and *B* before reaching *C*. The horizontal distances and vertical heights of *A* and *B* are shown below.

Point	Horizontal Distance from *O* (km)	Vertical Height (km)
A	0.15	0.1
B	0.3	0.125

It is given that the flight path of the drone is cubic in nature. Taking *O* to be the origin and *O*, *A*, *B* and *C* all lie on the same vertical plane, find the cartesian equation of the flight path. [3]

The drone manufacturer is highly confident of the ability of the drone to keep to its flight path despite external factors such as wind, temperature or humidity. It claims that

> "The drone on this particular programmed flight path will pass through the point at a vertical height of 0.1125 km and a horizontal distance of 0.45 km eastwards from *O*."

By considering the flight path of the drone, comment on the accuracy of the manufacturer's claim. [1]

[Turn over

3 **(a)** A point Q has position vector $\mathbf{q}$ and a line l has equation $\mathbf{r}=\mathbf{a}+\lambda\mathbf{d}$, $\lambda\in$ ¡ . A vector $\mathbf{p}$, where $|\mathbf{p}|=1$, is in the direction of $\mathbf{d}$. Give a geometrical meaning of $|\mathbf{p}\times(\mathbf{a}-\mathbf{q})|$. [1]

(b) The vectors $\mathbf{a}$ and $\mathbf{b}$ form two adjacent sides of a triangle *OAB*, where $\mathbf{a}=\overrightarrow{OA}$ and $\mathbf{b}=\overrightarrow{OB}$. The angle between $\mathbf{a}$ and $\mathbf{b}$ is θ. By considering $(\mathbf{a}-\mathbf{b}).(\mathbf{a}-\mathbf{b})$ and the cosine rule for the triangle *OAB*, show that $\mathbf{a}.\mathbf{b}=|\mathbf{a}||\mathbf{b}|\cos\theta$. [4]

4 **(i)** Show that $\frac{3}{2}-\sqrt{a+\frac{1}{4}}<1$, where a is a positive constant. [1]

(ii) Solve the inequality $\frac{x^2-x-a}{x-1}\le 2$, where a is a positive constant. [4]

(iii) Hence solve $\frac{a+x-x^2}{x}\le 2$. [2]

5 Let $\mathrm{f}(r)=\cos\left[\alpha+\left(r+\frac{1}{2}\right)\beta\right]$, $\beta\neq 2k\pi$, $k\in$ ¢ .

(i) Find $\mathrm{f}(r)-\mathrm{f}(r-1)$. [2]

(ii) Show that

$$\sin\alpha+\sin(\alpha+\beta)+\sin(\alpha+2\beta)+...+\sin(\alpha+n\beta)=\frac{\sin(\alpha+p\beta)\sin q\beta}{\sin\frac{1}{2}\beta},$$

where p and q are constants to be determined in terms of n . [4]

(iii) Deduce an expression for

$$\cos\alpha+\cos\left(\alpha+\frac{\pi}{2}\right)+\cos(\alpha+\pi)+\cos\left(\alpha+\frac{3\pi}{2}\right)+...+\cos\left(\alpha+\frac{n\pi}{2}\right).$$ [2]

6 A curve C has parametric equations

$$x=2\sin t,\qquad y=1+\cos t,\qquad 0<t<\pi.$$

(i) Show that the equation of the tangent to C at the point $P(2\sin p, 1+\cos p)$ is $2y+x\tan p=2(1+\sec p)$. [4]

(ii) The tangent at P meets the x-axis at the point A and the y-axis at the point B. The point M is the midpoint of AB. Find the cartesian equation of the curve traced by M as p varies. [5]

7 Mr Wang decides to paint the walls of his room which has a total surface area of 40 m^2. He has two plans:

Plan A: He paints 7 m^2 on the first day, and on each subsequent day, he paints 0.5 m^2 less than the previous day.

Plan B: He paints 7 m^2 on the first day, and on each subsequent day, he paints 20% less than the previous day.

(i) Find an expression for the area painted on the *n*th day according to Plan A. Give your answer in terms of *n*. [1]

(ii) Find the number of days required for him to complete painting his room according to Plan A. [2]

(iii) Explain why Plan A cannot be used to paint a wall of arbitrary size. Find the largest area that Plan A can be applied to. [2]

(iv) Find algebraically the number of days needed to paint at least 70% of his room according to Plan B. [3]

(v) State, with a reason, which plan A or B Mr Wang should choose to complete the paint job. [1]

8 The curve G has equation

$$y = \frac{x+2}{x(x+k)},$$

where k is a real constant.

(i) Find the range of values of k for which G has no stationary points. [3]

In the rest of the parts of the question, let $k = -2$.

(ii) Sketch G, stating clearly the equations of any asymptotes, the coordinates of the stationary points and the points where G crosses the axes. [3]

(iii) State the values of m for which the line $y = m$ intersects G once. [1]

(iv) By sketching a suitable curve on the diagram in part **(ii)**, show that the equation $x^4 - 4x^3 + 3x^2 + x - 2 = 0$ has exactly two real roots. [2]

9 **(a)** Showing your working clearly, find the complex numbers z and w which satisfy the simultaneous equations

$$4\mathrm{i}z - w = 9\mathrm{i} - 13\ ,$$
$$(4+2\mathrm{i})w^* = z + 3\mathrm{i}\ . \qquad [4]$$

(b) The complex numbers u and v are such that $u = 5\mathrm{e}^{\frac{7}{12}\pi \mathrm{i}}$ and $v = 6\mathrm{i}\,\mathrm{e}^{-\frac{1}{3}\pi \mathrm{i}}$ respectively.

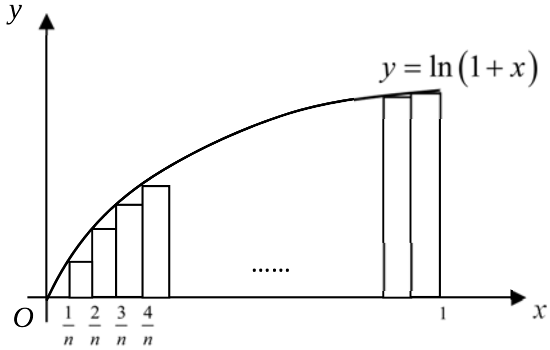
(i) Find an exact expression of $\dfrac{u^2}{v^*}$, giving your answer in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \le \pi$. [4]

(ii) Find the three smallest positive integer values of n for which $\dfrac{u^n}{v^*}$ is purely imaginary. [2]

10 **(a)** **(i)** Find $\int x\sqrt{x-1}\ \mathrm{d}x$ using integration by parts. [2]

(ii) The shape of a metal sculpture is formed by rotating the region bounded by the curve $y = \sqrt{a + x\sqrt{x-1}}$, where a is a positive integer, the lines $x = 1$ and $y = \sqrt{a+30}$, through 2π radians about the x-axis. Find the exact volume of the metal sculpture, giving your answer in terms of π. [4]

(b) **(i)** The diagram below shows a sketch of the graph of $y = \ln(1+x)$ for $0 \le x \le 1$. Rectangles each of width $\dfrac{1}{n}$ are drawn under the curve for $0 \le x \le 1$.

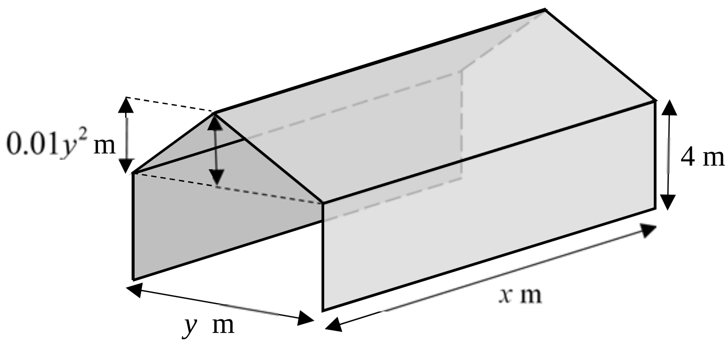


Show that A, the total area of all the rectangles, is given by

$$A = \frac{1}{n}\ln\left[\frac{(n+1)(n+2)(n+3)\mathrm{K}\ (2n-1)}{n^{n-1}}\right]. \qquad [2]$$

(ii) Find the exact value of $\lim\limits_{n\to\infty} A$. [3]

11 To celebrate Singapore's Bicentennial in 2019, the organising committee plans to hold a banquet at the Padang. A tent for the banquet consisting of two rectangular vertical sides and two pieces of the roof is to be constructed as shown in the diagram below. The tent has a length of *x* m and width *y* m, and a total floor area of 4000 m^2. The vertical sides of the tent are 4 m tall, and the roof adds another $0.01y^2$ m to the overall height of the tent.



(i) Show that *A*, the total external surface area of the tent is given by

$$A = 8x + 4000\sqrt{\frac{6400}{x^2} + 1}.$$ [3]

(ii) Show that if *A* has a stationary value for some *x*, then *x* satisfies the equation $\frac{a}{x^6} = \frac{6400}{x^2} + 1$, where *a* is a constant to be determined. [3]

(iii) If the material for the tent costs \$2.50 per m^2, estimate the minimum total cost of the material for the whole tent. [3]

(iv) A scale model of the tent is to be 3D-printed in a process where the total external surface area of this model always satisfies the equation $A = 8x + 4000\sqrt{\frac{6400}{x^2} + 1}$.

If *x* increases at a rate of 2 units per minute, use differentiation to find the rate of change in the total external surface area at the instant when $x = 200$ units. [3]

12 An experiment is conducted at room temperature where two substances, *A* and *B*, react in a chemical reaction to form *X* as shown below:

$$A + B \rightarrow X\ .$$

The initial concentrations in mol/dm^3 of substances *A* and *B* are *a* and *b* respectively. At time *t* seconds, the concentration of *A* and *B* are each reduced by x, where x denotes the concentration of *X* at time *t*.

(i) State the concentrations of *A* and *B* at time *t*. [1]

(ii) It is known that the rate of change of concentration of *X* at time *t* is proportional to the product of concentration of *A* and *B* at time *t* with a constant of proportionality k. Write down a differential equation involving x, a, b, t and k. [1]

(iii) State the maximum value of *x* if $a \le b$. Justify your answer. [2]

In the rest of the parts of the question, assume $a = b$.

(iv) The initial concentration of *X* is zero. Solve the differential equation in part **(ii)**, leave your answer in terms of x, a and t.

Express the solution in the form $x = \mathrm{f}(t)$ and sketch $x = \mathrm{f}(t)$ relevant in this context. Label the graph as S_1. [5]

It is known that the rate of change of concentration of *X* is doubled with every $10^{\circ}\mathrm{C}$ rise in the temperature. The experiment above is repeated but at a temperature $20^{\circ}\mathrm{C}$ above the room temperature. The concentration of *X* for this 2nd experiment at time *t* is now denoted by x_2. Let S_2 be the solution curve for the 2nd experiment.

(v) State an equation relating the rate of change of concentration of x and the rate of change of concentration of x_2 at time *t*. [1]

There is no need to solve for S_2 for the rest of the parts of the question.

(vi) On the same diagram as in part **(iv)**, sketch the solution curve for S_2. Show clearly the relative positions of S_1 and S_2 and their behaviour when $t \to \infty$. [2]

(vii) It is given that S_1 passes through the point $(1, \alpha)$ and S_2 passes through the point $(1, \beta)$. Using the rate of change of concentration of *X* for the two experiments, state the inequality relating α and β in this context, justifying your answer. [2]

Section A: Pure Mathematics [40 marks]

1 **(a)** Find $\displaystyle\int \frac{x+1}{x^2+3x+9}\,\mathrm{d}x$. [4]

(b) Use the substitution $x=\cos\theta$ to find the exact value of $\displaystyle\int_0^{\frac{1}{\sqrt{2}}} \frac{x^2}{\sqrt{1-x^2}}\,\mathrm{d}x$. [4]

2 **(i)** Using the standard series from the list of Formulae (MF26), show that the series expansion for $\dfrac{1}{1+\sin x}$ can be approximated to $1-x+x^2-\dfrac{5}{6}x^3$ when x is sufficiently small . [3]

(ii) Let $\mathrm{f}(x)=\dfrac{1}{1+\sin x^3}$. Using the expansion in part **(i)**, find $\mathrm{f}^{(9)}(0)$. [2]

(iii) " The series expansion of $\left(1+\dfrac{x}{a}\right)^b$ is equal to the series expansion of $\dfrac{1}{1-\sin x}$ as far as the term in x^3 where a and b are constants. "

Write down the series expansion of $\left(1+\dfrac{x}{a}\right)^b$ and $\dfrac{1}{1-\sin x}$ as far as the term in x^3.

Hence justify if the above statement is valid. [4]

3 The function f is defined as

$$\mathrm{f}: x\ \mathrm{a}\ \begin{cases} a\mathrm{e}^{a-x} & \text{for } 0\le x<a, \\ \left|\dfrac{1}{a}(x-a)^2-a\right| & \text{for } x\ge a. \end{cases}$$

(i) Sketch the graph of $y=\mathrm{f}(x)$, indicating clearly the axial intercepts. Show that f^{-1} does not exist. [4]

(ii) If the domain of f is restricted to $[0,k]$, determine the largest value of k in terms of a such that f^{-1} exists. [1]

Use the domain found in part **(ii)** for the rest of the question.

(iii) Define f^{-1} in similar form. [4]

(iv) Show that $\mathrm{ff}\left(\dfrac{a}{2}\right)=a\mathrm{e}^{\frac{a}{2}}\left(2-e^{\frac{a}{2}}\right)$ where $0<a\le\ln 2$. [2]

4 A line l has equation $x-1=\frac{y}{2}=z-3$, and a plane p has equation

$$\mathbf{r}=\begin{pmatrix}5\\-2\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-3\end{pmatrix}+\mu\begin{pmatrix}k\\1\\4\end{pmatrix},$$

where k is a real constant, $\lambda, \mu \in$ ¡ .

(i) Given that l and p are parallel, show that $k=-1$. [3]

Use $k=-1$ for the rest of the parts of the question.

(ii) Hence show that l and p do not intersect. [2]

(iii) Find the exact distance between l and p. [3]

(iv) A point A on l has coordinates $(2, 2, 4)$ and N is the foot of the perpendicular from A to p. Find the coordinates of N. Hence find the coordinates of the reflection of N in l. [4]

Section B: Statistics [60 marks]

5 A discrete random variable Y takes non-negative integer values with probabilities given as follows:

y	0	1	2	…	n	…
$\mathrm{P}(Y=y)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	…	$\frac{1}{2^{n+1}}$	…

(i) Find the probability that Y is odd. [2]

(ii) Y_1 and Y_2 are two independent observations of Y. Find the probability that the sum of Y_1 and Y_2 is less than 4, given that their sum exceeds 2. [4]

6 Six married couples are to be seated in a row at a concert. Find the number of ways they can sit if

(i) each couple is to sit together, [2]

(ii) all women are next to one another and all men are next to one another, such that no man can sit next to his wife. [3]

After the concert, one particular married couple leaves. The rest go to a restaurant where they sit at a round table. Find the probability that each man sits next to his wife, and men and women alternate. [2]

[Turn over

7 An engineering team from a car manufacturer wants to test their cars' braking system. The car travels along a stretch of road with speed v km/h. When the brakes are applied, the car comes to rest after travelling a further distance of d metres. A random sample of 6 pairs of values of v and d collected by a trainee mechanic from the engineering team is shown below.

v	30	40	50	60	70	80
d	5.00	5.30	6.45	8.50	17.00	25.90

(i) Draw a scatter diagram for these values, labelling the axes clearly. [2]

It is thought that the distance travelled d can be modelled by one of the following models.

Model I : $d = av + b$ or

Model II : $d = e^{pv+q}$

where a, b, p and q are constants.

(ii) Find the value of the product moment correlation coefficient between

(a) v and d ,

(b) v and $\ln d$. [2]

(iii) The trainee mechanic proposed that Model II is a better model than Model I. Use your answers to parts **(i)** and **(ii)** to explain why the trainee mechanic is right. [2]

(iv) Find the equation of the regression line of $\ln d$ on v. [1]

(v) Using the regression line in part **(iv)**, find the value of v if the driver applies his brakes immediately upon seeing an obstacle that is 10 metres away and stops just in time before crashing into it. [1]

(vi) The original data set contains 7 pairs of data with regression line $d = 0.4256v - 11.74$. The trainee mechanic found that he does not have the value of d when $v = 75$ from his record. Find the missing value of d correct to 2 decimal places. [3]

8 Mrs Lee claimed that the mean time taken by students to finish a meal during recess is not more than 20 minutes. Two students, Jack and Jill, decided to work together to test if Mrs Lee's claim is true. A total of 50 students were selected. The time, x minutes, by each of the 50 students to finish a meal during recess was recorded. The results are summarised by

$$\sum x = 1380, \quad \sum x^2 = 83000.$$

(i) Find unbiased estimates of the population mean and variance of the time taken by a student to finish a meal during recess. [2]

(ii) Stating a necessary assumption, carry out a test of Mrs Lee's claim at the 5% level of significance. [5]

(iii) Explain, in the context of the question, the meaning of "at the 5% level of significance". [1]

(iv) Jack and Jill have just learnt hypothesis testing. Jack carried out the test as in part **(ii)** while Jill performed a 2-tail test at 5% level of significance. Without performing any further test, explain whether Jill has the same conclusion as Jack. [2]

9 A bag contains 2 red balls, 3 yellow balls and 1 blue ball. Sue and Ben play a game where each takes turns to draw a ball from the bag, with replacement. The number of red balls obtained in n fixed draws from the bag is denoted by R.

(i) State, in context, an assumption satisfied by R for it to be well modelled by a binomial distribution. [1]

(ii) Sue and Ben each draws n times from the bag. Find the least n such that the probability of both getting a total of at most 10 red balls is not more than 0.5. [3]

(iii) Sue and Ben each draws 5 times from the bag. The player with more red balls drawn wins. Otherwise, the game ends in a draw. Find the probability that Sue wins the game if she draws more than 3 red balls. [3]

In a variation of the game, Sue draws balls at random from the bag, one at a time without replacement, and stops when she obtains 2 yellow balls. The total number of balls Sue has to draw from the bag before she stops is denoted by T. Find $\mathrm{E}(T)$ and $\mathrm{Var}(T)$. [5]

10 Each morning, Tony drives from his home to his office and has to pass 4 traffic lights on his way. He has to reach his office by 8.50 am. The driving time to his office and the time held up at a traffic light junction, in minutes, may be assumed to follow normal distributions with means and standard deviations as summarized below:

	Mean	Standard deviation
Driving time	14	2.1
Time held up at a traffic light junction	μ	0.2

Tony leaves home at 8.30 am. If the probability that Tony is not late is 0.713, show that $\mu = 1.2$, correct to 1 decimal place. State an assumption needed in your calculations. [4]

(i) For 10 mornings, Tony leaves home at 8.30 am. Find the probability that he arrives late at his office for the third time on the 10th day. [2]

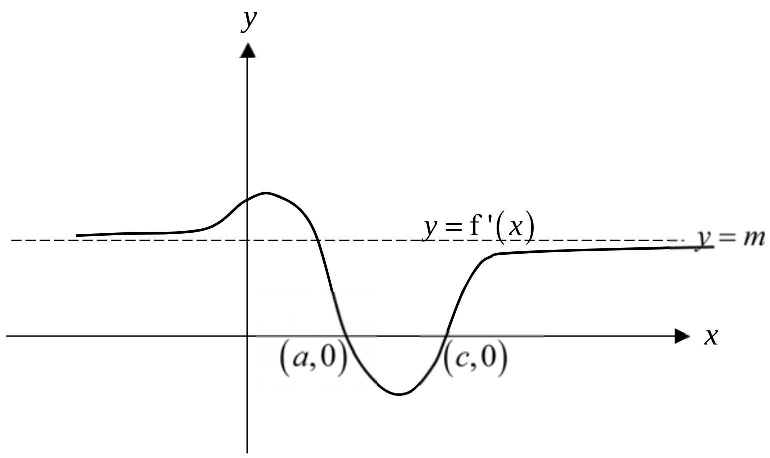
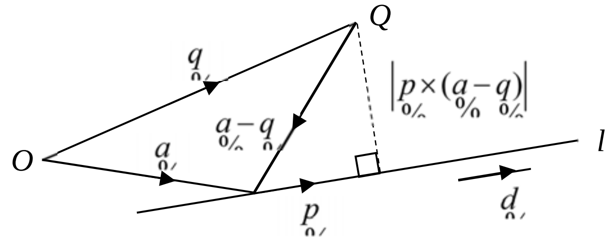
(ii) Find the probability that Tony's driving time to his office is less than 10 times the time he is held up at a traffic light junction. [2]

(iii) Find the probability that Tony's driving time to his office and the total time he is held up at the 4 traffic light junctions differs by more than 8 minutes. [3]

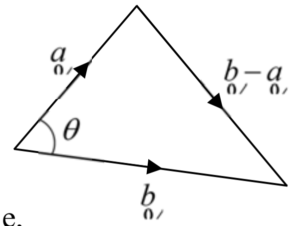
Assume μ is unknown.

(iv) The time held up at a traffic light junction is recorded on *n* randomly chosen occasions. Find the smallest *n* so that it is at least 98% certain that the sample mean time Tony is held up at a traffic light junction is within 5 seconds of μ. [3]

Marking Scheme for HCI 2018 Prelim Paper 1

Qn	Suggested Solution	Marking Scheme
1 **[2]**		**G1 –** Shape with at least 2 features correct **G1** – all features correct **SR: The maximum point could be in the first or second quadrant** 1. x-intercept $(a,0)$ 2. x-intercept $(c,0)$ 3. H.A $y=m$ (above x-axis) 4. y-intercept above H.A $y=m$
2 **[4]**	Let required equation be $y=ax^3+bx^2+cx+d$, $a,b,c,d\in\mathbb{R}$. Substitute $(0,0)$ into equation, $\therefore\ d=0$ $0.15^3a+0.15^2b+0.15c=0.1$ $0.3^3a+0.3^2b+0.3c=0.125$ $0.6^3a+0.6^2b+0.6c=0.1$ Using GC, $a=\frac{50}{27}$, $b=-\frac{5}{2}$, $c=1$ Required equation is $y=\frac{50}{27}x^3-\frac{5}{2}x^2+x \quad \ldots(*)$ Substitute $x=0.45$ into (*), $y=0.1125$ $\therefore$ the manufacturer's claim is accurate.	**B1** – $d=0$ [SOI] **M1** – Formulate system of equations with at least 1 equation correct **A1** – Correct equation **B1** – Correct conclusion with substitution of $x=0.45$ into (*) (SOI)
3(a) **[1]**	 $\left\|\underset{\sim}{p}\times(\underset{\sim}{a}-\underset{\sim}{q})\right\|$ is the shortest distance (or perpendicular distance) from Q to l.	**B1** – Correct Geometrical Interpretation

Marking Scheme for HCI 2018 Prelim Paper 1

Qn	Suggested Solution	Marking Scheme
	OR $\left\|\underset{\sim}{p}\times(\underset{\sim}{a}-\underset{\sim}{q})\right\|$ is the area of the parallelogram with adjacent vectors $\underset{\sim}{p}$ and $\underset{\sim}{a}-\underset{\sim}{q}$.	
3(b) **[4]**	 By cosine rule, $\left\|\underset{\sim}{b}-\underset{\sim}{a}\right\|^2=\left\|\underset{\sim}{a}\right\|^2+\left\|\underset{\sim}{b}\right\|^2-2\left\|\underset{\sim}{a}\right\|\left\|\underset{\sim}{b}\right\|\cos\theta$ $(\underset{\sim}{b}-\underset{\sim}{a}).(\underset{\sim}{b}-\underset{\sim}{a})=\underset{\sim}{a}.\underset{\sim}{a}+\underset{\sim}{b}.\underset{\sim}{b}-2\left\|\underset{\sim}{a}\right\|\left\|\underset{\sim}{b}\right\|\cos\theta$ $\underset{\sim}{b}.\underset{\sim}{b}-\underset{\sim}{b}.\underset{\sim}{a}-\underset{\sim}{a}.\underset{\sim}{b}+\underset{\sim}{a}.\underset{\sim}{a}=\underset{\sim}{a}.\underset{\sim}{a}+\underset{\sim}{b}.\underset{\sim}{b}-2\left\|\underset{\sim}{a}\right\|\left\|\underset{\sim}{b}\right\|\cos\theta$ $-2\underset{\sim}{a}.\underset{\sim}{b}=-2\left\|\underset{\sim}{a}\right\|\left\|\underset{\sim}{b}\right\|\cos\theta$ $\underset{\sim}{a}.\underset{\sim}{b}=\left\|\underset{\sim}{a}\right\|\left\|\underset{\sim}{b}\right\|\cos\theta$ (shown)	**B1** – Writing 3rd side of Δ as $\left\|\underset{\sim}{b}-\underset{\sim}{a}\right\|$ **M1** – Apply cosine rule to obtain $\left\|\underset{\sim}{b}-\underset{\sim}{a}\right\|^2=\left\|\underset{\sim}{a}\right\|^2+\left\|\underset{\sim}{b}\right\|^2-2\left\|\underset{\sim}{a}\right\|\left\|\underset{\sim}{b}\right\|\cos\theta$ **M1** – Write $\left\|\underset{\sim}{b}-\underset{\sim}{a}\right\|^2$ as $(\underset{\sim}{b}-\underset{\sim}{a}).(\underset{\sim}{b}-\underset{\sim}{a})$ and expand **AG1** – Obtain $-2\underset{\sim}{a}.\underset{\sim}{b}=-2\left\|\underset{\sim}{a}\right\|\left\|\underset{\sim}{b}\right\|\cos\theta$ and show that $\underset{\sim}{a}.\underset{\sim}{b}=\left\|\underset{\sim}{a}\right\|\left\|\underset{\sim}{b}\right\|\cos\theta$
4(i) **[7]**	$\sqrt{\frac{1}{4}}=\frac{1}{2}$ Since $a>0$, $a+\frac{1}{4}>\frac{1}{4}>0$ $\sqrt{a+\frac{1}{4}}>\frac{1}{2}$ $-\sqrt{a+\frac{1}{4}}<-\frac{1}{2}$ $\therefore\ \frac{3}{2}-\sqrt{a+\frac{1}{4}}<1$ (shown) $a+\frac{1}{4}>\frac{1}{4}>0$	**AG1** – Make use of $a>0$, compare $\sqrt{a+\frac{1}{4}}>\frac{1}{2}$ to show $\frac{3}{2}-\sqrt{a+\frac{1}{4}}<1$
(ii)	$\frac{x^2-x-a}{x-1}\le 2$ $\frac{x^2-x-a}{x-1}-2\le 0$	**M1** – Place all terms on one side and take common denominator (Algebraic method)

Marking Scheme for HCI 2018 Prelim Paper 1

<table>
<tr><th>Qn</th><th>Suggested Solution</th><th>Marking Scheme</th></tr>
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$$\frac{x^2-x-a-2x+2}{x-1}\le 0$$

$$\frac{x^2-3x+2-a}{x-1}\le 0$$

<u>Method 1</u>: (Algebraic method)

$$\frac{(x-\frac{3}{2})^2-(\frac{3}{2})^2+2-a}{x-1}\le 0$$

$$\frac{(x-\frac{3}{2})^2-(\frac{1}{4}+a)}{x-1}\le 0$$

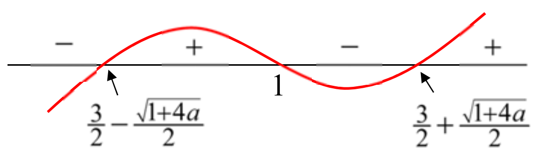
$$\frac{(x-\frac{3}{2})^2-(\sqrt{\frac{1+4a}{4}})^2}{x-1}\le 0$$

$$\frac{(x-\frac{3}{2}+\frac{\sqrt{1+4a}}{2})(x-\frac{3}{2}-\frac{\sqrt{1+4a}}{2})}{x-1}\le 0$$

$$\frac{\left[x-\left(\frac{3}{2}-\sqrt{\frac{1}{4}+a}\right)\right]\left[x-\left(\frac{3}{2}+\sqrt{\frac{1}{4}+a}\right)\right]}{x-1}\le 0$$

Since a is a positive constant, $\left(\frac{3}{2}-\sqrt{\frac{1}{4}+a}\right)<1$ (from (i))

And $\left(\frac{3}{2}+\sqrt{\frac{1}{4}+a}\right)>\frac{3}{2}>1$

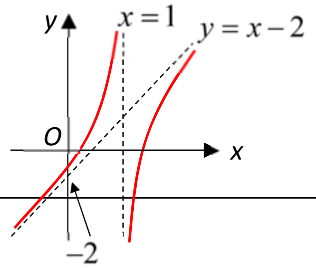


$\therefore\ x\le\frac{3}{2}-\frac{\sqrt{1+4a}}{2}$ or $1<x\le\frac{3}{2}+\frac{\sqrt{1+4a}}{2}$

<u>Method 2</u>: (Graphical method)

$$x-2-\frac{a}{x-1}\le 0$$

$$\begin{array}{r} x-2 \\ x-1\,\big)\,\overline{x^2-3x+2-a} \\ \underline{x^2-x} \\ -2x+2-a \\ \underline{-2x+2} \\ -a \end{array}$$



</td>
<td>

M1 – Attempt to factorise $x^2-3x+2-a$ by completing the square or using formula

M1 – Find all the critical values and attempt to using a number line determine range of x

A1 – Correct answer

(Graphical method)

M1 – Attempt to sketch graph to locate parts of the graph below x-axis

M1 – Attempt to find x-intercepts of graph

A1 – Correct answer

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Marking Scheme for HCI 2018 Prelim Paper 1

Qn	Suggested Solution	Marking Scheme
	Let $\frac{x^2-3x+2-a}{x-1}=0$ $\therefore\ x^2-3x+2-a=0$ $\therefore\ x=\frac{3\pm\sqrt{9-4(2-a)}}{2}$ $=\frac{3}{2}\pm\frac{\sqrt{1+4a}}{2}$ Since $\frac{x^2-3x+2-a}{x-1}\le 0$, $\therefore\ x\le\frac{3}{2}-\frac{\sqrt{1+4a}}{2}$ or $1<x\le\frac{3}{2}+\frac{\sqrt{1+4a}}{2}$	
	Replace x in $\frac{x^2-x-a}{x-1}\le 2$ with $1-x$, we obtain $\frac{(1-x)^2-(1-x)-a}{(1-x)-1}\le 2$ $\frac{1-2x+x^2-1+x-a}{-x}\le 2$ $\frac{x^2-x-a}{-x}\le 2$ $\frac{a+x-x^2}{x}\le 2$ Hence $1-x\le\frac{3}{2}-\frac{\sqrt{1+4a}}{2}$ or $1<1-x\le\frac{3}{2}+\frac{\sqrt{1+4a}}{2}$ $\therefore\ x\ge-\frac{1}{2}+\frac{\sqrt{1+4a}}{2}$ or $-\frac{1}{2}-\frac{\sqrt{1+4a}}{2}\le x<0$	**M1** – Replace x with $1-x$ **A1** – Correct answer
5(i) **[8]**	$\mathrm{f}(r)-\mathrm{f}(r-1)=\cos[\alpha+(r+\frac{1}{2})\beta]-\cos[\alpha+(r-\frac{1}{2})\beta)]$ $=-2\sin(\alpha+r\beta)\sin\frac{1}{2}\beta$ (By factor formula)	**M1** – Apply factor formula **A1** – Correct answer
5(ii)	$\sum_{r=0}^{n}[\mathrm{f}(r)-\mathrm{f}(r-1)]=\sum_{r=0}^{n}[-2\sin(\alpha+r\beta)\sin\frac{1}{2}\beta]$ $=-2\sin\frac{1}{2}\beta\sum_{r=0}^{n}\sin(\alpha+r\beta)$	**M1** – Able to use part **(i)**

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Qn	Suggested Solution	Marking Scheme
	$\sin\alpha+\sin(\alpha+\beta)+\sin(\alpha+2\beta)+...+\sin(\alpha+n\beta)$ $=\sum_{r=0}^{n}\sin(\alpha+r\beta)$ $\sum_{r=0}^{n}\sin(\alpha+r\beta)$ $=-\frac{1}{2\sin\frac{1}{2}\beta}\sum_{r=0}^{n}[\mathrm{f}(r)-\mathrm{f}(r-1)]$ $=-\frac{1}{2\sin\frac{1}{2}\beta}\begin{bmatrix}\mathrm{f}(0)-\mathrm{f}(1)\\+\mathrm{f}(1)-\mathrm{f}(2)\\+...\\+\mathrm{f}(n-1)-\mathrm{f}(n-2)\\+\mathrm{f}(n)-\mathrm{f}(n-1)]\end{bmatrix}$ $=-\frac{1}{2\sin\frac{1}{2}\beta}[\mathrm{f}(n)-\mathrm{f}(-1)]$ $=-\frac{1}{2\sin\frac{1}{2}\beta}\{\cos[\alpha+(n+\frac{1}{2})\beta]-\cos(\alpha-\frac{1}{2}\beta)\}$ $=-\frac{1}{2\sin\frac{1}{2}\beta}[-2\sin(\alpha+\frac{n}{2}\beta)\sin(\frac{n+1}{2})\beta]$ $=\frac{\sin(\alpha+\frac{n}{2}\beta)\sin(\frac{n+1}{2})\beta}{\sin\frac{1}{2}\beta}---(1)$ Where $p=\frac{n}{2}$ and $q=\frac{n+1}{2}$.	**M1** – Attempt to use MOD, must show the cancellation and at least 2 rows in front and at the end **M1** – Apply factor formula **A1** – Obtain *a* and *b* correctly
5(iii)	Sub $\beta=\pi$ Differentiate (1) wrt α : $\cos\alpha+\cos(\alpha+\pi)+\cos(\alpha+2\pi)+...+\cos(\alpha+n\pi)$ $=\frac{\cos\left(\alpha+\frac{n\pi}{2}\right)\sin\left[\left(\frac{n+1}{2}\right)\pi\right]}{\sin\frac{\pi}{2}}$ $=\cos\left(\alpha+\frac{n\pi}{2}\right)\sin\left[\left(\frac{n+1}{2}\right)\pi\right]$	**M1** – Sub $\beta=\pi$ and attempt to differentiate (1) wrt α **A1** – Show all steps clearly with the correct answer

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Qn	Suggested Solution	Marking Scheme
	Alternative Solution: $\cos\alpha+\cos(\alpha+\pi)+\cos(\alpha+2\pi)+...+\cos(\alpha+n\pi)$ $=\sum_{r=0}^{n}\cos(\alpha+r\beta)$ $=\sum_{r=0}^{n}\sin\left(\frac{\pi}{2}-\alpha-r\beta\right)$ $=\frac{\sin\left(\frac{\pi}{2}-\alpha-\frac{n\pi}{2}\right)\sin\left[-\pi\left(\frac{n+1}{2}\right)\right]}{\sin\left(-\frac{\pi}{2}\right)}$ $=\sin\left(\frac{\pi}{2}-\alpha-\frac{n\pi}{2}\right)\sin\left[\pi\left(\frac{n+1}{2}\right)\right]$ $=\cos\left(\alpha+\frac{n\pi}{2}\right)\sin\left[\pi\left(\frac{n+1}{2}\right)\right]$	**M1 –** Use of $\cos\theta=\sin\left(\frac{\pi}{2}-\theta\right)$ **,** $\beta=-\pi$ **A1** – Show all steps clearly with the correct answer
6(i) **[4]**	$\frac{dx}{dt}=2\cos t$ $\frac{dy}{dt}=-\sin t$ $\frac{dy}{dx}=\frac{-\sin t}{2\cos t}=-\frac{\tan t}{2}$ Equation of tangent T is $y-1-\cos p=-\frac{\tan p}{2}(x-2\sin p)$ $y=1+\cos p-\frac{\tan p}{2}x+\frac{\sin^2 p}{\cos p}$ $=1+\frac{\cos^2 p+\sin^2 p}{\cos p}-\frac{\tan p}{2}x$ $=1+\sec p-\frac{\tan p}{2}x$ $2y+x\tan p=2(1+\sec p)$	**M1** – Either $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct **A1** – Correct $\frac{dy}{dx}$ in terms of t **M1** – Form equation of tangent **AG1** – Correct equation of T
6(ii) **[5]**	When $y=0$, $\frac{\tan p}{2}x=1+\sec p$ $x=\frac{2+2\sec p}{\tan p}$ When $x=0$, $y=1+\sec p$	**M1** – Substitute $x=0$, $y=0$ to find A and B. Also accept $x = 2\cot p + 2\,\text{cosec}\, p$ or $x=\frac{2}{\tan p}+\frac{2}{\sin p}$.

Marking Scheme for HCI 2018 Prelim Paper 1

Qn	Suggested Solution	Marking Scheme
	Method 1: Coordinates of $M = \left(\dfrac{1+\sec p}{\tan p}, \dfrac{1+\sec p}{2}\right)$. $y = \dfrac{1+\sec p}{2} \Rightarrow \sec p = 2y-1$ $x = \dfrac{1+\sec p}{\tan p} = \dfrac{2y}{\tan p}$ Using $1+\tan^2 p = \sec^2 p$, $1+\left(\dfrac{2y}{x}\right)^2 = (2y-1)^2$ $1+\dfrac{4y^2}{x^2} = 4y^2-4y+1$ $y = yx^2 - x^2$ $x^2 = y(x^2-1)$ $y = \dfrac{x^2}{x^2-1}$	**M1** – Apply mid-point formula $M = \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$ **A1** – Obtain *x* and *y* coordinates of *M*. **M1** – Form cartesian equation of locus of *M* using $1+\tan^2 p = \sec^2 p$ **A1** – Correct cartesian equation of locus of *M*.
	Method 2: Coordinates of $M = \left(\cot p + \operatorname{cosec} p, \dfrac{1+\sec p}{2}\right)$. $y = \dfrac{1+\sec p}{2} \Rightarrow \sec p = 2y-1$ $x = \cot p + \operatorname{cosec} p = \dfrac{\cos p + 1}{\sin p}$ Using $\sin^2 p + \cos^2 p = 1$,	**M1** – Express $M = \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$ **A1** – Obtain *x* and *y* coordinates of *M*. **M1** – Form cartesian equation of locus of *M* using $\sin^2 p + \cos^2 p = 1$

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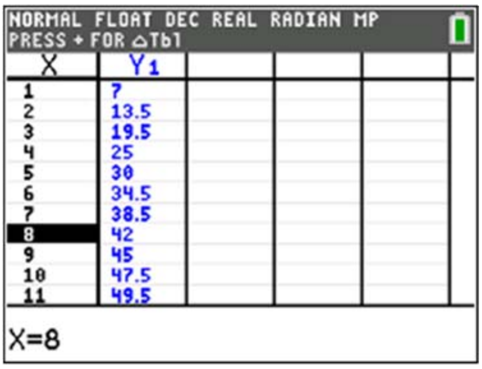
Qn	Suggested Solution	Marking Scheme
	$\left(\frac{\cos p+1}{x}\right)^2+\left(\frac{1}{2y-1}\right)^2=1$ $\frac{\left(\frac{1}{2y-1}+1\right)^2}{x^2}+\frac{1}{(2y-1)^2}=1$ $\frac{(2y)^2}{(2y-1)^2x^2}+\frac{1}{(2y-1)^2}=1$ $4y^2+x^2=(4y^2-4y+1)x^2$ $y^2=y^2x^2-yx^2$ $x^2=y(x^2-1)$ $y=\frac{x^2}{x^2-1}$	**A1** – Correct cartesian equation of locus of *M*.
7 [11] (i)	By Plan A, area painted on the *n*th day = 7 – (*n* – 1)0.5 = 7.5 – 0.5*n*	**B1** – Correct formula
(ii)	Total area painted on *n* days $=\frac{n}{2}\left[2(7)-(n-1)0.5\right]\geq 40$ **Method 1:**  NORMAL FLOAT DEC REAL RADIAN MP PRESS + FOR △Tbl X: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 Y1: 7, 13.5, 19.5, 25, 30, 34.5, 38.5, 42, 45, 47.5, 49.5 X=8 *n* | Area 7 | 38.5 < 40 8 | 42 > 40 He will finish painting his room on the 8th day.	**M1** – Form an inequality or use GC table involving the sum of AP formula SR: If students form equation instead of inequality, award 1 out of 2 marks. **A1** – Correct answer
	Method 2: Total area painted on *n* days =	**M1** – Form an inequality or use GC table involving the sum of AP formula

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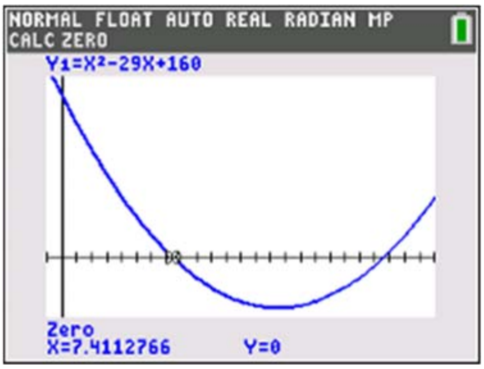
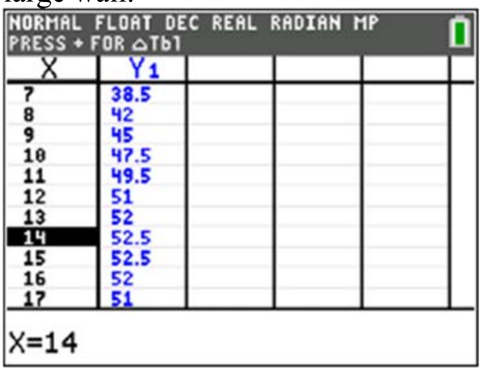
Qn	Suggested Solution	Marking Scheme
	$\frac{n}{2}[2(7)-(n-1)0.5]\geq 40$ $28n-n(n-1)\geq 160$ $n^2-29n+160\leq 0$  $7.41\leq n\leq 21.6$ But $7.5-0.5n>0\Rightarrow n<15$ So $8\leq n\leq 14$. Least $n=8$. He will finish painting his room on the 8th day.	SR: If students form equation instead of inequality, award 1 out of 2 marks. **A1** – Correct answer
(iii)	For $n\geq 15$, area painted on the nth day = 0 and painting stops. Therefore Plan A cannot be applied to an arbitrarily large wall.  By GC or using $\frac{14}{2}[2(7)-(14-1)0.5]=52.5$, largest area that Plan A can be applied to is 52.5 m^2.	**B1** – Correct reason **B1** – Correct largest area.

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Qn	Suggested Solution	Marking Scheme
(iv)	$7\frac{1-0.8^n}{1-0.8} \geq 0.7\times 40$ $\frac{1-0.8^n}{0.2} \geq 4$ $1-0.8^n \geq 0.8$ $0.8^n \leq 0.2$ $n\ln 0.8 \leq \ln 0.2$ $n \geq 7.21$ He needs 8 days to finish painting at least 70% of his room.	**M1** – Apply correct GP formula for S_n **M1** – Form inequality $S_n \geq 0.7\times 40$ and attempt to solve algebraically. **A1** – Correct number of days.
(v)	If he chooses Plan B, total area painted after an infinite number of days = $\frac{7}{1-0.8} = 35 < 40$. He cannot finish painting his room. Therefore he should choose Plan A.	**B1** – Correct conclusion with reason with correct formula for sum to infinity used and the answer is < 40
8[10] **(i)**	$y = \frac{x+2}{x(x+k)}$ $\frac{dy}{dx} = \frac{x(x+k)-(x+2)(2x+k)}{\left[x(x+k)\right]^2}$ $= \frac{x^2+kx-2x^2-kx-4x-2k}{\left[x(x+k)\right]^2}$ $= -\frac{x^2+4x+2k}{\left[x(x+k)\right]^2}$ Let $\frac{dy}{dx} = 0 \Rightarrow x^2+4x+2k=0$ For no stationary points, Discriminant < 0 $16-4(2k)(1)<0$ $k>2$ When $k=2$, $y=\frac{x+2}{x(x+2)}=\frac{1}{x}, x\neq -2$ $\Rightarrow$ No stationary points when $k=2$ $\therefore k \geq 2$	**M1** – Attempt to find $\frac{dy}{dx}$ using quotient/product rule **M1** – Use Discriminant < 0 to obtain a linear inequality involving k **A1** – correct answer $k \geq 2$

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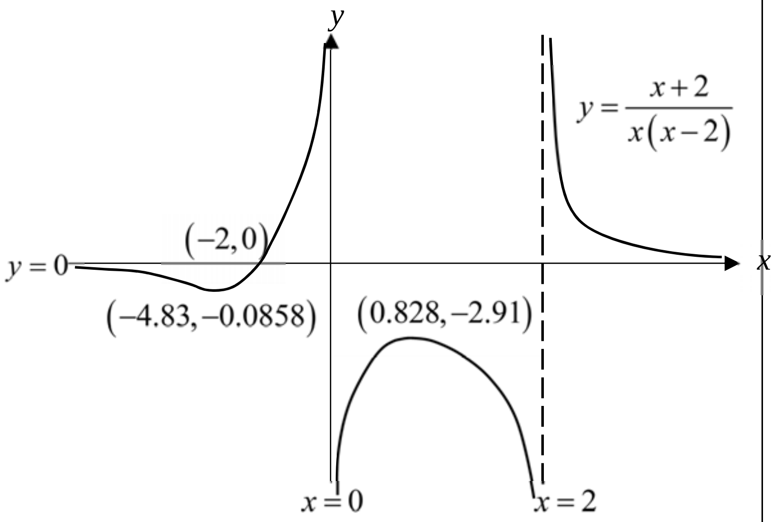
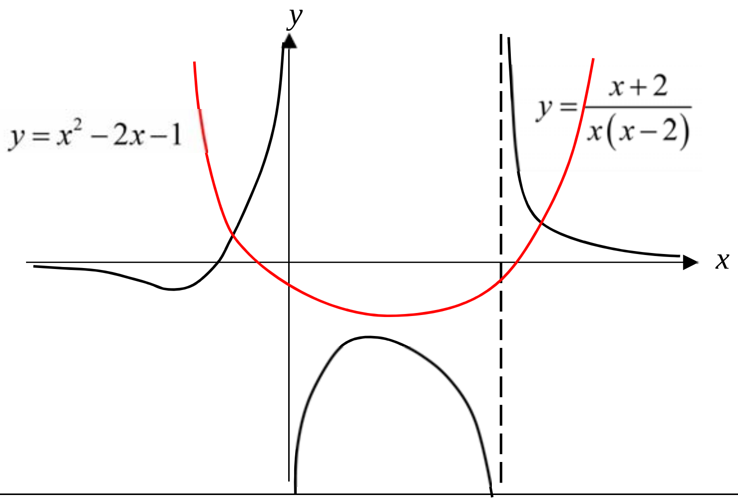
Qn	Suggested Solution	Marking Scheme
8(ii)		**G1** – Shape (with 2 turning points) **G1** – at least 4 out of 6 features **G1** – All correct 1. x-intercept $(-2,0)$ 2. min $(-4.83,-0.0858)$ 3. max $(0.828,-2.91)$ 4. V.A $x=2$ 5. V.A $x=0$ (**SOI**) 6. H.A $y=0$ (**SOI**)
8(iii)	$m=0,-0.0858$ or -2.91 (to 3s.f)	**B1** – All 3 values correct
8(iv)	$x^4-4x^3+3x^2+x-2=0$ $x^4-4x^3+3x^2+2x=x+2$ $x(x-2)(x^2-2x-1)=x+2$ $x^2-2x-1=\frac{x+2}{x(x-2)}$ Sketch the graph of $y=x^2-2x-1$ in **(ii)** 	**M1** – attempt to find the expression of the second curve by rearranging the terms And sketch the correct curve $y=x^2-2x-1$ **AG1 –** conclude with reference to diagram

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Qn	Suggested Solution	Marking Scheme
	Since the graph $y = x^2 - 2x - 1$ intersect the graph $y = \dfrac{x+2}{x(x-2)}$ at two distinct points, there are 2 real roots to the equation $x^4 - 4x^3 + 3x^2 + x - 2 = 0$.	
9[12] **(a)**	$4\text{i}z - w = 9\text{i} - 13$ ------- (1) $(4+2\text{i})w^* = z + 3\text{i}$ -----(2) Let $w = x + \text{i}y$ $4\text{i}[(4+2\text{i})(x - \text{i}y) - 3\text{i}] - (x + \text{i}y) = 9\text{i} - 13$ $4\text{i}[(4x+2x\text{i} - 4\text{i}y + 2y - 3\text{i}] - x - \text{i}y) = 9\text{i} - 13$ $16\text{i}x - 8x + 16y + 8y\text{i} + 12 - x - \text{i}y = 9\text{i} - 13$ Compare real and imaginary parts $16y - 9x = -25$ ------- (3) $16y + 8y - y = 9$ $16x + 7y = 9$ ---------- (4) From (3) $x = \dfrac{16y+25}{9}$ From (4) $16\left(\dfrac{16y+25}{9}\right) + 7y = 9$ $256y + 400 + 63y = 81$ $319y = -319$ $y = -1$ $x = \dfrac{16(-1)+25}{9} = 1$ $\therefore w = 1 - \text{i}$ $z = (4+2\text{i})(1+\text{i}) - 3\text{i} = 2+3\text{i}$	**M1** – Either let $z = x + \text{i}y,\ x \in \mathbb{R},\ y \in \mathbb{R}$ or $w = a + \text{i}b\ a \in \mathbb{R},\ b \in \mathbb{R}$ and attempt to solve the simultaneous equations **M1** – Use of $w^* = x - \text{i}y$ and attempt to compare real and imaginary parts to form two simultaneous equation **A1** – Either Obtain $w = 1 - \text{i}$ or $z = 2+3\text{i}$ **A1** – All the answers correct
9(bi)	$\left\|\dfrac{u^2}{v^*}\right\| = \dfrac{\|u\|^2}{\|v^*\|} = \dfrac{25}{6}$ $\arg\left(\dfrac{u^2}{v^*}\right) = 2\arg u - \arg v^*$ ----- (1) $v = 6\text{e}^{\text{i}\frac{\pi}{2}}\text{e}^{-\text{i}\frac{\pi}{3}} = 6\text{e}^{\text{i}\frac{\pi}{6}}$ $\arg v^* = -\dfrac{\pi}{6}$ From (1)	**B1** – Correct value of r i.e $\left\|\dfrac{u^2}{v^*}\right\| = \dfrac{25}{6}$ **B1** – Correct $\arg v^* = -\dfrac{\pi}{6}$ **M1** – Apply correct properties $\arg u^2 - \arg v^*$

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Qn	Suggested Solution	Marking Scheme
	$\arg\left(\frac{u^2}{v^*}\right)$ $=2\left(\frac{7\pi}{12}\right)-\left(-\frac{\pi}{6}\right)$ $=\frac{4\pi}{3}$ $\text{Arg}\left(\frac{u^2}{v^*}\right)=-2\pi+\frac{4\pi}{3}=-\frac{2\pi}{3}$ $\frac{u^2}{v^*}=\frac{25}{6}\text{e}^{-\text{i}\left(\frac{2\pi}{3}\right)}$	**A1** – Correct answer for $\frac{u^2}{v^*}$
9 (bii)	$\frac{u^n}{v^*}=\frac{5^n\text{e}^{\frac{7}{12}n\pi\text{i}}}{6\text{e}^{-\frac{\pi}{6}\text{i}}}$ $=\frac{5^n}{6}\text{e}^{\left(\frac{7}{12}n\pi+\frac{\pi}{6}\right)\text{i}}$ To be purely imaginary $\frac{7n\pi+2\pi}{12}=\frac{(2k+1)\pi}{2},\ k\in ¢$ $12k+4=7n$ $k=2,\ n=4$ $k=9,\ n=16$ $k=16,\ n=28$	**M1** – Apply properties of $\arg\left(\frac{u^2}{v^*}\right)$ using previous part and equate it to either $\frac{(2k+1)\pi}{2}$ or $\frac{(2k-1)\pi}{2}$ OR to compare it with $\pm\frac{\pi}{2},\pm\frac{3\pi}{2},\pm\frac{5\pi}{2}...$ or to equate real part to zero and solve for n **A1** – All three answers for n are correct by observation
10[14] (a)(i)	$\int x\sqrt{x+1}\ \text{d}x$ $=\frac{2x}{3}(x-1)^{\frac{3}{2}}-\int\frac{2}{3}(x-1)^{\frac{3}{2}}\ \text{d}x$ $=\frac{2x}{3}(x-1)^{\frac{3}{2}}+\frac{4}{15}(x-1)^{\frac{5}{2}}+c$ $u=x\ \Rightarrow\ u'=1$ $v'=\sqrt{x-1}\ \Rightarrow\ v=\frac{2}{3}(x-1)^{\frac{3}{2}}$	**M1** – Identify correct u and v' and use the correct formula for integration by parts **A1** – Correct answer

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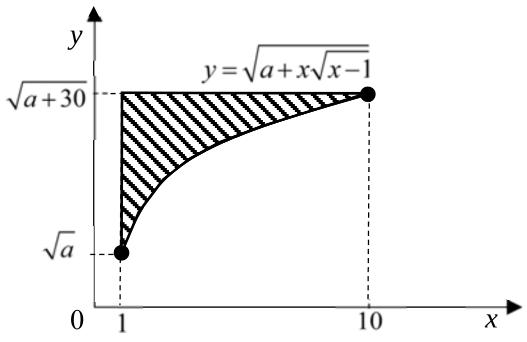
Qn	Suggested Solution	Marking Scheme
10 (a)(ii)	 $\sqrt{a+30}=\sqrt{a+x\sqrt{x-1}}$ $a+30=a+x\sqrt{x-1}$ $900=x^2(x-1)$ $x^3-x^2-900=0$ $x=10$	**B1** – Attempt to solve for $x=10$
	$V_c=\pi\int_1^{10} a+x\sqrt{x+1}\ \mathrm{d}x$ $=\pi[ax]_0^{10}+\int_1^{10} x\sqrt{x+1}\ \mathrm{d}x$ $=9a\pi+\int_1^{10} x\sqrt{x+1}\ \mathrm{d}x$	**M1** – Correct V_c or $9\pi\left(\sqrt{a+30}\right)^2$
	$=9a\pi+\left[\frac{2}{3}x(x-1)^{\frac{3}{2}}\right]_1^{10}-\frac{2}{3}\int_1^{10}(x-1)^{\frac{3}{2}}\ \mathrm{d}x$ $=9a\pi+\pi\left[\frac{2}{3}x(x-1)^{\frac{3}{2}}-\frac{4}{15}(x-1)^{\frac{5}{2}}\right]_1^{10}$ $=9a\pi+\pi\left[\frac{2700}{15}-\frac{972}{15}\right]$ $=\left(9a+\frac{1728}{15}\right)\pi$ units^3	
	$V=9\pi\left(\sqrt{a+30}\right)^2-V_c$ $=9\pi(a+30)-(9a+\frac{1728}{15})\pi$ $=\frac{774\pi}{5}$ units^3	**M1** – Find the required volume using $V=9\pi\left(\sqrt{a+30}\right)^2-V_c$ **A1** – Obtain Exact volume $\frac{774}{5}\pi$

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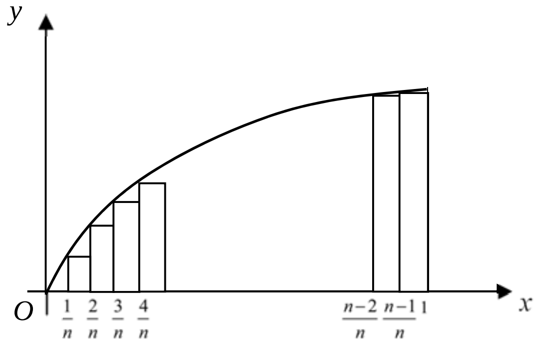
Qn	Suggested Solution	Marking Scheme
10 (c)(i)	 Area of 1st rectangle = $\frac{1}{n}\ln\left(1+\frac{1}{n}\right)$ Area of 2nd rectangle = $\frac{1}{n}\ln\left(1+\frac{2}{n}\right)$ Area of 3rd rectangle = $\frac{1}{n}\ln\left(1+\frac{3}{n}\right)$ M Area of $(n-1)$th rectangle = $\frac{1}{n}\ln\left(1+\frac{n-1}{n}\right)$ $A=\frac{1}{n}\left[\ln\left(1+\frac{1}{n}\right)+\ln\left(1+\frac{2}{n}\right)+\ln\left(1+\frac{3}{n}\right)+\text{L}\ +\ln\left(1+\frac{n-1}{n}\right)\right]$ $=\frac{1}{n}\ln\left[\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\text{L}\ \left(1+\frac{n-1}{n}\right)\right]$ $=\frac{1}{n}\ln\left[\left(\frac{n+1}{n}\right)\left(\frac{n+2}{n}\right)\left(\frac{n+3}{n}\right)\text{L}\ \left(\frac{2n-1}{n}\right)\right]$ $=\frac{1}{n}\ln\left[\frac{(n+1)(n+2)(n+3)\text{K}\ (2n-1)}{n^{n-1}}\right]$	**M1** – Attempt to find area of $n-1$ rectangles with the area of the first and the last rectangle correct **AG1** – Apply laws of logarithms to show the expression given
10 (c)(ii)	$u=\ln(1+x)\ \Rightarrow\ u'=\frac{1}{1+x}$ $v'=1\Rightarrow\ v=x$ $\lim_{n\to\infty}A=\int_0^1\ln(1+x)\,\mathrm{d}x$	**M1** – Use $\lim_{n\to\infty}A=\int_0^1\ln(1+x)\,\mathrm{d}x$ and apply integration by parts correctly

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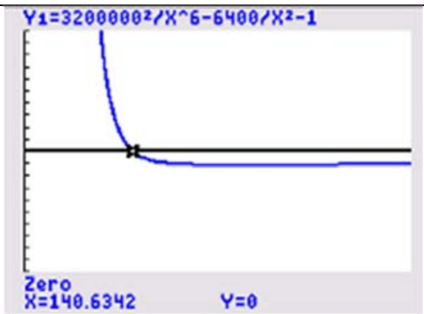
Qn	Suggested Solution	Marking Scheme
	$= \left[x\ln(1+x)\right]_0^1 - \int_0^1 1-\frac{1}{1+x}\,dx$ $= \left[x\ln(1+x)\right]_0^1 - \left[x-\ln(1+x)\right]_0^1$ $= \ln 2 - [1-\ln 2]$ $= 2\ln 2 - 1$ units^2	**M1 –** Rewrite as $\int_0^1 1-\frac{1}{1+x}\,dx$ and integrate correctly **A1** – Correct answer in exact form
11(i)	Floor area $xy = 4000 \Rightarrow y = \frac{4000}{x}$ Area $A = 2(4x) + 2x\sqrt{\left(0.01y^2\right)^2 + \left(\frac{y}{2}\right)^2}$ $= 8x + 2x\sqrt{\left(0.01\frac{4000^2}{x^2}\right)^2 + \left(\frac{4000}{2x}\right)^2}$ $= 8x + 2x\sqrt{\frac{160000^2}{x^4} + \frac{2000^2}{x^2}}$ $= 8x + 4000\sqrt{\frac{6400}{x^2} + 1}$	**M1** – Express A in terms of x and y **M1** – Substitute $y = \frac{4000}{x}$ **AG1** – Able to simplify to given expression
11(ii)	$\frac{dA}{dx} = 8 + 2000\left(\frac{6400}{x^2}+1\right)^{-1/2}\left(-\frac{12800}{x^3}\right) = 0$ $\left(\frac{6400}{x^2}+1\right)^{-1/2}\frac{25600000}{x^3} = 8$ $\frac{3200000}{x^3} = \sqrt{\frac{6400}{x^2}+1}$ $\frac{1.024\times10^{13}}{x^6} = \frac{6400}{x^2}+1$ $a = 1.024\times10^{13}$	**M1** – Apply Chain Rule to get $\frac{1}{2}(\mathrm{f}(x))^{-1/2}\left(-\frac{k}{x^3}\right)$. **M1** – Equate $\frac{dA}{dx} = 0$ and simplify to given expression **A1** – Correct value of a in any form e.g. $a = 3200000^2 = 20^{10}$.
11(iii)	 By GC, $x = 140.6342 = 141$ to 3 s.f.	**B1** – Correct value of x to 3 s.f.

Marking Scheme for HCI 2018 Prelim Paper 1

<table>
<tr><th>Qn</th><th>Suggested Solution</th><th>Marking Scheme</th></tr>
<tr><td></td><td>

Method 1:

<table>
<tr><td>x</td><td>140.63^{-}</td><td>140.63</td><td>140.63^{+}</td></tr>
<tr><td>$\frac{\mathrm{d}A}{\mathrm{d}x}$</td><td>–ve</td><td>0</td><td>+ve</td></tr>
<tr><td></td><td>\</td><td>–</td><td>/</td></tr>
</table>

Method 2:

$$\frac{\mathrm{d}^2A}{\mathrm{d}x^2}=\left(\frac{6400}{x^2}+1\right)^{-1/2}\left(\frac{76800000}{x^4}\right)$$
$$+\left(\frac{6400}{x^2}+1\right)^{-3/2}\left(-\frac{12800000}{x^3}\right)\left(-\frac{12800}{x^3}\right)$$

> 0 when $x = 140.6342$
Hence A is minimum when $x = 141$ to 3 s.f.

$$\text{Minimum Cost} = \left[8(140.6342)+4000\sqrt{\frac{6400}{140.6342^2}+1}\right]2.50$$

= \$14317.43 (nearest cents)
= \$14300 (to 3 s.f.)

</td><td>

B1 – Apply 1st or 2nd derivative test [only award mark if value of x is correct]

B1 – Correct minimum cost

</td></tr>
<tr><td>11(iv)</td><td>

From **(ii)**, $\frac{\mathrm{d}A}{\mathrm{d}x}=8-\left(\frac{6400}{x^2}+1\right)^{-1/2}\frac{25600000}{x^3}$

When $x = 200$, $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t}$

$$=\left[8-\left(\frac{6400}{200^2}+1\right)^{-1/2}\frac{25600000}{200^3}\right]2$$
$$=10.0577$$
$$=10.1\ \text{unit}^2/\text{min}\ \ (3\text{ s.f.})$$

</td><td>

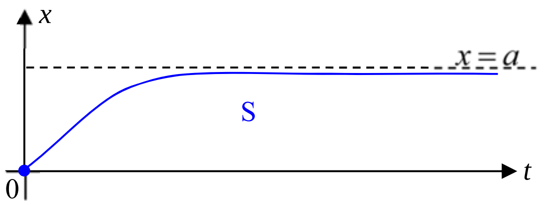
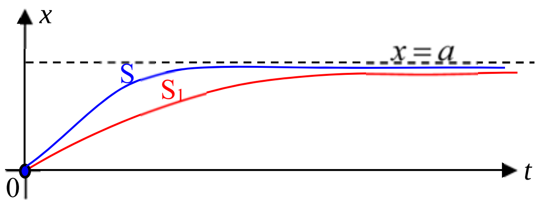
M1 – Attempt Chain rule to find $\frac{\mathrm{d}A}{\mathrm{d}x}$ or apply implicit differentiation to obtain an equation involving $\frac{\mathrm{d}A}{\mathrm{d}t}$ and $\frac{\mathrm{d}x}{\mathrm{d}t}$

M1 – Attempt to substitute $x = 200$ and $\frac{\mathrm{d}x}{\mathrm{d}t}=2$

A1 – Correct answer to 3 s.f.

</td></tr>
<tr><td>12(i)</td><td>the concentration of A and B at time t are $(a-x)$ and $(b-x)$ mol/dm^3 respectively</td><td>B1 – both answers correct</td></tr>
<tr><td>12(ii)</td><td>$\frac{\mathrm{d}x}{\mathrm{d}t}=k(a-x)(b-x),\ \ k\in ¡^{+}$</td><td>B1 – Correct answer</td></tr>
<tr><td>12(iii)</td><td>Max value for x is a, Q $\frac{\mathrm{d}x}{\mathrm{d}t}=0$ and after $x=a$ there is no more concentration of substance A for reaction to continue.</td><td>B1 – Correct Max value for x is a
B1 – state correct reason</td></tr>
</table>

Marking Scheme for HCI 2018 Prelim Paper 1

Qn	Suggested Solution	Marking Scheme
12(iv)	$\frac{\mathrm{d}x}{\mathrm{d}t} = k(a-x)^2$ $\int (a-x)^{-2}\mathrm{d}x = kt$ $(a-x)^{-1} = kt + C$ $x = a - \frac{1}{kt+C}$ ---(1) When $t=0, x=0 \Rightarrow c = \frac{1}{a}$ $x = a - \frac{1}{kt+\frac{1}{a}}$ $x = a - \frac{a}{akt+1}$ 	**M1** – correct method of integration $\int (a-x)^{-2}\mathrm{d}x$ **A1** – correct expression (1) accept Answer without constant **M1** – Use of initial value to find constant **A1** – Correct particular solution $x = a - \frac{a}{akt+1}$ **A1** – correct graph with asymptote and initial value labelled
12(v)	$\frac{\mathrm{d}x_1}{\mathrm{d}t} = 4\frac{\mathrm{d}x}{\mathrm{d}t}$	**B1**
12(vi)		**B2** – one mark for each curve showing clearly relative position Do not Accept graphs with *S* and S_1 overlapping each other at the tail ends
12(vii)	From the graph, $\alpha < \beta$. Q $\frac{\mathrm{d}x_1}{\mathrm{d}t}$ for S_1 > $\frac{\mathrm{d}x}{\mathrm{d}t}$ for S and both curves start from the origin.	**B1** – correct $\alpha < \beta$ **B1** – correct reason explain using the concentration of the

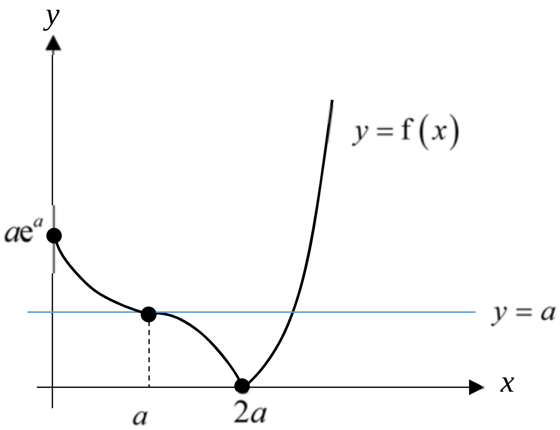
Marking Scheme for HCI 2018 Prelim Paper 2

Qn	Suggested Solution	Marking Scheme
1 [8] (a)	$\int \frac{x+1}{x^2+3x+9}\,\mathrm{d}x$ $=\frac{1}{2}\int \frac{2x+2+1-1}{x^2+3x+9}\,\mathrm{d}x$ $=\frac{1}{2}\int \frac{2x+3}{x^2+3x+9}\,\mathrm{d}x-\frac{1}{2}\int \frac{1}{x^2+3x+9}\,\mathrm{d}x$ $=\frac{1}{2}\ln\left\|x^2+3x+9\right\|-\frac{1}{2}\int \frac{1}{(x+\frac{3}{2})^2+9-\frac{9}{4}}\,\mathrm{d}x$ $=\frac{1}{2}\ln\left\|x^2+3x+9\right\|-\frac{1}{2}\int \frac{1}{(x+\frac{3}{2})^2+\left(\frac{\sqrt{27}}{4}\right)^2}\,\mathrm{d}x$ $=\frac{1}{2}\ln\left\|x^2+3x+9\right\|-\frac{1}{3\sqrt{3}}\tan^{-1}\left(\frac{2x+3}{3\sqrt{3}}\right)+C$	**M1** – Correct separation of terms **M1** – Apply $\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x$ or Complete the square and apply MF26 formula to obtain $\frac{1}{k}\tan^{-1}\left(\frac{2x+3}{k}\right)$ **A1** – Either $\frac{1}{2}\ln\left\|x^2+3x+9\right\|$ or $\frac{1}{3\sqrt{3}}\tan^{-1}\left(\frac{2x+3}{3\sqrt{3}}\right)$ **A1** – Correct answer with +C
1(b)	$\frac{\mathrm{d}x}{\mathrm{d}\theta}=-\sin\theta$, $\frac{1}{\sqrt{2}}=\cos\theta \Rightarrow \theta=\frac{\pi}{4}$ $0=\cos\theta \Rightarrow \theta=\frac{\pi}{2}$ $\int_0^{\frac{1}{\sqrt{2}}}\frac{x^2}{\sqrt{1-x^2}}\,\mathrm{d}x=\int_{\frac{\pi}{2}}^{\frac{\pi}{4}}\frac{\cos^2\theta}{\sqrt{1-\cos^2\theta}}\cdot(-\sin\theta)\,\mathrm{d}\theta$ $=\int_{\frac{\pi}{2}}^{\frac{\pi}{4}}\left(-\cos^2\theta\right)\mathrm{d}\theta$ $=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cos^2\theta\,\mathrm{d}\theta$ $=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{1+\cos 2\theta}{2}\,\mathrm{d}\theta$ $=\left[-\frac{1}{2}\theta+\frac{1}{4}\sin 2\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $=\frac{\pi}{4}-\left(\frac{\pi}{8}+\frac{1}{4}\right)$ $=\frac{\pi}{8}-\frac{1}{4}$	**B1** – Obtain $\theta=\frac{\pi}{4}$ and $\theta=\frac{\pi}{2}$ and differentiate correctly to get $\frac{\mathrm{d}x}{\mathrm{d}\theta}=-\sin\theta$ **M1** –and apply substitution to the integral and obtain $\int_{\theta_1}^{\theta_2}\cos^2\theta\,\mathrm{d}\theta$ **M1** – Attempt to apply double angle formula and able to integration $\cos 2\theta$ correctly **A1** – Correct answer in exact form

Marking Scheme for HCI 2018 Prelim Paper 2

Qn	Suggested Solution	Marking Scheme
2[9] (i)	$\frac{1}{1+\sin x}=(1+\sin x)^{-1}=1-(\sin x)+(\sin x)^2-(\sin x)^3+\ldots$ $=1-(x-\frac{x^3}{3!})+(x-\frac{x^3}{3!})^2-(x-\frac{x^3}{3!})^3+\ldots$ $=1-x+x^2-\frac{5}{6}x^3+\ldots$	**M1** – Correct binomial expansion $(1+\sin x)^{-1}$ **M1** – Substitute $\sin x \approx x-\frac{x^3}{3!}$ **A1** – Correct answer
2(ii)	Replace x by x^3, $\frac{1}{1+\sin x^3}=1-x^3+x^6-\frac{5}{6}x^9+\ldots$ $\Rightarrow \frac{f^9(0)}{9!}=-\frac{5}{6}$ $\Rightarrow f^9(0)=-\frac{5}{6}\times 9!=-302400$	**M1** – Replace x by x^3 **A1** – Correct answer
2(iii)	$(1+\frac{x}{a})^b=1+b(\frac{x}{a})+\frac{b(b-1)}{2}(\frac{x}{a})^2+\frac{b(b-1)(b-2)}{6}(\frac{x}{a})^3+\ldots$ $\frac{1}{1-\sin x}=1+x+x^2+\frac{5}{6}x^3+\ldots$ Coefficient of x: $\frac{b}{a}=1\Rightarrow a=b---(1)$ Coefficient of x^2 : $\frac{b(b-1)}{2a^2}=1---(2)$ Sub (1) into (2): $\frac{b(b-1)}{2b^2}=1$ $\Rightarrow \frac{(b-1)}{2b}=1$ $\Rightarrow b-1=2b$ $\Rightarrow b=1---(3)$ Coefficient of x^3 : $\frac{b(b-1)(b-2)}{6a^3}=-\frac{5}{6}---(4)$ Sub (1) and (2) into (4): $\frac{b(b-1)(b-2)}{6a^3}=\frac{5}{6}$ $\Rightarrow \frac{b(b-1)}{2a^2}\frac{b-2}{3a}=\frac{5}{6}$ $\Rightarrow \frac{b-2}{3b}=\frac{5}{6}$ $\Rightarrow b=\frac{4}{3}---(5)$	**B1** – Correct expansion for $(1+\frac{x}{a})^b$ **B1** – Correct expansion for $\frac{1}{1-\sin x}$ **M1** – or at least 2 correct equation out of 3 (eqs 1,2,4) And attempt for attempt to solve simult equations involving a and b **A1** – Correct conclusion

Marking Scheme for HCI 2018 Prelim Paper 2

Qn	Suggested Solution	Marking Scheme
	Results in (3) and (4) contradict each other. Hence the statement is invalid.	
3[11] (i)	 Since the line $y=a$ cuts the graph $y=\mathrm{f}(x)$ more than once, f is not a one-one function and therefore f^{-1} does not exist.	**G1** – Correct shape of graph for $y=ae^{a-x}$ for $0\le x<a$. **G1** – Correct shape of graph for $y=\left\lvert \frac{1}{a}(x-a)^2-a \right\rvert$ for $x\ge a$ **G1** – Axial intercepts **B1** – Give a counter example to show inverse does not exist
3(ii)	Largest value of $k=2a$	**B1**
3(iii)	Let $y=\mathrm{f}(x)\Rightarrow \mathrm{f}^{-1}(y)=x$ 1. $y=a-\frac{1}{a}(x-a)^2$ $(x-a)^2=a^2-ay$ $x=a\pm\sqrt{a^2-ay}$ $\left(\text{reject } x=a-\sqrt{a^2-ay}\ \because\ x\ge a\right)$ $\therefore x=a+\sqrt{a^2-ay}$ 2. $y=ae^{a-x}$ $a-x=\ln\frac{y}{a}$ $x=a-\ln\frac{y}{a}$ $\therefore \mathrm{f}^{-1}: x \mapsto \begin{cases} a+\sqrt{a^2-ax} & \text{for } 0\le x\le a \\ a-\ln\frac{x}{a} & \text{for } a<x\le ae^a \end{cases}$	**M1** – attempt to make x the subject for both expressions of $\mathrm{f}(x)$ **M1** – at least one of the expressions of $\mathrm{f}^{-1}(y)$ correct **A1** – correct answer with domain, in similar form Do not award full mark if student did not explain

Marking Scheme for HCI 2018 Prelim Paper 2

Qn	Suggested Solution	Marking Scheme
3(iv)	$\mathrm{f}\left(\frac{a}{2}\right)=a\mathrm{e}^{a-\frac{a}{2}}=a\mathrm{e}^{\frac{a}{2}}$ $\mathrm{ff}\left(\frac{a}{2}\right)=\mathrm{f}\left[\mathrm{f}\left(\frac{a}{2}\right)\right]$ $=\mathrm{f}\left(a\mathrm{e}^{\frac{a}{2}}\right)$ $=a-\frac{1}{a}\left(a\mathrm{e}^{\frac{a}{2}}-a\right)^2$ $=a-a\left(\mathrm{e}^{\frac{a}{2}}-1\right)^2$ $=a\mathrm{e}^{\frac{a}{2}}\left(2-\mathrm{e}^{\frac{a}{2}}\right)$ Since $0<a\le\ln 2$ $0<\frac{a}{2}\le\ln\sqrt{2}$ $1<\mathrm{e}^{\frac{a}{2}}\le\sqrt{2}$ $a<a\mathrm{e}^{\frac{a}{2}}\le a\sqrt{2}<2a$ Alternatively, $\mathrm{f}(x)=\begin{cases}a\mathrm{e}^{a-x} & \text{for } 0\le x<a\\ a-\frac{1}{a}(x-a)^2 & \text{for } a\le x\le 2a\end{cases}$ $\quad \mathrm{R_f}=\begin{cases}\left(a,a\mathrm{e}^{a}\right]\\ [0,a]\end{cases}$ $\therefore \mathrm{ff}(x)=\begin{cases}a-\frac{1}{a}\left(a\mathrm{e}^{a}-a\right)^2=a-a\left(\mathrm{e}^{a}-1\right)^2 & \text{for } 0\le x<a\\ a\mathrm{e}^{a-a+\frac{1}{a}(x-a)^2}=a\mathrm{e}^{\frac{1}{a}(x-a)^2} & \text{for } a\le x\le 2a\end{cases}$ $\mathrm{ff}\left(\frac{a}{2}\right)=a-a\left(\mathrm{e}^{\frac{a}{2}}-1\right)^2=a\mathrm{e}^{\frac{a}{2}}\left(2-\mathrm{e}^{\frac{a}{2}}\right)$	**M1** – attempt to substitute $a\mathrm{e}^{\frac{a}{2}}$ using the correct expression **A1** – correct answer **OR** **M1** – finding $\mathrm{ff}(x)$ with at least one of the expressions correct **A1** – correct answer

Marking Scheme for HCI 2018 Prelim Paper 2

<table>
<tr><th>Qn</th><th>Suggested Solution</th><th>Marking Scheme</th></tr>
<tr><td></td><td>Note for students:
for ff to exist, $R_f \subseteq D_f$
$R_f = \left[0, ae^a\right]$
$D_f = [0, 2a]$
$\Rightarrow \quad ae^a \le 2a$
$a\left(e^a - 2\right) \le 0$
Since $a > 0$, $e^a - 2 \le 0$
$a \le \ln 2$
$\therefore 0 < a \le \ln 2$</td><td></td></tr>
<tr><td>4[12]
(i)</td><td>Let $t = x - 1 = \frac{y}{2} = z - 3$, $t \in \mathbb{R}$.
$\therefore\ x = 1 + t$
$y = 0 + 2t$
$z = 3 + t$
Hence $l: \underset{\sim}{r} = \begin{pmatrix}1\\0\\3\end{pmatrix} + t\begin{pmatrix}1\\2\\1\end{pmatrix}$, $t \in \mathbb{R}$.
Normal of p is $\begin{pmatrix}2\\1\\-3\end{pmatrix} \times \begin{pmatrix}k\\1\\4\end{pmatrix} = \begin{pmatrix}7\\-8-3k\\2-k\end{pmatrix}$
Since l and p are parallel,
$\begin{pmatrix}7\\-8-3k\\2-k\end{pmatrix}$ and $\begin{pmatrix}1\\2\\1\end{pmatrix}$ are perpendicular.
Hence $\begin{pmatrix}7\\-8-3k\\2-k\end{pmatrix} \cdot \begin{pmatrix}1\\2\\1\end{pmatrix} = 0$
$7 - 16 - 6k + 2 - k = 0$
$\therefore\quad k = -1$ (shown)</td><td>B1: obtain correct direction vector of l

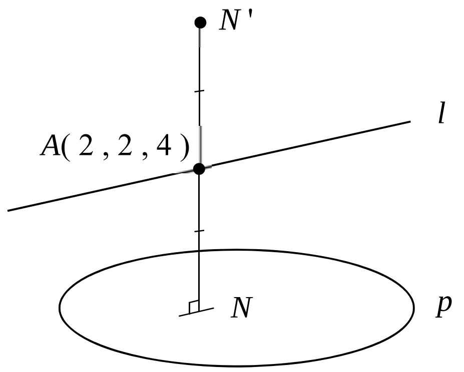
M1: apply cross product on 2 direction vectors

AG1: apply dot product $\begin{pmatrix}7\\-8-3k\\2-k\end{pmatrix} \cdot \begin{pmatrix}1\\2\\1\end{pmatrix} = 0$ and obtain $k = -1$</td></tr>
<tr><td>4(ii)</td><td><u>Method 1</u>: (using equation of p in dot product form)
$p: \underset{\sim}{r} \cdot \begin{pmatrix}7\\-5\\3\end{pmatrix} = \begin{pmatrix}5\\-2\\1\end{pmatrix} \cdot \begin{pmatrix}7\\-5\\3\end{pmatrix} = 48$</td><td>M1: obtain $p: \underset{\sim}{r} \cdot \begin{pmatrix}7\\-5\\3\end{pmatrix} = 48$</td></tr>
</table>

Marking Scheme for HCI 2018 Prelim Paper 2

Qn	Suggested Solution	Marking Scheme
	Substitute a point $(1,0,3)$ on l into p, $\begin{pmatrix}1\\0\\3\end{pmatrix}.\begin{pmatrix}7\\-5\\3\end{pmatrix}=16\neq 48$ Hence l does not lie on p. $\therefore$ l and p do not intersect. (shown) <u>Method 2</u>: (using equation of p in parametric form) If l and p intersect, $\begin{pmatrix}1\\0\\3\end{pmatrix}+t\begin{pmatrix}1\\2\\1\end{pmatrix}=\begin{pmatrix}5\\-2\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-3\end{pmatrix}+\mu\begin{pmatrix}-1\\1\\4\end{pmatrix}$ $\begin{pmatrix}1+t\\2t\\3+t\end{pmatrix}=\begin{pmatrix}5+2\lambda-\mu\\-2+\lambda+\mu\\1-3\lambda+4\mu\end{pmatrix}$ $\therefore$ $2\lambda-\mu-t=-4$ $\lambda+\mu-2t=2$ $-3\lambda+4\mu-t=2$ Solving using GC, no solution. $\therefore$ l and p do not intersect. (shown)	A1: show l does not lie on p and conclude l and p do not intersect. M1: equating l and p A1: show system of equations has no solution and conclude l and p do not intersect.
4(iii)	<u>Method 1</u>: (using $\left\|\underset{\sim}{a}.\underset{\sim}{b}\right\|$) Using points $(1,0,3)$ on l and $(5,-2,1)$ on p, $\begin{pmatrix}5\\-2\\1\end{pmatrix}-\begin{pmatrix}1\\0\\3\end{pmatrix}=\begin{pmatrix}4\\-2\\-2\end{pmatrix}$ Hence required distance $=\left\|\begin{pmatrix}4\\-2\\-2\end{pmatrix}.\frac{1}{\sqrt{83}}\begin{pmatrix}7\\-5\\3\end{pmatrix}\right\|$ $=\frac{32}{\sqrt{83}}=\frac{32\sqrt{83}}{83}$ units <u>Method 2</u>: (using intersection of $\perp$ line with p) Using the point $(1,0,3)$ on l, Equation of line through $(1,0,3)$ and perpendicular to p is	M1: attempt to find a vector from l to p M1: use $\left\|\underset{\sim}{a}.\underset{\sim}{b}\right\|$ to find distance A1: correct answer

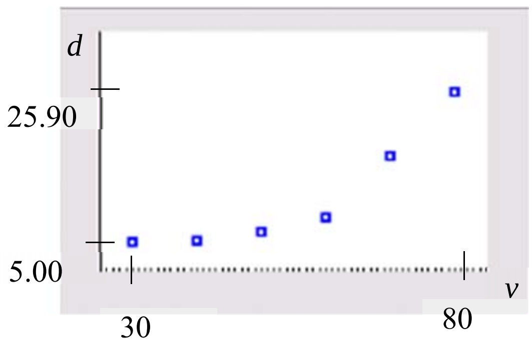
Marking Scheme for HCI 2018 Prelim Paper 2

Qn	Suggested Solution	Marking Scheme
	$\underset{\sim}{r} = \begin{pmatrix}1\\0\\3\end{pmatrix} + s\begin{pmatrix}7\\-5\\3\end{pmatrix}, \quad s \in \mathbb{R}$. When line intersects p, $\begin{pmatrix}1+7s\\-5s\\3+3s\end{pmatrix}.\begin{pmatrix}7\\-5\\3\end{pmatrix} = 48$ $7+49s+25s+9+9s=48$ $\therefore\ s=\frac{32}{83}$ Hence required distance $= \left\| \frac{32}{83}\begin{pmatrix}7\\-5\\3\end{pmatrix} \right\| = \frac{32\sqrt{83}}{83}$ units	M1: substitute equation of line into equation of p to find intersection M1: use $\left\| s\begin{pmatrix}7\\-5\\3\end{pmatrix} \right\|$ to find required distance A1: correct answer
4(iv)	 Method 1: (using part (iii)) From (iii), $\overrightarrow{AN} = \frac{32\sqrt{83}}{83}\left[\frac{1}{\sqrt{83}}\begin{pmatrix}7\\-5\\3\end{pmatrix}\right] = \frac{32}{83}\begin{pmatrix}7\\-5\\3\end{pmatrix}$ $\therefore\ \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AN} = \begin{pmatrix}2\\2\\4\end{pmatrix} + \frac{32}{83}\begin{pmatrix}7\\-5\\3\end{pmatrix} = \frac{1}{83}\begin{pmatrix}390\\6\\428\end{pmatrix}$ Hence $N\left(\frac{390}{83}, \frac{6}{83}, \frac{428}{83}\right)$ [or $N(4.70, 0.0723, 5.16)$] Method 2: (using intersection of $\perp$ line through A with p) Equation of line through $(2,2,4)$ and perpendicular to p is	M1: attempt to find $\overrightarrow{AN}$ using $\left[\begin{pmatrix}4\\-2\\-2\end{pmatrix}.\hat{\underset{\sim}{n}}\right]\hat{\underset{\sim}{n}}$ A1: correct coordinates of *N*.

Marking Scheme for HCI 2018 Prelim Paper 2

Qn	Suggested Solution	Marking Scheme
	$\underset{\sim}{r}=\begin{pmatrix}2\\2\\4\end{pmatrix}+\gamma\begin{pmatrix}7\\-5\\3\end{pmatrix},\quad \gamma\in\mathbb{R}$.	M1: substitute equation of line into equation of p to find intersection
	When line intersects p, $\begin{pmatrix}2+7\gamma\\2-5\gamma\\4+3\gamma\end{pmatrix}\cdot\begin{pmatrix}7\\-5\\3\end{pmatrix}=48$	
	$14+49\gamma-10+25\gamma+12+9\gamma=48$	
	$\therefore\ \gamma=\frac{32}{83}$	
	$\therefore\ \overrightarrow{ON}=\begin{pmatrix}2\\2\\4\end{pmatrix}+\frac{32}{83}\begin{pmatrix}7\\-5\\3\end{pmatrix}=\frac{1}{83}\begin{pmatrix}390\\6\\428\end{pmatrix}$	A1: correct coordinates of N.
	Hence $N\left(\frac{390}{83},\frac{6}{83},\frac{428}{83}\right)$ [or $N(4.70,0.0723,5.16)$ 3 s.f]	
	Let $N'(x,y,z)$ be the reflection of N in l .	M1: use mid-point theorem to find N'
	$\therefore\ \begin{pmatrix}2\\2\\4\end{pmatrix}=\frac{1}{2}\begin{pmatrix}x+\frac{390}{83}\\y+\frac{6}{83}\\z+\frac{428}{83}\end{pmatrix}$	A1: correct coordinates of N'
	$x=-\frac{58}{83}$, $y=\frac{326}{83}$, $z=\frac{236}{83}$	
	$\therefore\ N'(-\frac{58}{83},\frac{326}{83},\frac{236}{83})$ [or $N'(-0.699,3.93,2.84)$]	
5(i)	$\mathrm{P}(Y \text{ is odd})=\frac{\frac{1}{4}}{1-\frac{1}{4}}=\frac{1}{3}$	**M1** – Using correct GP sum formula with either *a* or *r* correct. **A1** – correct answer.

Marking Scheme for HCI 2018 Prelim Paper 2

Qn	Suggested Solution	Marking Scheme
5(ii)	$P(Y_1+Y_2<4 \mid Y_1+Y_2>2)$ $=\frac{P(2<Y_1+Y_2<4)}{P(Y_1+Y_2>2)}=\frac{P(Y_1+Y_2=3)}{1-P(Y_1+Y_2\le 2)}$ $=\frac{P((Y_1,Y_2)=(0,3),(1,2),(2,1),(3,0))}{1-P((Y_1,Y_2)=(0,0),(0,1),(0,2),(1,0),(1,1),(2,0))}$ $=\frac{2\left[\frac{1}{2}\frac{1}{16}+\frac{1}{4}\frac{1}{8}\right]}{1-\left[\frac{1}{2}\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\right)+\frac{1}{4}\left(\frac{1}{2}+\frac{1}{4}\right)+\frac{1}{8}\frac{1}{2}\right]}=\frac{\frac{1}{8}}{\frac{5}{16}}=\frac{2}{5}$	**M1** – Getting $\frac{P(Y_1+Y_2=3)}{P(Y_1+Y_2>2)}$. **M1** – Getting 6 cases for denominator in the complement method, or 10 cases for denominator in the direct method. **A1** – Getting numerator or denominator correctly. **A1** – Correct answer.
6(i)	No of ways = $2^6 6! = 46080$	**M1** – Getting 2^6 or 6! **A1** – Correct answer.
6(ii)	<u>Method 1</u> No of ways = $6!\left({}^5C_1\right)5!(2)=864000$ <u>Method 2</u> No of ways with all women next to one another and all men next to one another = 6! 6! 2 = 1036800 No of ways with couple seated at 6th and 7th seat together, and all women next to one another and all men next to one another = 5! 5!(6)(2) = 172800 Required no of ways = $1036800-172800=864000$	**M1** – Getting 6! 5! or 5! 5! **M1** – Multiply by 2. **A1** – Correct answer. **M1** – Complement method with either 6! 6! or 5! 5! 6 obtained. **M1** – Multiply by 2. **A1** – Correct answer.
	No of arrangement = $(5-1)!2=48$ Probability required = $\frac{48}{9!}=\frac{1}{7560}$ (or 0.000132 (3 s.f.))	**M1** – Getting $(5-1)!=24$ **A1** – Correct answer.
7(i)		**B1** – Correct relative positions of points for scatter plot **B1** – Correct labelling of axes.
7(ii) (a)	0.902 (3 s.f.)	**B1** – Correct *r* value

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7(ii) (b)	0.955 (3 s.f.)	**B1** – Correct *r* value
7(iii)	From the scatter plot of the data, it seems to fulfil the traits of exponential curve rather than a linear curve. For Model II, the *r* value is closer to 1 as compared to Model I. Thus Model II is better.	**B1** – Correct explanation
7(iv)	$d = e^{cv+d}$ $\ln d = cv + d$ Regression line of ln *d* on *v*: $\ln d = 0.0342758025v + 0.34294554177$ $\ln d = 0.0343v + 0.343$ (3 s.f.)	**B1** – Correct regression line of ln *d* on *v*.
7(v)	When $d = 10$, $\ln 10 = 0.0342758025v + 0.34294554177$ $v = 57.2$ km/h	**B1** – Correct answer
7(vi)	$d = 0.4256v - 11.74$ $(\bar{d}, \bar{v})$ satisfies the regression line. $\Rightarrow \bar{d} = 0.4256\bar{v} - 11.74$ Let d be the distance travelled after brakes are applied. $\Rightarrow \frac{(68.15+d)}{7} = 0.4256\frac{(330+75)}{7} - 11.74$ $\Rightarrow 68.15 + d = (0.4256)(405) - (11.74)(7)$ $\Rightarrow d = 22.04$ metres	**M1** – using $(\bar{d}, \bar{v})$ on regression line. **M1** – calculating $\bar{d}$ or $\bar{v}$ correctly. **A1** – correct answer
8(i)	Let *X* denote the time taken to finish a meal. $\bar{x} = \frac{1}{n}\sum x = \frac{1380}{50} = 27.6$ $s^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right)$ $= \frac{1}{49}\left(83000 - \frac{1380^2}{50}\right)$ $= \frac{44912}{49} = 916.57$ $= 917$ (3 s.f.)	**B1** – correct unbiased estimate of population mean **B1** – correct unbiased estimate of population variance
8(ii)	Let μ be the population mean time taken for a student to finish a meal during recess.	**B1** – Correct H_0 and H_1

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	$H_0 : \mu = 20$ $H_1 : \mu > 20$ Under H_0, test statistic, $Z = \dfrac{\overline{X} - 20}{\dfrac{\sqrt{44912/49}}{\sqrt{50}}} \sim \text{N}(0,\ 1)$ approximately by Central Limit Theorem, since n = 50 is sufficiently large. $p\text{-value} = 0.0379$ (3 s.f.) Since p-value = 0.0379 < 0.05, we reject H_0 at 5% level of significance. There is sufficient evidence to conclude that the time taken by students is more than 20 minutes. Assumption: Time taken by each student is independent of each other and that the students are chosen randomly.	**M1** – Doing Z-test with correct standardisation **A1** – Correct p-value **A1** – Correct conclusion (award only if p-value is correct) **B1** – Correct assumption
8(iii)	5% level of significance means there is a probability of 0.05 of concluding that the population mean time taken by a student to finish a meal during recess is more than 20 minutes when in fact, the population mean time is not more than 20 minutes.	**B1** – Correct definition in context. (accept if underlined phase is "population mean time is 20 minutes")
8(iv)	Jack did the Z-test with 1-tail test and rejected H_0. Jill did the Z-test with 2-tail test. p-value for 1-tail test = 0.0379, which is < 0.05. With the same test statistic, under Z-test 2-tail test, the p-value will be doubled, i.e. 0.0379 x 2 = 0.0758 > 0.05. Jill will not be rejecting H_0 and have a different conclusion from Jack.	**M1** – p-value in 2 tail test = 2(p-value in 1 tail test) **A1** – Correct conclusion
9(i)	Binomial distribution. The colour of a ball chosen is independent of another.	**B1** – Correct reason
9(ii)	Let Y denote no of red balls obtained in 2n draws from the bag. $Y \sim \text{B}\left(2n, \frac{1}{3}\right)$ $\text{P}(Y \le 10) \le 0.5$ When $n = 15$, $\text{P}(Y \le 10) = 0.5848 > 0.5$ When $n = 16$, $\text{P}(Y \le 10) = 0.4836 < 0.5$ Thus least $n = 16$.	**M1** – Distribution of Y and inequality $\text{P}(Y \le 10) \le 0.5$ **M1** – giving probabilities for 2 cases n=15 and n=16. **A1** – Correct answer.
9(iii)	Let S denote no of red balls obtained by Sue. $S \sim \text{B}\left(5, \frac{1}{3}\right)$ Let B denote no of red balls obtained by Ben. $B \sim \text{B}\left(5, \frac{1}{3}\right)$ Required probability	

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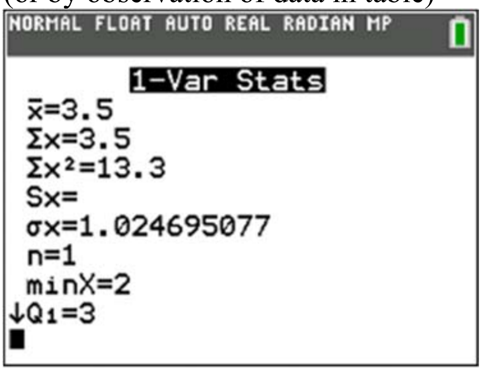
	$= \mathrm{P}(\text{Sue wins} \mid \text{gets} > 3 \text{ red balls})$ $= \dfrac{\mathrm{P}(\text{Sue wins and gets} > 3 \text{ red balls})}{\mathrm{P}(\text{gets} > 3 \text{ red balls})}$ $= \dfrac{\mathrm{P}(S=4)\mathrm{P}(B \le 3) + \mathrm{P}(S=5)\mathrm{P}(B \le 4)}{1-\mathrm{P}(S \le 3)}$ $= \dfrac{0.0433877}{0.045267}$ $= 0.958$	**M1** – apply conditional probability **M1** – correct numerator cases **A1** – correct answer (3 s.f.)
9(iv)	Let X denote number of balls drawn before Sue obtains 2 yellow balls. $\mathrm{P}(X=2) = \mathrm{P}(YY) = \frac{3}{6}.\frac{2}{5} = \frac{1}{5}$ $\mathrm{P}(X=3) = \mathrm{P}(YY'Y, Y'YY) = \frac{3}{6}.\frac{3}{5}.\frac{2}{4}.2 = \frac{3}{10}$ $\mathrm{P}(X=4) = \mathrm{P}(YY'Y'Y, Y'YY'Y, Y'Y'YY) = \frac{3}{6}.\frac{3}{5}.\frac{2}{4}.\frac{2}{3}.3 = \frac{3}{10}$ $\mathrm{P}(X=5)$ $= \mathrm{P}(YY'Y'Y'Y, Y'YY'Y'Y, Y'Y'YY'Y, Y'Y'Y'YY)$ $= \frac{3}{6}.\frac{3}{5}.\frac{2}{4}.\frac{1}{3}.\frac{2}{2}.4 = \frac{1}{5}$ Expected number $= (2)\left(\frac{1}{5}\right) + (3)\left(\frac{3}{10}\right) + (4)\left(\frac{3}{10}\right) + (5)\left(\frac{1}{5}\right)$ $= 3.5$ (or by observation of data in table)  Use $\mathrm{Var}(X) = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2$	 **B1** – correct probability for 2 out of the 4 **B1** – All probabilities correct **B1** – Correct Expectation **M1** – Correct method to get variance (SOI) **A1** – Var (X) correct

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	$\text{Var}(X) = 1.05 = \frac{21}{20}$	
10	Let D be the time taken to drive to office. $D \sim \text{N}(14, 2.1^2)$ Let T be the time held up at a traffic light junction. $T \sim \text{N}(\mu, 0.2^2)$ $D + T_1 + T_2 + T_3 + T_4 \sim \text{N}(14+4\mu, 4.57)$ $\text{P}(D + T_1 + T_2 + T_3 + T_4 \le 20) = 0.713$ $\text{P}(Z \le \frac{20-(14+4\mu)}{\sqrt{4.57}}) = 0.713$ $\frac{6-4\mu}{\sqrt{4.57}} = 0.5621702875$ $\mu = 1.19955$ $= 1.2$ (1 d.p.) The driving time and the time held up at the traffic light junctions are independent of one another.	**M1 –** getting $\text{N}(14+4\mu, 4.57)$ correctly and formulating $\text{P}(D + T_1 + T_2 + T_3 + T_4 \le 20) = 0.713$ **M1 –** attempt to solve by standardization or from GC. **AG1** – correct answer **B1** – correct assumption
10(i)	Probability Tony is late on 2 days in the first 9 days $= {}^9C_2 (0.713)^7 (1-0.713)^2 = 0.2777749021$ P(Tony is late at his office for the third time on the 10th day) $= {}^9C_2 (0.713)^7 (1-0.713)^2 (1-0.713)$ =0.0797 (3 s.f.)	**M1** – showing $\alpha(0.713)^7(1-0.713)^2$ or showing the use of $\text{B}(9,\ 1-0.713)$ **A1** – correct answer
10(ii)	$D - 10T \sim \text{N}(2, 8.41)$ $\text{P}(D < 10T) = \text{P}(D - 10T < 0) = 0.245$ (3 s.f.)	**M1** – getting distn of $D - 10T$ **A1** – correct answer
10(iii)	$D - (T_1 + T_2 + T_3 + T_4) \sim \text{N}(9.2, 4.57)$ $\text{P}(\lvert D - (T_1 + T_2 + T_3 + T_4)\rvert > 8)$ $= 1 - \text{P}(-8 < D - (T_1 + T_2 + T_3 + T_4) < 8)$ $= 1 - 0.28728$ $= 0.713$ (3 s.f.)	**M1** – getting $\text{N}(9.2, 4.57)$ with at least one of 2 parameters correct **M1 –** formulate correctly $\text{P}(\lvert D - (T_1 + T_2 + T_3 + T_4)\rvert > 8)$ and attempt to remove modulus **A1** – correct answer
10(iv)	Let μ be the population mean time held up at a traffic light junction, $\bar{X}$ be the mean time held up at a traffic light in a sample of size *n*. $\bar{X} \sim \text{N}\left(\mu, \frac{0.2^2}{n}\right)$	**M1 –** getting distn of $\bar{X}$ correctly and formulating $\text{P}\left(\lvert \bar{X} - \mu \rvert \le \frac{1}{12}\right) \ge 0.98$

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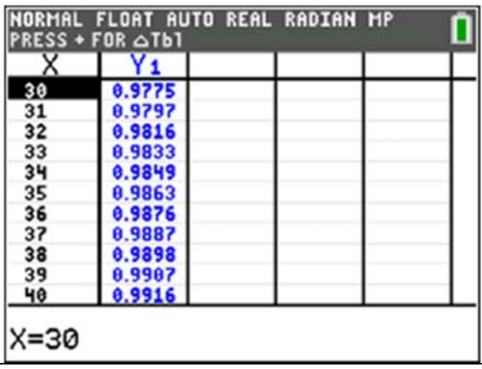
	Given $\mathrm{P}\left(\lvert \overline{X}-\mu \rvert \le \frac{1}{12}\right) \ge 0.98$ $\mathrm{P}\left(\lvert Z \rvert \le \frac{\frac{1}{12}}{\frac{0.2}{\sqrt{n}}}\right) \ge 0.98$ $\mathrm{P}\left(-\frac{5\sqrt{n}}{12} \le Z \le \frac{5\sqrt{n}}{12}\right) \ge 0.98$ $\frac{5\sqrt{n}}{12} \ge 2.326347877$ $n \ge 31.17$ Smallest $n = 32$ OR Using GC Tables, $n=31, \mathrm{P}\left(-\frac{5\sqrt{n}}{12} \le Z \le \frac{5\sqrt{n}}{12}\right) = 0.9797$ $n=32, \mathrm{P}\left(-\frac{5\sqrt{n}}{12} \le Z \le \frac{5\sqrt{n}}{12}\right) = 0.9816$ 	**M1** – attempt to solve by standardizing or showing list of GC table values **A1** – correct answer