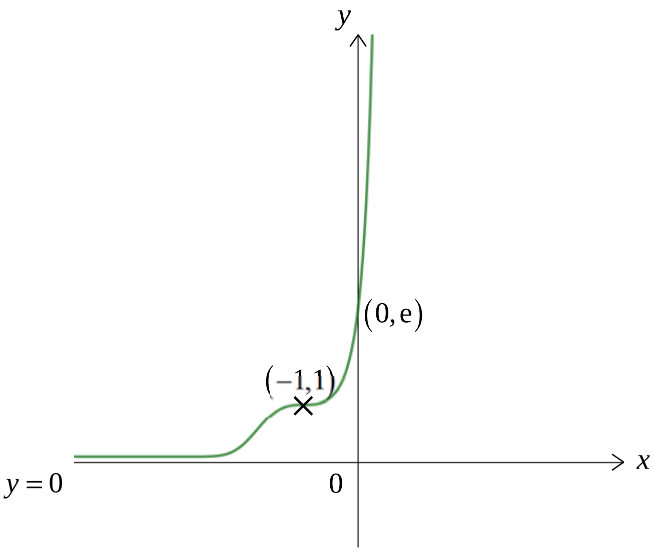
Qn	Solution
1	$\int_{-1}^{0}\left(\sqrt{e^{x}}-\frac{3}{\sqrt{e^{x}}}\right)^{2}dx=\int_{-1}^{0}\left(\left(e^{x}\right)^{\frac{1}{2}}-3\left(e^{x}\right)^{-\frac{1}{2}}\right)^{2}dx$ $=\int_{-1}^{0}\left(\left(e^{x}\right)-6+9\left(e^{-x}\right)\right)dx$ $=\left[e^{x}-6x-9e^{-x}\right]_{-1}^{0}$ $=\left[e^{0}-0-9e^{0}\right]-\left[e^{-1}-6(-1)-9e^{-1(-1)}\right]$ $=(1-0-9)-\left(e^{-1}+6-9e\right)$ $=-14-e^{-1}+9e$

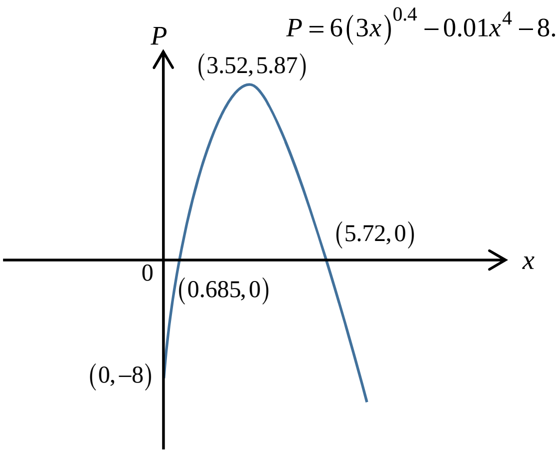
Qn	Solution
2(i)	$y = ax^2 + 2x - 1$ Since the curve has 2 real distinct roots, $b^2 - 4ac > 0,\ a \neq 0$ $4 - 4(a)(-1) > 0$ $a > -1,\ a \neq 0$
(ii)	$y = ax^2 + c$ **Method 1** Min Point of curve: $(0, c)$ Since the curve does not intersect the line $y = 2$ and $a > 0$, $c > 2$ **Method 2** $ax^2 + c = 2$ $ax^2 + (c-2) = 0$ Since the curve does not intersect the line, $b^2 - 4ac < 0$ $-4(a)(c-2) < 0$ $4(a)(c-2) > 0$ Since $a > 0,\ c - 2 > 0 \Rightarrow c > 2$

Qn	Solution
3(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln(x^2+9)\right] = \frac{2x}{x^2+9}$
(ii)	$\frac{A}{1-x} + \frac{Bx}{x^2+9} = \frac{A(x^2+9)+(1-x)(Bx)}{(1-x)(x^2+9)}$ $= \frac{Ax^2+9A+Bx-Bx^2}{(1-x)(x^2+9)}$ $= \frac{(A-B)x^2+Bx+9A}{(1-x)(x^2+9)}$

Qn	Solution
	Comparing coefficients with $\dfrac{2x^2-x+9}{(1-x)(x^2+9)}$: $x^2: A-B=2$ $x: B=-1$ $\text{constant}: 9A=9 \Rightarrow A=1$ Therefore, $A=1, B=-1$ Hence $\dfrac{2x^2-x+9}{(1-x)(x^2+9)}=\dfrac{1}{1-x}-\dfrac{x}{x^2+9}$ (shown)
(iii)	$\displaystyle\int \frac{2x^2-x+9}{(1-x)(x^2+9)}\,\mathrm{d}x=\int \frac{1}{1-x}-\frac{x}{x^2+9}\,\mathrm{d}x$ $\displaystyle=\int \frac{1}{1-x}\,\mathrm{d}x-\frac{1}{2}\int \frac{2x}{x^2+9}\,\mathrm{d}x$ $\displaystyle=-\ln\lvert 1-x\rvert-\frac{1}{2}\ln(x^2+9)+C$

Qn	Solution
4(i)	$y = e^{(x+1)^3}$ $\frac{dy}{dx} = e^{(x+1)^3} \times \frac{d}{dx}\left[(x+1)^3\right]$ $= 3(x+1)^2 e^{(x+1)^3}$ Since $(x+1)^2 \geq 0$ and $e^{(x+1)^3} > 0$ therefore $\frac{dy}{dx} \geq 0$.
(ii)	$\frac{dy}{dx} = 0$ $3(x+1)^2 e^{(x+1)^3} = 0$ Since $e^{(x+1)^3} > 0$ for all x, $(x+1)^2 = 0$ $x = -1$ When $x = -1$, $y = 1$ so the coordinates of the stationary point is $(-1,1)$.
(iii)	

(iv)	$\frac{\mathrm{d}y}{\mathrm{d}x}=3(x+1)^2\,\mathrm{e}^{(x+1)^3}$ when $x=-\frac{3}{2}$, $y=\mathrm{e}^{\left(-\frac{3}{2}+1\right)^3}=\mathrm{e}^{-\frac{1}{8}}$, $\frac{\mathrm{d}y}{\mathrm{d}x}=3\left(\frac{1}{4}\right)\mathrm{e}^{-\frac{1}{8}}=\frac{3}{4}\mathrm{e}^{-\frac{1}{8}}$ Equation of tangent at $x=-\frac{3}{2}$: $y-\mathrm{e}^{-\frac{1}{8}}=\frac{3}{4}\mathrm{e}^{-\frac{1}{8}}\left[x-\left(-\frac{3}{2}\right)\right]$ $y=\frac{3}{4}\mathrm{e}^{-\frac{1}{8}}\left(x+\frac{3}{2}\right)+\mathrm{e}^{-\frac{1}{8}}=\frac{3}{4}\mathrm{e}^{-\frac{1}{8}}x+\frac{9}{8}\mathrm{e}^{-\frac{1}{8}}+\mathrm{e}^{-\frac{1}{8}}$ $=\frac{3}{4}\mathrm{e}^{-\frac{1}{8}}x+\frac{17}{8}\mathrm{e}^{-\frac{1}{8}}$ Therefore, $m=\frac{3}{4}\mathrm{e}^{-\frac{1}{8}}$, $c=\frac{17}{8}\mathrm{e}^{-\frac{1}{8}}$
(v)	$\mathrm{e}^{(x+1)^3}\le\frac{3}{4}\mathrm{e}^{-\frac{1}{8}}x+\frac{17}{8}\mathrm{e}^{-\frac{1}{8}}$ By GC $-2.83\le x\le -0.173$

Qn	Solution
5(i)	Let M, D and W be the manufacturing cost (in $) of a 100 gram Milk, Dark and White chocolate bar respectively. 500 grams of Milk chocolate bars is equivalent to 5 Milk chocolate bars. $5M = D$ $10D - 8W = 16.7$ $6M + 5D + 3W = 25.08$ or $5M - D + 0W = 0$ $0M + 10D - 8W = 16.7$ $6M + 5D + 3W = 25.08$ By GC: $M = 0.63$ $D = 3.15$ $W = 1.85$
(ii)	 Maximum value of P is 5.87 (2.d.p.).
(iii)	By GC, $\frac{dP}{dx} = 1.77\,(2.d.p)$ at $x = 2.3$. When manufacturing cost to produce a Caramel chocolate bar is $2.30, every dollar spent on manufacturing cost will generate a profit of $1.77.
(iv)	$\text{Profit} = 6\left[3(2.3)\right]^{0.4} - 0.01(2.3)^4 - 8 = \$4.7126$ $\text{Selling price} = 4.7126 + 2.3 = \$7.01\,(2.d.p.)$

(v) When $x=6$, $P=-1.89397$. This suggests that when there is no manufacturing cost, the company will make a loss at \$1.89 dollars per Caramel chocolate bar, which is not advisable.

Or

$\dfrac{\mathrm{d}P}{\mathrm{d}x}$ at $x=6$ is $-7.37(2\text{d.p.})$ which suggest that the company will make a loss of \$7.37 for every dollar spent when the manufacturing cost to produce a Caramel chocolate bar is at \$6.

(vi) $C=\dfrac{1}{8}q^2(q-4)+3=\dfrac{1}{8}q^3-\dfrac{1}{2}q^2+3$

$\dfrac{\mathrm{d}C}{\mathrm{d}q}=\dfrac{3}{8}q^2-q=q\left(\dfrac{3}{8}q-1\right)$

To find the minimum value of C as q varies,

$\dfrac{\mathrm{d}C}{\mathrm{d}q}=q\left(\dfrac{3}{8}q-1\right)=0$

$q=0$ or $C=\dfrac{3}{8}q-1=0\Rightarrow q=\dfrac{8}{3}$

Reject $q=0$ since $q>0$

To calculate minimum value of C,

$C=\dfrac{1}{8}\left(\dfrac{8}{3}\right)^2\left(\dfrac{8}{3}-4\right)+3=\dfrac{49}{27}=1.81(2\text{d.p.})$

To show that it is minimum value,

$q=\dfrac{8}{3}-0.01$ $=2.6567$	$q=\dfrac{8}{3}$	$q=\dfrac{8}{3}+0.01$ $=2.6767$
$\dfrac{\mathrm{d}C}{\mathrm{d}q}=-0.00996$	$\dfrac{\mathrm{d}C}{\mathrm{d}q}=0$	$\dfrac{\mathrm{d}C}{\mathrm{d}q}=0.0101$
\	—	/

OR

	$\frac{\mathrm{d}^2C}{\mathrm{d}q^2} = \frac{3}{4}q - 1$ when $q = \frac{8}{3}, \frac{\mathrm{d}^2C}{\mathrm{d}q^2} = \frac{3}{4}\left(\frac{8}{3}\right) - 1 = 1 > 0$ Hence $C = 1.8148 = 1.81 (3.\text{s.f.})$ is a minimum value at $q = \frac{8}{3}$ The lowest marginal cost of \$1814.81 is achieved when the quantity of chocolate bar manufactured is about 2666.
(vii)	By GC $\int_0^1 \left[\frac{1}{8}q^2(q-4)+3\right] \mathrm{d}t = 2.864583 = 2.86 (3.\text{s.f.})$ The total cost of manufacture for the first thousand of Caramel chocolate bars is \$2864.58.

Qn	Solution
6(i)	Required probability $= \frac{3!\times 3\times 2}{5!} = \frac{36}{120} = \frac{3}{10}$ Or $\frac{3}{5}\times\frac{2}{4} = \frac{3}{10}$
(ii)	Number of ways such that all odd digits are together $= 3!\times 3! = 36$ Number of ways such that not all odd digits are together $= 5! - 36 = 84$ Required probability $= \frac{84}{5!} = \frac{7}{10}$
(iii)	Case 1: 3 or 5 as the first digit Probability $= \frac{2\times 2\times 3!}{5!} = \frac{1}{5}$ Case 2: 4 as the first digit Probability $= \frac{1\times 3\times 3!}{5!} = \frac{3}{20}$ Required probability $= \frac{1}{5} + \frac{3}{20} = \frac{7}{20}$

Qn	Solution
7(i)	Let Y denote the number of insurance agents with ADI out of 30 randomly chosen Prodential insurance agents. Then $$Y \sim \mathrm{B}(30,\ 0.052).$$ $\mathrm{P}(Y \geq 3) = 1 - \mathrm{P}(Y \leq 2)$ $= 0.2032366271$ $= 0.203 (3\text{s.f.})$
(ii)	In order for the number of insurance agents to follow a binomial distribution, the **probability of an agent chosen to have an ADI must be constant**. This can be assumed true only when the company is large.
(iii)	Let W denote the number of insurance agents with ADI out of 10 randomly chosen Avila insurance agents. Then $$W \sim \mathrm{B}(10,\ p).$$ $\mathrm{P}(W = 5) = 0.12294$ $\binom{10}{5} p^5 (1-p)^5 = 0.12294$ $p(1-p) = \left(\frac{0.12294}{252}\right)^{\frac{1}{5}} \approx 0.21760,$ i.e., $k = 0.21760 (5\text{s.f.})$ $p^2 - p + 0.21760 = 0$ $p = 0.68 \text{ or } 0.32$ Since $p < 0.5$, $p = 0.32$
(iv)	$\mathrm{E}(W) = 10 \times 0.24 = 2.4$ $\mathrm{Var}(W) = 10 \times 0.24 \times (1 - 0.24) = 1.824$
(v)	Since sample size, 40, is large, by Central Limit Theorem, $$\overline{W} \sim \mathrm{N}\left(2.4, \frac{1.824}{40}\right) \text{ approximately.}$$ Therefore, $\mathrm{P}(2.3 < \overline{W} < 2.5) \approx 0.36042 = 0.360$ (to 3 s.f.).

<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td>8</td><td>
<table>
<tr><th></th><th>Senior High</th><th>Junior High</th><th>Total</th></tr>
<tr><td>Woodwind</td><td>8</td><td>16</td><td>24</td></tr>
<tr><td>Brass</td><td>$\frac{2}{5}n$</td><td>$\frac{3}{5}n$</td><td>$\boldsymbol{n}$</td></tr>
<tr><td>Percussion</td><td>8</td><td>2</td><td>10</td></tr>
<tr><td>Total</td><td>$\frac{2}{5}n+16$</td><td>$\frac{3}{5}n+18$</td><td>$\boldsymbol{n+34}$</td></tr>
</table>
</td></tr>
<tr><td>8(i)</td><td>
P(brass player | SH student)
$= \frac{\text{P}(\text{brass player} \cap \text{SH student})}{\text{P}(\text{SH student})}$
$= \frac{\text{No. of Senior High brass players}}{\text{No. of Senior High concert band students}}$
$= \frac{\frac{2}{5}n}{\frac{1}{3}\times 24+\frac{2}{5}n+\frac{4}{5}\times 10}$
$= \frac{\frac{2}{5}n}{\frac{2}{5}n+16}$
$= \frac{n}{n+40}$
</td></tr>
<tr><td>(ii)</td><td>
P(neither SH student nor percussion player)
= P(either JH woodwind player or JH brass player)
$= \frac{\text{No. of JH woodwind players} + \text{No. of JH brass players}}{\text{Total no. of concert band students}}$
$= \frac{16+\frac{3}{5}n}{24+n+10}$
$= \frac{\frac{3}{5}n+16}{n+34}$
$= \frac{3n+80}{5n+170}$
</td></tr>
</table>

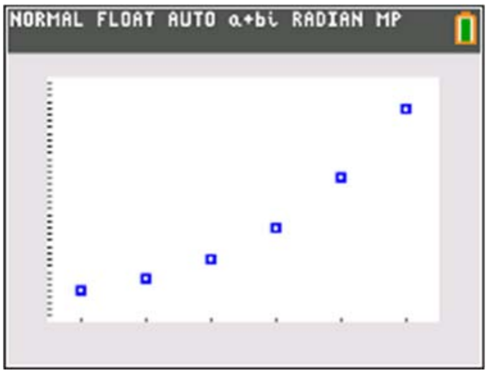
(iii)	$\mathrm{P}(\text{SH student or percussion player but not both})$ $= \mathrm{P}(\text{SH student or percussion player})$ $- \mathrm{P}(\text{SH student and percussion player})$ $= \dfrac{\left(\frac{2}{5}n + 8 + 8 + 2\right) - 8}{n + 34}$ $= \dfrac{\frac{2}{5}n + 10}{n + 34}$ $\dfrac{\frac{2}{5}n + 10}{n + 34} = \dfrac{1}{3}$ $\dfrac{6}{5}n + 30 = n + 34$ $6n + 150 = 5n + 170$ $n = 20$
(iv)	$\mathrm{P}(\text{JH student and brass player}) = \dfrac{12}{54} = \dfrac{2}{9}$ $\mathrm{P}(\text{JH student}) = \dfrac{30}{54} = \dfrac{5}{9}$ $\mathrm{P}(\text{brass player}) = \dfrac{20}{54} = \dfrac{10}{27}$ $\mathrm{P}(\text{JH student}) \times \mathrm{P}(\text{brass player}) = \dfrac{50}{243}$ $\mathrm{P}(\text{JH student}) \times \mathrm{P}(\text{brass player})$ $\neq \mathrm{P}(\text{JH student and brass player})$ Hence the two events are not independent.

Qn	Solution
9(i)	$\bar{x} = \frac{\sum x}{n} = \frac{23}{100} + 15 = 15.23$ $s^2 = \frac{1}{99}[113.26 - \frac{(23)^2}{100}] \approx 1.09061 = 1.09(3\text{s.f.})$
(ii)	Test $H_0 : \mu = 15$ against $H_1 : \mu \neq 15$ Level of significance = 5% (2-tailed) Under H_0, $\bar{X} \sim N\left(15, \frac{1.09061}{100}\right)$ approximately by Central Limit Theorem since sample size $n = 100$ is large. Hence $Z = \frac{\bar{X} - 15}{\sqrt{\frac{S^2}{100}}} : \; N(0,1)$ approximately. From GC, Critical region: $z \leq -1.95996$ or $z \geq 1.95996$ Test statistic, $z = \frac{15.23 - 15}{\sqrt{\frac{1.09061}{100}}} = 2.202 > 1.95996$ Or p-value = 0.0276 < 0.05 H_0 is rejected. Hence we conclude that there is sufficient evidence at the 5 % significance level to show that the manufacturer's claim is not valid.
(iii)	It means that there is a probability of 0.05 of wrongly rejecting the claim that the mean diameter of ball is 15 cm.
(iv)	No assumption about the diameter of the balls is required. Given that the sample size 100 (> 20) is large, the mean diameter of the balls will follow a Normal Distribution approximately by Central Limit Theorem.
Last part	Do not reject H_0 if p-value $> \frac{\alpha}{100}$

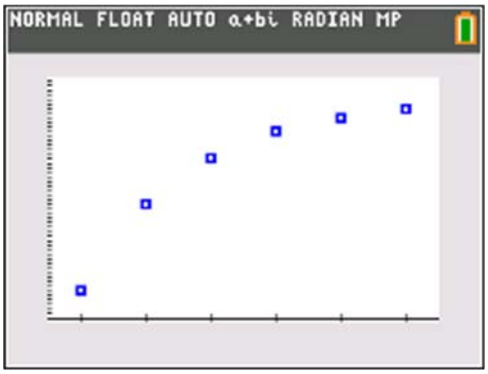
Qn	Solution
	From GC, p-value = 0.016670 $\therefore \frac{\alpha}{100} < 0.016670 \Rightarrow \alpha < 1.67 \text{ (3.s.f.)}$

<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td>10(i)</td><td>

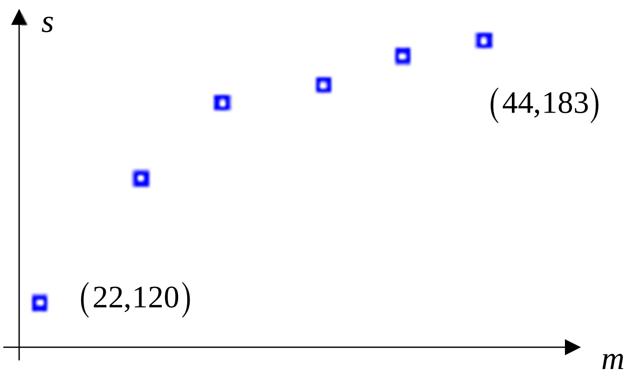
$y = a + bx^2$



$y = c + \frac{d}{x}$



</td></tr>
<tr><td>(ii)</td><td>



End points: (22, 120) & (44, 183)

</td></tr>
<tr><td>(iii)</td><td>

- The equation $y = c + \frac{d}{x}$, is more appropriate.

</td></tr>
</table>

Qn	Solution
	• From the scatter diagram, as m increases, s increases at an decreasing rate. $r = -0.982$
(iv)	Regression line of s on $\frac{1}{m}$: $s = 248.61 - 2731.4\left(\frac{1}{m}\right)$ $= 249 - \frac{2730}{m}\ (3\text{s.f.})$ Regression line of $\frac{1}{m}$ on s: $\frac{1}{m} = 0.088937 - 0.00035326\, s$ $= 0.0889 - 0.000353s\ (3\text{s.f.})$
(v)	Since systolic blood pressure depends on weight, we should use the regression line of s on $\frac{1}{m}$: $110 = 248.61 - 2731.4\left(\frac{1}{m}\right)$ $m \approx 19.706$ Since $s = 110$ lies outside the range of values of s, **extrapolation** is required which gives an **unreliable** estimate.
(vi)	• Too few patients were selected for the equation of the regression line to be reliable in estimation. • The data is only valid for estimating the blood pressure for people of a similar age profile. • The data is only valid for estimating the blood pressure of patients with similar medical conditions. • A person's blood pressure is not fixed and is influenced by other factors at time of measurement, such as physical activity and/or varying emotional states like anxiety.

Qn	Solution
11**(i)**	Let A be the random variable denoting the mass of a Red Prawn durian. $A \sim \mathrm{N}\left(0.25, k^2\right)$ $\mathrm{P}\left(A \le 0.15\right) = 0.2$ $\mathrm{P}\left(Z \le \frac{0.15-0.25}{k}\right) = 0.2$ $\mathrm{P}\left(Z \le \frac{-0.10}{k}\right) = 0.2$ $-\frac{0.10}{k} = -0.84162$ $k = 0.119\left(3\text{s.f.}\right)$
(ii)	$\mathrm{P}\left(A < 0.24\right)\mathrm{P}\left(0.24 \le A \le 0.26\right)\mathrm{P}\left(A \ge 0.26\right) \times 3! = 0.219$
(iii)	Let B be the random variable denoting the mass of a Black Gold durian. Let $T = \left(A_1 + A_2 + ... + A_6\right) - \left(B_1 + B_2 + ... + B_5\right)$ $\mathrm{E}\left(T\right) = 6\mathrm{E}\left(A\right) - 5\mathrm{E}\left(B\right) = 6\left(0.25\right) - 5\left(0.35\right) = -0.25$ $\mathrm{Var}\left(T\right) = 6\mathrm{Var}\left(A\right) + 5\mathrm{Var}\left(B\right) = 6\left(0.02\right)^2 + 5\left(0.03\right)^2 = 0.0069$ $\mathrm{P}\left(-0.2 < T < 0.2\right) = 0.274$
(iv)	Let \$$V$ and \$$W$ be the amount that Mr Phang and Mr Fong pay respectively.

$V = 1.5\left(A_{V1} + A_{V2} + A_{V3}\right) + 2.4\left(B_1 + B_2 + B_3\right)$

$W = 1.5\left(A_{W1} + A_{W2} + ... + A_{W10}\right)$

Need to compute $\mathrm{P}(V > W)$ or $\mathrm{P}(V - W > 0)$

$$
\begin{aligned}
&\mathrm{E}(V - W) = \mathrm{E}(V) - \mathrm{E}(W) \\
&= (1.5)(3)\,\mathrm{E}(A) + (2.4)(3)\,\mathrm{E}(B) - \left[(1.5)(10)\,\mathrm{E}(A)\right] \\
&= 3.645 - 3.75 \\
&= -0.105
\end{aligned}
$$

$$
\begin{aligned}
&\mathrm{Var}(V - W) = \mathrm{Var}(V) + \mathrm{Var}(W) \\
&= (1.5)^2(3)\,\mathrm{Var}(A) + (2.4)^2(3)\,\mathrm{Var}(B) + (1.5)^2(10)\,\mathrm{Var}(A) \\
&= 0.018252 + 0.009 \\
&= 0.027252
\end{aligned}
$$

$V - W : \mathrm{N}(-0.105, 0.027252)$

$\mathrm{P}(V - W > 0) = 0.262$

NATIONAL JUNIOR COLLEGE
SENIOR HIGH 2 PRELIMINARY EXAMINATION
Higher 1

MATHEMATICS 8865/01

Paper 1 **11 September 2018**

3 hours

Additional Materials: Answer Paper
List of Formulae (MF26)
Cover Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, registration number, subject tutorial group, on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in the brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

1 Find the exact value of $\int_{-1}^{0}\left(\sqrt{e^x}-\frac{3}{\sqrt{e^x}}\right)^2 dx$. [3]

2 The curve C has equation $y=ax^2+bx+c$.

(i) Given that $b=2$ and $c=-1$, find the range of values of a such that C has 2 real distinct roots for all real values of x. [2]

(ii) Given that $a>0$ and $b=0$, find the range of values of c such that C does not intersect the line $y=2$. [2]

3 **(i)** Differentiate $\ln\left(x^2+9\right)$. [1]

(ii) Express $\frac{2x^2-x+9}{(1-x)\left(x^2+9\right)}$ in the form $\frac{A}{1-x}+\frac{Bx}{x^2+9}$ where A and B where are integers to be determined. [2]

(iii) Hence find $\int \frac{2x^2-x+9}{(1-x)\left(x^2+9\right)} dx$. [3]

4 A curve C has equation $y=e^{(x+1)^3}$.

(i) Find $\frac{dy}{dx}$ and explain why $\frac{dy}{dx}\geq 0$ for all real values of x. [3]

(ii) Hence find the exact coordinates of the stationary point of C. [2]

(iii) Sketch C, stating the exact coordinates of any points of intersection with the axes and the equations of any asymptotes. Mark the point of inflexion with a cross. [3]

(iv) Find the equation of the tangent to C at the point where $x=-1.5$, giving your answer in the form of $y=mx+c$ where m and c are exact constants. [2]

(v) Using the values of m and c found in **(iv)**, find the range of values of x such that $e^{(x+1)^3}\leq mx+c$. [1]

[Turn over

5 A company produces three flavours of chocolate bars: Milk, Dark and White. Each chocolate bar weighs 100 grams. The manufacturing cost of 500 grams of Milk chocolate bars is the same as the manufacturing cost of a Dark Chocolate bar. The total manufacturing cost of 10 Dark chocolate bars is $16.70 more than the manufacturing cost of 8 White chocolate bars. The total manufacturing cost of 6 Milk, 5 Dark Chocolate and 3 White chocolate bars is $25.08.

(i) By writing down three linear equations, find the manufacturing cost of each flavour of chocolate bars. [3]

The company is trialling a new flavour of chocolate bar, Caramel. A financial consultant for the company predicts that the profit $*P*, generated by selling a Caramel chocolate bar, will be related to the manufacturing cost ($*x*) by the equation

$$P = 6(3x)^{0.4} - 0.01x^4 - 8, \ \ x \geq 0 .$$

(ii) Sketch the graph of P against x, stating the coordinates of the points where the graph crosses the x- and P-axes. Find the maximum value of P. [3]

(iii) Find the value of $\frac{\mathrm{d}P}{\mathrm{d}x}$ at $x = 2.3$ and give an interpretation of the value found in the context of the question. [2]

(iv) Given that the manufacturing cost of a Caramel chocolate bar is $2.30, state the selling price. $\left[\text{Profit } = \text{ selling price} - \text{manufacturing cost}\right]$ [1]

(v) If the manufacturing cost of a Caramel chocolate bar is increased to $6, would you advise the company to produce chocolate bar of this flavour? Justify your answer. [1]

The financial consultant predicts that the marginal cost of manufacturing a Caramel chocolate bar, C (in thousands) is related to the quantity manufactured, q (in thousands) by the equation

$$C = \frac{1}{8}q^2(q-4)+3, \ \ q > 0 .$$

[Marginal cost = additional cost incurred in the production of one more unit of a good or service]

(vi) Use differentiation to find the minimum value of C, justifying that the value is minimum. Give an interpretation of the value found in the context of the question. [4]

(vii) State the area bounded by the curve C, the line $q = 1$ and the axes. Give an interpretation of the value found in the context of the question. [2]

Section B: Statistics [60 marks]

6 The digits 1, 2, 3, 4 and 5 are arranged randomly to form a five-digit number. No digit is repeated. Find the probability that

(i) the last two digits are both odd, [2]

(ii) not all odd digits are together, [3]

(iii) the number is greater than 30 000 and odd. [3]

7 5.2% of all insurance agents from a large insurance company, Prodential, obtained an Advanced Diploma in Insurance (ADI). Each agent can only receive one ADI each. A sample that contains 30 randomly chosen agents from Prodential is obtained.

(i) Find the probability that at least three insurance agents obtained an ADI each. [2]

(ii) Explain the importance of choosing a sample from a 'large' company. [1]

100*p*% of all insurance agents from another large insurance company, Avila, obtained an Advanced Diploma in Insurance (ADI), where $p < 0.5$. A sample that contains 10 randomly chosen agents from Avila is obtained. It is given that the number of insurance agents with ADI in this sample can be modelled by a binomial distribution.

(iii) Given the probability that 5 agents from Avila obtained an ADI each is 0.12294, show that p satisfies an equation of the form $p(1-p)=k$ where k is a constant to be determined. Hence find the value of p correct to 2 decimal places. [3]

Given instead that $p = 0.24$.

(iv) Based on the same sample, state the mean and variance of the number of agents from Avila who have obtained an ADI. [1]

(v) Forty samples, where each sample consists of 10 randomly chosen agents from Avila, are chosen. Find the probability that the mean number of insurance agents with ADI is between 2.3 and 2.5. [2]

8 A school's concert band comprises 24 woodwind players, n brass players and 10 percussion players. $\frac{1}{3}$ of all woodwind players, $\frac{2}{5}$ of all brass players and $\frac{4}{5}$ of all percussion players are Senior High students, while the rest are Junior High students.

One student from the concert band is selected at random.

(i) Given that he or she is a Senior High student, show that the probability of selecting a brass player is $\frac{n}{n+40}$. [2]

(ii) Find, in terms of n, the probability that the student is neither a percussion player nor a Senior High student. [2]

(iii) The probability of selecting a Senior High student or a percussion player but not both is $\frac{1}{3}$. Find the value of n. [3]

(iv) Using the value of n found in **(iii)**, explain whether the event of selecting a Junior High student is independent of the event of selecting a brass player. [2]

9 A ball manufacturer claims that average diameter of the balls is 15 cm. To test this claim, a random sample of 100 balls is checked and the diameters, x cm, are summarised by

$$\sum(x-15)=23 \qquad \sum(x-15)^2=113.26.$$

(i) Find the unbiased estimates of the population mean and variance. [2]

(ii) Test, at the 5% significance level whether the manufacturer's claim is valid. [4]

(iii) Explain, in the context of the question, the meaning of "at the 5% significance level". [1]

(iv) State, giving a reason, whether any assumption is needed in order for the test to be valid. [1]

A new sample of a 100 balls is collected and the mean diameter of this sample is 15.25 cm. A test, at the $\alpha\%$ significance level, shows that the manufacturer's claim is justified. Find the set of values of α. [2]

10 **(i)** Sketch a scatter diagram that might be expected when x and y are related approximately as given in each of the cases (A) and (B) below. In each case your diagram should include 6 points, approximately equally spaced with respect to x, and with all x- and y- values positive.

(A) $y = a + bx^2$, where a and b are positive constants.

(B) $y = c + \dfrac{d}{x}$, where c is a positive constant and d is a negative constant. [2]

Obesity is becoming increasingly prevalent across the globe. To investigate the effects of obesity on one's health, a study was conducted to determine if the blood pressures of adults aged between 40 and 50 years old are dependent on their Body Mass Index (BMI). Data from six patients in this age-group from a hospital was collected. Their BMI, m, kg m^{-2} and systolic blood pressure, s, in mmHg, are as follows.

m	22	27	31	36	40	44
s	120	150	168	172	179	183

(ii) Sketch the scatter diagram for these values, labelling the axes clearly. [2]

(iii) Using your answer to part **(ii)**, explain why model (B) is more appropriate for modelling these values and calculate the product moment correlation coefficient for this case. [3]

(iv) Find the equation of the regression lines of s on $\dfrac{1}{m}$ and $\dfrac{1}{m}$ on s. [2]

(v) Choose an appropriate line found in part **(iv)** and use it to estimate the BMI of another patient (of a similar age profile) whose systolic blood pressure is 110 mmHg. Comment on the reliability of your estimate. [2]

(vi) State, in context, a limitation of using the regression equation in part **(v)** to estimate the systolic blood pressure of *other* people with known BMI in the interval $22 \le m \le 44$. [1]

11 In this question you should state clearly the values of the parameters of any normal distribution you use.

A supermarket sells two types of durians, Red Prawn and Black Gold. The masses, in kilograms, of the durians each have independent normal distributions. The means and standard deviations of these distributions, and the selling prices, in $ per kilogram, are shown in the following table.

	Mean mass (kg)	Standard deviation (kg)	Selling price ($ per kg)
Red Prawn	0.25	k	1.50
Black Gold	0.35	0.03	2.40

(i) Over a long period of time, it is found that 20% of Red Prawn durians have mass equal to or less than 0.15 kg. Find the value of k. [2]

Suppose that the true value of k is 0.02.

(ii) Three Red Prawn durians are randomly selected. Find the probability that exactly one of the durians has mass less than 0.24 kg and exactly one of the durians has mass more than 0.26 kg. [2]

(iii) Find the probability that the total mass of six randomly chosen Red Prawns durians is within 0.2 kg of the total mass of five randomly chosen Black Gold durians. [4]

Mr Phang buys three Red Prawn durians and three Black Gold durians. Mr Fong buys ten Red Prawn durians.

(iv) Find the probability that Mr Phang pays more than Mr Fong. [4]

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