ZHONGHUA SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2024
SECONDARY 4 EXPRESS/ 5 NORMAL (ACADEMIC)

Candidate's Name	Class	Register Number

ADDITIONAL MATHEMATICS — 4049/02

PAPER 2

9 September 2024

Candidates answer on the Question Paper.
No Additional Materials are required.

2 hours 15 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
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The total number of marks for this paper is **90**.

For Examiner's Use
90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 In Singapore, car owners need to secure a Certificate of Entitlement (COE) before they can register their vehicle and use it for 10 years.

David bought a Toyota Prius for \$170 000 in January 2022. The value of the car, C, can be modelled by the equation $C = k + 150000e^{nt}$, where k and n are constants, t is the number of years after 2022 and $0 \le t \le 10$.

(a) Show that $k = 20000$. [2]

It is given that the value of the car is worth \$140 000 in 2023.

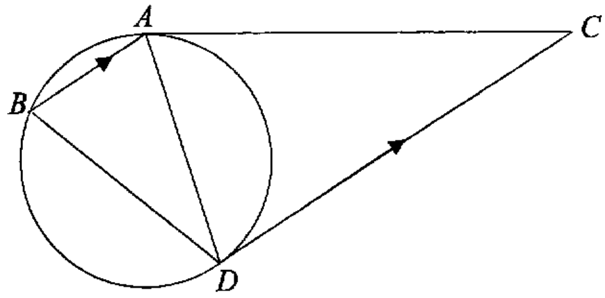
(b) Find the value of n. [3]

(c) David intends to sell the car before it depreciates below \$70 000. Which is the latest year that David has to sell the car? [4]

2 **(a)** By considering the general term in the binomial expansion of $\left(px+\frac{2}{x}\right)^7$, where p is a constant, explain why every term is dependent on x. [3]

(b) In the expansion of $\left(px+\frac{2}{x}\right)^7(5-2x)$, the term independent of x is -241920. Find the value of p. [4]

3



In the diagram, points A, B and D lie on a circle. The tangents at A and D meet at C and BA is parallel to DC.

(a) Prove that the triangle ABD is isosceles. [3]

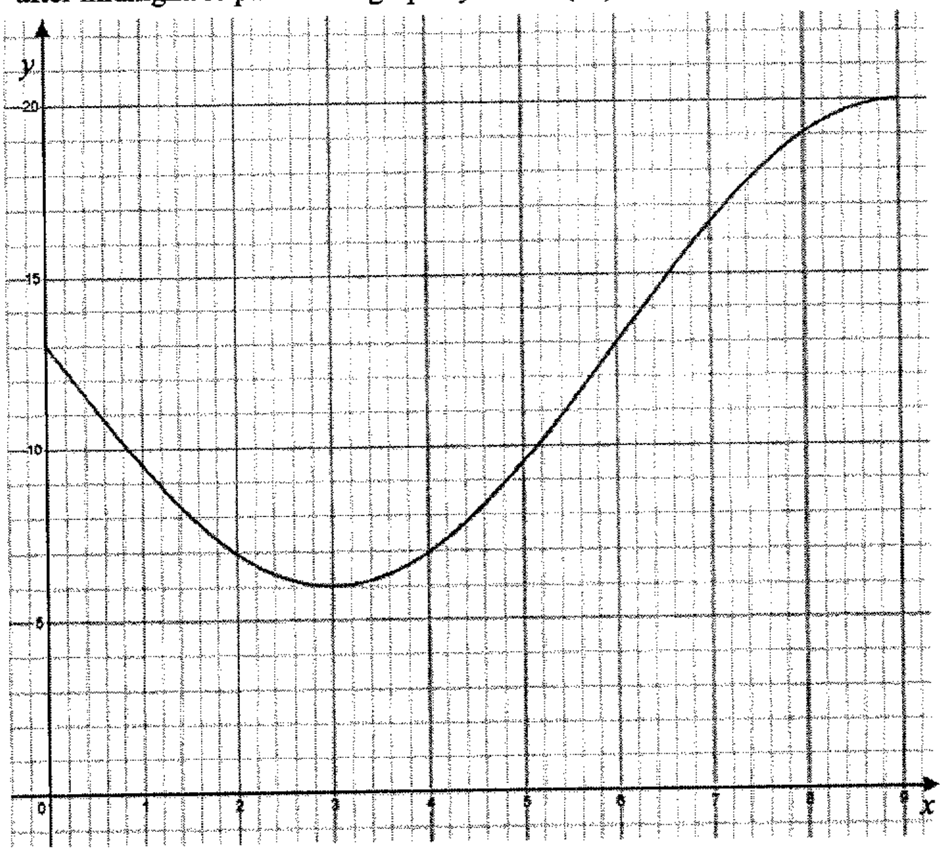
(b) Prove that angle BDA = angle DCA. [3]

4 (a) State the values between which each of the following must lie:

(i) The principal value of $\cos^{-1} x$. [1]

(ii) The principal value of $\tan^{-1} x$. [1]

(b) The depth of the water, y m, at a harbour on a particular day is given by $y = a\sin(bx) + c$, where a, b and c are constants and x is the time in hours after midnight. A part of the graph $y = a\sin(bx) + c$ is shown below.



(i) Find the value of each of the constants a, b and c. [3]

(ii) A cargo ship requires the depth of the water to be at least 7m in order to sail in to dock at the harbour. The cargo ship is expected to arrive after 7am. What would be the next time interval in which the ship would be unable to sail in to dock? Leave your answer to the nearest hour. [2]

5 **A calculator must not be used in the question.**

(a) Show that $\sec 105° = -\sqrt{2} - \sqrt{6}$. [4]

(b) The equation of a curve is $f(x)=\dfrac{e^{3x}}{x-1}$, where $x \neq 1$.

(i) Find an expression for $f'(x)$ in the form $\dfrac{e^{3x}(ax+b)}{(x-1)^2}$, where a and b are integers. [3]

(ii) The curve, $f(x)$, cuts the y axis at S, and the normal to the curve at S cuts the x axis at P. Find the coordinates of P. [4]

6 **(a)** Show that $2x+1$ is a factor of $10x^3-9x^2-3x+2$ and hence factorise $10x^3-9x^2-3x+2$ completely. [5]

(b) Solve the equation $5\left(9^{y}\right)+3^{-y}=\frac{3}{2}\left(3^{y+1}+1\right)$ and explain why there are only 2 real solutions. [7]

7 A particle, A, travelling along a straight road, passes a point O. The velocity of A, v m/s, t seconds after passing through O, is given by $v = 10\cos(5 - 2t) + 50$.

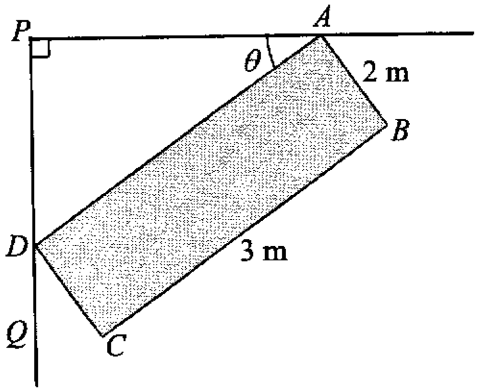
(a) Find the initial acceleration of A. [4]

Another particle, B, also travelling along the straight road, passes the point O with a velocity of 5 m/s. The acceleration of B, a m/s², t seconds after passing through O, is given by
$a = \dfrac{-24}{(t+2)^2}$.

(b) Find the value(s) of t at which B is at instantaneous rest. [4]

(c) Find the total distance travelled by B for the first 10 seconds of its journey. [6]

8 The diagram shows a rectangular table, $ABCD$, placed at the corner of the hall. It is given that the table has length $BC = 3\,\text{m}$, width $AB = 2\,\text{m}$, $\angle APD = \angle DQC = 90^\circ$ and $\angle PAD = \theta^\circ$.

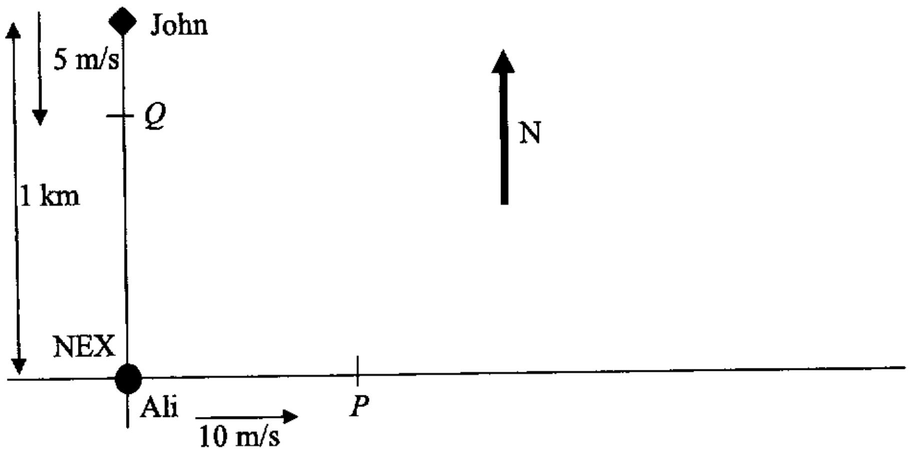


(a) Show that PQ can be expressed as $a\sin\theta + b\cos\theta$, where a and b are constants. [2]

(b) Express PQ in the form $R\sin(\theta+\alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [4]

(c) Find the maximum value of PQ and the corresponding value of θ. [3]

9 On a typical day at the local fast food outlet in NEX, a delivery rider, Ali, is heading East, away from NEX, at a constant speed of 10 m/s after collecting his order. Another delivery rider, John, 1km away from NEX, is heading South towards NEX at a constant speed of 5 m/s, to collect his order.



At time t s, Ali would have reached P and John would have reached the point Q.

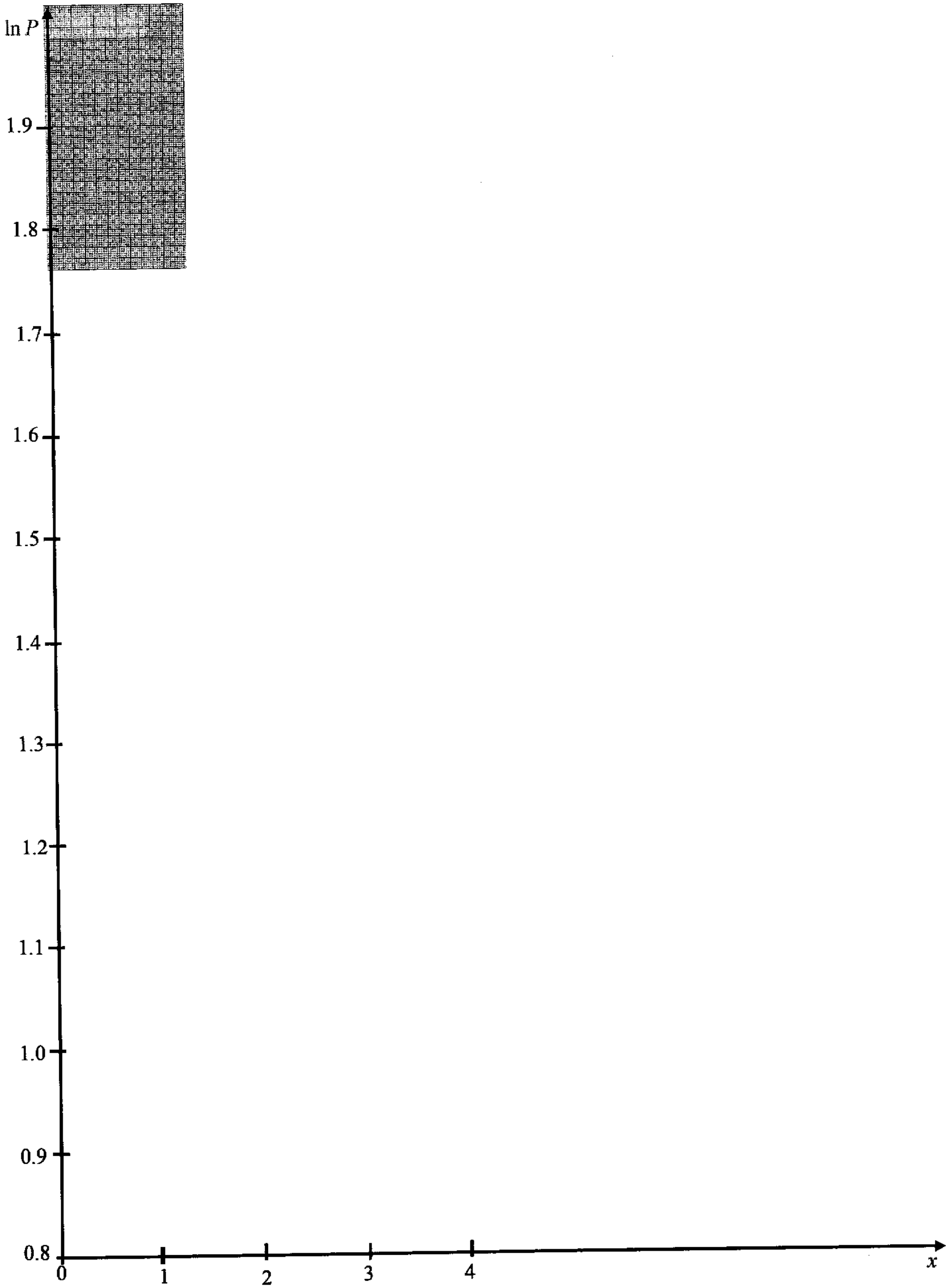
(a) Show that the distance between the two delivery riders, PQ, at time t is given by $s=\sqrt{1000000-10000t+125t^2}$. [3]

(b) Find the least distance between Ali and John. You are **not** required to justify that this time is the least. [4]

10 The table below shows, to 3 significant figures, the population, P, in millions of a country on January 1st at intervals of 10 years from 1980 to 2020. The variable x is measured in units of 10 years.

Year	1980	1990	2000	2010	2020
x	0	1	2	3	4
P	2.45	3.09	3.94	4.95	6.3

(a) On the grid below plot $\ln P$ (correct to 2 decimal places) against x and draw a straight line. [2]


ln P
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0
1
2
3
4
x

(b) Find the gradient of your straight line and hence express P in the form Ae^{kx}, where A and k are constants. [4]

(c) If this model for the population remains valid, find the first year of the interval in which the population exceeds 13 million. [2]

End of paper

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[Turn over

1	(a)	When $t = 0$, $170000 = k + 150000e^0$ $170000 = k + 150000$ $k = 20000$
	(b)	$140000 = 20000 + 150000e^n$ $120000 = 150000e^n$ $\frac{4}{5} = e^n$ $\ln\frac{4}{5} = n$ $n = -0.223144$ $n = -0.223$
	(c)	$70000 < 20000 + 150000e^{-0.223t}$ $50000 < 150000e^{-0.223t}$ $\frac{1}{3} < e^{-0.223t}$ $\ln\frac{1}{3} < -0.223t$ $t < 4.92$ $t = 4$ David must sell the car in 2026

2	(a)	General term $\left(px+\frac{2}{x}\right)^7$ $=\binom{7}{r}(px)^{7-r}\left(\frac{2}{x}\right)^r$ Consider the power of x $=7-r-r$ $=7-2r$ If there is an independent term, $7-2r=0$ $r=\frac{7}{2}$ Since r is not a whole number, there is no term independent of x and as such, every term is dependent on x
	(b)	$\left(px+\frac{2}{x}\right)^7(5-2x)$ Since there is no independent term in $\left(px+\frac{2}{x}\right)^7$, the only term to form the independent term is the $\frac{1}{x}$ in the expansion of $\left(px+\frac{2}{x}\right)^7(5-2x)$. Find the $\frac{1}{x}$ term Power of x: $7-2r=-1$ $r=4$ $\frac{1}{x}$ term $=\binom{7}{4}(px)^3\left(\frac{2}{x}\right)^4$ $=\frac{35p^3\times 16}{x}$ $=\frac{560p^3}{x}$

			Thus, the term independent of x $\frac{560p^3}{x}\times -2x = -241920$ $p^3 = 216$ $p = 6$	
3	(a)		Consider $\angle ABD = \angle ADC$ (by tangent chord theorem) $\angle ADC = \angle BAD$ (alternate angles) Thus, $\angle ABD = \angle BAD$. Since there are 2 equal angles in triangle ABD, triangle ABD is isosceles.	
	(b)		Let $\angle ABD = x$, Since triangle ABD is isosceles (part(a)), $\angle BDA = 180 - 2x$ (angle sum of triangle) by tangent chord theorem, $\angle ABD = \angle ADC = x$ Or $\angle ADC = 180 - [x + 180 - 2x]$ $= x$ (interior angles) Since tangents from an external point are equal, triangle CDA is also isosceles. Thus, $\angle ADC = \angle CAD = x$ And $\angle DCA = 180 - 2x = \angle BDA$ (shown)	
4	(a)	(i)	$0^\circ \le \cos^{-1} x \le 180^\circ$ or $0 \le \cos^{-1} x \le \pi$	
		(ii)	$-90^\circ < \tan^{-1} x < 90^\circ$ or $-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$	
	(b)	(i)	$a = -\left(\frac{20-6}{2}\right) = -7$ $b = \frac{2\pi}{12} = \frac{\pi}{6}$ $c = 13$	
		(ii)	$14 < x < 16$	

5	(a)		$\cos 105$ $= \cos(60+45)$ $= \cos 60 \cos 45 - \sin 60 \sin 45$ $= \frac{1}{2}\times\frac{\sqrt{2}}{2}-\frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{2}$ $= \frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}$ $= \frac{\sqrt{2}-\sqrt{6}}{4}$ $\sec 105°$ $= \frac{1}{\cos 105}$ $= \frac{4}{\sqrt{2}-\sqrt{6}}$ $= \frac{4}{\sqrt{2}-\sqrt{6}}\times\frac{\sqrt{2}+\sqrt{6}}{\sqrt{2}+\sqrt{6}}$ $= -\sqrt{2}-\sqrt{6}$
	(b)	(i)	$f(x)=\frac{e^{3x}}{x-1}$ $f'(x)=\frac{3e^{3x}(x-1)-e^{3x}}{(x-1)^2}$ $f'(x)=\frac{e^{3x}(3x-4)}{(x-1)^2}$ $a=3$ $b=-4$
		(ii)	At y axis, $x=0$. When $x=0$, $y=-1$ $f'(0)=-4$ Thus, gradient of normal $=\frac{1}{4}$ Equation of normal: $y=\frac{1}{4}x+c$ Sub $x=0$, $y=-1$ $c=-1$ $y=\frac{1}{4}x-1$ When $y=0$, $x=4$ $P(4,0)$

6	(a)	Let $f(x)=10x^3-9x^2-3x+2$ $f(-\frac{1}{2})=10\left(-\frac{1}{2}\right)^3-9\left(-\frac{1}{2}\right)^2-3\left(-\frac{1}{2}\right)+2$ $f(-\frac{1}{2})=0$ By factor theorem, $2x+1$ is a factor. $f(x)=10x^3-9x^2-3x+2$ $\begin{array}{r} 5x^2-7x+2 \\ 2x+1\overline{)10x^3-9x^2-3x+2} \\ -\quad 10x^3+5x^2 \\ \hline -14x^2-3x+2 \\ -\quad -14x^2-7x \\ \hline 4x+2 \\ -\quad 4x+2 \\ \hline 0 \end{array}$ $f(x)=(2x+1)(5x^2-7x+2)$ $\begin{array}{c\|cc} & x & -1 \\ \hline 5x & 5x^2 & -5x \\ -2 & -2x & 2 \end{array}$ $f(x)=(2x+1)(5x-2)(x-1)$
	(b)	Let $3^y=a$. $5(a^2)+\frac{1}{a}=\frac{3}{2}(3a+1)$ $10a^2+\frac{2}{a}=9a+3$ $10a^3+2=9a^2+3a$ $10a^3-9a^2-3a+2=0$ Comparing with (a) $(2a+1)(5a-2)(a-1)=0$ $a=-\frac{1}{2}$ or $a=\frac{2}{5}$ or $a=1$

		$3^y = -\frac{1}{2}$ or $3^y = \frac{2}{5}$ or $3^y = 1$ (reject as $3^y > 0$) or $y \ln 3 = \ln \frac{2}{5}$ $y = 0$ $y = -0.834$
7	(a)	$v = 10\cos(5-2t) + 50$ Acceleration = $\frac{dv}{dt}$. $\frac{dv}{dt} = 10(-\sin(5-2t) \times -2)$ $\frac{dv}{dt} = 20\sin(5-2t)$ When $t = 0$, $20\sin(5)$ $= -19.2\ \text{m/s}^2$
7	(b)	$v_c = \int \frac{-24}{(t+2)^2}\ dt$ $v_c = \int -24(t+2)^{-2}\ dt$ $v_c = 24(t+2)^{-1} + c$ When $t = 0$, $v_c = 5$ Thus, $5 = 12 + c$ $c = -7$ $v_c = 24(t+2)^{-1} - 7$ cyclist is instantaneously at rest $\Rightarrow v_c = 0$ $24(t+2)^{-1} - 7 = 0$ $\frac{24}{(t+2)} = 7$ $24 = 7(t+2)$ $24 = 7t + 14$ $t = \frac{10}{7}$
	(c)	Displacement, $s = \int v_c\ dt$ $s = \int 24(t+2)^{-1} - 7\ dt$ $s = 24\ln(t+2) - 7t + c$

When $t=0$, $s=0$

$c=-24\ln 2$

$s=24\ln(t+2)-7t-24\ln 2$

At $t=\dfrac{10}{7}$,

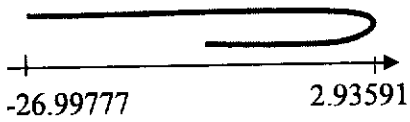
$s=24\ln\left(\dfrac{10}{7}+2\right)-7\left(\dfrac{10}{7}\right)-24\ln 2$

$s=2.93591\text{ m}$

At $t=10$,

$s=24\ln(10+2)-7(10)-24\ln 2$

$s=-26.99777\text{ m}$



Total distance

$=2.93591+(2.93591+26.99777)$

$=32.9\text{m}$

8 (a)

$PQ=PD+DQ$

Consider triangle APD,

$\dfrac{PD}{3}=\sin\theta$

$PD=3\sin\theta$

Consider triangle DCQ,

$\dfrac{DQ}{2}=\cos\theta$

$DQ=2\cos\theta$

Thus,

$PQ=3\sin\theta+2\cos\theta$

(c)

$PQ=\sqrt{13}\sin(\theta+33.7)$

Maximum value = $\sqrt{13}$

$\sqrt{13}\sin(\theta+33.7)=\sqrt{13}$

$\sin(\theta+33.7)=1$

$\theta+33.7=90$

$\theta=56.3^\circ$

9 (a)

At time t,

Distance from NEX to P = $10t$

Distance from NEX to Q = $1000-5t$

$PQ^2=(10t)^2+(1000-5t)^2$

$PQ^2 = 100t^2 + 1000000 - 10000t + 25t^2$

$PQ = \sqrt{125t^2 - 10000t + 1000000}$

$s = \sqrt{1000000 - 10000t + 125t^2}$

(b) Least distance occurs at minimum point

i.e. $\dfrac{ds}{dt} = 0$

$\dfrac{ds}{dt} = \dfrac{1}{2}\left(1000000 - 10000t + 125t^2\right)^{-\frac{1}{2}}(250t - 10000)$

$\dfrac{1}{2}\left(1000000 - 10000t + 125t^2\right)^{-\frac{1}{2}}(250t - 10000) = 0$

$250t - 10000 = 0$

$t = 40$

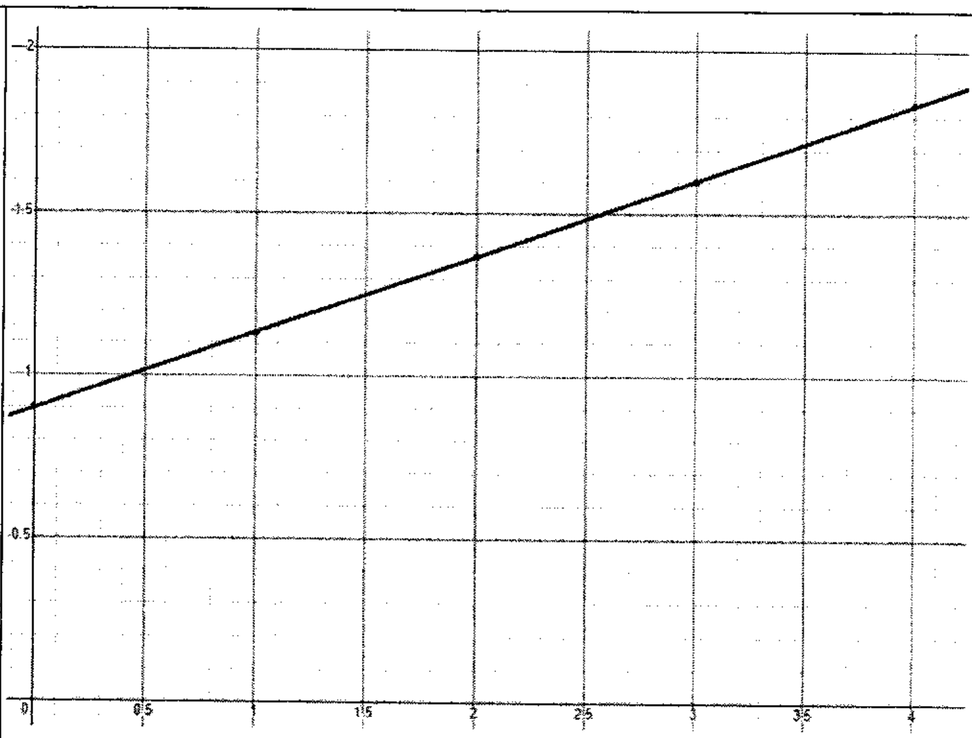
When $t = 40$,

$s = \sqrt{1000000 - 10000(40) + 125(40)^2}$

$s = 894.4$

$s = 894\,\text{m}$

10 (a)



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x	0	1	2	3	4
P	2.45	3.09	3.94	4.95	6.30
$\ln P$	0.90	1.13	1.37	1.60	1.84

(b) From the graph,

		$m = 0.235\ (\pm 0.2)$ $c = 0.90\ (\pm 0.2)$ $\ln P = 0.235x + 0.9$ $P = e^{0.235x+0.9}$ $P = 2.45e^{0.235x}$
	(c)	When $P = 13$ $13 = 2.45e^{0.235x}$ $t = 7.10$ First year in the interval would be 2050.

End of paper