PRELIMINARY EXAMINATION 2016
SECONDARY FOUR

4047/01 **ADDITIONAL MATHEMATICS**
PAPER 1

Thursday **11 August 2016** **2 h**

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper, and use a pencil for drawing graphs and diagrams.
Do not use staples, highlighters or correction fluid.

Answer **all** the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

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1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

*Answer **all** questions*

1 It is given that $\cos A = -\frac{1}{3}$ and $\sin B = \sqrt{\frac{2}{11}}$. A and B are in the same quadrant.
Without using a calculator, find the exact value of $\cot(90° - A - B)$. [5]

2 (i) Find the range of values of p for which $(x+1)(x-2) > p(x+2)$ for all real values of x. [4]

(ii) Deduce the number of points at which the line $y = p(x+2)$ intersects the curve $y = (x+1)(x-2)$ for $-1 \le p < 2$. [1]

3 2000 cm^3 of water is transferred from a rectangular tank to an empty inverted right circular cone in 10 seconds. The ratio of the radius of the cone to the height of the cone is 1 : 3.
Find the rate of change of the horizontal surface area, A cm^2, of the water in the cone, when the height, h cm, of the water in the cone is 12 cm. [6]

4 (i) Write down and simplify, the first 3 terms in the expansion of $(2-p)^7$ in ascending powers of p. [2]

(ii) Find the value of n where n is a positive integer, given that the coefficient of x^2 is 96 in the expansion of $(1+x)^n(2-x+x^2)^7$. [4]

5 A curve $y=\mathrm{f}(x)$ is such that $\mathrm{f}''(x)=48\sin 4x-8\cos 2x$. The curve intersects the x-axis at P. The x-coordinate of P is $\frac{\pi}{4}$ and the gradient of the curve at P is 8. Show that $\mathrm{f}''(x)+16\mathrm{f}(x)=24\cos 2x$. [7]

6 The table shows experimental values of two variables x and y.

x	2	4	6	7	8
y	1.33	2.29	3.27	3.77	6.12

It is known that x and y are related by an equation of the form $x^2+\frac{y}{a}=bxy$, where a and b are constants. An error was made in recording one of the values of y.

(i) Using a scale of 2 cm to represent 1 unit on the horizontal axis and 1 cm to represent 1 unit on the vertical axis, draw a straight line graph for the above given data. The straight line graph is to be drawn with variable x on the horizontal axis. [3]

(ii) Use the graph to estimate

(a) the correct value of y, [2]

(b) the values of a and b. [3]

7 **(i)** Express $\frac{4}{(x-3)x^2}$ in partial fractions. [4]

(ii) Hence evaluate $\int_4^7 \frac{1}{(x-3)x^2}\,\mathrm{d}x$. [4]

8 (i) Prove that $\frac{1-\sin x}{\cos x}+\frac{\cos x}{1-\sin x}=2\sec x$. [3]

(ii) In the equation

$$\frac{1-\sin x}{\cos x}+\frac{\cos x}{1-\sin x}+\tan^2 x=2,$$

$\cos x=a$ or b where a and b are constants, and $b<0$.

(a) Find the value of a and of b. [2]

(b) Solve the equation $\cos x=b$ for $-\pi \le x \le 2\pi$. [3]

9 The equation of a curve is $y=x\ln(2x-3)$ where $x>\frac{3}{2}$.

(i) Find the equation of the normal to the curve at $x=2$. [4]

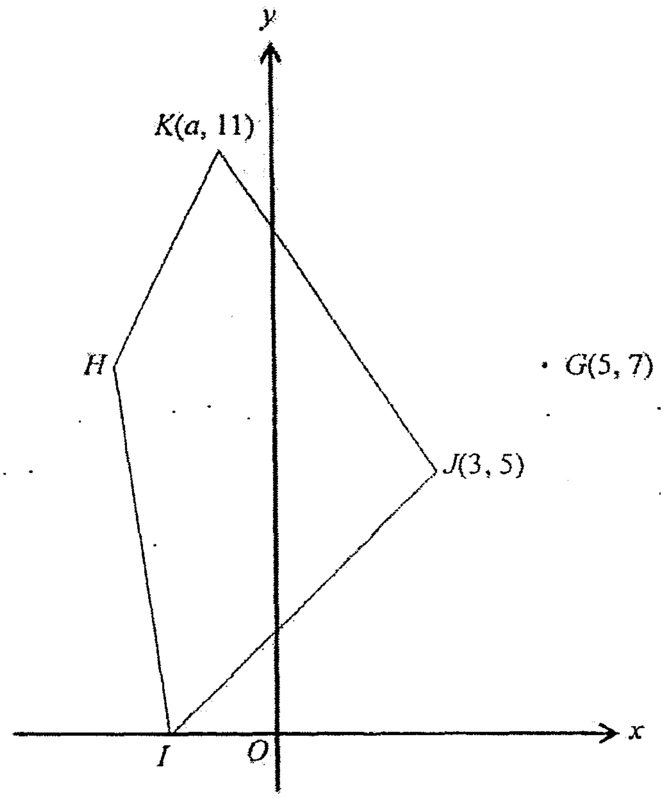
The normal to the curve $y=x\ln(2x-3)$ passes through the vertex of the graph of $y=k-4|2x+1|$ where k is a constant.

(ii) Determine the value of k. [2]

(iii) Sketch the graph of $y=k-4|2x+1|$ for the value of k in part **(ii)**.

Show the vertex and intercepts clearly. [2]

10 Solutions to this question by accurate drawing will not be accepted.



The diagram shows a quadrilateral $HIJK$. H is the reflection of point $G(5, 7)$ in the line $x = 1$. Point $K(a, 11)$ is such that the product of the gradients of HK and JK is -3. The perpendicular bisector of HJ intersects the x-axis at I.

(i) Deduce the coordinates of H. [1]

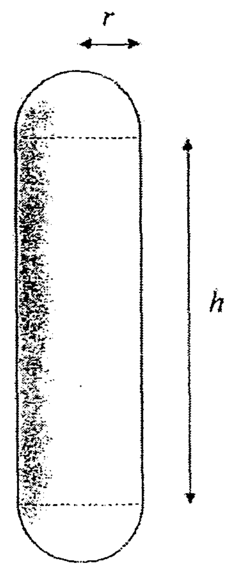
Find

(ii) the value of a given that $a < 0$, [2]

(iii) the equation of the perpendicular bisector of HJ, [3]

(iv) the area of quadrilateral $HIJK$. [3]

11



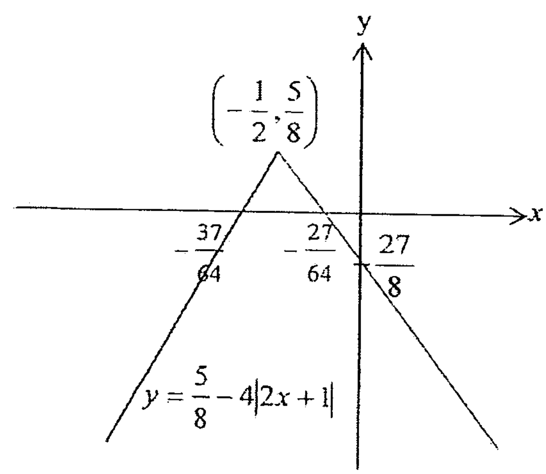
The diagram shows a capsule shaped object with surface area 18π cm^2. It comprised of 2 solid hemispheres of radius r cm joined to the 2 ends of a solid cylinder of radius r cm and height h cm.

(i) Show that the volume, V cm^3, of the object is given by $V = 9\pi r - \frac{2}{3}\pi r^3$. [4]

(ii) Find the stationary value of V, and determine if this stationary value is a maximum or minimum. [6]

THE END

Answer Key to Prelim 2016 Additional Mathematics Paper 1

1	$7\sqrt{2}$	8(i)	Proof
		(ii)(a)	$a=1$ and $b=-\frac{1}{3}$
2(i)	$-9<p<-1$	(ii)(b)	$-1.91, 1.91, 4.37$
2 (ii)	1 or 2 points		
		9(i)	$4y=-x+2$
3	$33\frac{1}{3}\text{ cm}^3/\text{s}$	(ii)	$\frac{5}{8}$
		(iii)	
4(i)	$128-448p+672p^2+\cdots$		
4(ii)	4		
5	proof		
		10(i)	$(-3, 7)$
6(ii)(a)	4.24	(ii)	-1
(b)	$a=1,\ b=2$	(iii)	$y=3x+6$
		(iv)	34 square units
7(i)	$\frac{4}{(x-3)x^2}=\frac{4}{9(x-3)}-\frac{4}{9x}-\frac{4}{3x^2}$	11(ii)	40.0 cm^3, Stationary value of V is a maximum.

2016 Sec 4 A Math Prelim Exam Paper 1 Marking Scheme

Qn	Solution	Marks	Teaching Points
1	A and B are in the same quadrant. $\therefore A$ and B are both in 2nd quadrant.		
	$\cos A = -\frac{1}{3}$ $\tan A = -\frac{\sqrt{8}}{1}$ $= -2\sqrt{2}$	B1	Understand how to find ratio of tan A from cos A.
	$\sin B = \sqrt{\frac{2}{11}}$ $= \frac{\sqrt{2}}{\sqrt{11}}$ $\tan B = -\frac{\sqrt{2}}{3}$	B1	Understand how to find tan B from sin B.
	$\cot(90° - A - B)$ $= \cot(90° - (A+B))$ $= \tan(A+B)$	B1	Know the relationships $\tan(90° - C) = \frac{1}{\tan C}$ and $\cot(90° - C) = \tan C$
	$= \frac{\tan A + \tan B}{1 - \tan A \tan B}$	M1	Know how to use the addition formula for $\tan(A+B)$
	$= \frac{-2\sqrt{2} - \frac{\sqrt{2}}{3}}{1 - \left(-2\sqrt{2}\right)\left(-\frac{\sqrt{2}}{3}\right)}$		

	$=\dfrac{\dfrac{-7\sqrt{2}}{3}}{1-\dfrac{4}{3}}$		Able to simplify surds
	$=7\sqrt{2}$	A1	Correct final answer

Qn	Solution	Marks	Teaching Points
2(i)	$(x+1)(x-2) > p(x+2)$ $x^2 - x - 2 > px + 2p$ $x^2 + (-1-p)x - 2 - 2p > 0$ $(x+1)(x-2) > p(x+2)$ for all x $\Rightarrow$ discriminant < 0 $\Rightarrow (-1-p)^2 - 4(1)(-2-2p) < 0$ $\Rightarrow 1 + 2p + p^2 + 8 + 8p < 0$ $\Rightarrow p^2 + 10p + 9 < 0$ $\Rightarrow (p+9)(p+1) < 0$ $\Rightarrow -9 < p < -1$	 B1 B1 M1 A1	Know that discriminant < 0 for inequality to be true for all x. Able to get expression for discriminant Able to solve quad inequality Correct answer
(ii)	Line $y = p(x+2)$ does not intersect curve $y = (x+1)(x-2)$ when p is in the range $-9 < p < -1$. For $p \geq -1$, line intersects curve at 1 or 2 points.	B1	Able to make a deduction from (i)

Qn	Solution	Marks	Teaching Points
3	V : volume of water in cone A : area of water surface on cone h : height of water in cone r : radius of the water surface t : time $\frac{dV}{dt} = \frac{2000}{10}$ cm^3/s $= 200$ cm^3/s	 B1	 Know how to get $\frac{dV}{dt}$

	$\frac{r}{h}=\frac{1}{3}$		
	$V=\frac{1}{3}\pi r^2 h$ $=\frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$ $=\frac{\pi}{27}h^3$	B1	Know how to express V in terms of h.
	$\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}$ $200=\frac{\pi}{9}h^2\frac{dh}{dt}$ $\frac{dh}{dt}=\frac{1800}{\pi h^2}$	M1	Know how to use Chain Rule to get a relationship between $\frac{dV}{dt}$, $\frac{dV}{dh}$ and $\frac{dh}{dt}$.
	$A=\pi r^2$ $=\pi\left(\frac{1}{3}h\right)^2$ $=\frac{\pi}{9}h^2$	B1	Know how to express A in terms of h.
	$\frac{dA}{dt}=\frac{dA}{dh}\times\frac{dh}{dt}$ $=\frac{2\pi h}{9}\left(\frac{1800}{\pi h^2}\right)$ $=\frac{400}{h}$	M1	Know how to use Chain Rule to get a relationship between $\frac{dA}{dt}$, $\frac{dA}{dh}$ and $\frac{dh}{dt}$.
	When $h=12$, $\frac{dA}{dt}=\frac{400}{12}$ $=33\frac{1}{3}$	A1	Correct final answer.
	Answer : Rate of change of the horizontal surface area of the water $33\frac{1}{3}\text{ cm}^3/\text{s}$.		

Qn	Solution	Marks	Teaching Points
4(i)	$(2-p)^7$ $2^7-\binom{7}{1}2^6p+\binom{7}{2}2^5p^2+...$ $=128-448p+672p^2+...$	M1 A1	Know formula for Binomial expansion Able to simplify
(ii)	$(1+x)^n(2-x+x^2)^7$ $=\left[1+\binom{n}{1}x+\binom{n}{2}x^2+...\right]\left[2-(x-x^2)\right]^7$ $=\left[1+nx+\frac{n(n-1))}{2\times1}x^2+...\right]\left[128-448(x-x^2)+672(x-x^2)^2+...\right]$ $=\left[1+nx+\frac{n(n-1))}{2}x^2+...\right]\left[128-448x+1120x^2+...\right]$ Coefficient of $x^2=96$ $1(1120)+n(-448)+\frac{n(n-1))}{2}(128)=96$ $64n^2-512n+1024=0$ $n^2-8n+16=0$ $(n-4)(n-4)=0$ $n=4$	B1 B1 M1 A1	Know $p=x-x^2$ Able to express $\binom{n}{1}$ and $\binom{n}{2}$ correctly in terms of n. Able to determine the terms in x^2 in the product of $(1+x)^n$ and $(2-x+x^2)^7$. Final answer

Qn	Solution	Marks	Teaching Points
5	$f''(x)=48\sin 4x-8\cos 2x$ $f'(x)=\int(48\sin 4x-8\cos 2x)dx$ $=-12\cos 4x-4\sin 2x+c_1$	B1	Know how to integrate $f''(x)$ correctly to get $f'(x)$
	$f'\left(\frac{\pi}{4}\right)=8$ $-12\cos 4\left(\frac{\pi}{4}\right)-4\sin 2\left(\frac{\pi}{4}\right)+c_1=8$ $-12(-1)-4(1)+c_1=8$ $c_1=0$	M1	Know how to use the grad at *P* to get $f'(x)$
	$f'(x)=-12\cos 4x-4\sin 2x$	A1	Correct expression for $f'(x)$
	$f(x)=\int(-12\cos 4x-4\sin 2x)dx$ $=-3\sin 4x+2\cos 2x+c_2$		Know how to integrate $f'(x)$ correctly to get $f(x)$
	$f\left(\frac{\pi}{4}\right)=0$ $-3\sin 4\left(\frac{\pi}{4}\right)+2\cos 2\left(\frac{\pi}{4}\right)+c_2=0$ $-3(0)+2(0)+c_2=0$ $c_2=0$	M1	Know how to use the *x*-coordinate of *P* to get $f(x)$
	$f(x)=-3\sin 4x+2\cos 2x$	A1	Correct expression for $f(x)$
	$f''(x)+16f(x)$ $=(48\sin 4x-8\cos 2x)+16(-3\sin 4x+2\cos 2x)$ $=24\cos 2x$ (Proved)	M1	sub. expressions for $f(x)$ and $f''(x)$
		A1	Able to get $24\cos 2x$

Qn	Solution	Marks	Teaching Points
6(i)	$x^2 + \frac{y}{a} = bxy$ $\frac{x^2}{y} + \frac{1}{a} = bx$ $\frac{x^2}{y} = bx - \frac{1}{a}$	B1	Able to transform given equation into a straight line form with x on horizontal axis.
	Graph	B1	Able to plot a straight line passing through all points
		B1	Graph cuts y-axis.
(ii)(a)	Correct reading of $\frac{x^2}{y} = 15.1$ $\frac{8^2}{y} = 15.1$	M1	Know the method to find the correct reading of y
	Correct reading of $y = 4.24$	A1	Correct final answer
(b)	$-\frac{1}{a} = \frac{x^2}{y}$ – intercept $= -1$ $a = 1$	B1	Understand how to get a using the vertical intercept
	b = Gradient $= \frac{11.01 - 3.01}{6 - 2}$	M1	Understand that b is the gradient
	$= 2$	A1	Correct value of b

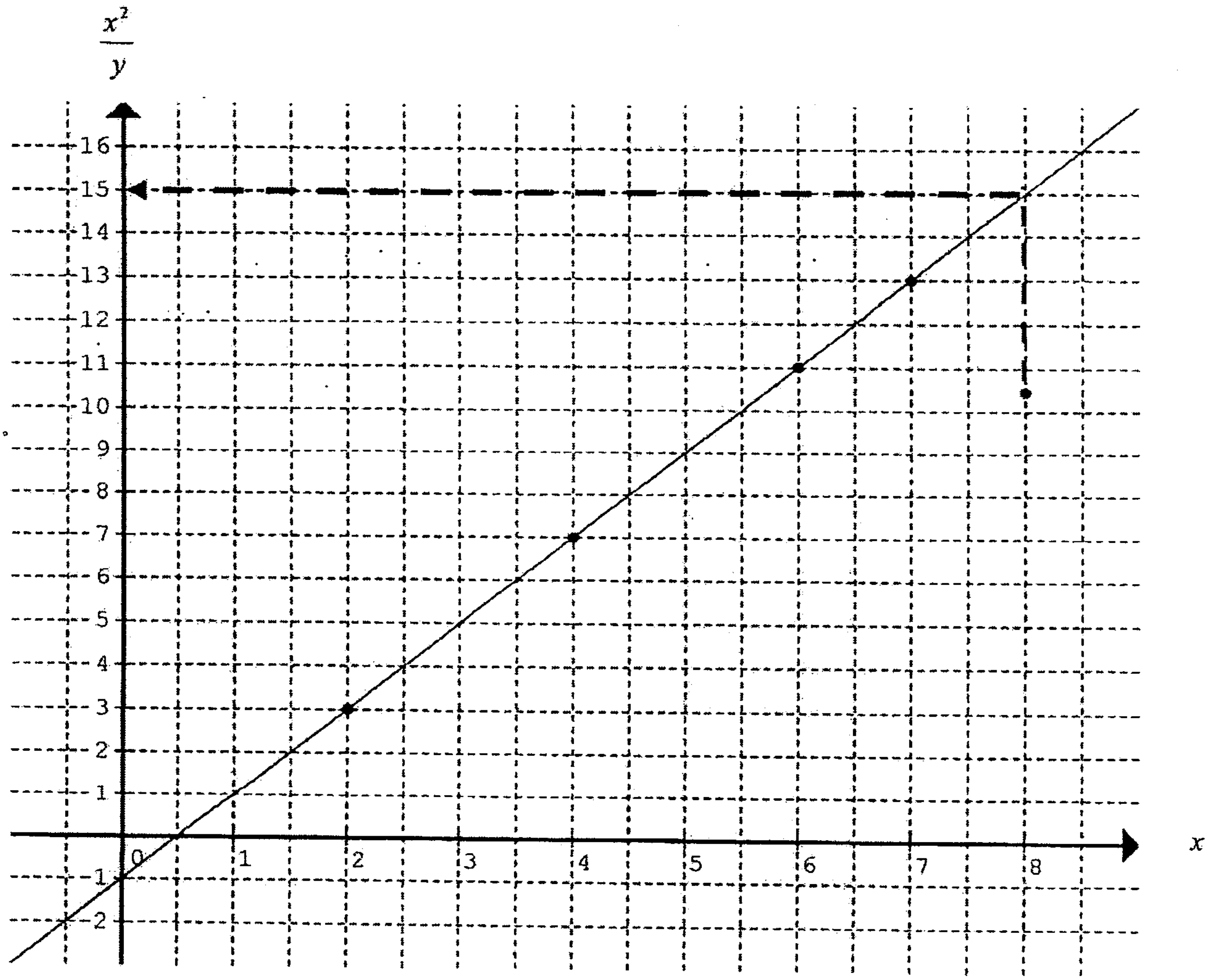
6(i)

x	2	4	6	7	8
y	1.33	2.29	3.27	3.77	6.12
$\frac{x^2}{y}$	3.01	6.99	11.01	13.00	10.46

Scale :

x-axis : 2 cm to 1 unit

$\frac{x^2}{y}$ axis : 1 cm to 1 unit



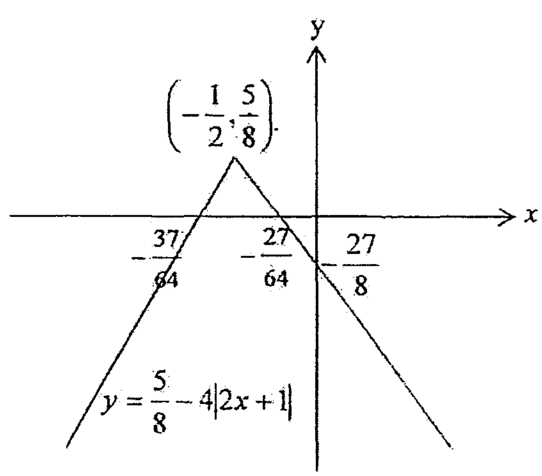
Qn	Solution	Marks	Teaching Points
7(i)	$\frac{4}{(x-3)x^2}=\frac{A}{x-3}+\frac{B}{x}+\frac{C}{x^2}$ $4=Ax^2+Bx(x-3)+C(x-3)$	B1	Know the various partial fraction forms.
	Consider $x=0$: $4=C(-3)$ $C=-\frac{4}{3}$	B1	Able to use suitable method to find C.
	Consider $x=3$: $4=9A$ $A=\frac{4}{9}$	B1	Able to use suitable method to find A.
	Compare coefficient of x^2: $0=A+B$ $B=-A$ $=-\frac{4}{9}$	B1	Able to use suitable method to find B.
	Hence $\frac{4}{(x-3)x^2}=\frac{4}{9(x-3)}-\frac{4}{9x}-\frac{4}{3x^2}$		Minus 1 mark if didn't write final line.

(ii)	$\int_4^7 \frac{1}{(x-3)x^2}\,dx$		
	$=\frac{1}{4}\int_4^7 \frac{4}{(x-3)x^2}\,dx$		
	$=\frac{1}{4}\int_4^7 \left(\frac{4}{9(x-3)}-\frac{4}{9x}-\frac{4}{3x^2}\right)dx$	B1	Know the formula $\int \frac{1}{ax+b}dx = \ln x + c$
	$=\frac{1}{4}\left[\frac{4}{9}\ln(x-3)-\frac{4}{9}\ln x-\frac{4}{3}\left(-x^{-1}\right)\right]_4^7$	B1	Know the formula $\int x^n dx = \frac{x^{n+1}}{n+1}+c$
	$=\frac{1}{4}\left(\frac{4}{9}\ln 4-\frac{4}{9}\ln 7+\frac{4}{3}\left(\frac{1}{7}\right)\right)-\frac{1}{4}\left(\frac{4}{9}\ln 1-\frac{4}{9}\ln 4+\frac{4}{3}\left(\frac{1}{4}\right)\right)$	M1	Know how to evaluate a definite integral
	$=0.0561$	A1	Correct final answer

Qn	Solution	Marks	Teaching Points
8(i)	$\dfrac{1-\sin x}{\cos x}+\dfrac{\cos x}{1-\sin x}$		
	$=\dfrac{(1-\sin x)^2+\cos^2 x}{\cos x(1-\sin x)}$		
	$=\dfrac{1-2\sin x+\sin^2 x+\cos^2 x}{\cos x(1-\sin x)}$		
	$=\dfrac{1-2\sin x+1}{\cos x(1-\sin x)}$	B1	Knows the identity $\sin^2 x+\cos^2 x=1$
	$=\dfrac{2(1-\sin x)}{\cos x(1-\sin x)}$	B1	Is aware of 'factorisation' as one technique used in proofs.
	$=\dfrac{2}{\cos x}$		
	$=2\sec x$	B1	Know the identity $\sec x=\dfrac{1}{\cos x}$

(ii)(a)	$\frac{1-\sin x}{\cos x}+\frac{\cos x}{1-\sin x}+\tan^2 x=2$ $2\sec x+\tan^2 x=2$ $2\sec x+\sec^2 x-1=2$ $\sec^2 x+2\sec x-3=0$ $(\sec x-1)(\sec x+3)=0$ $\sec x=1$ or $\sec x=-3$ $\cos x=1$ or $\cos x=-\frac{1}{3}$ Answer: $a=1$ and $b=-\frac{1}{3}$	B1 A1	Use identity $1+\tan^2 x=\sec^2 x$ Correct final answer
(b)	$\cos x=-\frac{1}{3}$ Basic ∠ = 1.2310 radians $x=-1.91,\ 1.91,\ 4.37$	B1,B1,B1	Correct angles

Qn	Solution	Marks	Teaching Points
9(i)	$y = x\ln(2x-3)$ $\frac{dy}{dx} = x\left(\frac{2}{2x-3}\right) + \ln(2x-3)$ At $x = 2$, $\frac{dy}{dx} = 2\left(\frac{2}{2(2)-3}\right) + \ln(2(2)-3)$ $= 4$ *and* $y = 2\ln(2(2)-3)$ $= 0$ Equation of normal : $\frac{y-0}{x-2} = -\frac{1}{4}$ $4y = -x + 2$	M1 M1 M1 A1	Use Product Rule to diff $x\ln(2x-3)$ Use Chain Rule to diff $\ln(2x-3)$ Know how to find gradient, y-coordinate and equation of normal Correct answer for equation of normal.

Qn	Solution	Marks	Teaching Points
9(ii)	Equation of normal : $4y = -x + 2$ $y = k - 4\|2x+1\|$ Coordinate of vertex : $\left(-\frac{1}{2}, k\right)$ When $x = -\frac{1}{2}$, $4y = -\left(-\frac{1}{2}\right) + 2$ $y = \frac{5}{8}$ $k = \frac{5}{8}$	M1 A1	Understand that the x-coordinate of vertex is $-\frac{1}{2}$ and that k is obtained when $\|2x+1\| = 0$. Correct value of k
(iii)		B1 B1	Shape Critical pts

Qn	Solution	Marks	Teaching Points
10(i)	Coordinates of H are $(-3, 7)$.	B1	Know how to get image of a point given the line of reflection
(ii)	Gradient of HK × Gradient of JK = –3	M1	Know how to get a relationship between the 2 gradients
	$\frac{11-7}{a+3}\times\frac{11-5}{a-3}=-3$		
	$\frac{24}{(a+3)(a-3)}=-3$		
	$a^2-9=-8$		
	$a^2=1$		
	$a=1$ (reject) or -1	A1	Correct value of a
(iii)	Midpoint of HJ $=\left(\frac{-3+3}{2},\frac{7+5}{2}\right)$ $=(0, 6)$	B1	Know formula for midpoint
	Gradient of HJ $=\frac{7-5}{-3-3}$ $=-\frac{1}{3}$		
	Equation of ⊥ bisector of HJ:		
	$\frac{y-6}{x-0}=3$	M1	Know how to get ⊥ bisector
	$y=3x+6$	A1	Correct answer

(iv)	$y = 3x + 6$ When $y = 0$, $0 = 3x + 6$ $x = -2$ Coordinates of $I = (-2, 0)$	B1	Know how to find coordinates of *I*
	Area of *HIJK* $= \frac{1}{2}\begin{vmatrix} -2 & 3 & -1 & -3 & -2 \\ 0 & 5 & 11 & 7 & 0 \end{vmatrix}$ $= \frac{1}{2}\{(-2)5 + 3(11) + (-1)7 + (-3)0 - 3(0) - (-1)5 - (-3)11 - (-2)7\}$ $= 34$ square units	M1 A1	Know the formula for area of polygon Correct final answer

Qn	Solution	Marks	Teaching Points
11(i)	$2\pi rh + 2(2\pi r^2) = 18\pi$	M1	Able to get a relationship between r, h and area.
	$h = \dfrac{18\pi - 4\pi r^2}{2\pi r}$		
	$= \dfrac{9}{r} - 2r$	A1	Correct expression for h in terms of r.
	$V = \pi r^2 h + \left(\dfrac{2}{3}\pi r^3\right)2$		
	$= \pi r^2 h + \dfrac{4}{3}\pi r^3$		
	$= \pi r^2\left(\dfrac{9}{r} - 2r\right) + \dfrac{4}{3}\pi r^3$	M1	Able to get V in terms of r and h.
	$= 9\pi r - 2\pi r^3 + \dfrac{4}{3}\pi r^3$		
	$= 9\pi r - \dfrac{2}{3}\pi r^3$		
		A1	Correct expression for V in terms of r.
(ii)	$V = 9\pi r - \dfrac{2}{3}\pi r^3$		
	$\dfrac{dV}{dr} = 9\pi - 2\pi r^2$	B1	Able to differentiate V
	For stationary value of V,		
	$\dfrac{dV}{dr} = 0$	M1	Know requirement for stationary pt.
	$9\pi - 2\pi r^2 = 0$		
	$r = \sqrt{\dfrac{9}{2}}$	A1	Able to get value of r at stationary pt.
	Stationary value of V		

	$= 9\pi\sqrt{\frac{9}{2}} - \frac{2}{3}\pi\left(\sqrt{\frac{9}{2}}\right)^3$ $= 40.0 \text{ cm}^3$ $\frac{d^2V}{dr^2} = -4\pi r$, When $r = \sqrt{\frac{9}{2}}$, $\frac{d^2V}{dr^2} = -4\pi\sqrt{\frac{9}{2}} < 0$ $\therefore$ Stationary value of V is a maximum.	B1 M1 A1	Correct stationary value of V Know the tests to determine nature of stationary pts Able to determine correctly the nature of the stationary value.

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SECONDARY FOUR

4047/02 **ADDITIONAL MATHEMATICS**
PAPER 2

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You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions

1. A man buys a new car. The value of the car depreciates with time so that its value, $\$V$, after t months' use is given by $V = 132\,000e^{-pt}$, where p is a constant.
The value of the car is expected to be \$122 000 after eight months' use.

(i) Find the value of the car, V when the man bought it. [1]

(ii) Show that $p = 0.01$. [2]

(iii) Using the value of $p = 0.01$, determine the age of the car to the nearest month, when its value reached half of the original value when the man bought it. [2]

2. The function $f(x) = 1 + 2x + Ax^2 - x^3$, where A is a constant, leaves a remainder of $1\frac{3}{8}$ when divided by $(2x-1)$.

(i) Find the value of A . [2]

(ii) Hence solve the equation $f(x) = 0$, giving your answers in the exact form. [4]

3. **(a)** **(i)** Solve $\sqrt{3x+2} - 3x = 0$. [2]

(ii) On the same axes, sketch the graphs of $y = \sqrt{3x+2}$ and $y = 3x$. Indicate clearly all the points of intersections. [2]

(b) The vertical height of a triangle is $\frac{8}{3-\sqrt{5}}$ cm.

Given that the area of the triangle is $\frac{20}{\sqrt{5}-1}$ cm^2, without using a calculator, find the length of the base of the triangle in the form $a + b\sqrt{5}$. [3]

4. The roots of the quadratic equation, $2x^2 + 4x + 5 = 0$ are $(\alpha + 1)$ and $(\beta + 1)$.

(i) Show that $\alpha + \beta = -4$ and hence find $\alpha\beta$. [3]

(ii) Find the quadratic equation in x with integer coefficients, whose roots are $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$. [5]

5. (a) Given that $\log_2(2x+1) - \log_4(3-x^2) = 1$, form a quadratic equation in x and explain with clear working why the roots of the quadratic equation are real and distinct. [5]

(b) Solve $3^{y+2} = 2(3^{-y}) + 17$. [4]

6. The curve $y = \frac{2x^2}{x^2+1}$ has one stationary point (p, q).

(i) Find the value of p and of q. [4]

(ii) Determine whether y is increasing or decreasing for

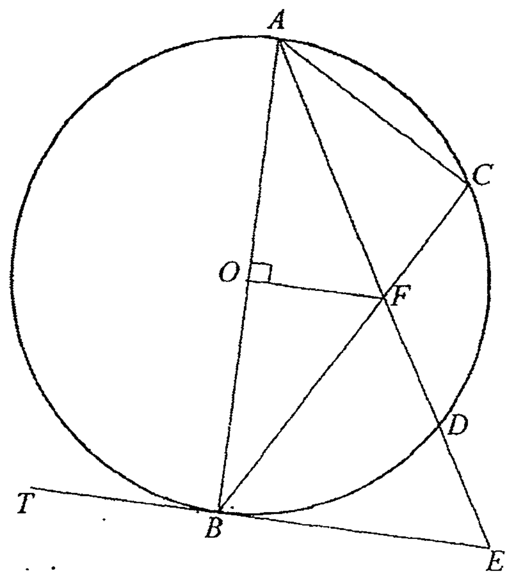
(a) $x > p$, [1]

(b) $x < p$. [1]

Hence state the nature of the stationary point. [1]

(iii) Find $\frac{d^2y}{dx^2}$ at the stationary point and explain how $\frac{d^2y}{dx^2}$ further supports your answer in part (ii). [2]

7.



In the figure, AB is a diameter of the circle with centre O. Chords AD and BC intersect at F. AD produced meets the tangent to the circle, TBE at E. AE is an angle bisector of angle BAC.

(i) Prove that $\angle CBD = \angle DBE$. [3]

Given that $\angle AOF = 90°$, prove that

(ii) triangle AOF is similar to triangle ADB. [2]

(iii) $2(AO)^2 = AF \times (AF + FD)$. [3]

8. A particle moving in a straight line passes through a fixed point O with a speed of 20 m/s. The acceleration, a m/s^2, of the particle, t s after passing through O is given by $a = -100e^{-3t}$. The particle comes to instantaneous rest at point N.

(i) Find the time the particle comes to instantaneous rest at point N. [5]

(ii) Calculate the distance ON. [4]

(iii) Show that the average speed of the particle in the first 2 seconds rounded off to a whole number is 10 m/s. [3]

9. (i) Solve the equation $2\sin 2P = 3\cos P$ for $0^\circ \le P \le 360^\circ$. [4]

(ii) On the same axes, sketch for $0^\circ \le x \le 720^\circ$, the graphs of

$$y = \sin x \quad \text{and} \quad y = \frac{3}{2}\cos\left(\frac{x}{2}\right).$$ [4]

(iii) Using the solutions to part (i), determine the x-coordinates of the points of intersection of the graphs of part (ii). [4]

10. A circle, C_1, has equation $x^2 + y^2 - 14x + 2y = -46$.

(i) Find the coordinates of the centre of the circle and the radius. [3]

The coordinates of the centre of a second circle, C_2, is $(-4, -2)$. The equation of the tangent to the circle, C_2 at a point P is $2y = -2x + 3$.

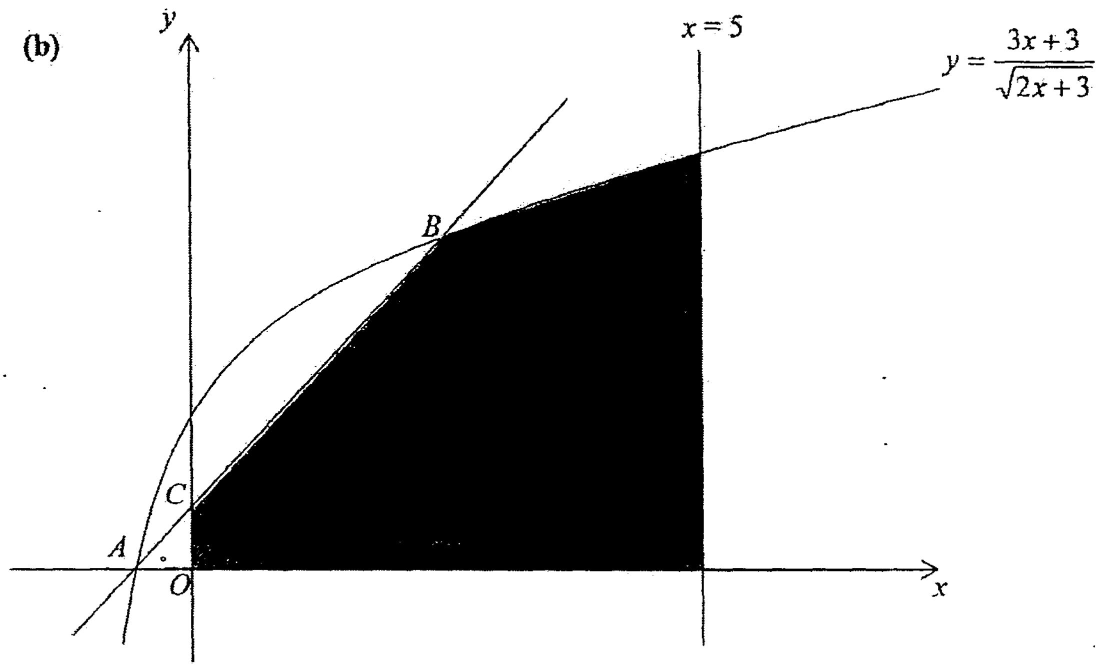
(ii) Find the coordinates of point P. [4]

(iii) Find the exact value of the radius of C_2 and the equation of the circle, C_2. [3]

(iv) Determine whether circles C_1 and C_2 will meet each other, showing your working clearly. [2]

11. **(a)** Show that $\dfrac{d}{dx}\left(2x\sqrt{2x+3}\right)=\dfrac{6x+6}{\sqrt{2x+3}}$. [3]

(b)



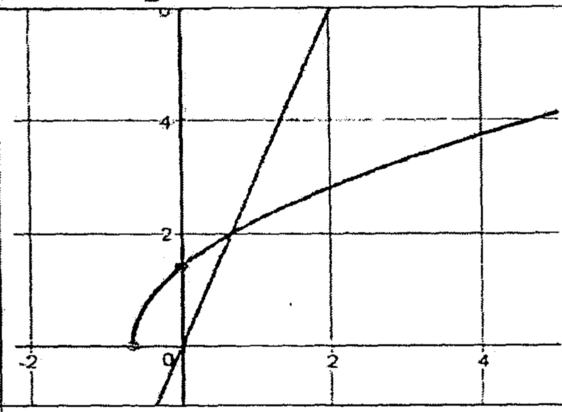
The diagram shows part of the curve $y=\dfrac{3x+3}{\sqrt{2x+3}}$. The curve intersects the x-axis at point A. The line through A and perpendicular to the line, $y+x=-7$ intersects the curve again at another point, B.

(i) Show that the y-coordinate of point B is 4. [5]

(ii) Given that the line AB intersects the y-axis at C, determine the area of the shaded region bounded by the line CB, the curve, the line $x=5$, the x-axis and the y-axis. [4]

End of Paper

S4 PRELIM 2016 Answer Key:

1(i)	$V = 132\ 000$	(ii)	show
(iii)	70 months		
2(i)	$A = -2$	(ii)	$x = 1\ ,\ \frac{-3 \pm \sqrt{5}}{2}$
3(a)i	$x = \frac{2}{3}$	Ii	
(b)	$\frac{5\sqrt{5}}{2} - \frac{5}{2}$	4(i)	$\alpha\beta = \frac{11}{2}$
4(ii)	$1331x^2 - 16x + 8 = 0$	5(a)	Discriminant = 368 Since discriminant > 0, the roots of the quadratic equation are real and distinct.
5(b)	$y = 0.631$	6(i)	$p = 0, q = 0$
6(ii)a	$\frac{dy}{dx} > 0$, y is increasing	6(ii)b	$\frac{dy}{dx} < 0$, y is decreasing
	Since the value of $\frac{dy}{dx}$ changes from negative to positive value, the stationary point is a minimum point.	6(iii)	$\frac{d^2y}{dx^2} = 4$, since $\frac{d^2y}{dx^2} > 0$, the stationary point is minimum, thus reiterating the result from part **(ii)**.
7	proof	8.(i)	$t = 0.305$ s
8(ii)	Distance = 2.59 m	8(iii)	show
9(i)	48.6°, 90°, 131.4°, 270°	(ii)	
9(iii)	97.2°,180°,262.8°,540°	10(i)	Centre(7, −1), radius = 2 units
10(ii)	$P(-\frac{1}{4}, \frac{7}{4})$	10(iii)	Radius = $\frac{15\sqrt{2}}{4}$ units

			$(x+4)^2+(y+2)^2=(\frac{15\sqrt{2}}{4})^2/\frac{225}{8}$
(iv)	Sum of radii(7.30 units) < distance between the centres (11.0 units) thus the circles will not meet.	11(a)	show
11(b)i	show	11(b)ii	16.5 units^2

4047/02 Prelim 2016 Suggested Solutions

1. A man buys a new car. The value of the car depreciates with time so that its value, $\$V$, after t months' use is given by $V = 132\,000e^{-pt}$, where p is a constant.
The value of the car is expected to be \$122 000 after eight months' use.

(i)

Find the value of the car when the man bought it.
$V = 132000e^{-pt}$ When the man bought the car, $t = 0$. Hence, $V = 132000e^{0}$, $e^{0} = 1$ $\therefore V = 132\,000$.

(ii)

Show that $p = 0.01$.
$V = 122000$ when $t = 8$ $122000 = 132000e^{-8p}$ $e^{-8p} = \dfrac{122000}{132000}$ $-8p = \ln\left(\dfrac{122000}{132000}\right)$ $p = -\dfrac{1}{8}\ln\left(\dfrac{122000}{132000}\right)$ $p = 0.009848$ $p = 0.01$ (1 sig fig) (shown)

(iii)

Using the value of $p = 0.01$, determine the age of the car to the nearest month, when its value reached half of the original value when the man bought it.
$132000e^{-0.01t} = \dfrac{1}{2} \times 132000$ $e^{-0.01t} = \dfrac{1}{2}$ $-0.01t = \ln\left(\dfrac{1}{2}\right)$ $t = 69.3147$ $t = 70$ months

2. The function $f(x)=1+2x+Ax^2-x^3$, where A is a constant, leaves a remainder of $1\frac{3}{8}$ when divided by $(2x-1)$.

(i) Find the value of A.

$f(x)=1+2x+Ax^2-x^3$

$f(\frac{1}{2})=1\frac{3}{8}$

$1+2(\frac{1}{2})+A(\frac{1}{2})^2-(\frac{1}{2})^3=\frac{11}{8}$

$\frac{1}{4}A=\frac{11}{8}-\frac{15}{8}$

$A=-2$

(ii) Hence, solve the equation $f(x)=0$, giving your answers in the exact form.

$f(x)=1+2x-2x^2-x^3$

$f(1)=1+2-2-1$

$f(1)=0$

$\therefore (x-1)$ is a factor

$f(x)=(x-1)(-x^2+ax-1)$

Compare coefficient of x:

$-1-a=2$

$a=-3$

$f(x)=(x-1)(-x^2-3x-1)$

$f(x)=0$,

$x=1$

$x^2+3x+1=0$

$$x=\frac{-3\pm\sqrt{(-3)^2-4(1)(1)}}{2(1)}$$

$$x=\frac{-3\pm\sqrt{5}}{2}$$

3.(a) (i) Solve $\sqrt{3x+2}-3x=0$.

$\sqrt{3x+2}-3x=0$

$\sqrt{3x+2}=3x$

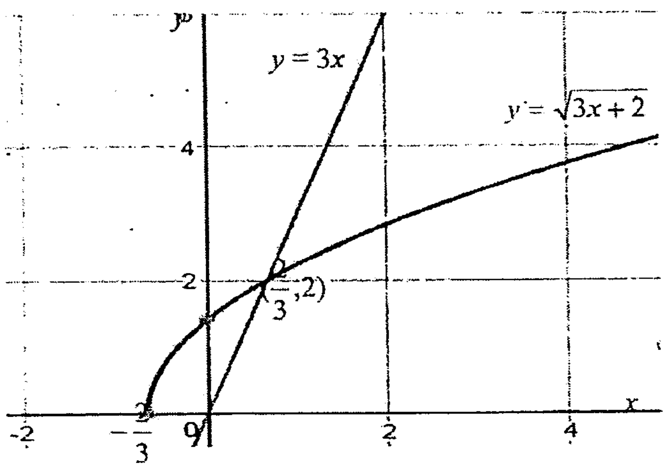
$3x+2=9x^2$

$9x^2-3x-2=0$

$(3x+1)(3x-2)=0$

$x=\frac{2}{3}$, $\quad x=-\frac{1}{3}$ (rejected)

(ii) On the same axes, sketch the graphs of $y=\sqrt{3x+2}$ and $y=3x$.
Indicate clearly all the points of intersections.



(b) The vertical height of a triangle is $\frac{8}{3-\sqrt{5}}$ cm. Given that the area of the triangle is $\frac{20}{\sqrt{5}-1}$ cm^2, without using a calculator, find the length of the base of the triangle in the form $a+b\sqrt{5}$.

$\frac{1}{2}\text{base of triangle}\times\frac{8}{3-\sqrt{5}}=\frac{20}{\sqrt{5}-1}$

$$\text{base of triangle}=\frac{20}{\sqrt{5}-1}\times\frac{3-\sqrt{5}}{4}$$

$$=\frac{5(3-\sqrt{5})}{\sqrt{5}-1}\times\frac{\sqrt{5}+1}{\sqrt{5}+1}$$

$$=\frac{5(2\sqrt{5}-2)}{5-1}$$

$$=\frac{5\times 2(\sqrt{5}-1)}{4}$$

$$=\frac{5}{2}\sqrt{5}-\frac{5}{2}$$

4. The roots of the quadratic equation, $2x^2+4x+5=0$ are $(\alpha+1)$ and $(\beta+1)$.

(i)

Show that $\alpha+\beta=-4$ and hence find $\alpha\beta$.
Sum of roots : $(\alpha+1)+(\beta+1)=-2$ $\alpha+\beta=-4$ (shown) Product of roots: $(\alpha+1)(\beta+1)=\frac{5}{2}$ $\alpha\beta+(\alpha+\beta)+1=\frac{5}{2}$ $\alpha\beta=\frac{5}{2}-1-(-4)$ $\alpha\beta=\frac{11}{2}$

(ii)

Find the quadratic equation in x with integer coefficients, whose roots are $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$.
Sum of roots of new equation: $\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{\alpha^3+\beta^3}{(\alpha\beta)^3}$ $=\frac{(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)}{(\alpha\beta)^3}$ $=\frac{(\alpha+\beta)[(\alpha+\beta)^2-2\alpha\beta-\alpha\beta]}{(\alpha\beta)^3}$ $=\frac{(-4)[(-4)^2-3(\frac{11}{2})]}{(\frac{11}{2})^3}$ $=\frac{16}{1331}$ Product of roots of new equation: $\frac{1}{(\alpha\beta)^3}$ $=\frac{1}{\left(\frac{11}{2}\right)^3}$ $=\frac{8}{1331}$ Equation is $1331x^2-16x+8=0$.

5(a)	Given that $\log_2(2x+1)-\log_4(3-x^2)=1$, form a quadratic equation in x and explain why the roots of the quadratic equation are real and distinct.
	$\log_2(2x+1)-\log_4(3-x^2)=1$ $\log_2(2x+1)-\dfrac{\log_2(3-x^2)}{\log_2 2^2}=1$ $\log_2(2x+1)-\dfrac{1}{2}\log_2(3-x^2)=1$ $\log_2\dfrac{(2x+1)}{\sqrt{3-x^2}}=1$ $\dfrac{2x+1}{\sqrt{3-x^2}}=2$ $2x+1=2\sqrt{3-x^2}$ $(2x+1)^2=4(3-x^2)$ $4x^2+4x+1=12-4x^2$ $8x^2+4x-11=0$ Discriminant $=4^2-4(8)(-11)$ $=368$ Since the discriminant is greater than 0, the roots of the quadratic equation are real and distinct.

5(b)	Solve $3^{y+2}=2(3^{-y})+17$.
	$3^{y+2}=2(3^{-y})+17$ $3^{2(y+1)}-17(3^y)=2$ $3^2(3^y)^2-17(3^y)=2$ Let $a=3^y$, $9a^2-17a-2=0$ $(9a+1)(a-2)=0$ $a=-\dfrac{1}{9}$ (rejected), $a=2$ $3^y=2$ $y=\dfrac{\lg 2}{\lg 3}$ $y=0.631$

6. The curve $y=\frac{2x^2}{x^2+1}$ has one stationary point (p, q).

(i)

Find the value of p and of q.
$y=\frac{2x^2}{x^2+1}$ $\frac{dy}{dx}=\frac{(x^2+1)(4x)-2x^2(2x)}{(x^2+1)^2}$ $=\frac{4x^3+4x-4x^3}{(x^2+1)^2}$ $=\frac{4x}{(x^2+1)^2}$ For a stationary point, put $\frac{dy}{dx}=0$ $(x^2+1)^2>0, \quad 4x=0, \therefore x=0$ $\therefore p=0$ and $q=0$

(ii) determine whether y is increasing or decreasing

(a)

for $x>p$,		
For $x>0$, $(x^2+1)^2>0$ and $4x>0, x>0$ $\therefore \frac{dy}{dx}>0$, y is increasing		

(b)

for $x<p$.
$x<0$, $(x^2+1)^2>0$ but $4x<0$, i.e. $x<0$ $\therefore \frac{dy}{dx}<0$, y is decreasing

Hence state the nature of the stationary point.
Since the value of $\frac{dy}{dx}$ changes from negative to positive, the stationary point is a minimum point.

(iii) Find $\frac{d^2y}{dx^2}$ at the stationary point and explain how $\frac{d^2y}{dx^2}$ further supports your answer to part **(ii)**.

$$\frac{dy}{dx} = \frac{4x}{(x^2+1)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(x^2+1)^2(4) - 4x(2)(x^2+1)(2x)}{(x^2+1)^4}$$

$$\frac{d^2y}{dx^2} = \frac{4(x^2+1)(x^2+1-4x^4-4x^2)}{(x^2+1)^4}$$

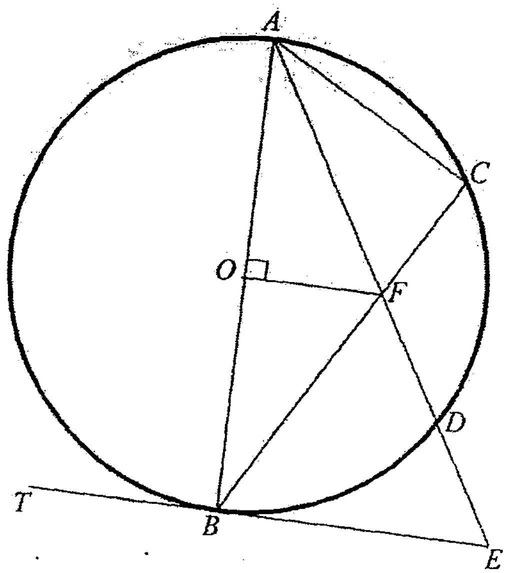
$$= \frac{4(1-3x^2-4x^4)}{(x^2+1)^3}$$

At the stationary point (0 , 0),

$$\frac{d^2y}{dx^2} = 4$$

Since, $\frac{d^2y}{dx^2} > 0$ the stationary point is minimum, thus reiterating the result from part **(ii)**

7.



In the figure, AB is a diameter of the circle with centre O. Chords AD and BC intersect at F. AD produced meets the tangent to the circle, TBE at E. AE is an angle bisector of angle BAC.

(i) Prove that $\angle CBD = \angle DBE$.

$\angle DBE = \angle BAD$ (Alternate segment Thm)

$\angle BAD = \angle CAD$ (given EA is bisector of $\angle BAC$)

$\therefore \angle DBE = \angle CAD$

$\angle CAD = \angle CBD$ (angles in the same segment)

$\angle DBE = \angle CBD$ (proven)

(ii) Given that $\angle AOF = 90°$, prove that triangle AOF is similar to triangle ADB.

$\angle A$ is a common angle.

$\angle ADB = 90°$ (angle in the semi-circle)

$\angle ADB = \angle AOF$

$\therefore \Delta AOF$ is similar to ΔADB (By AA similarity test)

(iii) $2(AO)^2 = AF \times (AF + FD)$.

Since ΔAOF is similar to ΔADB,

$$\frac{AO}{AD} = \frac{AF}{AB}$$

$$\frac{AO}{AF+FD} = \frac{AF}{AB} \quad (AD = AF + FD)$$

$$\frac{AO}{AF+FD} = \frac{AF}{2AO} \textit{(AO is radius and AB is diameter)}$$

$$2(AO)^2 = AF \times (AF + FD)$$

8. A particle moving in a straight line passes through a fixed point O with a speed of 20 m/s. The acceleration, a m/s^2, of the particle, t s after passing through O is given by $a = -100e^{-3t}$. The particle comes to instantaneous rest at point N.

(i)	Find the time the particle comes to instantaneous rest at point N.

$a = -100e^{-3t}$

velocity, $v = \int -100e^{-3t}\,dt$

$= \frac{100}{3}e^{-3t} + c$, where c is a constant

when $v = 20$ and $t = 0$,

$\frac{100}{3}e^{0} + c = 20$

$\therefore c = -\frac{40}{3}$

$v = \frac{100}{3}e^{-3t} - \frac{40}{3}$

at rest, $v = 0$

$\frac{100}{3}e^{-3t} - \frac{40}{3} = 0$

$e^{-3t} = \frac{40}{3} \times \frac{3}{100}$

$-3t \ln e = \ln\left(\frac{2}{5}\right)$

$t = -\frac{1}{3}\ln\left(\frac{2}{5}\right)$

$t = 0.30543$

The particle comes to rest at $t = 0.305$ s.

(ii)	Calculate the distance ON.

$v = \frac{100}{3}e^{-3t} - \frac{40}{3}$

displacement, $s = \int \frac{100}{3}e^{-3t} - \frac{40}{3}\ dt$

$s = -\frac{100}{9}e^{-3t} - \frac{40}{3}t + c$ where c is a constant

when $s = 0,\ t = 0\ \therefore\ c = \frac{100}{9}$

$\therefore s = -\frac{100}{9}e^{-3t} - \frac{40}{3}t + \frac{100}{9}$

when $t = 0.30543, s = -\frac{100}{9}e^{-3(0.30543)} - \frac{40}{3}(0.30543) + \frac{100}{9}$

$s = 2.5943$

Distance $ON = 2.59$ m

(iii)	Show that the average speed of the particle in the first 2 seconds rounded off to whole number is 10 metres per second.

At $t = 2$, $s = -\frac{100}{9}e^{-3(2)} - \frac{40}{3}(2) + \frac{100}{9}$

$= -15.583$ m

Total distance travelled in the first 2 seconds

$= (2)2.5943 + 15.583$

$=20.7716$

Average speed $= 20.7716 \div 2$

$= 10.3858$

$= 10$ m/s (whole number) (shown)

9(i)	Solve the equation $2\sin 2P = 3\cos P$ for $0^\circ \le P \le 360^\circ$.

$2\sin 2P - 3\cos P = 0$

$2(2\sin P\cos P) - 3\cos P = 0$

$\cos P(4\sin P - 3) = 0$

$\cos P = 0$, $\sin P = \dfrac{3}{4}$

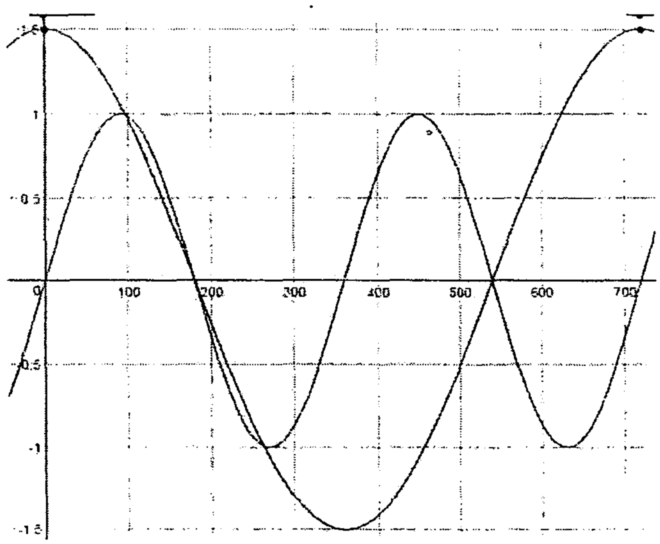
$P = 90°, 270°$ basic $\angle = 48.590°$

$P = 48.590°,\ 131.41°$

$\therefore$ Ans : $P = 48.6°,\ 90°,\ 131.4°, 270°$

(ii) On the same axes, sketch for $0^\circ \le x \le 720^\circ$, the graphs of

$y = \sin x$ and $y = \dfrac{3}{2}\cos\left(\dfrac{x}{2}\right)$.



(iii) Using solutions to part **(i)**, determine the x-coordinates of the points of intersections of the graphs of part **(ii)**.

$\sin x = \dfrac{3}{2}\cos\left(\dfrac{x}{2}\right)$

$2\sin 2\left(\dfrac{x}{2}\right) - 3\cos\left(\dfrac{x}{2}\right) = 0$

Let $P = \left(\dfrac{x}{2}\right)$, then

$\dfrac{x}{2} = 48.590°, 90°, 131.41°, 270°$

$x = 97.2°, 180°, 262.8°, 540°$

10. A circle, C_1, has equation $x^2+y^2-14x+2y=-46$.

(i)

Find the coordinates of the centre of the circle and the radius.
Centre $(7, -1)$ Radius $= \sqrt{7^2+(-1)^2-46}$ $= 2$ units

The coordinates of the centre of a second circle, C_2, is $(-4, -2)$. The equation of the tangent to the circle, C_2 at a point P is $2y=-2x+3$.

(ii)

Find the coordinates of point P.
Gradient of tangent to circle at $P=-1$ Equation of the normal at P is $\frac{y+2}{x+4}=1$ $y+2=x+4$ $y=x+2$ (1) $2y=-2x+3$ (2) substitute (1) into (2) $2(x+2)=-2x+3$ $2x+4=-2x+3$ $x=-\frac{1}{4}$, $y=-\frac{1}{4}+2$ $y=\frac{7}{4}$ $\therefore P(-\frac{1}{4},\frac{7}{4})$

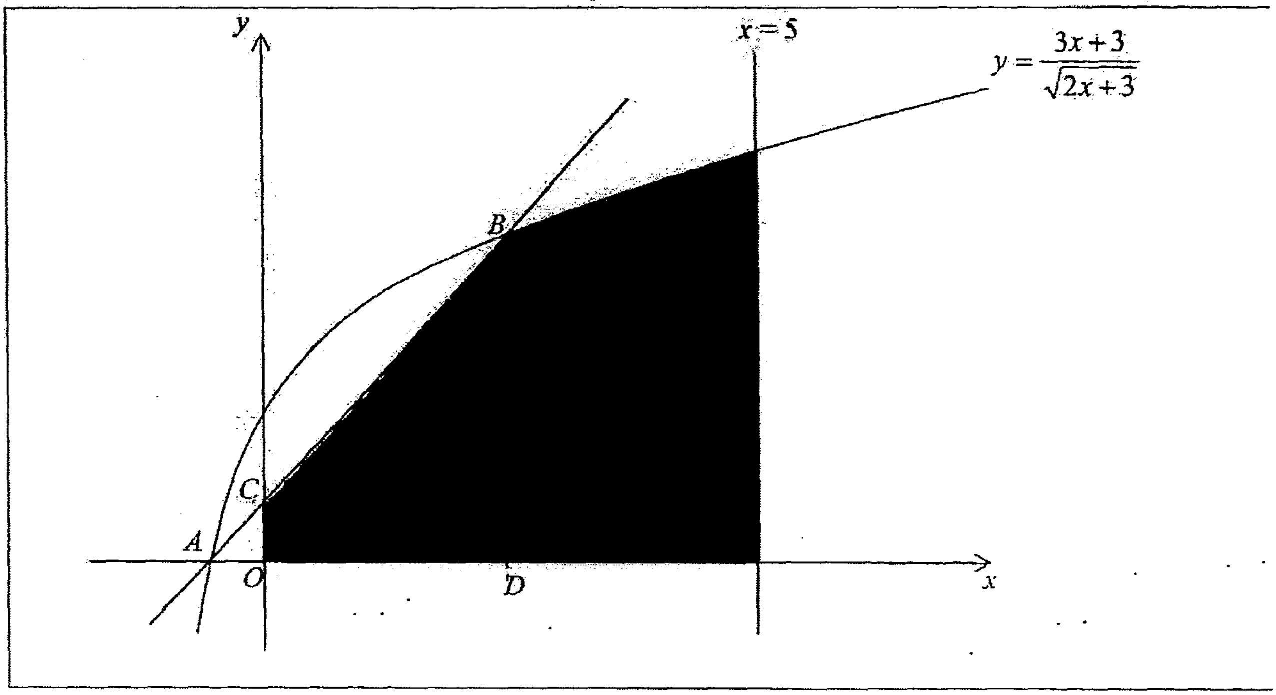
(iii)

Find the exact value of the radius of C_2 and the equation of the circle, C_2.
Radius of $C_2 = \sqrt{\left(-4+\frac{1}{4}\right)^2+\left(-2-\frac{7}{4}\right)^2}$ $=\frac{15\sqrt{2}}{4}$ units Equation of C_2 is $(x+4)^2+(y+2)^2=\frac{225}{8}$

(iv)	Determine whether circles C_1 and C_2 will meet each other, showing your working clearly.
	Distance between centres of C_1 and C_2 $=\sqrt{(7+4)^2+(-1+2)^2}$ $=\sqrt{122}$ $=11.0$ Sum of radii $= 2 + \dfrac{15\sqrt{2}}{4}$ $=7.30$ Since the sum of radii, 7.30 units, is less than the distance between the 2 centres, 11.0 units, the 2 circles C_1 and C_2 will not meet each other.

11(a)	Show that $\dfrac{d}{dx}\left(2x\sqrt{2x+3}\right)=\dfrac{6x+6}{\sqrt{2x+3}}$.
	$\dfrac{d}{dx}\left(2x\sqrt{2x+3}\right)$ $=2x.\dfrac{1}{2}(2x+3)^{-\frac{1}{2}}(2)+2\sqrt{2x+3}$ $=\dfrac{2(2x+3)+2x}{\sqrt{2x+3}}$ $=\dfrac{6x+6}{\sqrt{2x+3}}$ (shown)

(b)



The diagram shows part of the curve $y = \frac{3x+3}{\sqrt{2x+3}}$. The curve intersects the x-axis at point A. The line through A and perpendicular to the line $y + x = -7$ intersects the curve again at another point, B.

(i) Show that the y-coordinate of point B is 4.

When $y = 0$, $3x + 3 = 0$. $\therefore A(-1, 0)$
Gradient of the line $AB = 1$
Equation of line AB:

$$\frac{y-0}{x+1} = 1$$

$$y = x + 1 \qquad (1)$$

$$y = \frac{3x+3}{\sqrt{2x+3}} \qquad (2)$$

substitute (1) into (2)

$$x + 1 = \frac{3(x+1)}{\sqrt{2x+3}}$$

$$(x+1)\left[\frac{\sqrt{2x+3}-3}{\sqrt{2x+3}}\right] = 0$$

$$x = -1, \quad \sqrt{2x+3} - 3 = 0$$

$$2x + 3 = 9$$

$$x = 3$$

$$\therefore y = 3 + 1$$

$$y = 4$$

Hence the y-coordinate of $B = 4$ (shown)

(ii)	Given that the line AB intersects the y-axis at C, determine the area of the shaded region bounded by the line CB, the curve, the line $x = 5$, the x-axis and the y-axis.
	For $y = x + 1$ when $x = 0$, $y = 1$ $\therefore C(0,1)$ Area of shaded region = area of trapezium $OCBD$ + area under the curve $= \frac{1}{2}(1+4)\times 3 + \int_3^5 \frac{3x+3}{\sqrt{2x+3}}\,dx$ $= \frac{3}{2}(5) + \frac{1}{2}\int_3^5 \frac{6x+6}{\sqrt{2x+3}}\,dx$ $= 7.5 + \frac{1}{2}\left[2x\sqrt{2x+3}\right]_3^5$ $= 7.5 + \frac{1}{2}\left[2(5)\sqrt{2(5)+3} - 2(3)\sqrt{2(3)+3}\right]$ $= 16.5$ units^2

End of paper