	Class	Register Number
Name ______________________		

4047/01 16/S4PR2/AM/1

ADDITIONAL MATHEMATICS **PAPER 1**

Wednesday **3 August 2016** **2 hours**

PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

Additional Material: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

1 (i) Simplify $|21-14x|-\left|\frac{2}{3}x-1\right|$. [2]

(ii) Hence, solve $|21-14x|=\left|\frac{2}{3}x-1\right|+40-15x$. [3]

2 The range of solutions for x such that $a+bx-4x^2>0$ is $-\frac{1}{2}<x<3$. Find the value of a and of b, where a and b are real numbers. [4]

3 (a) Solve $4^x-20\left(4^{-x}\right)=1$. [3]

(b) Given that $\frac{5^{4-\frac{x}{2}}}{625\left(5^y\right)}=\frac{25^y}{\sqrt{125^x}}$, find the value of $\frac{x}{y}$. [3]

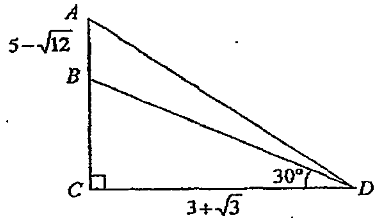
4 In the expansion of $\left(ax+\frac{1}{x}\right)^n$ in descending powers of x, where a and n are positive integers, the fourth term of the expansion is the constant term.

(i) Find the value of n and hence, express the constant term in terms of a. [4]

(ii) Using your value of n in (i), determine if a term in $\frac{1}{x^3}$ in the expansion $(1-x)\left(ax+\frac{1}{x}\right)^n$ exists. [2]

5 (a) Solve $\sqrt{1-2x}-3x=1$. [3]

(b)



In the diagram above, ACD is a triangle such that B lies on AC, $AB=\left(5-\sqrt{12}\right)$ cm, $CD=\left(3+\sqrt{3}\right)$ cm, $\angle BDC=30°$ and $\angle BCD$ is a right angle. Find AD^2 in the form $p+q\sqrt{3}$, where p and q are constants. [4]

6 (a) Differentiate $\ln\sqrt{\frac{1-3x}{e^{-x}}}$. [3]

(b) Given that $\int_1^4 f(x)\,dx=6$, $\int_3^6 f(x)\,dx=2$ and $\int_7^4 f(x)\,dx=-3$, find

(i) $\int_1^7 f(x)\,dx$, [1]

(ii) $\int_1^3 f(x)\,dx+\int_6^7 f(x)\,dx$, [1]

(iii) the value of h, where $\int_1^4 hx^2+2f(x)\,dx=180$. [2]

7 The equation of a curve is $y=3\left(\frac{x}{4}+a\right)^{\frac{5}{6}}$. The normal to the curve at $x=\frac{1}{2}$ is parallel to the line $5y+4x=2$.

(i) Show that $a=-\frac{7}{64}$. [4]

(ii) Find the equation of the tangent to the curve at $x=\frac{1}{2}$. [2]

(iii) Show that the curve is an increasing function for $x>\frac{7}{16}$. [2]

8 It is given that $\cos 140° = -k$ and $\tan A = -\frac{3}{4}$, where k is a positive number and A is a reflex angle. Without finding the value of A or of k,

(i) find the exact value of $\cos 2A$, [2]

(ii) express $\tan 50°$ in terms of k, [3]

(iii) express $\sin(40° + A)$ in terms of k. [3]

9 The points $P(1, -2)$ and $Q(1, 4)$ lie on the circumference of a circle with centre C. If the circle is reflected in a vertical line, P and Q remain unchanged in the reflection and the x-coordinate of the centre of the reflected circle is 5.

(i) State the equation of the vertical line of reflection. [1]

(ii) Show that the equation of the circle with centre C is $x^2 + y^2 + 6x - 2y - 15 = 0$. [3]

(iii) The line $3y + 4x = -9$ intersects the circle with centre C at two points, A and B. Find the coordinates of A and of B. [4]

(iv) Determine if AB is a diameter of the circle with centre C. [1]

10 A particle moves in a straight line such that at t seconds after passing point O, its velocity v m/s is given by $v = t - 7 + \frac{12}{t+1}$, where $t > 0$.

Find

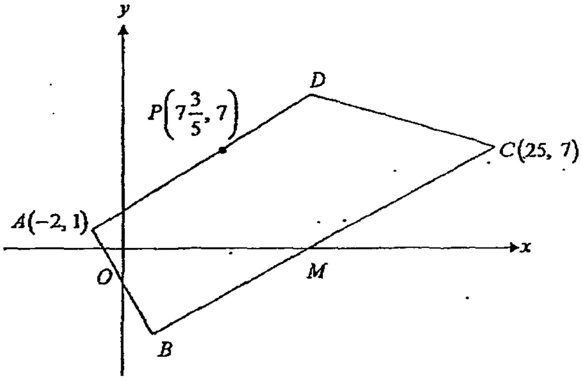
(i) the acceleration of the particle when it is first instantaneously at rest. [3]

(ii) an expression for the displacement of the particle from O. [3]

(iii) the total distance travelled by the particle from $t = 0$ to $t = 5$. [3]

11 **Solutions to this question by accurate drawing will not be accepted.**

The diagram shows the quadrilateral $ABCD$ in which point A is $(-2, 1)$ and point C is $(25, 7)$. The point $P\left(7\frac{3}{5}, 7\right)$ lies on AD such that $AP : PD = 3 : 2$. The midpoint of BC, point M, lies on the x-axis and directly below point D.



(i) Find the coordinates of points D, M and B. [6]

(ii) Determine if $\angle DAB$ is a right angle. [3]

(iii) Calculate the area of the quadrilateral $ABCD$. [2]

End of Paper

Answer key

1(i)	$\frac{20}{3}\lvert 3-2x\rvert$ or $\frac{20}{3}\lvert 2x-3\rvert$
1(ii)	$x=2\frac{2}{17}$ or 12 (NA)
2	$a=6,\ b=10$
3(a)	$x=1.16$
3(b)	$\frac{x}{y}=3$
4(i)	Constant term $=20a^3$
4(ii)	A term exists in $\frac{1}{x^3}$.
5(a)	$x=0$ or $-\frac{8}{9}$ (NA)
5(b)	$AD^2=51-6\sqrt{3}$
6(a)	$\frac{1}{2}\left(1-\frac{3}{1-3x}\right)$ OR $\frac{2+3x}{2(3x-1)}$
6(bi)	9
6(bii)	7
6(biii)	$h=8$
7(ii)	$y=\frac{5}{4}x-\frac{17}{32}$
7(iii)	Show $\frac{dy}{dx}>0$ for $x>\frac{7}{16}$.
8(i)	$\frac{7}{25}$
8(ii)	$\frac{k}{\sqrt{1-k^2}}$
8(iii)	$\frac{4}{5}\sqrt{1-k^2}-\frac{3}{5}k$
9(i)	$x=1$
9(iii)	$(0,-3)$ and $(-6,5)$
9(iv)	AB is a diameter of the circle.
10(i)	-2 m/s^2
10(ii)	$s=\frac{t^2}{2}-7t+12\ln\lvert t+1\rvert$
10(iii)	4.63 m
11(i)	$D(14,\ 11)$, $M(14,0)$, $B(3,\ -7)$
11(ii)	$\angle DAB$ is a right angle
11(iii)	210 units^2

S4 Prelim 2 AM Paper 1 Answers

1 (i) Simplify $|21-14x|-\left|\frac{2}{3}x-1\right|$. [2]

(ii) Hence, solve $|21-14x|=\left|\frac{2}{3}x-1\right|+40-15x$. [3]

(i) $|21-14x|-\left|\frac{2}{3}x-1\right|=7|3-2x|-\frac{1}{3}|2x-3|$ M1

$=\frac{20}{3}|3-2x|$ or $\frac{20}{3}|2x-3|$ A1

(ii) $|21-14x|-\left|\frac{2}{3}x-1\right|=40-15x$

$|21-14x|-\left|\frac{2}{3}x-1\right|=40-15x$

$\frac{20}{3}|3-2x|=40-15x$ M1

$|3-2x|=\frac{3}{20}(40-15x)$

$=6-\frac{9}{4}x$

$3-2x=6-\frac{9}{4}x$ or $3-2x=\frac{9}{4}x-6$ M1

$x=12$ (NA) or $x=2\frac{2}{17}$ A1

2 The range of solutions for x such that $a+bx-4x^2>0$ is $-\frac{1}{2}<x<3$. Find the value of a and of b, where a and b are real numbers. [4]

Given that $-\frac{1}{2}<x<3$, $\left(x+\frac{1}{2}\right)(x-3)<0$	B1 for correctly identifying factors $\left(x+\frac{1}{2}\right)(x-3)$
$x^2+\frac{1}{2}x-3x-\frac{3}{2}<0$	B1 for correct corresponding inequality OR
$x^2-\frac{5}{2}x-\frac{3}{2}<0$	$-\left(x+\frac{1}{2}\right)(x-3)>0$
$-4x^2+10x+6>0$	M1 for multiplication
$\therefore a=6,\ b=10$	A1

3 **(a)** Solve $4^x - 20(4^{-x}) = 1$. [3]

(b) Given that $\dfrac{5^{4-\frac{x}{2}}}{625(5^y)} = \dfrac{25^y}{\sqrt{125^x}}$, find the value of $\dfrac{x}{y}$. [3]

(a)	$4^x - 20(4^{-x}) = 1$	
	$4^{2x} - (4^x) - 20 = 0$	M1
	$(4^x + 4)(4^x - 5) = 0$	M1
	$4^x = -4$ (NA) or $4^x = 5$	
	$\therefore x = \dfrac{\lg 5}{\lg 4} \approx 1.16$	A1, with $4^x = -4$ (NA)
(b)	$\dfrac{5^{4-\frac{x}{2}}}{625(5^y)} = \dfrac{25^y}{\sqrt{125^x}}$	
	$\dfrac{5^{4-\frac{x}{2}}}{5^4(5^y)} = \dfrac{5^{2y}}{\sqrt{5^{3x}}}$	M1, changing all to base 5
	$5^{4-\frac{x}{2}} \times 5^{\frac{3}{2}x} = 5^{2y} \times 5^{4+y}$	
	$5^{4+x} = 5^{3y+4}$	M1, for simplifying indices, OR below.
	$x = 3y$	
	$\dfrac{x}{y} = 3$	A1
	OR	
	$\dfrac{5^{4-\frac{x}{2}}}{5^4(5^y)} = \dfrac{5^{2y}}{\sqrt{5^{3x}}}$	M1, changing all to base 5
	$4 - \dfrac{x}{2} - 4 - y = 2y - \dfrac{3x}{2}$	M1
	$x = 3y$	
	$\dfrac{x}{y} = 3$	A1

4 In the expansion of $\left(ax+\frac{1}{x}\right)^n$ in descending powers of x, where a and n are positive integers, the fourth term of the expansion is the constant term.

(i) Find the value of n and hence, express the constant term in terms of a. [4]

(ii) Using the value of n in (i), determine if a term in $\frac{1}{x^3}$ in the expansion $(1-x)\left(ax+\frac{1}{x}\right)^n$ exists. [2]

(i)

$$T_{r+1}=\binom{n}{r}(ax)^{n-r}\left(\frac{1}{x}\right)^r$$

$$=\binom{n}{r}a^{n-r}x^{n-2r}$$

M1 for general term

$$T_4=\binom{n}{3}a^{n-3}x^{n-6}$$

When $n-6=0$,

$n=6$

M1 for equating power of $x=0$

A1

Constant term $=\binom{6}{3}a^{6-3}$

$=20a^3$

A1

(ii)

$$(1-x)\left(ax+\frac{1}{x}\right)^n$$

Alternative method: Full or partial expansion of $(1-x)\left(ax+\frac{1}{x}\right)^6$

For a term in $\frac{1}{x^3}$, there must be a term in $\frac{1}{x^3}$ or $\frac{1}{x^4}$ in the expansion of $\left(ax+\frac{1}{x}\right)^n$.

General term $=\binom{6}{r}a^{6-r}x^{6-2r}$

When $6-2r=-4, r=5$

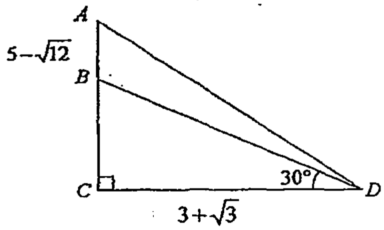
M1, for considering $\frac{1}{x^4}$ term

Hence, a term exists in $\frac{1}{x^3}$.

A1

5 **(a)** Solve $\sqrt{1-2x}-3x=1$. [3]

(b)



In the diagram above, ACD is a triangle such that B lies on AC, $AB=\left(5-\sqrt{12}\right)$cm, $CD=\left(3+\sqrt{3}\right)$cm, $\angle BDC=30°$ and $\angle BCD$ is a right angle. Find the exact length of AD^2 in the form $p+q\sqrt{3}$. [4]

(a) $\sqrt{1-2x}-3x=1$

$\sqrt{1-2x}=1+3x$

$1-2x=\left(1+3x\right)^2$ M1, for squaring both sides

$=1+6x+9x^2$

$9x^2+8x=0$

$x\left(9x+8\right)=0$ M1, for factorising

$x=0$ or $x=-\dfrac{8}{9}$ (NA) A1, must reject negative value

(b) $\tan 30°=\dfrac{BC}{3+\sqrt{3}}$ M1, to see $\tan 30°=\dfrac{1}{\sqrt{3}}$ and correct ratio

$\dfrac{1}{\sqrt{3}}=\dfrac{BC}{3+\sqrt{3}}$

$BC=\dfrac{3+\sqrt{3}}{\sqrt{3}}$

$=\dfrac{3\sqrt{3}+3}{3}$

$=\sqrt{3}+1$

$AC=5-\sqrt{12}+\sqrt{3}+1$ M1, to see addition and $\sqrt{12}=2\sqrt{3}$

$=5-2\sqrt{3}+\sqrt{3}+1$

$=6-\sqrt{3}$

$AD^2=\left(3+\sqrt{3}\right)^2+\left(6-\sqrt{3}\right)^2$ M1, for Pythagoras' Theorem

$=9+6\sqrt{3}+3+36-12\sqrt{3}+3$ A1

$=51-6\sqrt{3}$

6 **(a)** Differentiate $\ln\sqrt{\frac{1-3x}{e^{-x}}}$. [3]

(b) Given that $\int_1^4 f(x)dx = 6$, $\int_3^6 f(x)dx = 2$ and $\int_7^4 f(x)dx = -3$, find

(i) $\int_1^7 f(x)dx$, [1]

(ii) $\int_1^3 f(x)dx + \int_6^7 f(x)dx$, [1]

(iii) the value of h, where $\int_1^4 hx^2 + 2f(x)dx = 180$. [2]

(a)

$$\frac{d}{dx}\left(\ln\sqrt{\frac{1-3x}{e^{-x}}}\right) = \frac{1}{2}\frac{d}{dx}\left[\ln(1-3x) - \ln e^{-x}\right]$$

M1 for applying quotient law

$$= \frac{1}{2}\frac{d}{dx}\left[\ln(1-3x) + x\right]$$

M1 for applying power law

$$= \frac{1}{2}\left(1 - \frac{3}{1-3x}\right) \text{ OR } \frac{2+3x}{2(3x-1)}$$

A1

OR

$$\frac{d}{dx}\left(\ln\sqrt{\frac{1-3x}{e^{-x}}}\right) = \frac{\frac{1}{2}\left(\frac{1-3x}{e^{-x}}\right)^{-\frac{1}{2}}\left(\frac{e^{-x}(-3)-(1-3x)(-e^{-x})}{e^{-2x}}\right)}{\left(\frac{1-3x}{e^{-x}}\right)^{\frac{1}{2}}}$$

B1 for correct numerator

B1 for correct denominator

$$= \frac{\frac{-3+1-3x}{e^{-x}}}{2\left(\frac{1-3x}{e^{-x}}\right)}$$

$$= \frac{-2-3x}{2(1-3x)}$$

$$= \frac{2+3x}{2(3x-1)}$$

A1

(bi)

$$\int_1^7 f(x)dx = \int_1^4 f(x)dx + \int_4^7 f(x)dx$$
$$= 6-(-3)$$
$$= 9$$

A1, must see 6 + 3

(bii)	$\int_1^3 \mathrm{f}(x)\,\mathrm{d}x+\int_6^7 \mathrm{f}(x)\,\mathrm{d}x=\int_1^7 \mathrm{f}(x)\,\mathrm{d}x-\int_3^6 \mathrm{f}(x)\,\mathrm{d}x$	
	$=9-2$	
	$=7$	A1, must see 9 – 2
(biii)	$\int_1^4 hx^2+2\mathrm{f}(x)\,\mathrm{d}x=180$	
	$h\left[\frac{x^3}{3}\right]_1^4+2(6)=180$	M1
	$21h=168$	
	$h=8$	A1

7 The equation of a curve is $y=3\left(\frac{x}{4}+a\right)^{\frac{5}{6}}$. The normal to the curve at $x=\frac{1}{2}$ is parallel to the line $5y+4x=2$.

(i) Show that $a=-\frac{7}{64}$. [4]

(ii) Find the equation of the tangent to the curve at $x=\frac{1}{2}$. [2]

(iii) Show that the curve is an increasing function for $x>\frac{7}{16}$. [2]

(i)

$$y=3\left(\frac{x}{4}+a\right)^{\frac{5}{6}}$$

$$\frac{dy}{dx}=3\left(\frac{5}{6}\right)\left(\frac{x}{4}+a\right)^{-\frac{1}{6}}\left(\frac{1}{4}\right)=\frac{5}{8}\left(\frac{x}{4}+a\right)^{-\frac{1}{6}}$$

M1 for differentiation

When $x=\frac{1}{2}$, $\frac{dy}{dx}=\frac{5}{8}\left(\frac{1}{8}+a\right)^{-\frac{1}{6}}$

Gradient of normal = $-\frac{4}{5}$

Gradient of tangent = $\frac{5}{4}$

M1 for $\text{Grad}_{\text{tangent}}=-\frac{1}{\text{Grad}_{\text{normal}}}$

$$\frac{5}{8}\left(\frac{1}{8}+a\right)^{-\frac{1}{6}}=\frac{5}{4}$$

M1 for $\frac{dy}{dx}\Big|_{x=\frac{1}{2}}=\text{Grad}_{\text{tangent}}$

$$\left(\frac{1}{8}+a\right)^{-\frac{1}{6}}=2$$

$$\left(\frac{1}{8}+a\right)=2^{-6}=\frac{1}{64}$$

$$a=-\frac{7}{64}$$

A1

(ii)

When $x=\frac{1}{2}$, $y=3\left(\frac{1}{8}-\frac{7}{64}\right)^{\frac{5}{6}}=\frac{3}{32}$

M1 for finding y-coordinate

Equation of tangent is

$$y-\left(\frac{3}{32}\right)=\frac{5}{4}\left(x-\frac{1}{2}\right)$$

$y=\frac{5}{4}x-\frac{17}{32}$ OR $32y=40x-17$

A1

(iii) $\frac{dy}{dx}=\frac{5}{8}\left(\frac{x}{4}-\frac{7}{64}\right)^{-\frac{1}{6}}$

For $x>\frac{7}{16}$,

$\frac{x}{4}>\frac{7}{64}$

$\frac{x}{4}-\frac{7}{64}>0$ M1, for showing $\frac{x}{4}-\frac{7}{64}>0$

$\frac{5}{8}\left(\frac{x}{4}-\frac{7}{64}\right)^{-\frac{1}{6}}>0$

$\frac{dy}{dx}>0$

$\therefore$ the curve is an increasing function for $x>\frac{7}{16}$. A1, with conclusion $\frac{dy}{dx}>0$

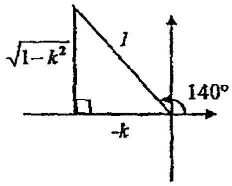
8 It is given that $\cos 140°=-k$ and $\tan A=-\frac{3}{4}$, where k is a positive number and A is a reflex angle. Without finding the value of A or of k,

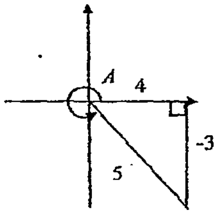
(i) find the exact value of $\cos\ 2A$, [2]

(ii) express $\tan 50°$ in terms of k, [3]

(iii) express $\sin(40°+A)$ in terms of k. [3]

(i)





(i) $\cos 2A=2\cos^2 A-1$ OR $\cos 2A=1-2\sin^2 A$

$=2\left(\frac{4}{5}\right)^2-1$ OR $=1-2\left(-\frac{3}{5}\right)^2$ M1, for $\cos A=4/5$ OR $\sin A=-3/5$

$=\frac{7}{25}$ $=\frac{7}{25}$ A1

(ii)	$\tan 50° = \tan(90° - 40°)$	M1, for $\tan(90° - 40°)$
	$= \frac{1}{\tan 40°}$	M1, for $\tan(90° - \theta) = \frac{1}{\tan\theta}$
	$= \frac{k}{\sqrt{1-k^2}}$	A1
	OR	
	$\tan 50° = \tan(90° - 40°)$	M1, for $\tan(90° - 40°)$
	$= \frac{1}{\tan 40°}$	M1, for $\tan(90° - \theta) = \frac{1}{\tan\theta}$
	$= -\frac{1}{\tan 140°}$	
	$= \frac{k}{\sqrt{1-k^2}}$	A1
(iii)	$\sin(40° + A)$	
	$= \sin 40° \cos A - \cos 40° \sin A$	M1, for identity
	$= \frac{4}{5}\sqrt{1-k^2} - \frac{3}{5}k$	A1 for $\frac{4}{5}$ and $-\frac{3}{5}$
		A1 for $\sqrt{1-k^2}$ and k
	OR	
	$\sin(40° + A) = \sin[180° - (140° - A)]$	
	$= \sin(140° - A)$	
	$= \sin 140° \cos A - \cos 140° \sin A$	M1, for identity
	$= \frac{4}{5}\sqrt{1-k^2} - \frac{3}{5}k$	A1 for $\frac{4}{5}$ and $-\frac{3}{5}$
		A1 for $\sqrt{1-k^2}$ and k

9 The points $P(1, -2)$ and $Q(1, 4)$ lie on the circumference of a circle with centre C. If the circle is reflected in a vertical line, P and Q remain unchanged in the reflection and the x-coordinate of the centre of the reflected circle is 5.

(i) State the equation of the vertical line of reflection. [1]

(ii) Show that the equation of the circle with centre C is $x^2+y^2+6x-2y-15=0$. [3]

(iii) The line $3y+4x=-9$ intersects the circle with centre C at two points, A and B. Find the coordinates of A and of B. [4]

(iv) Determine if AB is a diameter of the circle with centre C. [1]

(i) $x=1$ B1

(ii) y-coordinate of centre $=\frac{4-2}{2}=1$

x-coordinate of centre $=1-(5-1)=-3$

B1, either x or y coordinate of centre correct with working

Coordinate of centre $=(-3,1)$

Radius of circle $=\sqrt{(-3-1)^2+(1-4)^2}$

$=5$ units

M1, for finding radius

Equation of circle is

$(x+3)^2+(y-1)^2=5^2$

$x^2+6x+9+y^2-2y+1=25$

$x^2+y^2+6x-2y-15=0$ A1, with method

(iii) $3y+4x=-9$

$y=-\frac{4}{3}x-3$

$x^2+y^2+6x-2y-15=0$

$x^2+\left(-\frac{4}{3}x-3\right)^2+6x-2\left(-\frac{4}{3}x-3\right)-15=0$ M1, for substitution

$x^2+\frac{16}{9}x^2+8x+9+6x+\frac{8}{3}x+6-15=0$

$\frac{25}{9}x^2+\frac{50}{3}x=0$

$x^2+6x=0$

$x(x+6)=0$

$x=0$ or $x=-6$ M1, for method of solving quadratic equation

When $x=0, y=-3$
When $x=-6, y=5$

$\therefore (0,-3)$ and $(-6,5)$ A2

(iv) Substitute $(-3,1)$ in $3y+4x=-9$

$3(1)+4(-3)=-9$

Since the centre of the circle lies on the line, AB is a diameter of the circle. A1, with method

OR

Length of $AB=\sqrt{(-6-0)^2+(5+3)^2}$

$=10$ units A1, with method.

Since radius $=5$ units, $AB=$ twice the length of the radius. AB is a diameter of the circle.

10 A particle moves in a straight line such that at t seconds after passing point O, its velocity v m/s is given by $v = t - 7 + \frac{12}{t+1}$, where $t > 0$.

(i) Find the acceleration of the particle when it is first instantaneously at rest. [3]

(ii) Find an expression for the displacement of the particle from O. [3]

(iii) Find the total distance travelled by the particle from $t = 0$ to $t = 5$. [3]

(i) When $v = 0$,

$t - 7 + \frac{12}{t+1} = 0$ M1, for solving $v = 0$

$(t-7)(t+1) + 12 = 0$

$t^2 - 6t + 5 = 0$

$(t-5)(t-1) = 0$

$t = 1$ or 5

$v = t - 7 + \frac{12}{t+1}$

$\frac{dv}{dt} = 1 - 12(t+1)^{-2}$ M1, for differentiating v wrt t

When $t = 1$, $\frac{dv}{dt} = 1 - \frac{12}{2^2} = -2$ m/s^2 A1

(ii) $v = t - 7 + \frac{12}{t+1}$

$s = \frac{t^2}{2} - 7t + 12\ln|t+1| + c$ M1, for integrating v wrt t

When $t = 0$, $s = 0$, $c = 0$ M1, for finding c

$s = \frac{t^2}{2} - 7t + 12\ln|t+1|$ A1

(iii) When $t = 1$,

$s = \frac{1^2}{2} - 7(1) + 12\ln 2$ M1

$= 1.8178$ m

When $t = 5$,

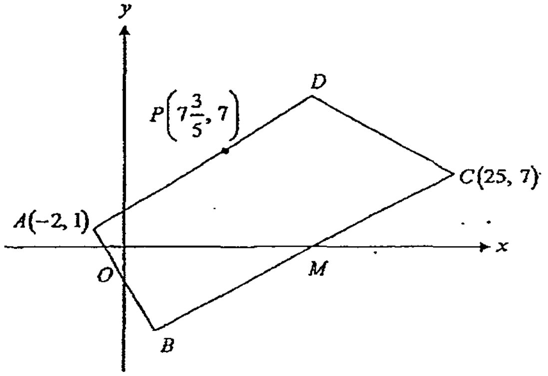
$s = \frac{5^2}{2} - 7(5) + 12\ln 6$ M1

$= -0.99889$m

Total distance travelled $= 2(1.8178) + 0.99889$
$= 4.63$ m A1

11 Solutions to this question by accurate drawing will not be accepted.

The diagram shows the quadrilateral $ABCD$ in which point A is $(-2, 1)$ and point C is $(25, 7)$. The point $P\left(7\frac{3}{5}, 7\right)$ lies on AD such that $AP : PD = 3 : 2$. The midpoint of BC, point M, lies on the x-axis and directly below point D.



(i) Find the coordinates of points D, M and B. [6]

(ii) Determine if $\angle DAB$ is a right angle. [3]

(iii) Calculate the area of the quadrilateral $ABCD$. [2]

(i) Let D be (x, y)

$$\frac{x-(-2)}{7\frac{3}{5}-(-2)}=\frac{5}{3} \quad \text{and} \quad \frac{y-1}{7-1}=\frac{5}{3}$$

$x=14$ $\quad$ $y=11$

M1 for x-coordinate
M1 for y-coordinate

$\therefore D(14, 11)$ $\quad$ A1

OR

$\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{OA}$ $\quad$ M1 for $\overrightarrow{AP}$

$$=\begin{pmatrix} 7\frac{3}{5}+2 \\ 7-1 \end{pmatrix}$$

$$=\begin{pmatrix} 9\frac{3}{5} \\ 6 \end{pmatrix}$$

	$\overline{AD}=\frac{5}{3}\overline{AP}$ $=\frac{5}{3}\begin{pmatrix}9\frac{3}{5}\\6\end{pmatrix}$ $=\begin{pmatrix}16\\10\end{pmatrix}$	M1 for $\overline{AD}=\frac{5}{3}\overline{AP}$
	$\overline{OD}=\overline{OD}+\overline{OA}$ $=\begin{pmatrix}16-2\\10+1\end{pmatrix}$ $=\begin{pmatrix}14\\11\end{pmatrix}$	
	$\therefore D(14,\ 11)$	A1
	$M(14, 0)$	B1
	Let B be (p, q)	
	$\frac{p+25}{2}=14$ and $\frac{q+7}{2}=0$ $x=3$ $\quad q=-7$	M1
	$\therefore B(3,\ -7)$	A1
(ii)	Gradient of AP $=\frac{7-1}{7\frac{3}{5}-(-2)}$ $=\frac{5}{8}$	M1 for finding both gradients
	Gradient of AB $=\frac{-7-1}{3-(-2)}$ $=\frac{-8}{5}$	
	Since Gradient of AP × Gradient of AB = $\frac{5}{8}\times\left(\frac{-8}{5}\right)=-1$, AP is perpendicular to AB and $\angle DAB$ is a right angle.	M1 for showing $m_1\times m_2$, or equivalent A1

(iii) Area of *ABCD*

$$=\frac{1}{2}\begin{vmatrix} -2 & 3 & 25 & 14 & -2 \\ 1 & -7 & 7 & 11 & 1 \end{vmatrix}$$

$$=\frac{1}{2}\left[(14+21+275+14)-(3-175+98-22)\right]$$ M1

$=210 \text{ units}^2$ A1

	Class	Register Number
Name ______________________		

4047/02 **16/S4PR2/AM/2**

ADDITIONAL MATHEMATICS **PAPER 2**

Thursday **4 August 2016** **2 hours 30 minutes**

PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

Additional Materials:	Answer Paper Graph paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
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Do not use paper clips, highlighters, glue or correction fluid.

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You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This paper consists of 7 printed pages, including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\text{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

1 The variables x and y are connected by the equation $y+a=-x(x+1)$, where a is a constant. Using experimental values of x and y, a graph was drawn in which $y+x^2$ was plotted on the vertical axis against x on the horizontal axis. The straight line obtained passes through the point $(-3, 1)$.

Calculate the

(i) value of a, [3]

(ii) coordinates of the point on the line at which $y=x(3-x)$. [2]

2 (a) Find the range of values of k such that the line $y=kx-4$ meets the curve $4x^2-(k-x)=2y+3x$. [3]

(b) The equation $2x^2-7x+6=0$ has roots $2\alpha-1$ and $2\beta-1$ where $\alpha>\beta$.

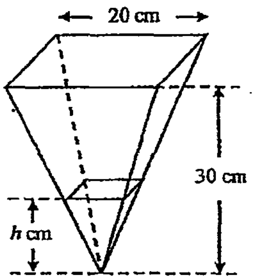
(i) Without solving for α and β, find the value of $\alpha-\beta$. [4]

Hence,

(ii) find the value of $\alpha^3-\beta^3$, [2]

(iii) state the quadratic equation whose roots are α^3 and $-\beta^3$. [1]

3



The diagram shows an inverted square-base pyramidal tank of height 30 cm and base length 20 cm. The tank is filled with water and is held fixed with its square rim horizontal. Water leaks out of the tank at a constant rate of 15 $\text{cm}^3\ \text{s}^{-1}$. After t seconds, the depth of water is h cm.

(i) Show that the volume of water in the tank, V cm^3, at time t is given by $V=\frac{4}{27}h^3$. [2]

(ii) Find the rate of change of depth when $h=5$. [3]

4 Given that $f(x) = a\left(x^4+1\right)+7x^3-10x^2+bx$ and that $4x^2+7x-2$ is a factor of $f(x)$.

(i) Show that $a=4$ and $b=-14$. [5]

(ii) Find the remainder when $f(x)$ is divided by $x+1$. [2]

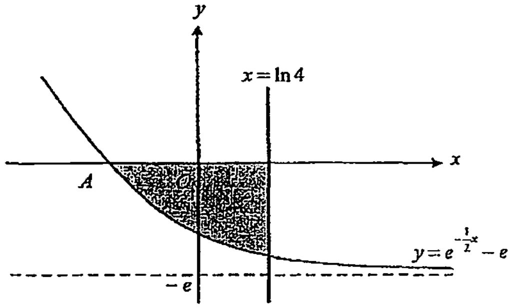
5 (a) The Population White Paper released by the Government of Singapore in 2013, projected that Singapore's population will hit 6.9 million by year 2030. The population of Singapore, P, increased from 5.399 million to 5.535 million from 2013 to 2015. Given that $P = Ae^{kt}$, where A and k are constants and t is the time in years from 2013.

(i) Find the value of A and of k. [3]

If the population continues to increase at the same rate,

(ii) determine if the population trajectory for year 2030 in the Population White Paper is accurate. [2]

(b)



The diagram shows the line $x = \ln 4$ and part of the curve $y = e^{-\frac{1}{2}x} - e$. The curve intersects the x-axis at the point A. Determine the area of the shaded region bounded by the curve, the line $x = \ln 4$ and the x-axis. [4]

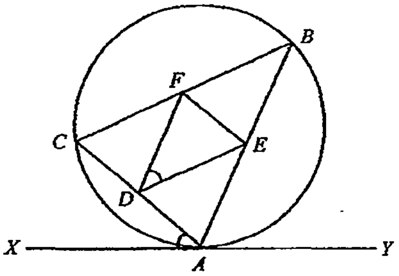
6 (i) Factorise completely the cubic polynomial x^3-x^2+3x-3. [2]

(ii) Express $\dfrac{2x^3-5x^2+10x-3}{x^3-x^2+3x-3}$ in partial fractions. [5]

(iii) Differentiate $\ln\left(x^2+3\right)$ with respect to x. Hence express $\int_2^5 \dfrac{2x^3-5x^2+10x-3}{x^3-x^2+3x-3}\,dx$ in the form $a+b\ln 2$, where a and b are integers. [5]

7 (a) Prove the identity $\frac{1}{\operatorname{cosec}\theta+\cot\theta}=\frac{1-\cos\theta}{\sin\theta}$. [3]

(b)



The diagram above shows a triangle ABC whose vertices lie on the circumference of a circle. D and E are the mid-points of AC and AB respectively. XY is a tangent to the circle at A. Given that CFB is a straight line and angle FDE = angle DAX.

Prove that

(i) DE is parallel to BC, [1]

(ii) ΔFDE is congruent to ΔEBF, [3]

(iii) $DEBF$ is a parallelogram. [2]

8 (i) On the same axes sketch, for $0 \le x \le 2\pi$, the graphs of

$$y_1 = 2\sin x + 1 \quad \text{and} \quad y_2 = -\cos x.$$ [4]

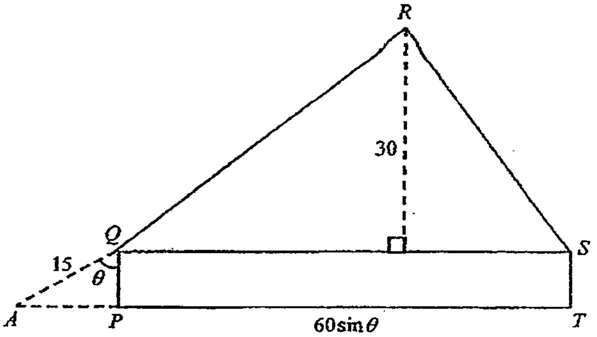
(ii) Given that $f(x) = y_1 + y_2$, express $f(x)$ in the form $p\sin(x-q)+r$, where p, q and r are constants to be found. [4]

(iii) State, in exact form, the

(a) greatest and least values of $f(x)$, [2]

(b) amplitude of $f(x)$. [1]

9



The diagram shows the vertical cross-section *PQRST* of a structure, consisting of a triangle *QRS* of height 30 m and a rectangle *PQST*. The structure rests with *PT* on horizontal ground. To hold the structure up, a 15 m rope is secured at *Q* to a point, *A*, on the ground. It is given that *QA* is inclined at an angle, θ radians, to *QP* and $PT = 60\sin\theta$ m.

(i) Show that the area, A m^2, of the cross-section *PQRST* is given by

$$A = 900\sin\theta + 450\sin 2\theta.$$ [4]

(ii) Given that θ can vary, find the value of θ for which the maximum amount of paint is required to colour this cross-section. [5]

(iii) Hence, find the maximum value of A. [1]

10 A trapezium of area, A cm^2, has parallel sides of length px^2 cm and q cm and its perpendicular height is x cm. Corresponding values of x and A are shown in the table below.

x	1	2	3	4
A	1.75	5	11.25	22

(i) Using suitable variables, draw, on a graph paper, a straight line graph and hence estimate the value of each of the constants p and q. [6]

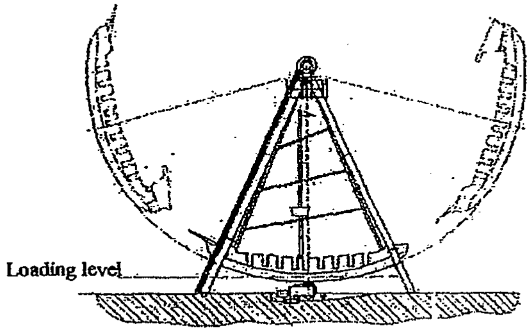
(ii) Using your values of p and q, calculate the value of x for which the trapezium is a rectangle. [2]

(iii) Explain how another straight line drawn on your diagram can lead to an estimate of the value of x for which the trapezium is a rectangle. Draw this line and hence verify your value of x found in part (ii). [3]

11 Gravitational potential energy, measured in kilojoules (kJ), is the energy a body has due to its position. It can be calculated by the following equation:

$$\text{Gravitational potential energy} = \frac{mgh}{1000}$$

where m is the mass of the body in kg, g is the gravitational field strength in N/kg and h is the height of the object in m. The gravitational field strength, g, on Earth is approximately 10 N/kg.



The gravitational potential energy, E, in kJ, of a pirate ship ride can be modelled by the equation, $E = 100(1-\cos kt)+a$, where k and a are constants and t is the time in seconds after starting the ride at loading level.

(i) Given that the mass of the ride is 1000 kg and at an initial loading level of 3 m, show that $a = 30$. [1]

(ii) Explain why this model suggests that the maximum gravitation potential energy possessed by the ride is 230 kJ. [1]

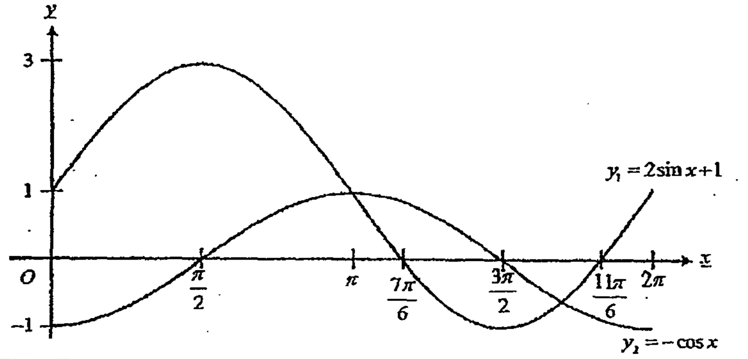
The ride takes 6 seconds to travel from one peak to another.

(iii) Show that the value of k is $\frac{\pi}{3}$ radians per second. [2]

(iv) Calculate the gravitation potential energy of the ride at $t = 8$ s. [2]

(v) If the ride continues for 60 seconds, find the exact duration for which the ride possesses more than 80 kJ of gravitational potential energy. [5]

End of Paper

1	i	$a=2$	8	ii	$2.24\sin(x-0.464)+1$
	ii	$\left(-\frac{1}{2},-1\frac{1}{2}\right)$		iii	a) Greatest value of $f(x)=1+\sqrt{5}$ Least value of $f(x)=1-\sqrt{5}$ b) Amplitude of $f(x)=\sqrt{5}$
2	a	$k\le -9.32$ or $k\ge 3.32$			
	bi	$\frac{1}{4}$	9	i	Show $QP=15\cos\theta$
	ii	$1\frac{27}{64}$		ii	$\frac{\pi}{3}$
	iii	$x^2-\frac{91}{64}x-6\frac{303}{512}=0$		iii	1170 m^2
3	i	Show base of similar pyramid $=\frac{2}{3}h$	10	i	Plot or $\frac{2A}{x}=px^2+q$ | $\frac{A}{x}=\frac{1}{2}px^2+\frac{1}{2}q$ $p=0.500$ $q=3$
	ii	-1.35 cm s^{-1}			
4	i	Let $f\left(\frac{1}{4}\right)=0$ and $f(-2)=0$			
	ii	5		ii	$x=2.45$
5	ai	$A=5.399\times10^6$ $k=0.0124$		iii	Draw or $\frac{2A}{x}=x^2$ or $\frac{2A}{x}=6$ | $\frac{A}{x}=0.5x^2$ or $\frac{A}{x}=3$ $x=2.45$
	ii	Show $t=17$, $P=6.67\times10^6$ or $P=6.9\times10^6$, $t=19.72$, Year $=2032$ Not accurate			
	b	4.77 units^2			
6	i	$(x-1)(x^2+3)$	11	i	Sub. $m=1000$, $g=10$, $h=3$
	ii	$2+\frac{1}{x-1}-\frac{4x}{x^2+3}$		ii	Sub. $\cos kt=-1$
	iii	$\frac{2x}{x^2+3}$; $6-2\ln 2$		iii	Period $=\frac{2\pi}{k}=6$
7	bi	Mid-Point Theorem		iv	180 kJ
	ii	$\Delta FDE\equiv\Delta EBF$ (AAS)		v	40 s
8	i				

	Class	Register Number
Name SOLUTION		

4047/02 16/S4PR2/AM/2

ADDITIONAL MATHEMATICS **PAPER 2**

Thursday 4 August 2016 2 hours 30 minutes

PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

Additional Materials: Answer Paper
Graph paper

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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Calculate the

(i) value of a, [3]

(ii) coordinates of the point on the line at which $y=x(3-x)$. [2]

(i) $y+a=-x(x+1)$

$y+a=-x^2-x$

$(y+x^2)=-(x)-a \quad \Rightarrow Y=mX+c \quad \longleftarrow$ M1

Sub $(-3,1)$, $1=-(-3)-a \quad \longleftarrow$ M1

$a=2 \quad \longleftarrow$ A1

Note: $Y=-X-2$

(ii) $y=x(3-x)$

$y=3x-x^2$

$y+x^2=3x \quad \Rightarrow Y=3X$

$\therefore \; 3X=-X-2 \quad \longleftarrow$ M1 or finding $x=-\frac{1}{2}$ or $y=-1\frac{3}{4}$

$X=-\frac{1}{2}$

$X=-\frac{1}{2}, \quad Y=3\left(-\frac{1}{2}\right)$

$=-1\frac{1}{2}$

coordinates of the point $\left(-\frac{1}{2},-1\frac{1}{2}\right) \quad \longleftarrow$ A1

2 **(a)** Find the range of values of k such that the line $y = kx - 4$ meets the curve $4x^2 - (k - x) = 2y + 3x$. [3]

(a) $4x^2 - (k - x) = 2(kx - 4) + 3x$ ⟵ M1 for substitution

$4x^2 - k + x - 2kx + 8 - 3x = 0$

$4x^2 + (-2 - 2k)x + (8 - k) = 0$

$b^2 - 4ac \geq 0$

$(-2 - 2k)^2 - 4(4)(8 - k) \geq 0$ ⟵ M1 for substituting $b^2 - 4ac$

$4(1 + k)^2 - 4(4)(8 - k) \geq 0$

$1 + 2k + k^2 - 32 + 4k \geq 0$

$k^2 + 6k - 31 \geq 0$

Let $k^2 + 6k - 31 = 0$

$$k = \frac{-6 \pm \sqrt{6^2 - 4(1)(-31)}}{2}$$

$$= \frac{-6 \pm \sqrt{160}}{2}$$

$= 3.32$ or -9.32 (3sf)

$\therefore k^2 + 6k - 31 \geq 0$

$k \leq -9.32$ or $k \geq 3.32$ ⟵ A1

2 (b) The equation $2x^2 - 7x + 6 = 0$ has roots $2\alpha - 1$ and $2\beta - 1$ where $\alpha > \beta$.

(i) Without solving for α and β, find the value of $\alpha - \beta$. [4]

Hence,

(ii) find the value of $\alpha^3 - \beta^3$, [2]

(iii) state the quadratic equation whose roots are α^3 and $-\beta^3$. [1]

(i) Sum of roots:

$$(2\alpha-1)+(2\beta-1)=\frac{7}{2} \longleftarrow \text{B1}$$

$$2(\alpha+\beta)-2=\frac{7}{2}$$

$$\alpha+\beta=\frac{11}{4}$$

Product of roots:

$$(2\alpha-1)(2\beta-1)=3 \longleftarrow \text{B1}$$

$$3=4\alpha\beta-2(\alpha+\beta)+1$$

$$3=4\alpha\beta-2\left(\frac{11}{4}\right)+1$$

$$\alpha\beta=\frac{15}{8}$$

$$\begin{aligned}(\alpha-\beta)^2 &= \alpha^2-2\alpha\beta+\beta^2\\ &=(\alpha+\beta)^2-2\alpha\beta-2\alpha\beta\\ &=(\alpha+\beta)^2-4\alpha\beta\\ &=\left(\frac{11}{4}\right)^2-4\left(\frac{15}{8}\right) \longleftarrow \text{M1}\\ &=\frac{1}{16}\end{aligned}$$

since $\alpha > \beta$, $(\alpha-\beta)=\frac{1}{4} \longleftarrow \text{A1}$

(ii)

$$\begin{aligned}\alpha^3-\beta^3 &= (\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)\\ &=(\alpha-\beta)\left[(\alpha+\beta)^2-\alpha\beta\right] \quad [\text{B1}]\\ &=\left(\frac{1}{4}\right)\left[\left(\frac{11}{4}\right)^2-\frac{15}{8}\right]\\ &=\frac{91}{64}\\ &=1\frac{27}{64} \quad [\text{A1}]\end{aligned}$$

or

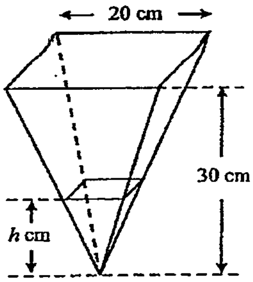
$$(\alpha-\beta)^3=\alpha^3-3\alpha^2\beta+3\alpha\beta^2-\beta^3$$

$$\begin{aligned}\alpha^3-\beta^3 &= (\alpha-\beta)^3+3\alpha^2\beta-3\alpha\beta^2\\ &=(\alpha-\beta)^3+3\alpha\beta(\alpha-\beta) \longleftarrow \text{B1}\\ &=\left(\frac{1}{4}\right)^3+3\left(\frac{15}{8}\right)\left(\frac{1}{4}\right)\\ &=\frac{91}{64}\\ &=1\frac{27}{64} \longleftarrow \text{A1}\end{aligned}$$

(iii) The equation is $x^2-\frac{91}{64}x-\left(\frac{15}{8}\right)^3=0$

$$x^2-\frac{91}{64}x-6\frac{303}{512}=0 \quad \text{or} \quad 512x^2-728x-3375=0 \longleftarrow \text{A1}$$

3



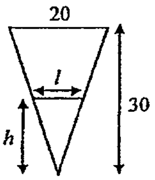
The diagram shows an inverted square-base pyramidal tank of height 30 cm and base length 20 cm. The tank is filled with water and is held fixed with its square rim horizontal. Water leaks out of the tank at a constant rate of 15 $\text{cm}^3\ \text{s}^{-1}$. After t seconds, the depth of water is h cm.

(i) Show that the volume of water in the tank, V cm^3, at time t is given by $V=\frac{4}{27}h^3$. [2]

(ii) Find the rate of change of depth when $h=5$. [3]

(i) Using similar triangles, $\frac{20}{l}=\frac{30}{h}$ ⟵ B1

$$l=\frac{2}{3}h$$



$$\left.\begin{aligned} V&=\frac{1}{3}\pi\left(\frac{2}{3}h\right)^2 h \\ &=\frac{4}{27}h^3 \end{aligned}\right\} \text{A1}$$

(ii) $\frac{dV}{dh}=\frac{4}{27}(3h^2)$ $\qquad \frac{dV}{dt}=-15$

$$=\frac{4}{9}h^2 \longleftarrow \text{B1}$$

$$\frac{dh}{dt}\times\frac{dV}{dh}=\frac{dV}{dt}$$

$$\frac{dh}{dt}=-15\div\frac{9}{4h^2} \longleftarrow \text{M1}$$

$$=-\frac{135}{4h^2}$$

when $h=5$, rate of change $=-\frac{135}{4(5)^2}$

$$=-1\frac{7}{20}\text{ cm s}^{-1} \text{ or } -1.35\text{ cm s}^{-1} \longleftarrow \text{A1}$$

4 Given that $f(x)=a(x^4+1)+7x^3-10x^2+bx$ and that $4x^2+7x-2$ is a factor of $f(x)$.

(i) Show that $a=4$ and $b=-14$. [5]

(ii) Find the remainder when $f(x)$ is divided by $x+1$. [2]

(i) $4x^2+7x-2=(4x-1)(x+2) \longleftarrow \text{B1}$

$$f\left(\frac{1}{4}\right)=a\left[\left(\frac{1}{4}\right)^4+1\right]+7\left(\frac{1}{4}\right)^3-10\left(\frac{1}{4}\right)^2+b\left(\frac{1}{4}\right)$$

$$0=\frac{257}{256}a+\frac{7}{64}-\frac{5}{8}+\frac{1}{4}b \longleftarrow \text{M1}$$

$$0=257a-132+64b \quad \cdots(1)$$

$$f(-2)=a\left[(-2)^4+1\right]+7(-2)^3-10(-2)^2+b(-2)$$

$$0=17a-96-2b \longleftarrow \text{M1}$$

$$0=544a-3072-64b \quad \cdots(2)$$

$$\therefore (1)+(2):\ 0=801a-3204 \longleftarrow \text{M1}$$

$$\left.\begin{array}{l} a=4 \quad \text{(shown)} \\ \\ (1):\ 0=257(4)-132+64b \\ b=-14 \quad \text{(shown)} \end{array}\right\} \text{A1}$$

(ii) $f(x)=4(x^4+1)+7x^3-10x^2-14x$

$$\left.\begin{array}{l} \text{Remainder}=f(-1) \\ =4\left[(-1)^4+1\right]+7(-1)^3-10(-1)^2-14(-1) \end{array}\right\} \text{B1 either line}$$

$$=5 \longleftarrow \text{A1}$$

5 **(a)** The Population White Paper released by the Government of Singapore in 2013, projected that Singapore's population will hit 6.9 million by year 2030. The population of Singapore, P, increased from 5.399 million to 5.535 million from 2013 to 2015. Given that $P = Ae^{kt}$, where A and k are constants and t is the time in years from 2013.

(i) Find the value of A and of k. [3]

If the population continues to increase at the same rate,

(ii) determine if the population trajectory for year 2030 in the Population White Paper is accurate. [2]

(a) (i) $5.399\times10^6 = Ae^{k(0)}$

$A = 5.399\times10^6$ or 5.399 million $\longleftarrow$ B1

$5.535\times10^6 = 5.399\times10^6 e^{k(2)}$ $\longleftarrow$ M1

$$e^{2k} = \frac{5.535}{5.399}$$

$$k = \frac{1}{2}\ln\left(\frac{5.535}{5.399}\right)$$

$= 0.0124389$

$= 0.0124$ (3sf) $\longleftarrow$ A1

(ii) $P = 5.399\times10^6 e^{0.0124389k}$

$6.9\times10^6 = 5.399\times10^6 e^{0.0124389t}$ [M1]

$$e^{0.0124389t} = \frac{6.9\times10^6}{5.399\times10^6}$$

$$t = \frac{1}{0.0124389}\ln\left(\frac{6.9}{5.399}\right)$$

$= 19.72$

[A1] $\left\{\begin{array}{l}\text{Year to reach population of 6.9 million} \\ = 2013 + 19.72 \\ = 2032 \text{ (nearest year)} \\ \therefore \text{ not accurate}\end{array}\right.$

OR:

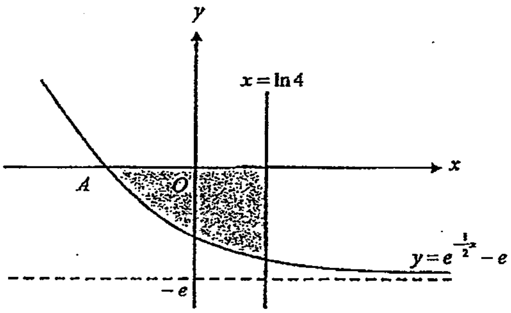
$t = 2030 - 2013 = 17$

$P = 5.399\times10^6 e^{0.0124389(17)}$ $\longleftarrow$ MI

$= 6.67$ million (3sf)

$\left.\begin{array}{l}6.67 \text{ million} < 6.9 \text{ million} \\ \text{Population did not hit 6.9 million.} \\ \therefore \text{ not accurate}\end{array}\right\}$ A1

5 (b)



The diagram shows the line $x=\ln 4$ and part of the curve $y=e^{-\frac{1}{2}x}-e$. The curve intersects the x-axis at the point A. Determine the area of the shaded region bounded by the curve, the line $x=\ln 4$ and the x-axis. [4]

(b) At A, $y=0$, $0=e^{-\frac{1}{2}x}-e$

$$e=e^{-\frac{1}{2}x}$$

$$1=-\frac{1}{2}x$$

$$x=-2 \longleftarrow \text{A1}$$

$$\text{Area}=\left|\int_{-2}^{\ln 4}\left(e^{-\frac{1}{2}x}-e\right)\text{d}x\right| \longleftarrow \text{M1 also } \int_{\ln 4}^{-2}\left(e^{-\frac{1}{2}x}-e\right)\text{d}x \text{ or } -\int_{-2}^{\ln 4}\left(e^{-\frac{1}{2}x}-e\right)\text{d}x$$

$$=\left|\left[\frac{e^{-\frac{1}{2}x}}{-\frac{1}{2}}-ex\right]_{-2}^{\ln 4}\right| \longleftarrow \text{A1 for integration}$$

$$=\left|\left[-2e^{-\frac{1}{2}x}-ex\right]_{-2}^{\ln 4}\right|$$

$$=\left|\left[-2e^{-\frac{1}{2}(\ln 4)}-e(\ln 4)\right]-\left[-2e^{-\frac{1}{2}(-2)}-e(-2)\right]\right|$$

$$=\left|(-1-e\ln 4)-(-2e+2e)\right|$$

$$=1+e\ln 4$$

$$=4.77 \text{ units}^2 \longleftarrow \text{A1}$$

6 (i) Factorise completely the cubic polynomial x^3-x^2+3x-3. [2]

(ii) Express $\dfrac{2x^3-5x^2+10x-3}{x^3-x^2+3x-3}$ in partial fractions. [5]

(iii) Differentiate $\ln(x^2+3)$ with respect to x. Hence express $\int_2^5 \dfrac{2x^3-5x^2+10x-3}{x^3-x^2+3x-3}\,dx$ in the form $a+b\ln 2$, where a and b are integers. [5]

(i)
$$x^3-x^2+3x-3=x^2(x-1)+3(x-1) \longleftarrow \text{M1}$$
$$=(x-1)(x^2+3) \longleftarrow \text{A1}$$

$$\begin{array}{r}
2 \\
x^3-x^2+3x-3\,\big)\overline{\,2x^3-5x^2+10x-3} \\
-\left(2x^3-2x^2+6x-6\right) \\
\hline
-3x^2+4x+3
\end{array}$$

(ii)
$$\frac{2x^3-5x^2+10x-3}{x^3-x^2+3x-3}=2+\frac{-3x^2+4x+3}{(x-1)(x^2+3)} \longleftarrow \text{M1}$$
$$=2+\frac{A}{x-1}+\frac{Bx+C}{x^2+3} \longleftarrow \text{B1}$$
$$=2+\frac{A(x^2+3)+(Bx+C)(x-1)}{(x-1)(x^2+3)}$$
$$\therefore -3x^2+4x+3=A(x^2+3)+(Bx+C)(x-1)$$

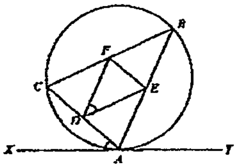
A2
$$x=1,\quad -3+4+3=4A \qquad A=1$$
$$x=0,\quad 3=3-C \qquad C=0$$
$$x=-1,\quad -3-4+3=4+2B \qquad B=-4$$
$$\therefore \frac{2x^3-5x^2+10x-3}{x^3-x^2+3x-3}=2+\frac{1}{x-1}-\frac{4x}{x^2+3} \longleftarrow \text{A1}$$

(iii)
$$\frac{d}{dx}\ln(x^2+3)=\frac{2x}{x^2+3} \longleftarrow \text{B1}$$
$$\int_2^5 \frac{2x^3-5x^2+10x-3}{x^3-x^2+3x-3}\,dx=\int_2^5 2+\frac{1}{x-1}-\frac{4x}{x^2+3}\,dx$$
$$=\int_2^5 2+\frac{1}{x-1}-2\left(\frac{2x}{x^2+3}\right)dx$$
$$=\left[2x+\ln(x-1)-2\ln(x^2+3)\right]_2^5 \longleftarrow \text{A1 M2}$$
$$=\left[2(5)+\ln(4)-2\ln(28)\right]-\left[2(2)+\ln(1)-2\ln(7)\right]$$
$$=6+\ln 4-\ln 28^2+\ln 7^2$$
$$=6+\ln\frac{4\times 7^2}{28^2}$$
$$=6+\ln\left(\frac{1}{4}\right)$$
$$=6+\ln 2^{-2}$$
$$=6-2\ln 2 \longleftarrow \text{A1}$$

7 **(a)** Prove the identity $\frac{1}{\operatorname{cosec}\theta+\cot\theta}=\frac{1-\cos\theta}{\sin\theta}$. [3]

(b) The diagram above shows a triangle ABC whose vertices lie on the circumference of a circle. D and E are the mid-points of AC and AB respectively. XY is a tangent to the circle at A. Given that CFB is a straight line and angle FDE = angle DAX.

Prove that



(i) DE is parallel to BC, [1]

(ii) ΔFDE is congruent to ΔEBF, [3]

(iii) $DEBF$ is a parallelogram. [2]

(a)
$$\frac{1}{\operatorname{cosec}\theta+\cot\theta}=1\div\left(\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}\right)$$

B1 change to sin and cos
$$=1\div\frac{1+\cos\theta}{\sin\theta}$$
$$=\frac{\sin\theta}{1+\cos\theta}\times\frac{1-\cos\theta}{1-\cos\theta}$$
$$=\frac{\sin\theta(1-\cos\theta)}{1-\cos^2\theta}$$

B1 use of identity $\longrightarrow$
$$=\frac{\sin\theta(1-\cos\theta)}{\sin^2\theta}$$
$$=\frac{1-\cos\theta}{\sin\theta}\longleftarrow\text{A1}$$

$$\frac{1}{\operatorname{cosec}\theta+\cot\theta}$$
$$=\frac{\operatorname{cosec}^2\theta-\cot^2\theta}{\operatorname{cosec}\theta+\cot\theta}\quad\text{[B1]}$$
$$=\frac{(\operatorname{cosec}\theta+\cot\theta)(\operatorname{cosec}\theta-\cot\theta)}{\operatorname{cosec}\theta+\cot\theta}$$
$$=\operatorname{cosec}\theta-\cot\theta$$
$$=\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\quad\text{[B1]}$$
$$=\frac{1-\cos\theta}{\sin\theta}\quad\text{[A1]}$$

(b) (i) Since D and E are the mid-points of AC and AB respectively,
By Mid-Point Theorem, DE // BC. } B1 Mid-Point Theorem

(ii) In ΔFDE and ΔEBF,
$\angle FDE=\angle DAX$ (given)
$\angle EBF=\angle DAX$ (Alt. Segment Theorem)
$\therefore \angle FDE=\angle EBF$ } B1 must state reason

$\angle DEF=\angle BFE$ (alt. ∠s, DE//BC) ⟵ B1
$FE=FE$ (common side)
$\Delta FDE\equiv\Delta EBF$ (AAS) } A1

(iii) From (i), DE // BC and since F is a point on BC, ∴ DE // BF.
From (ii), since $\Delta FDE\equiv\Delta EBF$, ∴ $DE=BF$. } A1 either condition
Since DE // BF and $DE=BF$, ∴ $DEBF$ is a parallelogram. ⟵ A1

(iii) Possible Proofs
- ✓ 1 pair = & // lines
- ✓ 1 pair = angles & 1 pair // lines
- ✓ 2 pairs // lines
- ✓ 2 pairs = angles

8 (i) On the same axes sketch, for $0 \le x \le 2\pi$, the graphs of

$$y_1 = 2\sin x + 1 \quad \text{and} \quad y_2 = -\cos x.$$ [4]

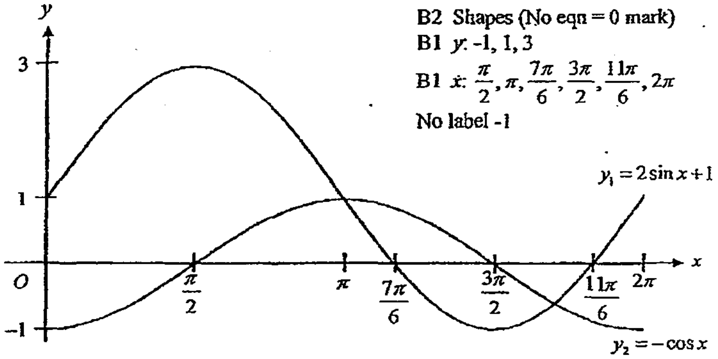
(ii) Given that $f(x) = y_1 + y_2$, express $f(x)$ in the form $p\sin(x-q)+r$, where p, q and r are constants to be found. [4]

(iii) State, in exact form, the

(a) greatest and least values of $f(x)$, [2]

(b) amplitude of $f(x)$. [1]

(i)



B2 Shapes (No eqn = 0 mark)
B1 y: -1, 1, 3
B1 x: $\frac{\pi}{2}, \pi, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}, 2\pi$
No label -1

(ii)

$$\begin{aligned} f(x) &= 2\sin x + 1 - \cos x \\ &= 2\sin x - \cos x + 1 \\ &= p\sin(x-q)+1 \\ &= p\sin x\cos q - p\cos x\sin q + 1 \end{aligned}$$

$p\cos q = 2 \cdots(1) \qquad p\sin q = 1 \cdots(2)$ B1

$(1)^2 + (2)^2$:

$p^2\cos^2 q + p^2\sin^2 q = 2^2 + 1^2$

$p^2 = 5$

$p = \sqrt{5}$

$\frac{(2)}{(1)}$: $\frac{p\sin q}{p\cos q} = \frac{1}{2} \Rightarrow \tan q = \frac{1}{2}$

$q = \tan^{-1}\left(\frac{1}{2}\right)$

$= 0.4636$ (4sf) M2

$\therefore f(x) = \sqrt{5}\sin(x - 0.4636) + 1$

$= 2.24\sin(x - 0.464) + 1$ (3sf) ← A1

(iii) (a) Greatest value of $f(x) = 1 + \sqrt{5}$ ← A1

Least value of $f(x) = 1 - \sqrt{5}$ ← A1

(b) Amplitude of $f(x) = \sqrt{5}$ ← A1

9 The diagram shows the vertical cross-section $PQRST$ of a structure, consisting of a triangle QRS of height 30 m and a rectangle $PQST$. The structure rests with PT on horizontal ground. To hold the structure up, a 15 m rope is secured at Q to a point, A, on the ground. It is given that QA is inclined at an angle, θ radians, to QP and $PT = 60\sin\theta$ m.

(i) Show that the area, A m^2, of the cross-section $PQRST$ is given by

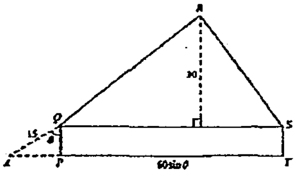
$$A = 900\sin\theta + 450\sin 2\theta. \qquad [4]$$

(ii) Given that θ can vary, find the value of θ for which the maximum amount of paint is required to colour this cross-section. [5]

(iii) Hence, find the maximum value of A. [1]

(i) In ΔAQP, $\cos\theta = \dfrac{QP}{15}$

$QP = 15\cos\theta$ m



Area, $A = \frac{1}{2}(30)(QS) + QS(QP)$

$= \left(\frac{1}{2}\times 30\times 60\sin\theta\right) + 60\sin\theta(15\cos\theta) \longleftarrow$ M2

$= 900\sin\theta + 900\sin\theta\cos\theta$

$= 900\sin\theta + 450(2\sin\theta\cos\theta) \longleftarrow$ M1 implied $\sin 2\theta$

$= 900\sin\theta + 450\sin 2\theta$ (shown) $\longleftarrow$ A1

(ii) $\dfrac{dA}{d\theta} = 900\cos\theta + 450(\cos 2\theta)(2)$

$= 900\cos\theta + 900\cos 2\theta \longleftarrow$ B1

At stationary point, $\dfrac{dA}{d\theta} = 0 \longleftarrow$ M1

$900\cos\theta + 900\cos 2\theta = 0$

$\cos\theta + (2\cos^2\theta - 1) = 0$

$2\cos^2\theta + \cos\theta - 1 = 0$

$(\cos\theta + 1)(2\cos\theta - 1) = 0$

$\cos\theta = -1$ or $\cos\theta = \dfrac{1}{2}$

$\theta = \pi$ (reject) $\qquad \alpha = \dfrac{\pi}{3}$

$\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ (reject)

$\theta = \dfrac{\pi}{3} \longleftarrow$ A1

$\dfrac{d^2A}{d\theta^2} = 900(-\sin\theta)1500 + 900(-\sin 2\theta)(2)$

$= -900\sin\theta - 1800\sin 2\theta \longleftarrow$ B1

$$\left.\begin{array}{l} \text{When } \theta = \dfrac{\pi}{3}, \quad \dfrac{d^2A}{d\theta^2} < 0 \\ \text{Since } \dfrac{d^2A}{d\theta^2} < 0, \\ \therefore A \text{ is maximum when } \theta = \dfrac{\pi}{3} \end{array}\right\} \text{A1 only if } \dfrac{d^2A}{d\theta^2} \text{ is correct}$$

(iii) Maximum value of A

$= 900\sin\left(\dfrac{\pi}{3}\right) + 450\sin 2\left(\dfrac{\pi}{3}\right)$

$= 900\left(\dfrac{\sqrt{3}}{2}\right) + 450\left(\dfrac{\sqrt{3}}{2}\right)$

$= 675\sqrt{3}$

$= 1170$ m^2 (3sf) $\longleftarrow$ A1

10 A trapezium of area, A cm^2, has parallel sides of length px^2 cm and q cm and its perpendicular height is x cm. Corresponding values of x and A are shown in the table below.

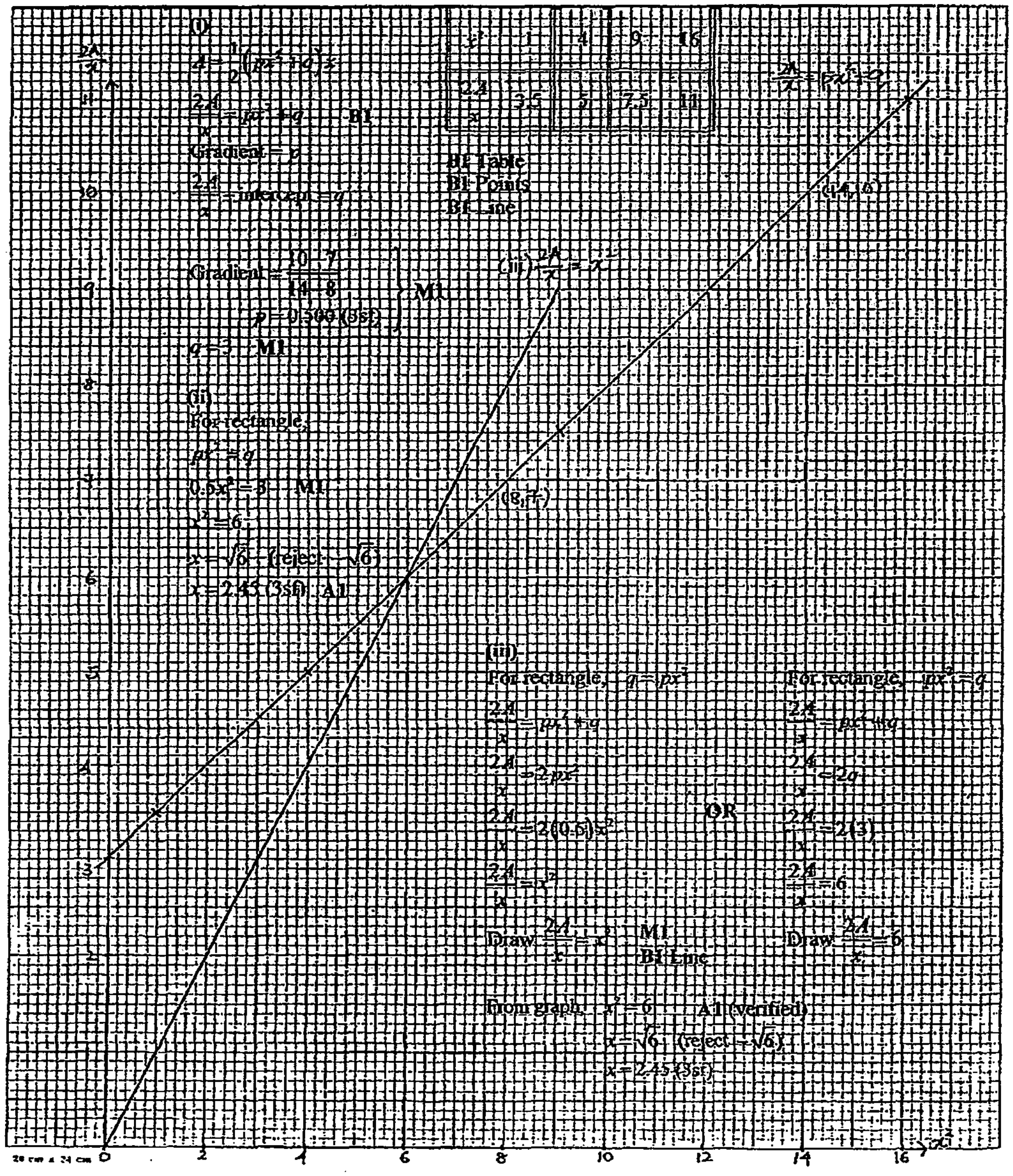
x	1	2	3	4
A	1.75	5	11.25	22

(i) Using suitable variables, draw, on a graph paper, a straight line graph and hence estimate the value of each of the constants p and q. [6]

(ii) Using your values of p and q, calculate the value of x for which the trapezium is a rectangle. [2]

(iii) Explain how another straight line drawn on your diagram can lead to an estimate of the value of x for which the trapezium is a rectangle. Draw this line and hence verify your value of x found in part (ii). [3]

Plot $\frac{2A}{x}$ against x^2



(i)

$A=\frac{1}{2}(px^2+q)x$

$\frac{2A}{x}=px^2+q$ **B1**

Gradient $= p$

$\frac{2A}{x}$ – intercept $= q$

x^2	1	4	9	16
$\frac{2A}{x}$	3.5	5	7.5	11

B1 Table
B1 Points
B1 Line

Gradient $=\frac{10-7}{14-8}$
$p=0.500$ (3sf) **M1**

$q=3$ **M1**

(ii)

For rectangle,

$px^2=q$

$0.5x^2=3$ **M1**

$x^2=6$

$x=\sqrt{6}$ (reject $-\sqrt{6}$)

$x=2.45$ (3sf) **A1**

(iii)

For rectangle, $q=px^2$

$\frac{2A}{x}=px^2+q$

$\frac{2A}{x}=2px^2$

$\frac{2A}{x}=2(0.5)x^2$

$\frac{2A}{x}=x^2$

Draw $\frac{2A}{x}=x^2$ **M1**
B1 Line

OR

For rectangle, $px^2=q$

$\frac{2A}{x}=px^2+q$

$\frac{2A}{x}=2q$

$\frac{2A}{x}=2(3)$

$\frac{2A}{x}=6$

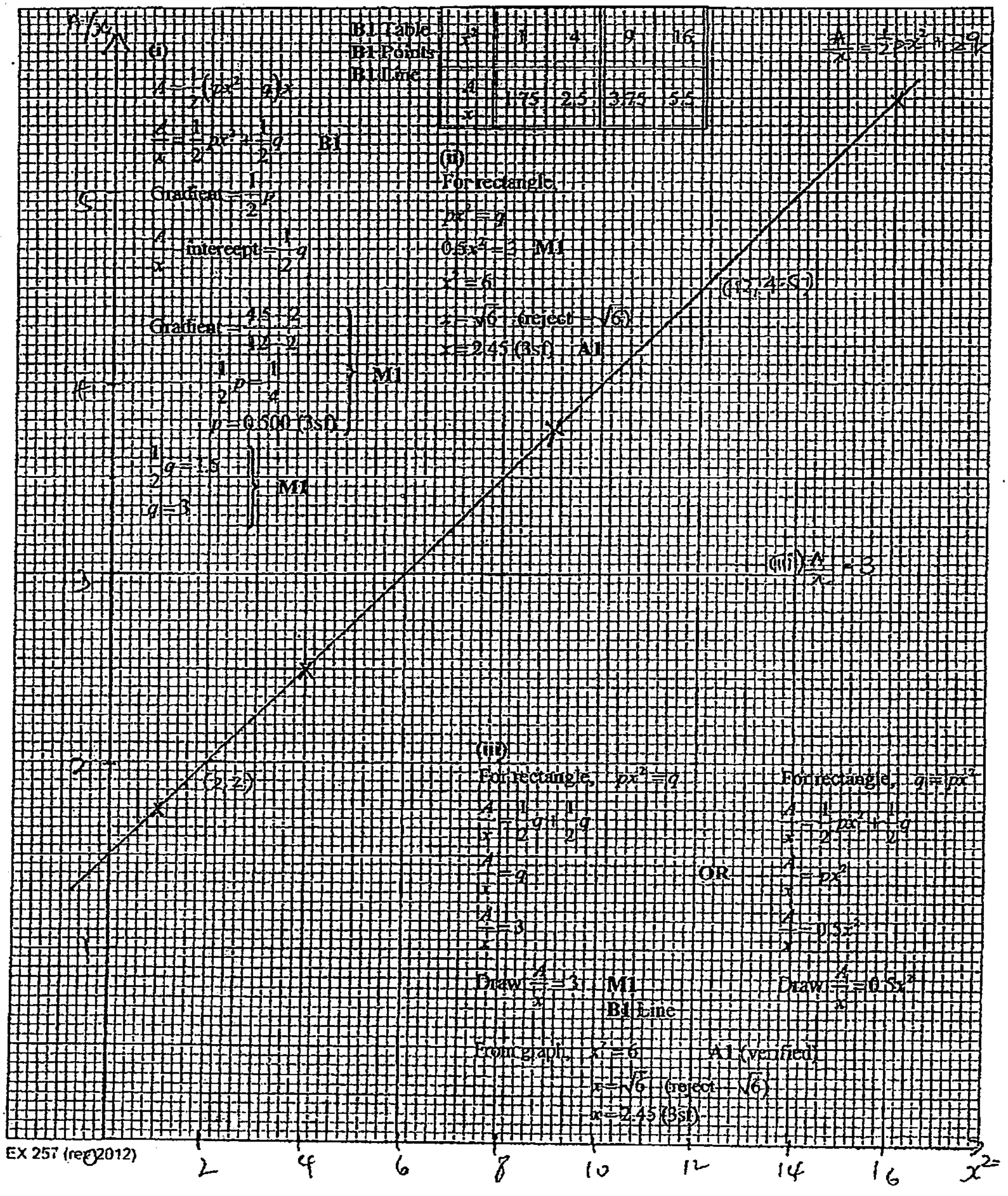
Draw $\frac{2A}{x}=6$

From graph, $x^2=6$ **A1** (verified)

$x=\sqrt{6}$ (reject $-\sqrt{6}$)

$x=2.45$ (3sf)

Plot $\frac{A}{x}$ against x^2



B1 Table
B1 Points
B1 Line

x^2	1	4	9	16
$\frac{A}{x}$	1.75	2.5	3.75	5.5

(i)

$A=\frac{1}{2}(px^2+q)x$

$\frac{A}{x}=\frac{1}{2}px^2+\frac{1}{2}q$ **B1**

Gradient $=\frac{1}{2}p$

$\frac{A}{x}$ intercept $=\frac{1}{2}q$

Gradient $=\frac{4.5-2}{12-2}$

$\frac{1}{2}p=\frac{1}{4}$

$p=0.500$ (3sf) **M1**

$\frac{1}{2}q=1.5$

$q=3$ **M1**

(ii)

For rectangle,

$px^2=q$

$0.5x^2=3$ **M1**

$x^2=6$

$x=\sqrt{6}$ (reject $-\sqrt{6}$)

$x=2.45$ (3sf) **A1**

(iii)

For rectangle, $px^2=q$

$\frac{A}{x}=\frac{1}{2}q+\frac{1}{2}q$

$\frac{A}{x}=q$

$\frac{A}{x}=3$

Draw $\frac{A}{x}=3$ **M1**

B1 Line

OR

For rectangle, $q=px^2$

$\frac{A}{x}=\frac{1}{2}px^2+\frac{1}{2}q$

$\frac{A}{x}=px^2$

$\frac{A}{x}=0.5x^2$

Draw $\frac{A}{x}=0.5x^2$

From graph, $x^2=6$ **A1** (verified)

$x=\sqrt{6}$ (reject $-\sqrt{6}$)

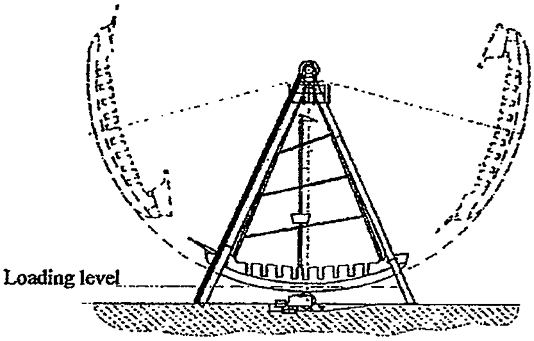
$x=2.45$ (3sf)

EX 257 (rev 2012)

11 Gravitational potential energy, measured in kilojoules (kJ), is the energy a body has due to its position. It can be calculated by the following equation:

$$\text{Gravitational potential energy} = \frac{mgh}{1000}$$

where m is the mass of the body in kg, g is the gravitational field strength in N/kg and h is the height of the object in m. The gravitational field strength, g, on Earth is approximately 10 N/kg.



The gravitational potential energy, E, in kilojoules (kJ), of a pirate ship ride can be modelled by the equation, $E = 100(1 - \cos kt) + a$, where k and a are constants and t is the time in seconds after starting the ride at loading level.

(i) Given that the mass of the ride is 1000 kg and at an initial loading level of 3 m, show that $a = 30$. [1]

(ii) Explain why this model suggests that the maximum gravitation potential energy possessed by the ride is 230 kJ. [1]

(i) $a = \dfrac{1000 \times 10 \times 3}{1000}$

$= 30 \text{ kJ} \longleftarrow \text{A1}$

(ii) Since $-1 \le \cos kt \le 1$,

When $\cos kt = -1$, Max. $E = 100[1 - (-1)] + 30$

$= 230 \text{ kJ}$ } A1

11 The ride takes 6 seconds to travel from one peak to another.

(iii) Show that the value of k is $\frac{\pi}{3}$ radians per second. [2]

(iv) Calculate the gravitation potential energy of the ride at $t=8$ s. [2]

(v) If the ride continues for 60 seconds, find the exact duration for which the ride possesses more than 80 kJ of gravitational potential energy. [5]

(iii) $\text{Period}=\frac{2\pi}{k}=6 \longleftarrow$ B1 or sub $t=6$ & $E=230$

$k=\frac{\pi}{3} \longleftarrow$ A1

(iv) $E=100\left(1-\cos\frac{\pi}{3}t\right)+30$

$t=8,\quad E=100\left(1-\cos\frac{\pi}{3}(8)\right)+30 \longleftarrow$ B1

$=180\text{ kJ} \longleftarrow$ A1

(v) $80=100\left(1-\cos\frac{\pi}{3}t\right)+30 \longleftarrow$ B1 sub $E=80$

$\frac{1}{2}=1-\cos\frac{\pi}{3}t$

$\cos\frac{\pi}{3}t=\frac{1}{2}$

$\alpha=\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3} \longleftarrow$ A1

$\frac{\pi}{3}t=\frac{\pi}{3},2\pi-\frac{\pi}{3} \longleftarrow$ M1

$t=1,\ 5\text{ s}$

Duration where $E>80$ in 1 swing $=5-1=4$ s $\longleftarrow$ M1

No. of swings in 60 s $=\frac{60}{6}=10$

Total time $=10\times4=40$ s $\longleftarrow$ A1

End of Paper