Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

1 Show that the equation $x^2 + (2 - k)x + k = 3$ has real roots for all real values of k. [4]

2 (i) Sketch the graph of $y = |4 - x|$. [2]

(ii) Determine the number of intersections of the line $y = \frac{1}{2}x$ with $y = |4 - x|$, justifying your answer. [2]

3 Express $\frac{3x^2 + 2x - 28}{x^2 - 5x + 6}$ in partial fractions. [5]

4 A rectangular block has a square base. The length of each side of the base is $(2\sqrt{2} - \sqrt{3})$ m and the volume of the block is $(21\sqrt{2} - 13\sqrt{3})$ m^3. Find, **without using a calculator**, the height of the block in the form $(a\sqrt{2} + b\sqrt{3})$ m, where a and b are integers. [5]

5 Find the range of values of x which satisfy both of the inequalities

$$3 - 2x < 5,$$
$$2x^2 + 3x < 2.$$

[4]

6 Given that $\log_b(x^3 y) = p$ and $\log_b\left(\frac{y}{x^2}\right) = q$, express $\log_b(xy)$ in terms of p and q. [4]

7 Solve the equation $e^{2x} = e^x + 12$. [5]

8 Given that the expansion of $(a + x)(1 - 3x)^n$ in ascending powers of x is $2 - 47x + bx^2 + \ldots$ find the values of the constants n, a and b. [6]

[Turn over

9 A collector bought a painting in the beginning of 1990. The value V dollars of the painting is given by the formula $V = 2800\ e^{kt}$, where t is the time in years since the beginning of 1990 and k is a constant.

(i) Find the value of the painting when the collector bought it. [1]

The value of the painting in the beginning of 2010 was 10000 dollars.

(ii) Find the expected value of the painting in the beginning of 2020. [3]

(iii) Find the year in which the expected value of the painting first crosses 40000 dollars. [2]

10 Solve $\log_{16}(3x-1) = \log_4(3x) - \frac{1}{2}$. [5]

11 The expression $ax^3 + 2ax^2 - 15x + b$ is exactly divisible by $x+3$ but leaves a remainder of -12 when divided by $x-1$.

(i) Find the value of a and of b. [4]

(ii) Using the values of a and b found in part (i), factorise the expression completely and hence solve the equation $ax^3 + 2ax^2 - 15x + b = 0$ [4]

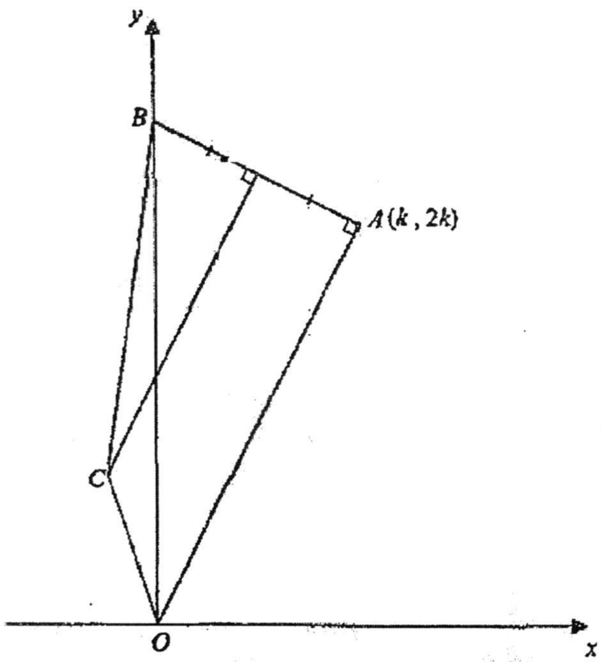
12 The roots of the equation $2x^2 - 3x + 4 = 0$ are α and β.

(i) Form a quadratic equation whose roots are $\alpha - 2\beta$ and $\beta - 2\alpha$. [6]

(ii) Show that $4\alpha^3 = \alpha - 12$. [3]

(iii) Find the value of $\alpha^3 + \beta^3$. [3]

13



The diagram shows the quadrilateral $OABC$.
The coordinates of A are $(k, 2k)$, where $k > 0$, and the length of OA is $\sqrt{80}$ units.

(i) Show that $k = 4$. [2]

AB is perpendicular to OA and B lies on the y-axis.

(ii) Find the equation of AB and the coordinates of B. [4]

The point C lies on the line through O parallel to $y + 3x = 5$ and also on the perpendicular bisector of AB.

(iii) Calculate the coordinates of C. [4]

(iv) Calculate the area of the quadrilateral $OABC$. [2]

1 A curve is such that $\frac{dy}{dx} = \frac{4}{(2x+3)^2}$. Given that the curve passes through the point $(1, -2)$, find the coordinates of the point where the curve crosses the $x-$axis. [4]

2 The radius, r cm, of a sphere is increasing at a constant rate of 0.5 cm/s.
Find, in terms of π, the rate at which the volume is increasing at the instant when the volume is 972π cm^3. [4]

[Volume of sphere, $V = \frac{4}{3}\pi r^3$]

3 An experiment on the topic of Optics in Physics was carried out to find the focal length, f cm, of a certain type of lens. The experiment requires the student to place an object at a distance, u cm, from the lens and to record the distance, v cm, at which the image can be seen on the other side of the lens. The data below shows some of the tabulated experimental results.

u	15	20	30	40	50	55
v	52.5	27.8	18.9	18.3	15.2	14.7

It is known that u, v and f are related by the equation $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$.
It is believed that an error was made in recording one of the values of v.

(i) Plot $\frac{1}{v}$ against $\frac{1}{u}$ and draw a straight line graph which represents the experimental values in the table above. [2]

(ii) Determine which value of v is the incorrect reading.
Use the straight line graph obtained in part (i) to estimate the correct value that should replace the incorrect value of v. [2]

(iii) Estimate the focal length of the lens, f cm, from the graph. [2]

(iv) Verify the accuracy of the straight line graph drawn by evaluating the gradient. [1]

4 A polynomia $= 12x + 4$, $f'(-1) = 1$ and $f(2) = 19$.

(i) Using integration, show that $f(x) = 2x^3 + 2x^2 - x - 3$. [4]

(ii) Show that $x = 1$ is the only solution to $f(x) = 0$. [3]

5 Variables x and y are connected by the equation $y = a^{b+x}$, where a and b are constants. Using experimental values of x and y, a graph was drawn in which $\lg y$ was plotted on the vertical axis against x on the horizontal axis. The straight line which was obtained passed through the points $(0.48, 0.7)$ and $(0.6, 0.82)$. Find

(i) the values of a and b, [4]

(ii) the coordinates of the point on the line at which $y = 0.1^x$. [3]

6 A curve has the equation $y = (x-1)\sqrt{2x+1}$.

(i) Show that $\frac{dy}{dx} = \frac{kx}{\sqrt{2x+1}}$, where k is a constant and state the value of k. [4]

(ii) Hence, evaluate $\int_0^{12} \frac{6x}{5\sqrt{2x+1}}\, dx$. [4]

7 The point $A(-1, 2)$ lies on a circle with centre $(3, -1)$.

(i) Given that AB is the diameter of the circle, find the coordinates of B. [2]

(ii) Find the radius of the circle and hence, state its equation. [2]

Another point $C(3, 4)$ lies on the circle.
A line which passes through the point A, cuts the circle at point D and is parallel to BC.

(iii) Find the equation of the straight line BD. [4]

8 The equation of a curve is $y = x^2(x-2)^2$.

(i) Show that $\frac{dy}{dx} = 4x(x-1)(x-2)$ and hence state the number of stationary points of the curve. [3]

(ii) Find the coordinates of the stationary points of the curve. [3]

(iii) Find an exp dx ce determine the nature of these stationary points. [4]

9 The curve $y = f(x)$ is such that $f(x) = \dfrac{2x+6}{x+1}$ where $x \neq a$.

(i) State the value of a. [1]

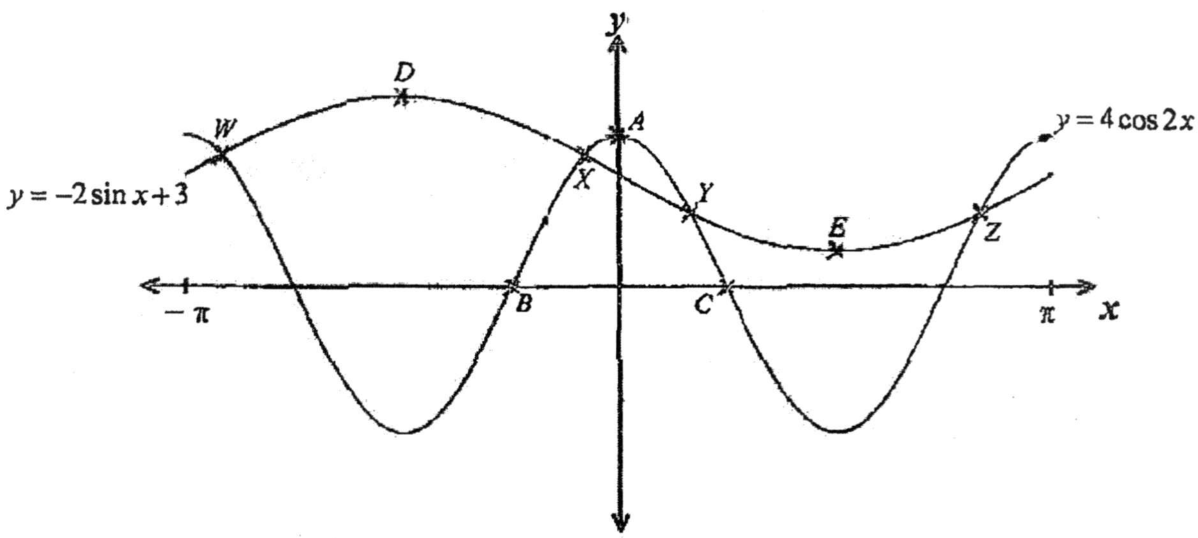
(ii) Find $f'(x)$ and explain why the curve $y = f(x)$ is a decreasing function. [4]

The curve intersects the x – axis at the point A.

(iii) Find the equation of the tangent at A. [3]

(iv) If the normal to the curve at A meets the y – axis at B, show that the area of ΔAOB, where O is the origin, is 4.5 square units. [3]

10 The diagram shows the curves $y = 4\cos 2x$ and $y = -2\sin x + 3$ for $-\pi \leq x \leq \pi$ radians. A, B and C are the axes intercepts of the curve $y = 4\cos 2x$. D and E are the turning points of the curve $y = -2\sin x + 3$. The curves intersect at the points W, X, Y and Z.



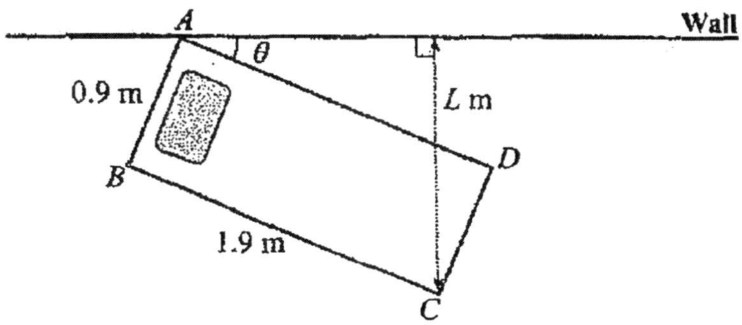
(i) State the coordinates of A, B, C, D and E. [5]

(ii) Show that the equation $4\cos 2x = -2\sin x + 3$ can be expressed as

$\sin x - 1 = 0$. [2]

(iii) Hence, find in radians, the x – coordinate of W, X, Y and Z. [4]

11 The diagram shows a rectangular single bed with wheels, $ABCD$, which is hinged to the wall at A. It is given that the dimensions of the bed is 1.9 m by 0.9 m and L m is the perpendicular distance from the wall to C. The bed can be rolled such that the angle between the wall and the side, AD, of the bed is θ and that $0° \leq \theta < 90°$.



(i) Show that the length, L m, can be expressed as $L = 1.9\sin\theta + 0.9\cos\theta$. [3]

(ii) Express L in the form $R\sin(\theta + \alpha)$ where $R > 0$ and α is an acute angle. [3]

(iii) Hence, find the maximum value of L and the corresponding value of θ. [3]

(iv) Find the value of θ when $L = 1.3$ m. [2]

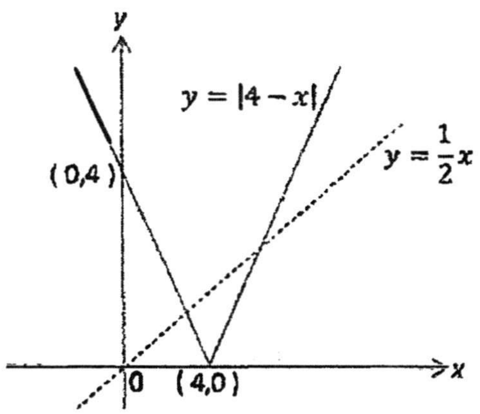
12 (i) Prove the identity $\cos^4 x - \sin^4 x + 2\cos^2 x - 1 = 2\cos 2x$. [3]

(ii) Solve the equation $2\cos^4 x - 2\sin^4 x = \sqrt{2}$, for $0 < x < \pi$, giving your answers in terms of π. [4]

(iii) Given that $3\cos^4 x - 3\sin^4 x = \sqrt{3}\sin 2x$, and **without using a calculator**,

(a) deduce that $\tan 2x = \sqrt{3}$, [2]

(b) find the possible values of $\tan x$. [3]

1	$x^2+(2-k)x+k-3=0$ $b^2-4ac=(2-k)^2-4(1)(k-3)$ $=4-4k+k^2-4k+12$ $=k^2-8k+16$ $=(k-4)^2$ ≥ 0 $\therefore$ The equation has real roots for all real values of k.	M1 M1A1 A1	[4]	>0 is not acceptable
2(i)		B2|0		
(ii)	No. of solutions = 2 Justification: Correct sketch/ Explanation/ Solving	A1 A1	[4]	
3	$x^2-5x+6 \overline{)\,3x^2+2x-28}$ quotient 3 $3x^2-15x+18$ $17x-46$ $x^2-5x+6=(x-3)(x+2)$ Let $\frac{17x-46}{(x-3)(x-2)}=\frac{A}{x-2}+\frac{B}{x-3}$ $17x-46=A(x-3)+B(x-2)$ $34-46=-A$ (Taking $x=2$) $A=12$ [illegible] (Taking $x=3$) $\therefore \frac{[illegible]}{x^2-5x+6}=3+\frac{12}{x-2}+\frac{5}{x-3}$	B1 B1 M1 M1A1	[5]	

4	Height $=\dfrac{21\sqrt{2}-13\sqrt{3}}{(2\sqrt{2}-\sqrt{3})^2}$	B1		
	$=\dfrac{21\sqrt{2}-13\sqrt{3}}{8-4\sqrt{6}+3}$			
	$=\dfrac{21\sqrt{2}-13\sqrt{3}}{11-4\sqrt{6}}$	M1		
	$=\dfrac{(21\sqrt{2}-13\sqrt{3})}{(11-4\sqrt{6})}\times\dfrac{(11+4\sqrt{6})}{(11+4\sqrt{6})}$	M1		
	$=\dfrac{231\sqrt{2}+84\sqrt{12}-143\sqrt{3}-52\sqrt{18}}{121-96}$	M1		
	$=\dfrac{75\sqrt{2}+25\sqrt{3}}{25}$			
	$=(3\sqrt{2}+\sqrt{3})$ m	A1	[5]	
5	$3-2x<5$ $-2x<2$ $x>-1$	B1		
	$2x^2+3x-2<0$ $(2x-1)(x+2)<0$ $-2<x<\dfrac{1}{2}$	M1A1		
	$\therefore -1<x<\dfrac{1}{2}$	A1	[4]	
6	$\log_b(x^3y)=p$ $3\log_b x+\log_b y=p$ ....(1)	M1		
	$\log_b\left(\dfrac{y}{x^2}\right)=q$ $\log_b y-2\log_b x=q$ ....(2)	M1		
	$(1)-(2) \Rightarrow 5\log_b x=p-q$ $\log_b x=\frac{p-q}{5}$ $\log_b y=q+2\frac{(p-q)}{5}$ $\log_b xy=\log_b x+\log_b y$ $=\frac{p-q}{5}+q+2\frac{(p-q)}{5}$	M1A1	[4]	

7	$e^{2x} - e^{x} - 12 = 0$ $let\ y = e^{x}$ $y^2 - y - 12 = 0$ $(y-4)(y+3) = 0$ $y = 0$ or $y = -3$ (N.A) $e^{x} = 4$ $x = \ln 4$ $= 1.39$ (3 s.f.)	 M1 M1A1 M1A1	 [5]	
8	$(a+x)(1-3x)^n$ $= (a+x)\left[1 - \binom{n}{1}(3x) + \binom{n}{2}(3x)^2 + \cdots\right]$ $= (a+x)\left[1 - 3nx + \binom{n}{2}9x^2 + \cdots\right]$ $= a - 3anx + \binom{n}{2}9ax^2 + x - 3nx^2 + \cdots$ $= a + (1-3an)x + \left[\binom{n}{2}9a - 3n\right]x^2 + \cdots$ By comparison, $a = 2$ $1 - 3 \times 2n = -47$ $-6n = -48$ $n = 8$ $b = \binom{8}{2} \times 9 \times 2 - 3 \times 8 = 480$	 M1A1 M1 A1 A1 A1	 [6]	
9	(i) when $t = 0$ V = \$2800 ii) $t = 20$ $10000 = 2800e^{20k}$ $e^{20k} = \frac{10000}{2800}$ $k = \frac{\ln\left(\frac{10000}{2800}\right)}{20} = 0.063648$ In the beginning of 2020, $t = 30$ $V = 2800e^{30 \times 0.063648}$ = \$18898.06 (Accept) = \$18900 (iii) $40000 = 2800e^{0.063648t}$ $t = \frac{\ln\left(\frac{40000}{2800}\right)}{0.063648}$ The year is 2031.	 B1 M1A1 A1 M1 A1	 [6]	

10	$\log_{16}(3x-1)=\log_4(3x)-\frac{1}{2}$ $\frac{\log_4(3x-1)}{\log_4 16}=\log_4 3x-\log_4 2$ $\frac{\log_4(3x-1)}{2}=\log_4\left(\frac{3x}{2}\right)$ $\log_4(3x-1)=2\log_4\left(\frac{3x}{2}\right)$ $\log_4(3x-1)=\log_4\left(\frac{3x}{2}\right)^2$ $3x-1=\frac{9x^2}{4}$ $9x^2-12x+4=0$ $(3x-2)^2=0$ $\Rightarrow 3x-2=0$ $x=\frac{2}{3}$	B1 M1 M1A1 A1	[5]	B1 for correct application of change of base
11	(i) Let $f(x)=ax^3+2ax^2-15x+b$ $f(-3)=0$ $-27a+18a+45+b=0$ $-9a+b=-45$ ——— (1) $f(1)=-12$ $a+2a-15+b=-12$ $3a+b=3$ ——— (2) (1)−(2) $\Rightarrow -12a=-48$ $a=4$ $12+b=3$ $b=-9$ $\therefore a=4,\ b=-9$ (ii) $$\begin{array}{r\|l} & 4x^2-4x-3 \\ \hline x+3 & 4x^3+8x^2-15x-9 \\ & 4x^3+12x^2 \\ \hline & -4x^2-15x \\ & -4x^2-12x \\ \hline & -3x-9 \\ & -3x-9 \\ \hline & 0 \end{array}$$ $(x+3)(4x^2-4x-3)=0$ $(x+3)(2x-3)(2x+1)=0$ $\therefore x=-3,-\frac{1}{2},\frac{3}{2}$	M1 M1 A2 M1 M1A1 A1	[8]	

12	(i) $\alpha+\beta=\frac{3}{2}$, $\alpha\beta=2$	B1B1		
	$\alpha-2\beta+\beta-2\alpha=-(\alpha+\beta)=-\frac{3}{2}$	A1		
	$(\alpha-2\beta)(\beta-2\alpha)$ $=\alpha\beta-2\alpha^2-2\beta^2+4\alpha\beta$ $=5\alpha\beta-2(\alpha^2+\beta^2)$ $=5\alpha\beta-2[(\alpha+\beta)^2-2\alpha\beta]$ $=9\alpha\beta-2(\alpha+\beta)^2$ $=9\times2-2\times\left(\frac{3}{2}\right)^2$ $=\frac{27}{2}$	M1A1		
	$\therefore$ The equation is $x^2+\frac{3}{2}x+\frac{27}{2}=0$ *ie,* $2x^2+3x+27=0$	A1		
	(ii) Since α is a root, $2\alpha^2-3\alpha+4=0$	B1		
	$2\alpha^2=3\alpha-4$ $4\alpha^3=6\alpha^2-8\alpha$ $=3(2\alpha^2)-8\alpha$ $=3(3\alpha-4)-8\alpha$ $=\alpha-12$ (shown)	M1A1		
	(iii) $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$	B1		
	$=\frac{3}{2}[(\alpha+\beta)^2-3\alpha\beta]$ $=\frac{3}{2}\left[\left(\frac{3}{2}\right)^2-3\times2\right]$ $=-\frac{45}{8}$	M1A1	[12]	
13.	1. (i) $(2k)^2+k^2=80$ $5k^2=80$ $k^2=16$ $k=4$	M1A1		
	(ii) $A(4,8)$ $M_{OA}=2$ $M_{AB}=-\frac{1}{2}$	M1		
	Equation of AB is $y-8=-\frac{1}{2}(x-4)$	M1A1		
	$\therefore B(0,10)$	A1		

	(iii) Equation of OC is $y = -3x$ ———— (1) Midpoint of AB is (2,9) Equation of the perpendicular of AB is $y - 9 = 2(x - 2)$ $y = 2x + 5$ ———— (2) Solving (1) and (2), C is (-1, 3) (iv) Area $= \frac{1}{2}\begin{vmatrix} 0 & 4 & 0 & -1 & 0 \\ 0 & 8 & 10 & 3 & 0 \end{vmatrix}$ $= \frac{1}{2}[40 + 10]$ $= 25 \text{ units}^2$	B1 M1 M1A1 M1A1	 [12]	

Geylang Methodist School (Secondary)
Mid – Year Examination 2017

ADDITIONAL MATHEMATICS 4047 / 02
Paper 2

4 Express /
5 Normal (Academic)

Additional Materials provided: Cover Sheet (1 sheet) 2 hours 30 minutes
Writing Paper ([illegible] sheets)
Graph Paper (1 sheet)

Setter: Mrs Nair [illegible] Monday, 8 May 2017

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the Writing Papers provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

This document consists of **19** printed pages including the cover page and 1 blank page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$
$$\sec^2 A=1+\tan^2 A$$
$$\operatorname{cosec}^2 A=1+\cot^2 A$$
$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$
$$a^2=b^2+c^2-2bc\cos A$$
$$\Delta=\frac{1}{2}ab\sin C$$

1 A curve is such that $\frac{dy}{dx}=\frac{4}{(2x+3)^2}$. Given that the curve passes through the point $(1,-2)$, find the coordinates of the point where the curve crosses the x-axis. [4]

1 Given: $\frac{dy}{dx}=\frac{4}{(2x+3)^2}$ and $(1,-2)$

$y=\int\frac{4}{(2x+3)^2}\,dx=\int 4(2x+3)^{-2}\,dx=4\int(2x+3)^{-2}\,dx$

$=4\left[\frac{(2x+3)^{-2+1}}{(-2+1)(2)}\right]+c=4\left[\frac{(2x+3)^{-1}}{(-1)(2)}\right]+c$

$y=-2\left[\frac{1}{(2x+3)}\right]+c$ [M1]

At $(1,-2)$, $(x=1, y=-2)$

$-2=-2\left[\frac{1}{(2(1)+3)}\right]+c$

$-2=-2\left[\frac{1}{5}\right]+c$

$-2+\frac{2}{5}=c$

$c=-\frac{8}{5}$

$\Rightarrow y=-\frac{2}{(2x+3)}-\frac{8}{5}$ [M1]

Crosses the x – axis $\Rightarrow y=0$

$0=-\frac{2}{(2x+3)}-\frac{8}{5}$ [M1]

$\frac{2}{(2x+3)}=-\frac{8}{5}$

$(2)(5)=-8(2x+3)$

$10=-16x-24$

$16x=-24-10$

$16x=-34$

$x=-\frac{34}{16}=-\frac{17}{8}=-2\frac{1}{8}=-2.125$

$\therefore$ Coordinates: $\left(-\frac{17}{8},0\right)$; $\left(-2\frac{1}{8},0\right)$; $(-2.125,0)$ [A1]

2 The radius, r cm, of a sphere is increasing at a constant rate of 0.5 cm/s.

Find, in terms of π, the rate at which the volume is increasing at the instant when the volume is 972π cm^3. [4]

[Volume of sphere, $V = \frac{4}{3}\pi r^3$]

2 Given: $V = \frac{4}{3}\pi r^3$; $V = 972\pi$ cm^3 ; $\frac{dr}{dt} = 0.5$ cm s^{-1}

$V = \frac{4}{3}\pi r^3$

$972\pi = \frac{4}{3}\pi r^3$

$972 = \frac{4}{3}r^3$

$972 \div \frac{4}{3} = r^3$

$r^3 = 972 \times \frac{3}{4}$

$r^3 = 729$

$r = 9$ cm [M1]

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = \frac{4}{3}(3)\pi r^{3-1}$

$\frac{dV}{dr} = 4\pi r^2$ [M1]

$\left.\frac{dV}{dr}\right|_{r=9} = 4\pi(9)^2$

$\left.\frac{dV}{dr}\right|_{r=9} = 4 \times 81\pi$

$\left.\frac{dV}{dr}\right|_{r=9} = 324\pi$ [M1]

$\left(\frac{dV}{dr} = 324\pi \ ; \ \frac{dr}{dt} = 0.5 \text{ cm s}^{-1}\right)$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$\frac{dV}{dt} = 324\pi \times 0.5$

$\therefore \frac{dV}{dt} = 162\pi$ [A1]

3 An experiment on the topic of Optics in Physics was carried out to find the focal length, f cm, of a certain type of lens. The experiment requires the student to place an object at a distance, u cm, from the lens and to record the distance, v cm, at which the image can be seen on the other side of the lens. The data below shows some of the tabulated experimental results.

u	15	20	30	40	50	55
v	52.5	27.8	18.9	18.3	15.2	14.7

It is known that u, v and f are related by the equation $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$.

It is believed that an error was made in recording one of the values of v.

(i) Plot $\frac{1}{v}$ against $\frac{1}{u}$ and draw a straight line graph which represents the experimental values in the table above. [2]

(ii) Determine which value of v is the incorrect reading. Use the straight line graph obtained in part (i) to estimate the correct value that should replace the incorrect value of v. [2]

(iii) Estimate the focal length of the lens, f cm, from the graph. [2]

(iv) Verify the accuracy of the straight line graph drawn by evaluating the gradient. [1]

3 (i) Given: $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ ⇒ $\frac{1}{v}=-\frac{1}{u}+\frac{1}{f}$ ⇒ $\frac{1}{v}=(-1)\frac{1}{u}+\frac{1}{f}$

$Y = m\,X + c$

Y vs X ⇒ $\frac{1}{v}$ vs $\frac{1}{u}$; Gradient, $m=-1$; y-intercept, $c=\frac{1}{f}$

	u	15	20	30	40	50	55
X	$\frac{1}{u}$	0.067	0.050	0.033	0.025	0.020	0.018
	v	52.5	27.8	18.9	18.3	15.2	14.7
Y	$\frac{1}{v}$	0.019	0.036	0.053	0.055	0.066	0.068

Correct Plot / X, Y – axes / Label (graph, coordinates etc) → [M1]
Suitable Scale / Best Straight Line / Cuts y–axis (need not cut x axis) → [A1]

(ii) Incorrect value: $\frac{1}{v}=0.055$ ⇒ $v=18.3$ [M1]

Correct value (from Straight-Line Graph): $\frac{1}{v}=0.061$ ⇒ $v=16.4$ (3sf) [A1]

(Accept: $\frac{1}{v}=0.061 \pm 0.001$; $v=16.4 \pm 0.3$)

(iii) From graph,

y-intercept, $c=\frac{1}{f}=0.086$ [M1]

(Accept: $c=0.086 \pm 0.001$)(3sf)

$f=\frac{1}{0.086}$ ⇒ $f=11.6279$ cm [A1]

(Accept:

(iv) Comparing gradient, $m=-1$ (from equation)

From graph: $(0.067, 0.019)$; $(0, 0.086)$

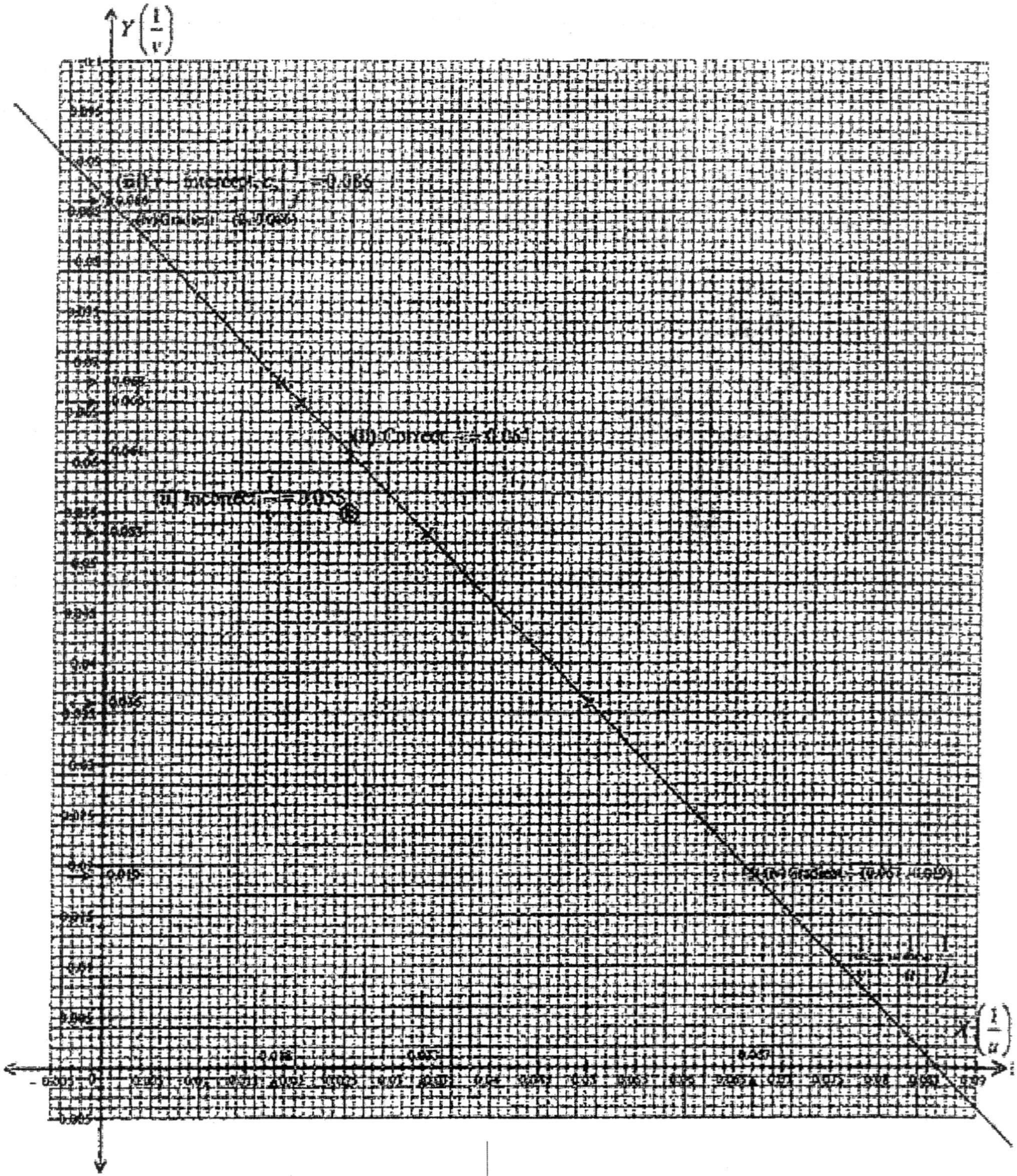
$m=\frac{y_2-y_1}{x_2-x_1}=\frac{0.086-0.019}{0-0.067}=\frac{0.067}{-0.067}$

$m=-1$ $(=-1)$ (Graph is accurate) [B1]

(Graph gentler/steeper than what is required depending on student's answer)

[Turn over

(i) Graph of $\frac{1}{v} = -\frac{1}{u} + \frac{1}{f}$ → ***Scale***: Horizontal & Vertical axis, 2 cm to 0.01 units



4 A polynomial $f(x)$ is such that $f''(x)=12x+4$, $f'(-1)=1$ and $f(2)=19$.

(i) Using integration, show that $f(x)=2x^3+2x^2-x-3$. [4]

(ii) Show that $x=1$ is the only solution to $f(x)=0$. [3]

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4 (i) Given: $f''(x)=12x+4$, $f'(-1)=1$

$$f'(x)=\int f''(x)\,dx$$
$$f'(x)=\int (12x+4)\,dx$$
$$=12\left[\frac{x^{1+1}}{1+1}\right]+4\left[\frac{x^{0+1}}{0+1}\right]+c$$
$$f'(x)=12\left[\frac{x^2}{2}\right]+4\left[\frac{x^1}{1}\right]+c$$
$$f'(x)=6x^2+4x+c \quad \text{[M1]}$$
$$f'(-1)=1,$$
$$f'(-1)=6(-1)^2+4(-1)+c$$
$$1=6-4+c$$
$$c=1-6+4$$
$$c=-1$$
$$\Rightarrow f'(x)=6x^2+4x-1 \quad \text{[M1]}$$

Since: $f'(x)=6x^2+4x-1$, $f(2)=19$

$$f(x)=\int f'(x)\,dx$$
$$f(x)=\int (6x^2+4x-1)\,dx$$
$$=6\left[\frac{x^{2+1}}{2+1}\right]+4\left[\frac{x^{1+1}}{1+1}\right]-1\left[\frac{x^{0+1}}{0+1}\right]+c$$
$$=6\left[\frac{x^3}{3}\right]+4\left[\frac{x^2}{2}\right]-1\left[\frac{x^1}{1}\right]+c$$
$$f(x)=2x^3+2x^2-x+c \quad \text{[M1]}$$
$$f(2)=19,$$
$$f(2)=2(2)^3+2(2)^2-(2)+c$$
$$19=16+8-2+c$$
$$c=19-16-8+2$$
$$c=-3$$
$$\therefore f(x)=2x^3+2x^2-x-3 \text{ (Shown)} \quad \text{[A1]}$$

(ii) Show: $x=1$ is the <u>only</u> solution to $f(x)=0$

$(x-1)$ is a factor of $f(x)=2x^3+2x^2-x-3$.

Try: $f(1)=2(1)^3+2(1)^2-(1)-3$
$=2+2-1-3$
$=0$

$\Rightarrow$ $f(1)=0$ $\Rightarrow x=1$ is a solution to $f(x)=2x^3+2x^2-x-3=0$ [B1]

$$f(x)=2x^3+2x^2-x-3=0$$
$$\rightarrow 2x^3+2x^2-x-3=0$$
$$\Rightarrow (x-1)(ax^2+bx+c)=0$$
$$(x-1)(2x^2+4x+3)=0$$

$(x-1)=0$; $(2x^2+4x+3)=0$

$\therefore x=1$; $(a=2, b=4, c=3)$

$$b^2-4ac=4^2-4(2)(3)$$
$$=16-24$$
$$b^2-4ac=-8<0 \quad \text{[M1]}$$

No solution for:
$(2x^2+4x+3)=0$

Hen... to $f(x)=0$

$$\begin{array}{r|l} & 2x^2+4x+3 \\ \hline x-1 & 2x^3+2x^2-x-3 \\ & -(2x^3-2x^2) \\ \hline & 4x^2-x \\ & -(4x^2-4x) \\ \hline & 3x-3 \\ & -(3x-3) \\ \hline & 0 \end{array} \quad \text{[M1]}$$

<u>OR</u> By Synthetic Division:

	[illegible]	[illegible]	[illegible]	[illegible]
	2	2	-1	-3
1		2	4	3
	2	4	3	0
	[illegible]	[illegible]	[illegible]	

5 Variables x and y are connected by the equation $y = a^{b+x}$ where a and b are constants. Using experimental values of x and y, a graph was drawn in which $\lg y$ was plotted on the vertical axis against x on the horizontal axis. The straight line which was obtained passed through the points $(0.48, 0.7)$ and $(0.6, 0.82)$. Find

(i) the values of a and b, **[4]**

(ii) the coordinates of the point on the line at which $y = 0.1^x$. **[3]**

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5 (i) Given: $y = a^{b+x}$

$\lg y = \lg a^{b+x}$

$\lg y = (b+x)\lg a$

$\lg y = b\lg a + x\lg a$

$\lg y = (\lg a)x + b\lg a$ **[M1]**

$Y = m\,X + c$

Y vs $X \Rightarrow \lg y$ vs x

Gradient, $m = \lg a$

y-intercept, $c = b\lg a$

$(0.48, 0.7)$ and $(0.6, 0.82)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\lg a = \frac{0.82 - 0.7}{0.6 - 0.48} = \frac{0.12}{0.12}$$

$\lg a = 1$ **[M1]**

$\log_{10} a = 1$

$a = 10^1$

$\therefore a = 10$ **[M1]**

$(x, \lg y) \Rightarrow (0.48, 0.7)$, $\lg a = 1$

$\lg y = (\lg a)\;x + b\;\lg a$

$0.7 = (1)\;(0.48) + b\;(1)$

$b = 0.7 - 0.48$

$\therefore b = 0.22$ **[A1]**

Hence, Straight-Line equation:

$\lg y = (1)x + 0.22$

$Y = m\,X + c$

(ii) $y = a^{b+x}$, $(a = 10, b = 0.22)$

$y = 10^{0.22+x} \rightarrow (1)$

$y = 0.1^x \rightarrow (2)$

$(1) = (2)$: $10^{0.22+x} = 0.1^x$ **[M1]**

$10^{0.22+x} = \left(\frac{1}{10}\right)^x$

$10^{0.22+x} = (10^{-1})^x$

$10^{0.22+x} = 10^{-x}$

$\Rightarrow 0.22 + x = -x$

$x + x = -0.22$

$2x = -0.22$

$\therefore x = -0.11$

Sub $x = -0.11$ in Straight-Line equation:

$\Rightarrow \lg y = (1)\;x + 0.22$

$\lg y = (1)(-0.11) + 0.22$ **[M1]**

$\lg y = -0.11 + 0.22$

$\therefore \lg y = 0.11$

Hence, point on the straight line

$\Rightarrow \lg y = x + 0.22$

$(x, \lg y) \rightarrow (-0.11, 0.11)$ **[A1]**

6 A curve has the equation $y=(x-1)\sqrt{2x+1}$

(i) Show that $\frac{dy}{dx}=\frac{kx}{\sqrt{2x+1}}$, where k is a constant and state the value of k. [4]

(ii) Hence, evaluate $\int_0^{12}\frac{6x}{5\sqrt{2x+1}}\,dx$. [4]

6 (i) Given:

$$y=(x-1)\sqrt{2x+1}$$

$$\frac{dy}{dx}=u\,dv+v\,du$$

$$=(x-1)\left(\frac{1}{\sqrt{2x+1}}\right)+\left(\sqrt{2x+1}\right)(1)$$

[M1] [M1]

$$=\frac{x-1}{\sqrt{2x+1}}+\sqrt{2x+1}$$

$$=\frac{x-1}{\sqrt{2x+1}}+\frac{\left(\sqrt{2x+1}\right)\left(\sqrt{2x+1}\right)}{\sqrt{2x+1}}$$ [M1]

$$=\frac{x-1}{\sqrt{2x+1}}+\frac{(2x+1)}{\sqrt{2x+1}}$$

$$=\frac{x-1+(2x+1)}{\sqrt{2x+1}}$$

$$\therefore\frac{dy}{dx}=\frac{3x}{\sqrt{2x+1}}\quad\left(\frac{kx}{\sqrt{2x+1}};\ k=3\right)$$ [A1]

Let: $u=x-1$

$$\frac{du}{dx}=1$$

$$v=\sqrt{2x+1}$$

$$v=(2x+1)^{\frac{1}{2}}$$

$$\frac{dv}{dx}=\frac{1}{2}(2x+1)^{\frac{1}{2}-1}(2)$$

$$\frac{dv}{dx}=(2x+1)^{-\frac{1}{2}}$$

$$\frac{dv}{dx}=\frac{1}{(2x+1)^{\frac{1}{2}}}$$

$$\frac{dv}{dx}=\frac{1}{\sqrt{2x+1}}$$

(ii) From (i):

$$\int\frac{3x}{\sqrt{2x+1}}\,dx=(x-1)\sqrt{2x+1}+c\quad\rightarrow\quad(*)$$

$$\int\frac{6x}{5\sqrt{2x+1}}\,dx=\int\left(\frac{2}{5}\right)\frac{3x}{\sqrt{2x+1}}\,dx$$

$$=\left(\frac{2}{5}\right)\int\frac{3x}{\sqrt{2x+1}}\,dx$$ [M1]

$$=\left(\frac{2}{5}\right)\left[(x-1)\sqrt{2x+1}+c\right]\quad\rightarrow\text{ From }(*)$$ [M1]

$$\int_0^{12}\frac{6x}{5\sqrt{2x+1}}\,dx=\left(\frac{2}{5}\right)\left[(x-1)\sqrt{2x+1}\right]_0^{12}$$

$$=\left(\frac{2}{5}\right)\left[\left((12-1)\sqrt{2(12)+1}\right)-\left((0-1)\sqrt{2(0)+1}\right)\right]$$ [M1]

$$=\left(\frac{2}{5}\right)\left[(11\sqrt{25})-\left(-\sqrt{1}\right)\right]=\left(\frac{2}{5}\right)\left[(11)(5)-(-1)\right]$$

$$=\left(\frac{2}{5}\right)\left[55+1\right]=\frac{2\times56}{5}$$

$$\therefore\int_0^{12}\frac{6x}{5\sqrt{2x+1}}\,dx=\frac{112}{5};\ 22\frac{2}{5};\ 22.4$$ [A1]

7 The point $A(-1,2)$ lies on a circle with centre $(3,-1)$.

(i) Given that AB is the diameter of the circle, find the coordinates of B. [2]

(ii) Find the radius of the circle and hence, state its equation. [2]

Another point $C(3,4)$ lies on the circle.

A line which passes through the point A, cuts the circle at point D and is parallel to BC.

(iii) Find the equation of the straight line BD. [4]

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7 (i) Given: Centre $(3,-1)$; $A(-1,2)$

Let: $B(x,y)$

Midpoint of AB is centre $(3,-1)$ { AB is diameter}

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=(3,-1)$$

$$\left(\frac{-1+x}{2},\frac{2+y}{2}\right)=(3,-1) \quad \text{[M1]}$$

Comparing $x-$ component.

$$\Rightarrow \frac{-1+x}{2}=3$$
$$-1+x=6$$
$$x=7$$

Comparing $y-$ component.

$$\Rightarrow \frac{2+y}{2}=-1$$
$$2+y=-2$$
$$y=-4$$

$\therefore B(7,-4)$ [A1]

(ii) Centre $(3,-1)$; $A(-1,2)$

$$\text{Radius} = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$
$$=\sqrt{(3-(-1))^2+(-1-2)^2}=\sqrt{(3+1)^2+(-3)^2}=\sqrt{(4)^2+(-3)^2}$$
$$=\sqrt{16+9}=\sqrt{25}$$

Radius = 5 units [A1]

Equation of circle: Centre $(3,-1)$; Radius = 5 units

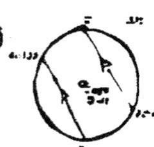
Standard form: $(x-a)^2+(y-b)^2=r^2$

$$(x-3)^2+(y-(-1))^2=5^2$$
$$(x-3)^2+(y+1)^2=5^2$$
$$(x-3)^2+(y+1)^2=25 \quad \text{[B1]}$$

General form: $x^2+y^2+2gx+2fy+c=0$

$$(x-3)^2+(y+1)^2=25$$
$$x^2-6x+9+y^2+2y+1-25=0$$
$$x^2+y^2-6x+2y-15=0 \quad \text{[B1]}$$

(iii)



$$m_{BC}=\frac{y_2-y_1}{x_2-x_1}=\frac{-4-4}{7-3}$$
$$m_{BC}=\frac{-8}{4}=-2 \quad \text{[M1]}$$

$m_{AD}=m_{BC}=-2$ $(AD // BC)$

AD is diameter, $\angle ACB=\angle ADB=90°$

($\angle$ in semicircle)

$(BD \perp BC)$, $(m_{BD})(m_{BC})=-1$

$$(m_{BD})(-2)=-1$$
$$m_{BD}=\frac{-1}{-2}=\frac{1}{2} \quad \text{[M1]}$$

Equation of BD: $B(7,-4)$, $m_{BD}=\frac{1}{2}$

$$y-y_1=m(x-x_1)$$
$$y-(-4)=\frac{1}{2}(x-7) \quad \text{[M1]}$$
$$y+4=\frac{1}{2}(x-7)$$
$$y=\frac{1}{2}x-\frac{7}{2}-4$$
$$\therefore y=\frac{1}{2}x-7\frac{1}{2} \quad \text{[A1]}$$

8 The equation of a curve is $y = x^2(x-2)^2$.

(i) Show that $\dfrac{dy}{dx} = 4x(x-1)(x-2)$ and hence state the number of stationary points of the curve. [3]

(ii) Find the coordinates of the stationary points of the curve. [3]

(iii) Find an expression for $\dfrac{d^2y}{dx^2}$ and hence determine the nature of these stationary points. [4]

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8 (i) Given: $y = x^2(x-2)^2$

Let: $u = x^2$, $\dfrac{du}{dx} = 2x$

$v = (x-2)^2$, $\dfrac{dv}{dx} = 2(x-2)(1)$, $\dfrac{dv}{dx} = 2(x-2)$

$\dfrac{dy}{dx} = u\,dv + v\,du$

$= (x^2)(2)(x-2) + (x-2)^2(2x)$ **[M1]**

$= (2x)(x-2)[(x)+(x-2)]$

$= (2x)(x-2)[2x-2] = (2x)(x-2)(2)(x-1)$

$\therefore \dfrac{dy}{dx} = 4x(x-1)(x-2)$ (Shown) **[A1]**

Stationary points: $\dfrac{dy}{dx} = 0$

$4x(x-1)(x-2) = 0$

3 solutions for x for $\dfrac{dy}{dx} = 0$

$\therefore$ Number of stationary points: 3 (Shown) **[A1]**

(ii) From (i):

$4x(x-1)(x-2) = 0$

$\rightarrow 4x = 0 \Rightarrow x = 0$ | $\rightarrow (x-1) = 0 \Rightarrow x = 1$ | $\rightarrow (x-2) = 0 \Rightarrow x = 2$

Substitute in $y = x^2(x-2)^2$,

$\rightarrow y = 0^2(0-2)^2 \Rightarrow y = 0$ | $\rightarrow y = 1^2(1-2)^2 \Rightarrow y = 1$ | $\rightarrow y = 2^2(2-2)^2 \Rightarrow y = 0$

$\therefore$ Coordinates of stationary points:

$(0, 0)$ **[B1]** $(1, 1)$ **[B1]** $(2, 0)$ **[B1]**

(iii) $\dfrac{d^2y}{dx^2}$ $= 4x(x-1)(x-2) = 4x(x^2-3x+2) = 4x^3 - 12x^2 + 8x$

$\dfrac{d^2y}{dx^2} = 4(3)x^{3-1} - 12(2)x^{2-1} + 8 = 12x^2 - 24x + 8$ **[A1]**

$\left.\dfrac{d^2y}{dx^2}\right|_{x=0} = 12(0)^2 - 24(0) + 8 = 8$ $\left(\dfrac{d^2y}{dx^2} > 0\right) \rightarrow (0, 0)$ (Min. pt.) **[B1]**

$\left.\dfrac{d^2y}{dx^2}\right|_{x=1} = 12(1)^2 - 24(1) + 8 = 12 - 24 + 8 = -4$ $\left(\dfrac{d^2y}{dx^2} < 0\right) \rightarrow (1, 1)$ (Max. pt.) **[B1]**

$\dfrac{d^2y}{dx^2} = 12(2)^2 - 24(2) + 8 = 48 - 48 + 8 = 8$ $\left(\dfrac{d^2y}{dx^2} > 0\right) \rightarrow (2, 0)$ (Min. pt.) **[B1]**

9 The curve $y = f(x)$ is such that $f(x) = \dfrac{2x+6}{x+1}$ where $x \neq a$.

(i) State the value of a. [1]

(ii) Find $f'(x)$ and explain why the curve $y = f(x)$ is a decreasing function. [4]

The curve intersects the x-axis at the point A.

(iii) Find the equation of the tangent at A. [3]

(iv) If the normal to the curve at A meets the y-axis at B, show that the area of ΔAOB, where O is the origin, is 4.5 square units. [3]

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9 (i) $f(x)$ is not defined when $x+1=0$

Hence, $x+1 \neq 0 \rightarrow x \neq -1 \rightarrow x \neq a$

Value of a is $\underline{-1}$ [B1]

(ii) $f(x) = \dfrac{2x+6}{x+1}$

Let: $u = 2x+6$, $\dfrac{du}{dx} = 2$

$v = x+1$, $\dfrac{dv}{dx} = 1$

$f'(x) = \dfrac{v\,du - u\,dv}{v^2}$

$= \dfrac{(x+1)(2)-(2x+6)(1)}{(x+1)^2} = \dfrac{2x+2-2x-6}{(x+1)^2}$ [M1]

$f'(x) = \dfrac{-4}{(x+1)^2}$ [A1]

$(x+1)^2 > 0 \quad (x \neq -1)$

$\dfrac{-4}{(x+1)^2} < 0$ [M1]

$\therefore f'(x) < 0$

$\therefore$ Hence $y = f(x)$ is a decreasing function [A1]

(iii) Curve intersects the x-axis $\Rightarrow y = 0$

$y = f(x) = 0$

$f(x) = \dfrac{2x+6}{x+1}$

$0 = \dfrac{2x+6}{x+1}$

$2x+6=0$

$2x=-6$

$x=-3$

$A(-3,0)$ [M1]

m_{tan} at $x=-3$

$f'(x) = \dfrac{-4}{(x+1)^2}$

$f'(-3) = \dfrac{-4}{(-3+1)^2} = \dfrac{-4}{(-2)^2}$

$f'(-3) = -1$

$m_{tan} = -1$ at $x=-3$ [M1]

Equation of tangent:

$A(-3,0)$; $m_{tan} = -1$

$y - y_1 = m_{tan}(x - x_1)$

$y - 0 = -1(x-(-3))$

$y = -1(x+3)$

$\therefore y = -x-3$ [A1]

(iv) $(m_{tan})(m_{nor}) = -1$

$(-1)(m_{nor}) = -1$

$(m_{nor}) = 1$

Equation of normal:

$A(-3,0)$; $m_{nor} = 1$

$y - y_1 = m_{nor}(x - x_1)$

$y - 0$

$y = x$

Meets the y-axis:

$x = 0$

$y = 0+3$

$y = 3$

$B(0,3)$ [M1]

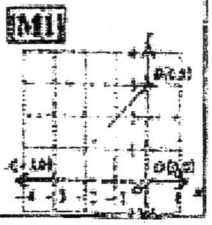
$\therefore$ Area of ΔAOB

$= (0.5)(\text{base})(\perp \text{height})$

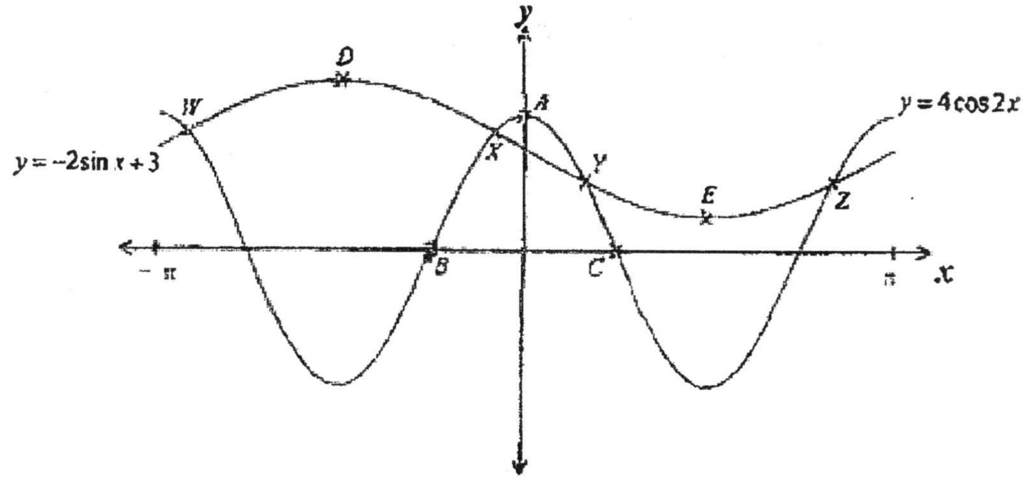
$= (0.5)(3)(3)$

$= 4.5$ square units [M1]

(Shown)



10 The diagram shows the curves $y = 4\cos 2x$ and $y = -2\sin x + 3$ for $-\pi \le x \le \pi$ radians.
A, B and C are the axes intercepts of the curve $y = 4\cos 2x$.
D and E are the turning points of the curve $y = -2\sin x + 3$.
The curves intersect at the points W, X, Y and Z.



(i) State the coordinates of A, B, C, D and E. [5]

(ii) Show that the equation $4\cos 2x = -2\sin x + 3$ can be expressed as

$$8\sin^2 x - 2\sin x - 1 = 0.$$ [2]

(iii) Hence, find in radians, the x-coordinate of W, X, Y and Z. [4]

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10 (i) $A \rightarrow y-$ intercept of $y = 4\cos 2x \quad \rightarrow x = 0$

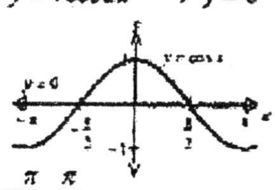
$y = 4\cos 2(0) = 4\cos 0 = 4(1) = 4$ $\qquad \therefore A(0, 4)$ [B1]

$B, C \rightarrow x-$ intercepts of $y = 4\cos 2x \quad \rightarrow y = 0$

$0 = 4\cos 2x$

$\cos 2x = 0$

$2x = -\dfrac{\pi}{2}, \dfrac{\pi}{2}$



$x = \left(-\dfrac{\pi}{4} \div 2\right), \left(\dfrac{\pi}{4} \div 2\right) = -\dfrac{\pi}{4}, \dfrac{\pi}{4}$

$\therefore B\left(-\dfrac{\pi}{4}, 0\right)$ [B1]

$\therefore C\left(\dfrac{\pi}{4}, 0\right)$ [B1]

$D, E \rightarrow$ turning points of $y = -2\sin x + 3$

$x-$coordinate of $D = -\dfrac{\pi}{2}$; $y-$coordinate of $D = 2 + 3 = 5$ $\therefore D\left(-\dfrac{\pi}{2}, 5\right)$ [B1]

$x-$coordinate of $E = \dfrac{\pi}{2}$; $y-$coordinate of $E = -2 + 3 = 1$ $\therefore E\left(\dfrac{\pi}{2}, 1\right)$ [B1]

(ii) $4\cos 2x = -2\sin x + 3$ $\qquad (\cos 2x = 1 - 2\sin^2 x)$

$4(1 - 2\sin^2 x) = -2\sin x + 3$ [M1]

$4 - 8\sin^2 x = -2\sin x + 3$

$0 = -2\sin x + 3 - 4 + 8\sin^2 x$

8sin … wn) [A1]

[Turn over

(iii) $y = 4\cos 2x \quad \rightarrow \quad (1)$

$y = -2\sin x + 3 \quad \rightarrow \quad (2)$

(Intersection points)

$(1) = (2)$:

$4\cos 2x = -2\sin x + 3$

$8\sin^2 x - 2\sin x - 1 = 0$ [From (ii)]

$(4\sin x + 1)(2\sin x - 1) = 0$

$4\sin x$	$+1$	$+2\sin x$
$2\sin x$	-1	$-4\sin x$
$8\sin^2 x$	-1	$-2\sin x$

$(4\sin x + 1) = 0$

$4\sin x = -1$

$\sin x = -\frac{1}{4}$

[$\sin x$ is negative → Quadrants 3 and 4]

Basic angle, $\alpha = \sin^{-1}\left(\frac{1}{4}\right) \Rightarrow \alpha = 0.25268$

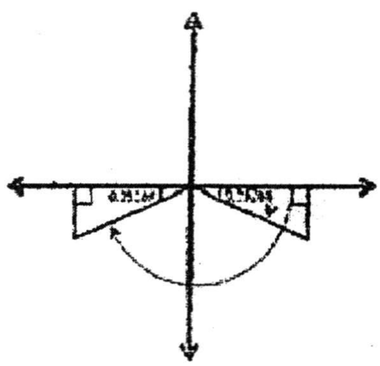
$(2\sin x - 1) = 0$

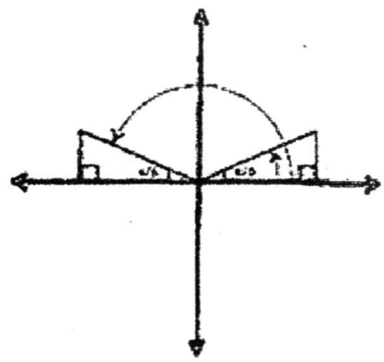
$2\sin x = 1$

$\sin x = \frac{1}{2}$

[$\sin x$ is positive → Quadrants 1 and 2]

Basic angle, $\alpha = \sin^{-1}\left(\frac{1}{2}\right) \Rightarrow \alpha = \frac{\pi}{6}$





$\Rightarrow \quad x = -0.25268\ ,\ -(\pi - 0.25268)$

$\Rightarrow \quad x = -0.253\ ,\ -2.89$ (3sf)

$\Rightarrow \quad x = \frac{\pi}{6}\ ,\ \left(\pi - \frac{\pi}{6}\right)$

$\Rightarrow \quad x = \frac{\pi}{6}\ ,\ \frac{5\pi}{6}$

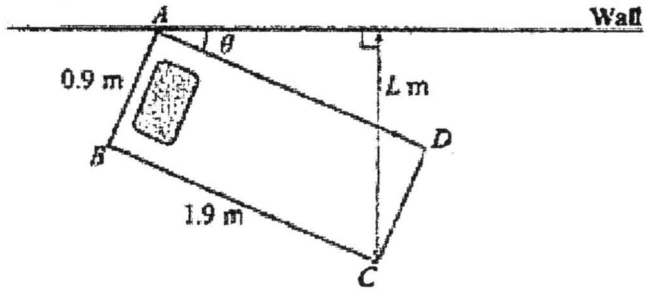
$\therefore x -$ coordinate of $W = -2.89$ **[B1]**

$\therefore x -$ coordinate of $X = -0.253$ **[B1]**

$\therefore x -$ coordinate of $Y = \frac{\pi}{6}$ **[B1]**

$\therefore x -$ coordinate of $Z = \frac{5\pi}{6}$ **[B1]**

11 The diagram shows a rectangular single bed with wheels, $ABCD$, which is hinged to the wall at A. It is given that the dimensions of the bed is 1.9 m by 0.9 m and L m is the perpendicular distance from the wall to C. The bed can be rolled such that the angle between the wall and the side, AD, of the bed is θ and that $0° \le \theta < 90°$.



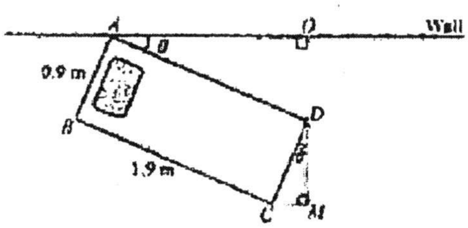
(i) Show that the length, L m, can be expressed as $L = 1.9\sin\theta + 0.9\cos\theta$. [3]

(ii) Express L in the form $R\sin(\theta+\alpha)$ where $R>0$ and α is an acute angle. [3]

(iii) Hence, find the maximum value of L and the corresponding value of θ. [3]

(iv) Find the value of θ when $L = 1.3$ m. [2]

11 (i)



- In ΔOAD, $\angle OAD = \theta$
- $\angle ODA = 180° - 90° - \theta$
  $\angle ODA = 90° - \theta$
- $\angle ADC = 90°$
- $\angle CDM = 180° - \angle ADC - \angle ODA$
  $= 180° - 90° - (90° - \theta)$
  $\angle CDM = \theta$

In ΔODA,

$\sin\theta = \dfrac{OD}{AD}$

$\sin\theta = \dfrac{OD}{1.9}$

$OD = 1.9\sin\theta$ [M1]

In ΔCDM,

$\cos\theta = \dfrac{DM}{DC}$

$\cos\theta = \dfrac{DM}{0.9}$

$DM = 0.9\cos\theta$ [M1]

$L = OD + DM$

$\therefore L = 1.9\sin\theta + 0.9\cos\theta$ [A1]

(Shown)

(ii) The R-Formula:

$(a\sin\theta + b\cos\theta) = R\sin(\theta+\alpha)$, $R = \sqrt{a^2+b^2}$ and $\tan\alpha = \dfrac{b}{a}$

$(1.9\sin\theta + 0.9\cos\theta) \Rightarrow a = 1.9, b = 0.9$

$R = \sqrt{a^2+b^2} = \sqrt{(1.9)^2+(0.9)^2} = \sqrt{4.42}$ [M1]

$\tan\alpha = \dfrac{b}{a} = \dfrac{0.9}{1.9} \Rightarrow \alpha = \tan^{-1}\left(\dfrac{0.9}{1.9}\right) \Rightarrow \alpha = 25.34618°$ [M1]

$\therefore$ $\sqrt{4.42}\sin(\theta + 25.346°)$ [A1]

(iii) From **(ii)**: $L=(1.9\sin\theta+0.9\cos\theta)=\sqrt{4.42}\sin(\theta+25.346°)$

Maximum value of L occurs when:

$\sin(\theta+25.346°)=1$

Hence, Maximum value of $L=\sqrt{4.42}$ (1)

$\therefore L=2.10\text{m}$ **[A1]**

This occurs when $\sin(\theta+25.346°)=1$ **[M1]**

$(\theta+25.346°)=\sin^{-1}1$

$(\theta+25.346°)=90°$

$\theta=90°-25.346°$

$\Rightarrow \theta=64.654°$

$\therefore \theta=64.7°$ (1dp) **[A1]**

(iv) When $L=1.3\text{m}$,

$L=(1.9\sin\theta+0.9\cos\theta)=\sqrt{4.42}\sin(\theta+25.346°)$

$1.3=\sqrt{4.42}\sin(\theta+25.346°)$

$\frac{1.3}{\sqrt{4.42}}=\sin(\theta+25.346°)$

$(\theta+25.346°)=\sin^{-1}\left(\frac{1.3}{\sqrt{4.42}}\right)$ **[M1]**

$\theta=\sin^{-1}\left(\frac{3}{\sqrt{4.42}}\right)-25.346°$

$\theta=38.19551968°-25.34618°$

$\theta=12.84933968°$

$\therefore \theta=12.8°$ (1dp) **[A1]**

12 (i) Prove the identity $\cos^4 x - \sin^4 x + 2\cos^2 x - 1 = 2\cos 2x$ [3]

(ii) Solve the equation $2\cos^4 x - 2\sin^4 x = \sqrt{2}$, for $0 < x < \pi$,

giving your answers in terms of π. [4]

(iii) Given that $3\cos^4 x - 3\sin^4 x = \sqrt{3}\sin 2x$, and without using a calculator,

(a) deduce that $\tan 2x = \sqrt{3}$, [2]

(b) find the possible values of $\tan x$. [3]

---

12 (i) Given: $\cos^4 x - \sin^4 x + 2\cos^2 x - 1 = 2\cos 2x$.

$LHS = \cos^4 x - \sin^4 x + 2\cos^2 x - 1$ ($\cos 2x = 2\cos^2 x - 1$)

$= (\cos^2 x)^2 - (\sin^2 x)^2 + (2\cos^2 x - 1)$ [M1] ($a^2 - b^2 = (a+b)(a-b)$)

$= (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) + (\cos 2x)$ [M1] ($\cos 2x = \cos^2 x - \sin^2 x$)

$= (1)(\cos 2x) + (\cos 2x)$ ($\cos^2 x + \sin^2 x = 1$)

$= 2\cos 2x$ [A1]

$= RHS$

(ii) Solve: $2\cos^4 x - 2\sin^4 x = \sqrt{2}$, for $0 < x < \pi$

$2(\cos^4 x - \sin^4 x) = \sqrt{2}$

$2((\cos^2 x)^2 - (\sin^2 x)^2) = \sqrt{2}$

$(\cos^2 x)^2 - (\sin^2 x)^2 = \dfrac{\sqrt{2}}{2}$ [M1]

$(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) = \dfrac{\sqrt{2}}{\sqrt{2} \times \sqrt{2}}$

$(1)(\cos 2x) = \dfrac{1}{\sqrt{2}}$

$\cos 2x = \dfrac{1}{\sqrt{2}}$ {$\cos 2x$ is positive → Quadrants 1 and 4} [M1]

Basic angle, $\alpha = \cos^{-1}\left(\dfrac{1}{\sqrt{2}}\right) \Rightarrow \alpha = \dfrac{\pi}{4}$

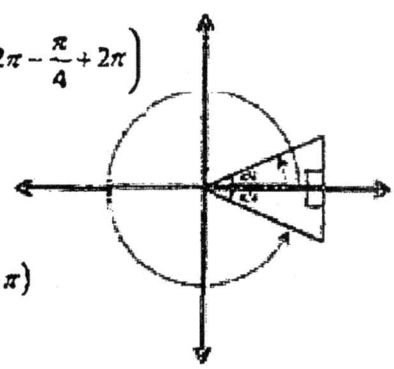
$\Rightarrow \quad 2x = \dfrac{\pi}{4}, \left(2\pi - \dfrac{\pi}{4}\right), \left(\dfrac{\pi}{4} + 2\pi\right), \left(2\pi - \dfrac{\pi}{4} + 2\pi\right)$

$\Rightarrow \quad 2x = \dfrac{\pi}{4}, \dfrac{7\pi}{4}, \dfrac{9\pi}{4}, \dfrac{15\pi}{4}$

$\Rightarrow \quad x = \dfrac{\pi}{8}, \dfrac{7\pi}{8}, \dfrac{9\pi}{8}, \dfrac{15\pi}{8}$

$\therefore \quad x = \dfrac{\pi}{8}, \dfrac{7\pi}{8}$ $(0 < x < \pi)$

[A1] [A1]



(iii) (a) Given:

$$3\cos^4 x - 3\sin^4 x = \sqrt{3}\sin 2x$$

$$3(\cos^4 x - \sin^4 x) = \sqrt{3}\sin 2x$$

$$3((\cos^2 x)^2 - (\sin^2 x)^2) = \sqrt{3}\sin 2x$$

$$3((\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)) = \sqrt{3}\sin 2x$$

$$3((1)(\cos 2x)) = \sqrt{3}\sin 2x$$

$$3\cos 2x = \sqrt{3}\sin 2x$$

$$\frac{3}{\sqrt{3}} = \frac{\sin 2x}{\cos 2x} \qquad \text{[M1]}$$

$$\tan 2x = \frac{\sqrt{3}\sqrt{3}}{\sqrt{3}}$$

$$\therefore \tan 2x = \sqrt{3} \quad \text{(Deduced)} \qquad \text{[A1]}$$

(b) From (iii)(a): $\tan 2x = \sqrt{3} \rightarrow (1)$

And, $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x} \rightarrow (2)$

$\Rightarrow$ (1) = (2): $\sqrt{3} = \dfrac{2\tan x}{1-\tan^2 x}$ [M1]

$$\sqrt{3}(1-\tan^2 x) = 2\tan x$$

$$\sqrt{3} - \sqrt{3}\tan^2 x = 2\tan x$$

$$0 = \sqrt{3}\tan^2 x + 2\tan x - \sqrt{3}$$

$$\sqrt{3}\tan^2 x + 2\tan x - \sqrt{3} = 0$$

$$(\sqrt{3}\tan x - 1)(\tan x + \sqrt{3}) = 0$$

$\sqrt{3}\tan x$	-1	$-\tan x$
$\tan x$	$+\sqrt{3}$	$+3\tan x$
$\sqrt{3}\tan^2 x$	$-\sqrt{3}$	$2\tan x$

$\rightarrow (\sqrt{3}\tan x - 1) = 0$

$\sqrt{3}\tan x = 1$

$\therefore \tan x = \dfrac{1}{\sqrt{3}}$ [A1]

$\rightarrow (\tan x + \sqrt{3}) = 0$

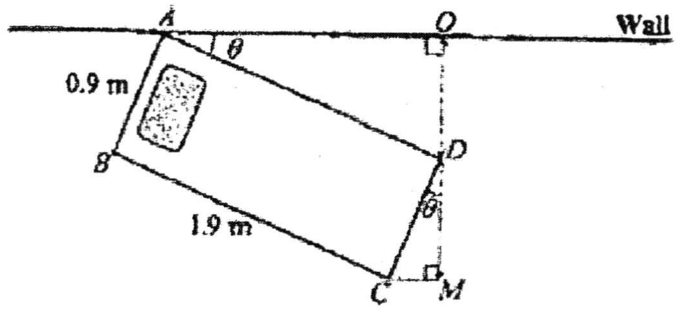
$\therefore \tan x = -\sqrt{3}$ [A1]

- End Of Paper -

GMS(S) 2017 MYE 4E + 5N(A) A Maths Paper 2 [redacted] Answers

1	$\left(-\frac{17}{8},0\right)$; $\left(-2\frac{1}{8},0\right)$; $(-2.125,0)$
2	$\frac{dV}{dt}=162\pi\ cm^3\ s^{-1}=162\pi\ cm^3/s$
3	(i) On graph (ii) Incorrect value: $\frac{1}{v}=0.055 \Rightarrow v=18.3$, Correct value: $\frac{1}{v}=0.061 \Rightarrow v=16.4$ (3sf) $\left(\text{Accept: } \frac{1}{v}=0.061 \pm 0.001 \ ; \ v=16.4 \pm 0.3\right)$ (iii) y – intercept, $c=\frac{1}{f}=0.086$ $\quad$ (Accept: $c=0.086 \pm 0.001$) (3sf) $f=\frac{1}{0.086} \Rightarrow f\approx 11.6279$ cm $\quad$ (Accept: $f=11.6 \pm 0.2$) (3sf) (iv) Comparing gradient, $m=-1$ (from equation) (Graph gentler/steeper than what is required depending on student's answer)
4	(i) Use integration $\quad$ (ii) Show: $b^2-4ac<0$
5	(i) $a=10$, $b=0.22$ $\quad$ (ii) $(x,\lg y)\rightarrow(-0.11,0.11)$
6	(i) $\frac{dy}{dx}=\frac{3x}{\sqrt{2x+1}}$ $\left(\frac{kx}{\sqrt{2x+1}} \ ; \ k=3\right)$ $\quad$ (ii) $\frac{112}{5}$; $22\frac{2}{5}$; 22.4
7	(i) $B(7,-4)$ $\quad$ (ii) Radius = 5 units $(x-3)^2+(y+1)^2=5^2$ OR $x^2+y^2-6x+2y-15=0$ $\quad$ (iii) $y=\frac{1}{2}x-7\frac{1}{2}$
8	(i) Use differentiation (Product rule) $\quad$ (ii) $(0,0)$, $(1,1)$, $(2,0)$ $\quad$ (iii) $\frac{d^2y}{dx^2}=12x^2-24x+8$ $(0,0)$ (Minimum point) $(1,1)$ (Maximum point) $(2,0)$ (Minimum point)
9	(i) $a=-1$ $\quad$ (ii) $f'(x)=\frac{-4}{(x+1)^2}$ Show: $f'(x)<0$ $\quad$ (iii) $y=-x-3$ $\quad$ (iv) Show: Area = 4.5 units^2
10	(i) $A(0,4)$; $B\left(-\frac{\pi}{4},0\right)$; $C\left(\frac{\pi}{4},0\right)$ $D\left(-\frac{\pi}{2},5\right)$; $E\left(\frac{\pi}{2},1\right)$ $\quad$ (ii) Use: $(\cos 2x=1-2\sin^2 x)$ $\quad$ (iii) x – coordinate of: $x_{R'}=-2.89$; $x_{S'}=-0.253$ $x_1=\frac{\pi}{6}$; $x_2=\frac{5\pi}{6}$
11	(i) Show: $L=OD+DM$ $\quad$ (ii) $\sqrt{4.42}\sin(\theta+25.346°)$ $\quad$ (iii) Max L = 2.10 m, $\theta=64.7°$ (1dp) $\quad$ (iv) L = 1.3 m, $\theta=12.8°$ (1dp)
12	(ii) $x=\frac{\pi}{}$ $\frac{7\pi}{}$ $\quad$ (iii)(b) $\tan x=\frac{1}{\sqrt{3}}$; $\tan x=-\sqrt{3}$

[Turn over



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