Geylang Methodist School (Secondary)
Preliminary Examination 2019

Candidate Name			
Class		Index No	

ADDITIONAL MATHEMATICS **4047 / 01**

Paper 1 **4 Express / 5 Normal (Academic)**

2 hours

Setter : Mrs Goh Heng Mei 30 August 2019

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers in the space below the question.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use
80

This document consists of 21 printed pages including the cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

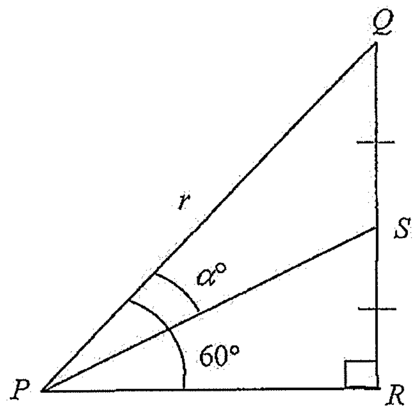
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Given that $\sqrt{27^x} = \dfrac{3^{1-x}}{9}$, find the value of $\sqrt{27^x}$. [4]

2 The diagram shows a triangle PQR in which $PQ = r$, angle $QPR = 60°$ and angle $QRP = 90°$. S is the midpoint of QR and angle $QPS = \alpha°$.



Show that

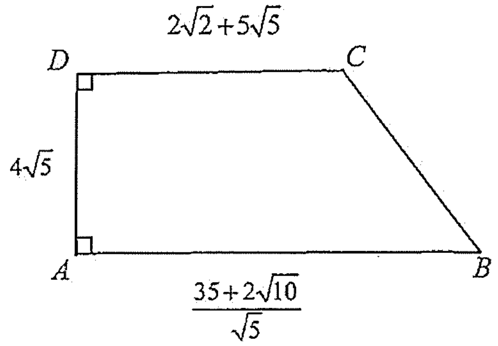
(i) the length of PS is exactly $\frac{\sqrt{7}}{4}r$, [4]

(ii) $\alpha° = 60° - \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$. [3]

3 Express $\dfrac{7x^2-12x+16}{x\left(x^2+4\right)}$ in partial fractions. [5]

4 In the diagram below, angle BAD = angle $ADC = 90°$, $AD = 4\sqrt{5}$ cm,

$AB = \dfrac{35+2\sqrt{10}}{\sqrt{5}}$ cm and $DC = 2\sqrt{2}+5\sqrt{5}$ cm.



Without using a calculator, find the length of BC. [5]

5 The graphs of $y = x^n$ and $y^2 = kx$, where k and n are integers, intersect at the point $\left(\frac{1}{2}, 2\right)$.

(i) Find the value of k and of n. [2]

(ii) Sketch these graphs on the same diagram. [2]

6 **(a)** Given that $4 \lg \left(x\sqrt{y}\right) = 1.5 + \lg x - \lg y$, where x and y are both positive, express, in its simplest form, y in terms of x. [4]

(b) Solve $\log_4 2 + \log_9 (2x+5) = \log_8 64$. [3]

7 A liquid is poured onto a flat surface. It forms a circular patch which grows at a steady rate of 5 cm^2 / s. Find, in terms of π ,

(i) the radius of the patch 30 seconds after pouring has begun, [3]

(ii) the rate of increase of the radius at this instant. [3]

8 **(i)** Prove that $\dfrac{1}{\tan A + \cot A} = \dfrac{1}{2}\sin 2A$. [3]

Hence,

(ii) state the period and amplitude of the graph $y = \dfrac{1}{\tan A + \cot A}$, [2]

(iii) sketch the graph of $y = \dfrac{1}{\tan A + \cot A}$ for $0 \le A \le 2\pi$ radians. [2]

9 **Solutions to this question by accurate drawing will not be accepted.**

The coordinates of the points A and B are $(3,5)$ and $(7,13)$ respectively.

Find

(i) the equation of the line parallel to AB and passing through the point $(-2,4)$, giving your answer in the form $ax+by+c=0$, [3]

(ii) the equation of the perpendicular bisector of AB. [3]

A point C is at a distance of $\sqrt{15}$ from the midpoint of AB and lies on the perpendicular bisector of AB.

(iii) Find the area of triangle ABC, expressing your answer in the form $k\sqrt{3}$, where k is an integer. [3]

10 A factory makes closed right circular cylinders each with a volume of 32π cm^3. The material for the top and bottom of the cylinder costs 2 cents per cm^2 while that of the sides of the cylinder costs 1 cent per cm^2. If the radius of the circular top is r cm and the height of cylinder is h cm,

(i) express h in terms of r. [2]

To be economically competitive, the manufacturing cost must be as low as possible.

(ii) Find the dimensions of each cylinder so that the cost of manufacturing is a minimum and show that the cost is a minimum. [5]

(iii) Find the minimum cost of manufacturing each cylinder, giving your answer in dollars. [2]

11 It is given that f (x) is defined for $x > \frac{2}{3}$, and $f'(x) = \frac{6x+1}{3x-2}$.

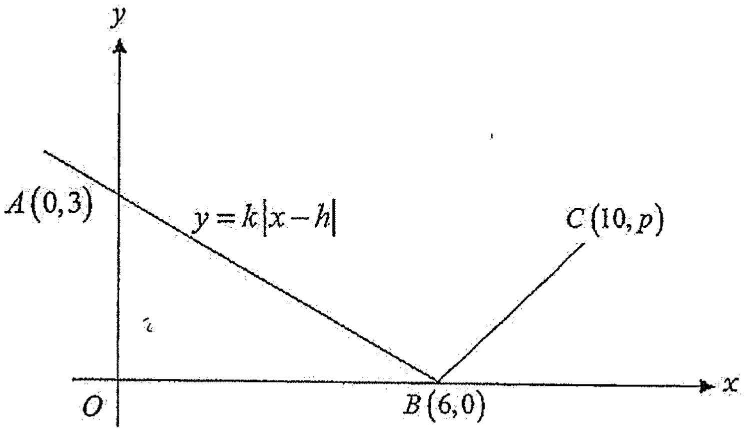
(i) Express $f'(x)$ in the form $p + \frac{q}{3x-2}$, where p and q are integers. [2]

(ii) Determine whether f (x) is an increasing or a decreasing function. [2]

(iii) Given that $f(1) = 6$, obtain an expression for $f(x)$. [3]

(iv) Find the equation of the tangent at the point (1, 6). [3]

12



The diagram shows the graph of $y = k|x-h|$, where k and h are positive constants. The graph crosses the y-axis at $A(0,3)$, meets the x-axis at $B(6,0)$ and passes through point $C(10, p)$.

(i) State the value of h. [1]

(ii) Find the value of k and of p. [3]

(iii) With the values of k and h from **(i)** and **(ii)** above, find the range of values of m such that the equation $k|x-h| = mx+2$ has two distinct solutions. [3]

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Geylang Methodist School (Secondary)
Preliminary Examination 2019

Candidate Name			
Class		Index Number	

ADDITIONAL MATHEMATICS **4047/02**

Paper 2

Sec 4 Express
Sec 5 Normal (Academic)

Additional Materials : Graph Paper (1 sheet) **2 hours 30 minutes**

Setter : **Mr Johney Joseph** **29 Aug 2019**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the space provided at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use
100

This document consists of **17** printed pages and **1** blank page.

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Mathematical Formulae

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Quadratic Equation

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

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$$\Delta = \frac{1}{2}ab\sin C$$

1 The roots of the quadratic equation $3x^2 - 2x + 1 = 0$ are α and β.

(i) Show that $\alpha^2 + \beta^2 = -\frac{2}{9}$. [3]

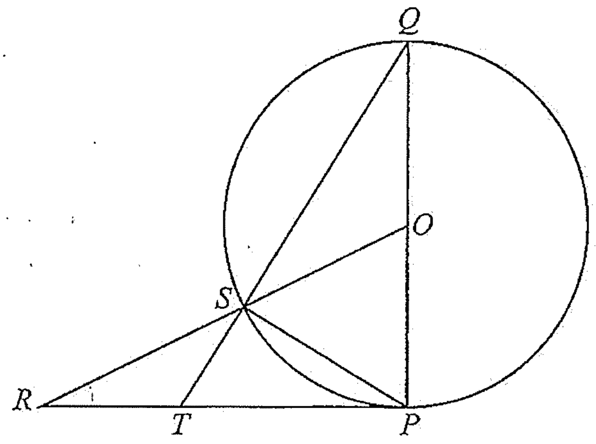
(ii) Find a quadratic equation whose roots are $3\alpha^2 + 2\beta$ and $3\beta^2 + 2\alpha$. [5]

2 **(i)** Write down the general term in the binomial expansion of $\left(px+\frac{q}{x}\right)^n$. [1]

(ii) Given that the fourth term is independent of x, find the value of n. [2]

(iii) It is given further that the fourth term is equal to 160, both p and q are positive and $p-q=1$. Calculate the value of p and of q. [5]

3



In the diagram above, PQ is the diameter of a circle with centre O. PR is a tangent to the circle at P and $PQ = PR$. The line OR cuts the circle at S and QS produced meets PR at T.

(i) Show that triangle PRS is similar to triangle SRT. [3]

(ii) Show that triangle PST is similar to triangle QPT. [3]

(iii) Hence deduce that $RS = PT$. [3]

4 The table shows experimental values of two variables x and y.

x	0.2	0.6	1.0	1.4
y	0.5	1.4	1.5	4.0

It is known that x and y are related by the equation $y = ax\sqrt{x} + b\sqrt{x}$, where a and b are constants.

(i) Explain how a straight line graph can be obtained using the above data. [2]

(ii) Draw this graph for the given data and use it to estimate the value of a and of b. (Use the graph paper provided) [5]

(iii) Use your graph to find the value of y when $x = 1.2$. [2]

5 **(i)** Differentiate $\tan^3 x$ with respect to x. [2]

(ii) Show that $\tan^4 x = \tan^2 x \sec^2 x - \sec^2 x + 1$. [3]

(iii) Hence evaluate $\int_0^{\frac{\pi}{4}} \tan^4 x \, dx$.

6 The equation of a circle is $x^2 + y^2 - 4x + 6y = 12$.

(i) Find the coordinates of the centre of the circle and the radius of the circle. [3]

(ii) Explain why (2, 2) is the highest point on the circle. [2]

$P(-2, k)$ is a point on the circle where $k < 0$.

(iii) Show that $k = -6$. [2]

(iv) Find the equation of the tangent to the circle at P. [3]

7 The equation of a polynomial is given by $f(x) = x^3 + (k-4)x^2 + (k-9)x - 4$.

(i) Show that $f(x)$ is divisible by $x + 1$ for all values of k. [2]

(ii) Find the value of k for which $f(x)$ has a remainder of 12 when divided by $x - 2$. [2]

(iii) With the value of k found in **(ii)**, factorise $f(x) - 12$. [2]

(iv) Explain why $f(x) - 12 = 0$ has only one real root. [3]

8 The equation of a curve is $y = x^2 + (3k - 1)x + 2k + 10$.

(i) In the case when $k = 3$, show that the x-axis is a tangent to the curve. [3]

(ii) Find the range of values of k for which y is always positive. [4]

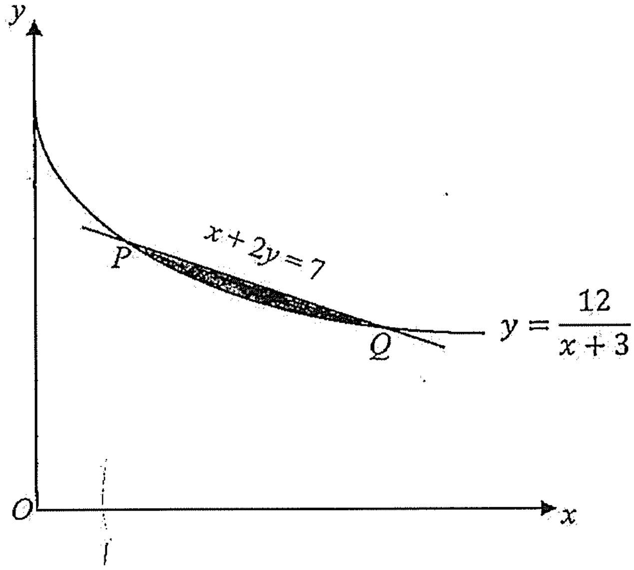
9 A particle moves in a straight line such that, at time t seconds after leaving a fixed point O, its velocity v m/s is given by $v = \frac{27}{(2t+1)^2} - 3$.

(i) Find the initial acceleration. [2]

(ii) Find the displacement of the particle from O when $t = 4$ seconds. [4]

(iii) Write a statement about the position of the particle when $t = 4$ seconds. [1]

10



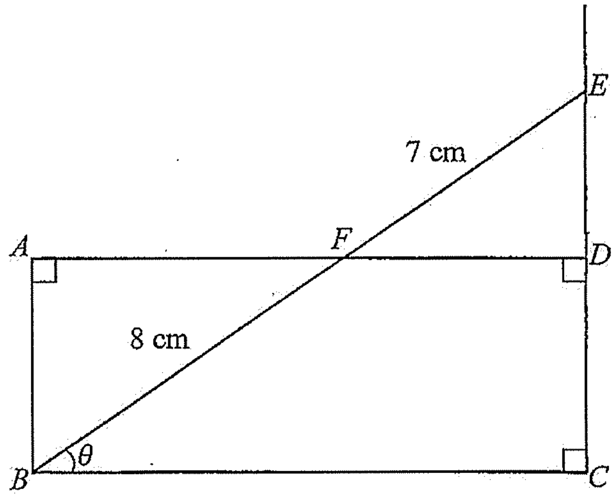
The diagram shows part of the curve $y = \dfrac{12}{x+3}$ and the line $x + 2y = 7$ intersecting at points P and Q. Find

(i) the coordinates of P and of Q, [4]

(ii) the equation of the normal to the curve at point P, [3]

(iii) the area of the shaded portion. [5]

11



In the diagram above, the line through B meets AD at F and CD produced at E. The angle FBC is θ degrees. The lengths of BF and FE are 8 cm and 7 cm respectively.

(i) Show that the perimeter, P cm of the rectangle $ABCD$ can be expressed as $P = a \sin\theta + b \cos\theta$ where the value of a and of b are to be found. [3]

(ii) Find the maximum value of P and the corresponding value of θ. [5]

(iii) Find the value of θ when $P = 20$ cm. [3]

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Geylang Methodist Sec Sch : 2019 AM I

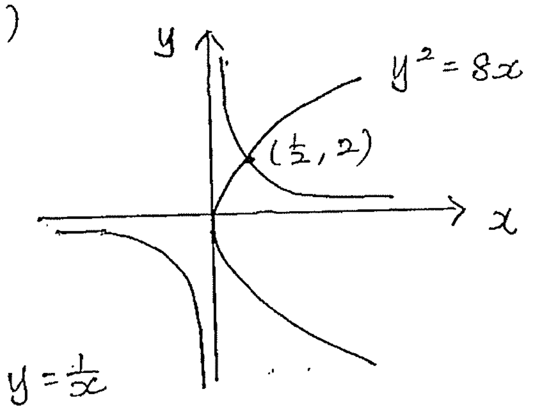
1) $x = -\frac{2}{5}$; 0, 517

3) $\frac{4}{x} + \frac{3x-12}{x^2+4}$

4) 10 cm

5i) $k = 8$; $n = -1$

(ii)

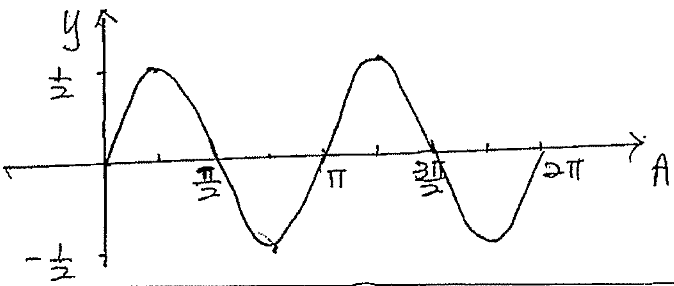


6a) $y = \frac{\sqrt{10}}{x}$

(b) $x = 11$

7i) $r = 5\sqrt{\frac{6}{\pi}}$ cm (ii) $\frac{1}{2\sqrt{6\pi}}$ cm²/s

8(ii) Period = π ; amplitude = $\frac{1}{2}$



9i) $2x - y + 8 = 0$

ii) $y = -\frac{1}{2}x + \frac{23}{2}$

iii) $10\sqrt{3}$ units²

10i) $h = \frac{32}{r^2}$

(ii) $r = 2$ cm ; $h = 2$ cm

(iii) $C \simeq \$151$

11(i) $f'(x) = 2 + \frac{5}{3x-2}$

(ii) Increasing function

(iii) $f(x) = 2x + \frac{5}{3}\ln(3x-2) + 4$

(iv) $y = 7x - 1$

12i) $h = 6$

ii) $k = \frac{1}{2}$, $p = 2$

iii) $-\frac{1}{3} < m < \frac{1}{2}$

Geylang Methodist Sec Sch : 2019 AM2

1 ii) $x^2 - \frac{2}{3}x + \frac{1}{9} = 0$

2 i) ${}^nC_r (p)^{n-r} q^r x^{n-2r}$

ii) $n = 6$ (iii) $p = 2$; $q = 1$

3 i) ∡PRS = ∡SRT (common ∡)
∡SPR = ∡PQS (alt seg thm)
∡PQS = ∡OSQ (isos △)
∡OSQ = ∡TSR (vert. opp ∡s)
∴ ∡SPR = ∡TSR
∴ △PRS and △SRT are similar

ii) ∡PTS = ∡QTP (common ∡)
∡TPS = ∡TQP (alt seg. thm)
∴ △PST and △QPT are similar

(iii) PQ = PR

$\frac{PQ}{PS} = \frac{PT}{ST}$; $\frac{PR}{SR} = \frac{PS}{ST}$

$PQ = \frac{PS \times PT}{ST}$ $PR = \frac{PS \times SR}{ST}$

$\therefore \frac{PS \times PT}{ST} = \frac{PS \times SR}{ST}$

∴ PT = SR

4 i) Draw $\frac{y}{\sqrt{x}}$ against x
with gradient = a
vertical intercept = b

$\frac{y}{\sqrt{x}} = ax + b$

ii) $a = 2.27$; $b = 0$

iii) $y = 2.95$

5 i) $3\tan^2 x \sec^2 x$

ii) 0.119

6 i) (2, −3) ; radius = 5 units

(ii) It is 5 units above the centre of the circle

(iv) $y = -\frac{4}{3}x - \frac{26}{3}$

7 i) $f(-1) = 0$

(ii) $k = 7$ (iii) $(x-2)(x^2+5x+8)$

(iv) $x = 2$; $x^2 + 5x + 8 = 0$
$b^2 - 4ac = -7 < 0$
No real roots
∴ $f(x) - 12$ has only 1 root

8 i) $\frac{dy}{dx} = 2x + 8$
when $x = -4$, $\frac{dy}{dx} = 0$

(ii) $-\frac{13}{9} < k < 3$

9 i) $a = -108\ m/s^2$

ii) Displacement = 0

(iii) The particle is at O

10 i) P(1, 3) ; Q(3, 2)

(ii) $y = \frac{4}{3}x + \frac{5}{3}$

(iii) 0.134 units²

11 i) $P = 16\sin\theta + 30\cos\theta$

ii) max P = 34 ; $\theta = 28.1°$

(iii) $\theta = 82.0°$ #