Geylang Methodist School (Secondary)
Preliminary Examination 2024

Candidate Name			
Class		**Index Number**	

ADDITIONAL MATHEMATICS
Paper 1

4049 / 01

4 Express/5 Normal(A)

Candidates answer on the Question Paper.

No Additional Materials are required.

Setter: **Mr Johney Joseph**

2 hours 15 minutes

12 August 2024

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets **[ ]** at the end of each question or part question.
The total score for this paper is **90.**

For Examiner's Use
90

This document consists of **19** printed pages and **1** blank page.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ \ldots\ (n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}ab\sin C$$

1 Find the range of values of p for which the curve $y = px^2 + 2(p+2)x + p + 7$ has no x-intercepts. [4]

2 Given that $y = 3e^{2x} + 2e^{-x}$, and that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} + y = Ae^{2x} + Be^{-x}$, find the values of each of the constants A and B. [5]

3 The equation of a curve is $y = 4 - 3\cos 2x$.

(a) State the period and amplitude of y. [2]

(b) Sketch the graph of $y = 4 - 3\cos 2x$ for $0 \le x \le 2\pi$. [3]

4 For a particular curve $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=3x-2$.

The tangent to the curve at the point $A(4,-40)$ is parallel to the line $3x-y=2$.

Find the equation of the curve. [6]

5 The triangle ABC is such that its area is $\frac{16+7\sqrt{10}}{2}$ cm^2, the length of AB is $\left(3\sqrt{2}+\sqrt{5}\right)$ cm, and AB is perpendicular to BC.

(a) Find the length, in cm, of BC in the form $\left(a\sqrt{2}+b\sqrt{5}\right)$, where a and b are integers. [3]

(b) Find an expression, in cm^2, for AC^2 in the form $c+d\sqrt{10}$, where c and d are integers. [3]

6 The function f is defined by $\mathrm{f}(x) = 3\sin x - 4\cos 2x,\ 0 < x < \frac{\pi}{2}$.

(a) Explain with working, whether f is an increasing or a decreasing function. [4]

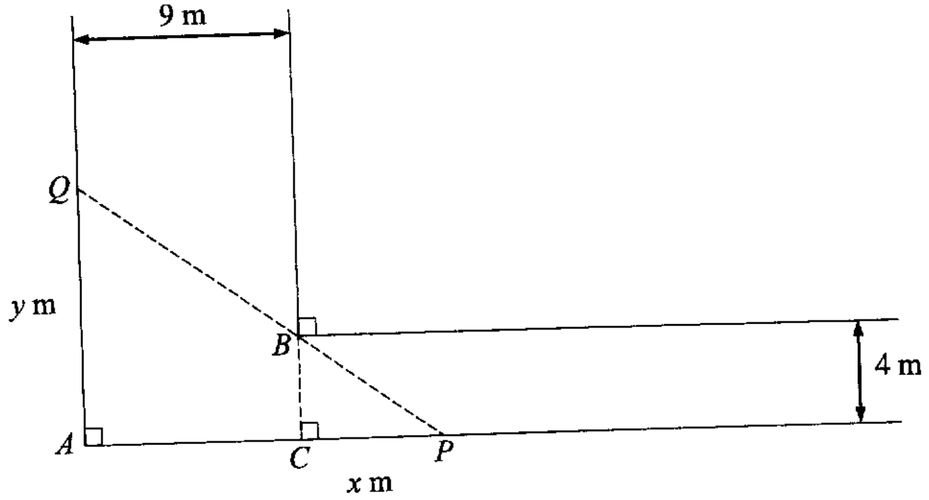
A point P moves along the curve $y = \mathrm{f}(x)$ in such a way that the x-coordinate of P is increasing at the rate of 2 units per second.

(b) Show that the y-coordinate of P increases at the rate of $11\sqrt{3}$ units per second when $x = \frac{\pi}{6}$. [3]

7 **(a)** By considering the general term in the expansion of $\left(\frac{2}{x^3}-x^2\right)^{10}$, explain why there is no term in x^6. [3]

(b) Find the coefficient of x^6 in the expansion of $\left(\frac{2}{x^3}-x^2\right)^{10}\left(3-\frac{x^6}{8}\right)$. [4]

8

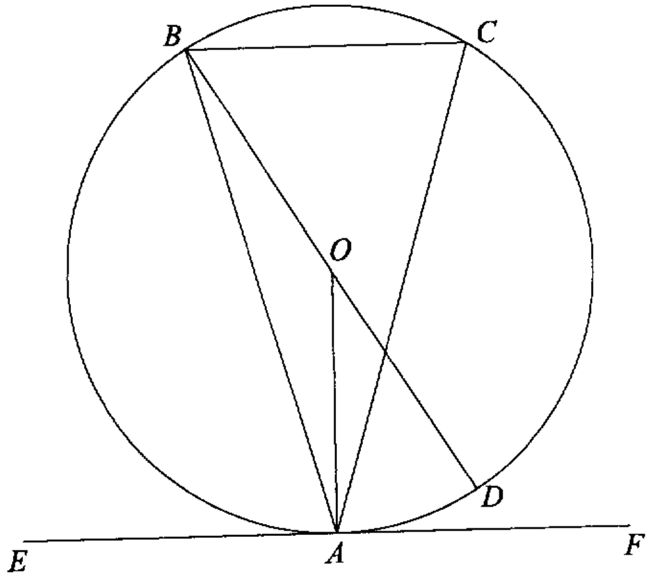


The diagram shows the junction of two corridors of width 9 m and 4 m which are at right angles. *P* and *Q* are variable points and *PBQ* is a straight line.

(a) Given that the length of *CP* is *x* m and the length of *AQ* is *y* m, show that $y = 4 + \dfrac{36}{x}$. [3]

(b) Find the least possible area of triangle APQ. [4]

9



In the diagram, *A*, *B*, *C* and *D* lie on the circumference of a circle with centre *O* such that *AB* = *AC* and *EAF* is a tangent to the circle at *A*.

(a) Show that angle *BCA* = angle *CAF*. [3]

(b) Show that *OA* bisects angle *BAC*. [5]

10 **(a)** Solve the equation $\log_3(x-8)=2-\log_3 x$. [4]

(b) Given $\log_b a=c$, show that $\log_{\frac{1}{b}} a=-c$. [4]

11 The equation of a curve is $y=16-ax-x^2$, where a is positive constant.

(a) Given that y can be expressed in the form $25-(b+x)^2$, where b is a positive constant, find the values of a and b. [4]

(b) State the maximum value of y and the corresponding value of x. [2]

(c) Find the range of values of x when y is positive. [3]

12 **(a)** Prove the identity $\sin\left(\frac{\pi}{3}+x\right)-\sin\left(\frac{\pi}{3}-x\right)=\sin x$. [4]

(b) Hence solve the equation $\sin\left(\frac{\pi}{3}+2x\right)-\sin\left(\frac{\pi}{3}-2x\right)-2=4\sin 2x$ for $0 \le x \le \pi$. [5]

13 The table shows the experimental values of two variables x and y.

x	0.5	1.0	1.5	2.0	2.5	3
y	6.2	4.7	3.7	2.9	2.2	1.7

It is known that x and y are connected by the equation $y = ab^x$, where a and b are constants.

(a) Express the equation in a form suitable to draw a straight line graph. [1]

(b) On the grid on **page 19**, draw the straight line graph to represent the above data. [3]

(c) Use your graph to estimate the values of a and b. [3]

(d) Use your graph to find the value of y when $x = 0.8$. [2]

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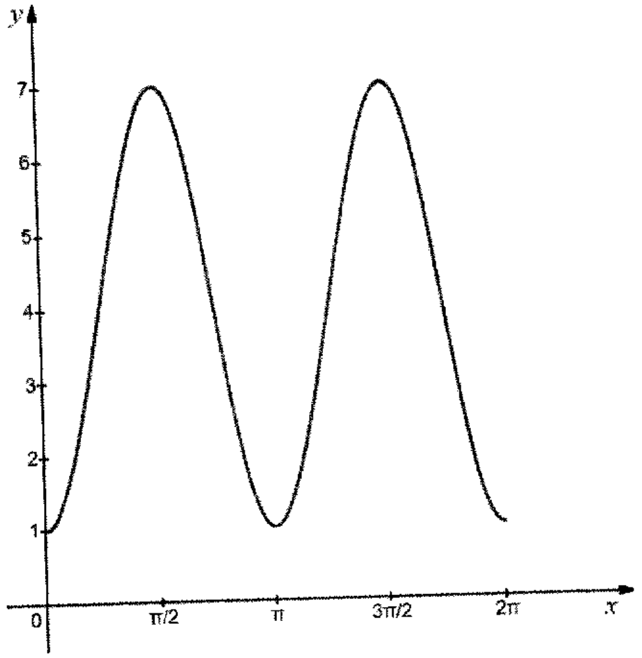
Marking Scheme AM P1 (4049/01)- Prelim 2024

Qn	Answer	Marks	Partial Marks	Guidance
1	$b^2-4ac<0$ $[2(p+2)]^2-4p(p+7)<0$ $4(p^2+4p+4)-4p^2-28p<0$ $4p^2+16p+16-4p^2-28p<0$ $-12p+16<0$ $12p-16>0$ $p>\frac{4}{3}$			
2	$y=3e^{2x}+2e^{-x}$ $\frac{dy}{dx}=6e^{2x}-2e^{-x}$ $\frac{d^2y}{dx^2}=12e^{2x}+2e^{-x}$ $\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=12e^{2x}+2e^{-x}+6e^{2x}-2e^{-x}+3e^{2x}+2e^{-x}$ $=21e^{2x}+2e^{-x}$ $A=21$ $B=2$			
3(a)	Period $=180^\circ$ $(or\ \pi)$ Amplitude $=3$			

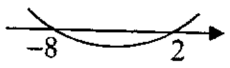
3(b)				
4	$\frac{d^2y}{dx^2}=3x-2$ $\frac{dy}{dx}=\int(3x-2)dx$ $=\frac{3x^2}{2}-2x+c$ $c=-13$ $\frac{dy}{dx}=\frac{3x^2}{2}-2x-13$ $y=\int\frac{3x^2}{2}-2x-13\ dx$ $=\frac{x^3}{2}-x^2-13x+d$ $-40=\frac{(4)^3}{2}-(4)^2-13(4)+d$ $d=-4$ $y=\frac{x^3}{2}-x^2-13x-4$			

5(a)	$\frac{1}{2}(3\sqrt{2}+\sqrt{5})BC=\frac{16+7\sqrt{10}}{2}$ $BC=\frac{16+7\sqrt{10}}{3\sqrt{2}+\sqrt{5}}$ $=\frac{\left(16+7\sqrt{10}\right)\left(3\sqrt{2}-\sqrt{5}\right)}{\left(3\sqrt{2}+\sqrt{5}\right)\left(3\sqrt{2}-\sqrt{5}\right)}$ $=\frac{48\sqrt{2}-16\sqrt{5}+21\sqrt{20}-7\sqrt{50}}{13}$ $=\frac{48\sqrt{2}-16\sqrt{5}+42\sqrt{5}-35\sqrt{2}}{13}$ $=\frac{13\sqrt{2}+26\sqrt{5}}{13}$ $=\sqrt{2}+2\sqrt{5}$			
5(b)	$AC^2=\left(3\sqrt{2}+\sqrt{5}\right)^2+\left(\sqrt{2}+2\sqrt{5}\right)^2$ $=18+6\sqrt{10}+5+2+4\sqrt{10}+20$ $=45+10\sqrt{10}$			
6(a)	$f'(x)=3\cos x+8\sin 2x$ $\cos x>0$ and $\sin 2x>0$ for $0<x<\frac{\pi}{2}$ $\therefore 3\cos x+8\sin 2x>0$ for $0<x<\frac{\pi}{2}$ $f'(x)>0$ for $0<x<\frac{\pi}{2}$ Hence f is an increasing function			
6(b)	$\frac{dy}{dx}=3\cos\frac{\pi}{6}+8\sin\left(2\times\frac{\pi}{6}\right)$ $=3\times\frac{\sqrt{3}}{2}+8\times\frac{\sqrt{3}}{2}$ $=11\frac{\sqrt{3}}{2}$ $\frac{dy}{dt}=\frac{11\sqrt{3}}{2}\times 2$ $=11\sqrt{3}$			

7(a)	General term $=\begin{pmatrix}10\\r\end{pmatrix}\left(\frac{2}{x^3}\right)^{10-r}\left(-x^2\right)^r$ $=\begin{pmatrix}10\\r\end{pmatrix}2^{10-r}\left(-1\right)^r x^{5r-30}$ when $5r-30=6$ $r=\frac{36}{5}$ Hence there is no term in x^6			
7(b)	$5r-30=0$ $r=6$ $\begin{pmatrix}10\\6\end{pmatrix}2^{10-6}\left(-1\right)^6=3360$ $\left(....+3360+...\right)\left(3-\frac{x^6}{8}\right)$ Coefficient of $x^6=3360\times\frac{-1}{8}$ $=-420$			

8(a)	$\frac{4}{y}=\frac{x}{x+9}$ (similar triangles) $xy=4x+36$ $y=4+\frac{36}{x}$			
8(b)	$A=\frac{1}{2}(x+9)y$ $=\frac{1}{2}(x+9)\left(4+\frac{36}{x}\right)$ $=2x+36+\frac{162}{x}$ $\frac{dA}{dx}=2-\frac{162}{x^2}$ when $\frac{dA}{dx}=0,\quad 2-\frac{162}{x^2}=0$ $2=\frac{162}{x^2}$ $x^2=81$ $x=9$ Minimum Area $=2\times9+36+\frac{162}{9}$ $=72\text{ m}^2$			
9(a)	$\angle BCA=\angle ABC$ (given $AB=AC$) $\angle ABC=\angle CAF$ (Alternate segment theorem) $\therefore \angle BCA=\angle CAF$			
9(b)	$\angle OAE=\angle OAF=90^\circ$ (Radius perpendicular to tangent) $\angle BAE=\angle BCA$ (Alternate segment theorem) $\angle BAE=\angle CAF$ (Using part a) $\angle OAB=\angle OAC$ (Both 90° – Equal angles) Hence OA bisects angle BAC			
10	$\log_3(x-8)+\log_3 x=2$			

(a)	$\log_3 x(x-8)=2$ $x(x-8)=3^2$ $x^2-8x-9=0$ $(x-9)(x+1)=0$ $x=9 \quad or \quad x=-1$ (NA) $\therefore x=9$			
10 (b)	$\log_{\frac{1}{b}} a=\dfrac{1}{\log_a\left(\dfrac{1}{b}\right)}$ $=\dfrac{1}{\log_a 1-\log_a b}$ $=\dfrac{1}{-\log_a b}$ $=\dfrac{-1}{\left(\dfrac{1}{\log_b a}\right)}$ $=\dfrac{-1}{\left(\dfrac{1}{c}\right)}$ $=-c$			
11 (a)	$16-ax-x^2=16-\left(x^2+ax\right)$			

	$=16-\left[\left(x+\frac{a}{2}\right)^2-\frac{a^2}{4}\right]$ $=16+\frac{a^2}{4}-\left(\frac{a}{2}+x\right)^2$ $16+\frac{a^2}{4}=25$ $\frac{a^2}{4}=9$ $a^2=36$ $a=6$ $b=\frac{a}{2}$ $=3$ **Alternate solution** $16-ax-x^2=25-\left(b^2+2bx+x^2\right)$ $16-ax-x^2=25-b^2-2bx-x^2$ **B2** $16=25-b^2$ $b^2=9$ $b=3$			
11 (b)	$y=25-(3+x)^2$ Max value of $y=25$ Corresponding value of $x=-3$			
11 (c)	$y>0$ $16-6x-x^2>0$ $x^2+6x-16<0$ $(x+8)(x-2)<0$  $-8<x<2$			
12 (a)	$\sin\left(\frac{\pi}{3}+x\right)-\sin\left(\frac{\pi}{3}-x\right)$			

<table>
<tr><td></td><td>$= \sin\frac{\pi}{3}\cos x + \cos\frac{\pi}{3}\sin x - \left(\sin\frac{\pi}{3}\cos x - \cos\frac{\pi}{3}\sin x\right)$
$= \sin\frac{\pi}{3}\cos x + \cos\frac{\pi}{3}\sin x - \sin\frac{\pi}{3}\cos x + \cos\frac{\pi}{3}\sin x$
$= 2\cos\frac{\pi}{3}\sin x$
$= 2\times\frac{1}{2}\times\sin x$
$= \sin x$</td><td></td><td></td><td></td></tr>
<tr><td>12
(b)</td><td>$\sin\left(\frac{\pi}{3}+2x\right) - \sin\left(\frac{\pi}{3}-2x\right) = \sin 2x$
$\sin 2x - 2 = 4\sin 2x$
$3\sin 2x = -2$
$\sin 2x = -\frac{2}{3}$
Basic angle $= 0.72973$
$2x = \pi + 0.72973,\ \ 2\pi - 0.72973$
$x = 1.94,\ \ 2.78$</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>13
(a)</td><td>$\lg y = (\lg b)x + \lg a$</td><td></td><td></td><td></td></tr>
<tr><td>13
(b)</td><td colspan="3"><table><tr><td>lg y</td><td>0.792</td><td>0.672</td><td>0.568</td><td>0.462</td><td>0.342</td><td>0.230</td></tr></table>
Straight line graph</td><td></td></tr>
<tr><td>13
(c)</td><td>$\lg a \approx 0.9$
$a \approx 7.94$

$\lg b \approx \frac{0.3-0.5}{2.7-1.8} \approx -0.222$
$b = 0.599$</td><td></td><td></td><td></td></tr>
<tr><td>13
(d)</td><td>when $x = 0.8$
$\lg y = 0.72$
$y = 5.23$</td><td></td><td></td><td></td></tr>
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