NCHS

PAPER-1

1 Find the range of values of the constant m for which the curve $y = (m - 6)x^2 - 8x + m$ lies completely above the x-axis. [4]

2 Show that the line $x + y = m$ will intersect the curve $x^2 + 2y^2 = 2x + 3$ if $m^2 \leq 2m + 5$. [4]

3 It is given that $f(x) = (b - 3x)e^{2-3x}$. Find the value of the constant b if f(x) is a decreasing function when $x < \frac{4}{3}$. [4]

4 Prove that $\frac{2-\operatorname{cosec}^2\theta}{\operatorname{cosec}^2\theta+2\cot\theta}=\frac{1-\cot\theta}{1+\cot\theta}$. [4]

5 Express $\frac{4x^2+5x-32}{(x+2)(x^2-9x-22)}$ in partial fractions. [5]

6 (a) Show that $3\cos x = 2\operatorname{cosec} x \cot x$ can be written as $\cos x\,(3\sin^2 x - 2) = 0$. [3]

(b) Hence, solve the equation $3\cos(0.6y - 1.4) = 2\operatorname{cosec}(0.6y - 1.4)\cot(0.6y - 1.4)$ for values of y between -3 and 4. [5]

7 (a) Given that $y = \frac{1+\sin x}{\cos x}$, find $\frac{dy}{dx}$. [2]

(b) Hence, without using a calculator, find the value of each of the constants p and q for which $\int_0^{\frac{\pi}{3}} \frac{3 + 3\sin x - 10\cos^3 x}{5\cos^2 x}\, dx = p + q\sqrt{3}$. [6]

8 The height of Jeremiah above the surface of the water, h metres, can be modelled by the equation $h = -4.9t^2 + 8t + 5$, where t is the time in seconds after he leaves the diving board.

(a) State the height of the diving board above the surface of the water. [1]

(b) Express h in the form $k - a(x - b)^2$, where k, a and b are constants to be determined. [3]

(c) State the greatest height reached by Jeremiah and the corresponding time when the greatest height occurs. [2]

(d) Using your answer obtained in (b), calculate the duration which Jeremiah stay in the air. [2]

9 Solve the simultaneous equations.

$\frac{5^p}{25} = 125^q$

$\log_3 7 = 1 + \log_3(11q - 2p)$ [5]

10 The equation of a curve is $y = x^3 \ln x$.

(a) Find the exact coordinates of the stationary point(s) of the curve. [4]

(b) Determine the nature of the stationary point(s) of the curve. [2]

A particle moves along the curve $y = x^3 \ln x$. At point M, the x-coordinate of the particle is increasing at a rate of 0.06 units/s and the y-coordinate is increasing at a rate of $0.3x^2$ units/s.

(c) Find the exact coordinate of M. [4]

11 A circle, C_1, has equation $x^2 + y^2 + kx - 6y = h$, where k and h are constants.

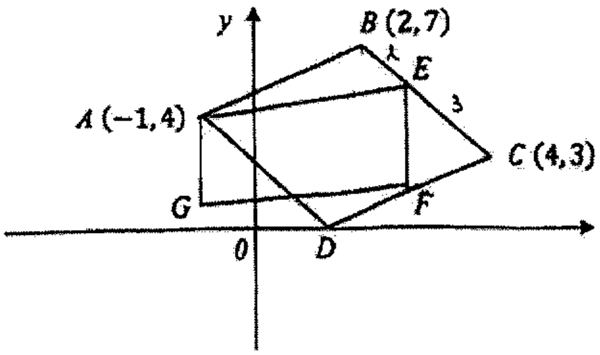
(a) Given that the radius C_1 of is 7 units and the coordinates of the centre is $(-2, m)$, find the values of k, m and h. [4]

(b) Another circle, C_2, has diameter PQ. The point P is $(3, n)$ and Q is $(-5, 5)$. The equation PQ is $4y + 3x = 5$.

(i) Find the equation of C_2. [4]

(ii) Explain, with appropriate working, why the point $S\ (4, 5)$ only lies inside the circle C_1 but not C_2. [2]

12 The diagram shows a parallelogram $ABCD$ in which D lies on the x-axis. Point A is $(-1, 4)$, $B\ (2, 7)$ and $C\ (4, 3)$. The point E lies on BC such that $5BE = 2BC$.



(a) Find the coordinates of D, E and F. [8]

Continuation of working space for question **12(a)**.

(b) *AG* and *EF* are two vertical lines. The *y*-coordinate of *G* is $\frac{2}{5}$. *E* and *F* lie on *BC* and *DC* respectively. Explain, with an appropriate working, what is the name of the special quadrilateral *AEFG*. [2]

13 A container in the shape of a right pyramid, has a height of 45 cm and a square base of side 20 cm, was initially empty. Sand is then allowed to flow into the container through a small hole at the top. After t seconds, the height of the sand in the container is $(45 - x)$ cm and the volume of the sand in the container is V cm^3.

(a) Show that $V = 6000 - \frac{16}{243}x^3$. [3]

(b) Given that the rate of flow of the sand into the container is bx^2 cm^3/s, where b is a constant. Find the numerical value of the rate of change of x if the height of the sand in the container is 36 cm after 24 seconds. [7]

End of paper

Qn	Solutions
1	$m-6>0$ and $b^2-4ac<0$ $m>6$ $(-8)^2-4(m-6)(m)<0$ $64-4m^2+24m<0$ $m^2-6m-16>0$ $(m+2)(m-8)>0$ $m<-2$ or $m>8$  Hence, $m>8$
2	$y=m-x$ ....(1) Sub (1) into $x^2+2y^2=2x+3$, $x^2+2(m-x)^2-2x-3=0$ $3x^2-x(4m+2)+2m^2-3=0$ Discriminant $=[-(4m+2)]^2-4(3)(2m^2-3)$ $=16m^2+16m+4-24m^2+36$ $=-8(m^2-2m-5)$ Given $m^2\le 2m+5 \Rightarrow m^2-2m-5\le 0$ Hence $-8(m^2-2m-5)\ge 0$ Since, discriminant ≥ 0, hence the line will intersect the curve.
3	$f(x)=(b-3x)\,e^{2-3x}$ $f'(x)=-3e^{2-3x}+(b-3x)(-3e^{2-3x})$ $=-3e^{2-3x}(1+b-3x)$ $=3e^{2-3x}(3x-1-b)$ For decreasing function, $3e^{2-3x}(3x-1-b)<0$ Since $3e^{2-3x}>0 \Rightarrow 3x-1-b<0$ $x<\frac{1+b}{3}$ Given $x<\frac{4}{3}$, $\therefore 1+b=4$ $b=3$

4	LHS $=\frac{2-\operatorname{cosec}^2\theta}{\operatorname{cosec}^2\theta+2\cot\theta}$ $\Rightarrow\frac{2-(1+\cot^2\theta)}{1+\cot^2\theta+2\cot\theta}$ $=\frac{1-\cot^2\theta}{(1+\cot\theta)^2}$ $=\frac{(1+\cot\theta)(1-\cot\theta)}{(1+\cot\theta)^2}$ $=\frac{1-\cot\theta}{1+\cot\theta}$
5	$x^2-9x-22=(x-11)(x+2)$ $\frac{4x^2+5x-32}{(x-11)(x+2)^2}=\frac{A}{x-11}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$ $4x^2+5x-3=A(x+2)^2+B(x-11)(x+2)+C(x-11)$ Let $x=11$, $4(11)^2+5(11)-32=A(13)^2$ $A=3$ Let $x=-2$, $4(-2)^2+5(-2)-32=C(-13)$ $C=2$ Let $x=0$, $-32=A(4)+B(-11)(2)+C(-11)$ $-32=12-22B-22$ $B=1$ $\frac{3}{x-11}+\frac{1}{x+2}+\frac{2}{(x+2)^2}$
6(a)	$3\cos x=2(\frac{1}{\sin x})(\frac{\cos x}{\sin x})$ $3\cos x\sin^2 x=2\cos x$ $3\cos x\sin^2 x-2\cos x=0$ $\cos x(3\sin^2 x-2)=0$ (Shown)

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(b) From (a), $\cos x(3\sin^2 x - 2) = 0$

$3\sin^2 x - 2 = 0$ or $\cos x = 0$

$\sin^2 x = \frac{2}{3}$

$\sin x = \pm\sqrt{\frac{2}{3}}$

Basic angle, $\alpha = \sin^{-1}\sqrt{\frac{2}{3}}$ or Basic angle, $\alpha = \frac{\pi}{2}$

≈ 0.95532

New range: $-3.2 < 0.6y - 1.4 < 1$

Replaced x by $(0.6y - 1.4)$,

$0.6y - 1.4 = 0.95532,$ or $0.6y - 1.4 = -\frac{\pi}{2}$

$-0.95532, -(\pi - 0.95532)$ $\therefore y = -0.284$ (3sf)

$\therefore y = -1.31, 0.741, 3.93$ (3sf)

Answer: $-1.31, 0.741, 3.93$

7(a) $\frac{dy}{dx} = \frac{(\cos x)(\cos x) - (1 + \sin x)(-\sin x)}{\cos^2 x}$

$= \frac{\cos^2 x + \sin x + \sin^2 x}{\cos^2 x}$

$= \frac{1 + \sin x}{\cos^2 x}$

(b) From (a),

$\int_0^{\frac{\pi}{3}} \frac{1 + \sin x}{\cos^2 x}\, dx = \left[\frac{1 + \sin x}{\cos x}\right]_0^{\frac{\pi}{3}}$

$= \frac{1 + \sin(\frac{\pi}{3})}{\cos(\frac{\pi}{3})} - \frac{1 + \sin 0}{\cos 0}$

$= \frac{1 + \frac{\sqrt{3}}{2}}{\frac{1}{2}} - 1$

$= 1 + \sqrt{3}$

Suggested Solutions for 2024 NCHS Prelim Exam Add Math Paper 1 (4049/1)

$\int_0^{\frac{\pi}{3}} \frac{3 + 3\sin x - 10\cos^3 x}{5\cos^2 x}\, dx$

$= \int_0^{\frac{\pi}{3}} \frac{3 + 3\sin x}{5\cos^2 x} - 2\cos x\, dx$

$= \frac{3}{5}\int_0^{\frac{\pi}{3}} \frac{1 + \sin x}{\cos^2 x}\, dx - \int_0^{\frac{\pi}{3}} 2\cos x\, dx$

$= \frac{3}{5}(1 + \sqrt{3}) - [2\sin x]_0^{\frac{\pi}{3}}$

$= \frac{3}{5} + \frac{3}{5}\sqrt{3} - (\sqrt{3} - 0)$

$= \frac{3}{5} - \frac{2}{5}\sqrt{3}$

$p = \frac{3}{5}, q = -\frac{2}{5}$

8(a) Sub $t = 0$, $h = 5$ m

(b) $h = -4.9\left(t^2 - \frac{80}{49}t\right) + 5$

$= -4.9\left[\left(t - \frac{40}{49}\right)^2 - \left(-\frac{40}{49}\right)^2\right] + 5$

$= -4.9\left(t - \frac{40}{49}\right)^2 + \frac{160}{49} + 5$

$= 8\frac{13}{49} - 4.9\left(t - \frac{40}{49}\right)^2$

(c) Greatest height $= 8\frac{13}{49}$ m; $t = \frac{40}{49}$ s

(d) Sub $h = 0$,

$8\frac{13}{49} - 4.9\left(t - \frac{40}{49}\right)^2 = 0$

$\left(t - \frac{40}{49}\right)^2 = 1.68680$

$t = 2.12$ s or -0.482 (rejected)

9 $5^p = 5^2 \times 5^{3q}$

$p = 2 + 3q$ ………(1)

$\log_3 7 - \log_3(11q - 2p) = 1$

$\log_3 \frac{7}{11q - 2p} = 1$

	$\frac{7}{11q-2p}=3^1$ $7=33q-6p$ ....(2) Sub (1) into (2), $7=33q-6(2+3q)$ $q=\frac{19}{15}$ Sub $q=\frac{19}{15}$ into (1), $p=5\frac{4}{5}$
10 (a)	$\frac{dy}{dx}=x^3\left(\frac{1}{x}\right)+(\ln x)(3x^2)$ $=3x^2\ln x+x^2$ $=x^2(3\ln x+1)$ At stationary point, $\frac{dy}{dx}=0$ $x^2(3\ln x+1)=0$ $x^2=0$ or $3\ln x+1=0$ $x=0$ (rejected) $\ln x=-\frac{1}{3}$ $x=e^{-\frac{1}{3}}$ Sub $x=e^{-\frac{1}{3}}$ into $y=x^3\ ln\ x$, $y=e^{-1}\times\ln e^{-\frac{1}{3}}$ $=-\frac{1}{3e}$ Coordinates $=\left(e^{-\frac{1}{3}},-\frac{1}{3e}\right)$
(b)	$\frac{d^2y}{dx^2}=2x(3\ln x+1)+x^2$ ($=6x\ ln\ x+5x$ Sub $x=e^{-\frac{1}{3}}$, $\frac{d^2y}{dx^2}=6e^{-\frac{1}{3}}ln\ e^{-\frac{1}{3}}+5e^{-\frac{1}{3}}=2.15>0$ Hence, $\left(e^{-\frac{1}{3}},-\frac{1}{3e}\right)$ is a minimum point.
(c)	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $0.3x^2=\frac{dy}{dx}\times 0.06$ $\frac{dy}{dx}=5x^2$ $3x^2\ln x+x^2=5x^2$

	$x^2(3\ln x-4)=0$ $x^2=0$ or $3\ln x-4=0$ $x=0$ (rejected) or $x=e^{\frac{4}{3}}$ Sub $x=e^{\frac{4}{3}}$ into y, $y=e^4\ln e^{\frac{4}{3}}$ $=\frac{4}{3}e^4$ $M=(e^{\frac{4}{3}},\frac{4}{3}e^4)$
11 (a)	Centre $=(\frac{k}{-2},\frac{-6}{-2})$ $(-2,m)=(\frac{k}{-2},3)$ By comparing, $k=4$ $m=3$ Radius $=\sqrt{(-2)^2+(3)^2-(-h)}$ $7=\sqrt{13+h}$ $h=36$ Alternative method: $(x+2)^2+(y-m)^2=7^2$ $x^2+y^2+4x-2my=49-4-m^2$ Compare with $x^2+y^2+kx-6y=h$ $\therefore k=4$ $-2m=-6\Rightarrow m=3$ $h=49-4-m^2$ $=49-4-3^2$ $=36$
(bi)	Sub $x=3$, $4y+9=5$ $y=-1$ $P=(3,-1)$ Centre = midpoint of PQ $=\left(\frac{3+(-5)}{2},\frac{-1+5}{2}\right)$ $=(-1,2)$ Radius $=\frac{1}{2}\sqrt{(3+5)^2+(-1-5)^2}$ $=5$ Equation of C_1: $(x+1)^2+(y-2)^2=25$

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(biii) Distance of S from centre of C_1 (-2, 3)

$= \sqrt{(4+2)^2 + (5-3)^2}$

$= 6.32 < 7$ (radius of C_1)

Distance of S from centre of C_2 (-1, 2)

$= \sqrt{(4+1)^2 + (5-2)^2}$

$= 5.83 > 5$ (radius of C_2)

Hence S only lies inside C_1 but not C_2.

12 (a) Let D be $(x, 0)$

Midpoint of AC = Midpoint of BD

$\left(\frac{x+2}{2}, \frac{0+7}{2}\right) = \left(\frac{-1+4}{2}, \frac{4+3}{2}\right)$

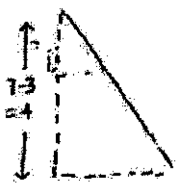
$x = 1 \Rightarrow D = (1, 0)$

By similar triangles,

$\frac{x-2}{2} = \frac{2}{5}$

$5x - 10 = 4$

$x = 2\frac{4}{5}$



$\frac{7-y}{4} = \frac{2}{5}$

$35 - 5y = 8$

$y = 5\frac{2}{5}$

$E = (2\frac{4}{5}, 5\frac{2}{5})$

Equation of DC:

$m_{DC} = m_{AB} = \frac{7-4}{2-(-1)} = 1$

Sub (4, 3), m = 1,

$y - 3 = 1(x - 4)$

$y = x - 1$ ………(1)

Sub $x = 2\frac{4}{5}$ into (1),

$y = 1\frac{4}{5}$

$F = (2\frac{4}{5}, 1\frac{4}{5})$

(b) $G = (-1, \frac{2}{5})$

$m_{AB} = \frac{5.4-4}{2.8+1} = \frac{7}{19}$

$m_{GF} = \frac{1.8-\frac{2}{5}}{2.8+1} = \frac{7}{19}$

Hence $m_{AB} = m_{GE}$ and $m_{AG} = m_{EF}$ (Vertical lines)

Since there are 2 pairs of opposite parallel lines, $AEFG$ is a parallelogram.

13 (a) By similar triangles,

$\frac{s}{20} = \frac{x}{45}$

$s = \frac{4}{9}x$

Volume of sand, V

= Volume of big pyramid – volume of small pyramid

$= \frac{1}{3}(20)^2(45) - \frac{1}{3}\left(\frac{4}{9}x\right)^2 (x)$

$= 6000 - \frac{16}{243}x^3$ (Shown)

(b) $\frac{dV}{dx} = -\frac{16}{81}x^2$

$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$

$bx^2 = -\frac{16}{81}x^2 \times \frac{dx}{dt}$

$\frac{dx}{dt} = -\frac{81b}{16}$

$x = \int -\frac{81b}{16}\, dt$

$= -\frac{81b}{16}t + c$

Sub $t = 0, x = 45$

$c = 45$

Hence $x = -\frac{81b}{16}t + 45$

Given $t = 24$, $45 - x = 36 \Rightarrow x = 9$

Sub $t = 24, x = 9$,

$9 = -\frac{81b}{16} \times 24 + 45$

$b = \frac{8}{27}$

$\therefore \frac{dx}{dt} = -\frac{81b}{16}$

$= -\frac{81}{16} \times \frac{8}{27}$

$= -1.5$ cm/s