2016 JC2 PRELIMINARY EXAMINATIONS
MATHEMATICS

Higher 1

8864

Thursday **15 September 2016**
3 Hours

Additional materials: Writing paper

List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and class on the cover page and on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total marks for this paper is **95**.

At the end of the examination, fasten all your work securely together.

This question paper consists of **6** printed pages and **2** blank pages.

[Turn Over

Section A: Pure Mathematics (35 Marks)

1 Find the exact value of $\int_1^4 \left(\frac{x^2+1}{\sqrt{x}} - e^{-2x} \right) dx$ [3]

2 **(i)** Differentiate the expression $\ln \sqrt{\frac{2x-1}{1-x}}$ with respect to x. [2]

(ii) The curve $y = \ln \sqrt{\frac{2x-1}{1-x}}$ has gradient $\frac{1}{2k}$ at a point (x, y) where k is a non-zero constant.

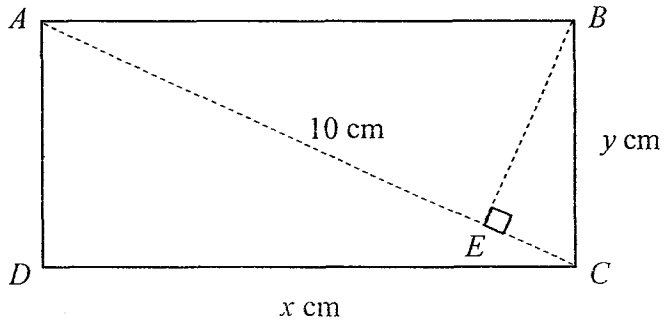
(a) Show that $2x^2 - 3x + k + 1 = 0$. [1]

(b) Find the range of values of k if there are two points on the curve where the gradient is $\frac{1}{2k}$. [3]

3 Sketch the graph of the function $y = 3 - e^{2x}$ indicating clearly the equation(s) of asymptote(s), if any, and the exact coordinates of points of intersection with the axes. [3]

Without the use of a graphic calculator, find the exact equation of the tangent to the curve $y = 3 - e^{2x}$ at the point where $x = 1$. [4]

4



The diagram shows a piece of paper $ABCD$ in the shape of a rectangle of length x cm and width y cm, where $x > y$. The perimeter of the rectangle $ABCD$ is 28 cm and the length of the diagonal AC is 10 cm.

(i) Find the values of x and y. [4]

(ii) Hence, calculate the area of the triangle ABC. [1]

The point E is on AC such that BE is perpendicular to AC.

(iii) Find the length of BE. [1]

5 The curve C has equation $y=(k-1)x^3+\frac{3}{2}x^2-6x+h$, where k and h are constants and $k\neq 1$. The stationary points of C are at A and B.

(i) Given that A has coordinates $\left(1,\frac{1}{2}\right)$, show that $k=2$ and find the value of h. [3]

(ii) Hence, find the exact value of the coordinates of B. [2]

(iii) Sketch the graph of C, stating the coordinates of A and B. [3]

(iv) Hence, by sketching a suitable graph on the same diagram as **(iii)**, solve the inequality $(k-1)x^3+\frac{3}{2}x^2-6x+h\geq\frac{1}{2}x$. [2]

(v) For $x>0$ and $y>0$, use a non-calculator method to find the exact area of the region bounded by the curve C, the line $y=\frac{1}{2}x$ and the $y-$axis. [3]

Section B: Statistics (60 Marks)

6 The random variable X follows a normal distribution with mean μ and variance σ^2. Given that $P(X<24)=P(X>30)=0.256$, write down the value of μ. Hence, calculate the value of σ, correct to 3 decimal places. [4]

7 There are 1000 students in a particular educational institution. For each student, the monthly stationery shopping is done either by going to the school bookstore or by ordering online and having the order delivered. The numbers using each method of shopping are recorded, according to the age in years, of the student responsible for the shopping. The data is summarised in the following table.

Age	School Bookstore	Online
7 – 12 years	280	20
13 – 18 years	250	450

A teacher carries out a survey to investigate the amount spent on stationery per month. She decides to use a sample of size 100 from these students.

(i) Describe how she might obtain a systematic sample. [2]

(ii) Describe how she might obtain a stratified sample, identifying the strata and finding the size of the sample taken from each of the strata. [2]

(iii) State, with a reason, whether a systematic sample or a stratified sample would be more appropriate in this context. [1]

8 **(i)** In a large population, the proportion having blood type C is $p\%$. A random sample of 15 people is taken and their blood specimens are to be tested. It is known that 3 people are expected to have blood type C. Show that the value of p is 20. [2]

(ii) Another random sample of 25 people is taken. Show that the probability that more than 24% of the people have blood type C is 0.21996, corrected to 5 decimal places. [2]

(iii) One hundred samples of 25 people are taken. Estimate the probability that there are at least 30 samples but less than 55 samples with more than 24% of the people having blood type C. You should state clearly the mean and variance of any distribution that you use. [4]

9 A researcher recorded the water temperature T, in °C, and the depth D, in metres, at noon on a certain day at each of the eight locations in a lake. The results are summarized in the table below.

D (m)	10	50	80	120	200	250	320	400
T (°C)	25.0	23.0	22.2	20.0	16.4	24.2	8.0	4.0

(i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]

(ii) One pair of data was incorrectly recorded. Identify this pair of data by writing down its coordinates. [1]

The incorrect pair of data is now removed from the data set.

(iii) Calculate the linear product moment correlation coefficient for the revised data and comment on its value in the context of the question. [2]

(iv) Find the equation of the regression line T on D and sketch this line on your scatter diagram. [2]

(v) Hence, estimate the water temperature when the depth of the water is 600 metres. Comment on the reliability of this estimate. [2]

(vi) Another researcher uses this set of revised data for his research. The depth of the water was, however, recorded in kilometres. Without any further calculations, state with a reason, if you would expect a change in the value of the linear product moment correlation coefficient found in part **(iii)**. [1]

10 A particular dairy product company produces two types of yogurt namely Brand A and Brand B. The mass of a cup of Brand A yogurt follows a normal distribution with mean 240 grams and standard deviation 5 grams. The mass of a cup of Brand B yogurt follows a normal distribution with mean 500 grams and standard deviation 7 grams.

(i) Find the probability that the mass of five cups of Brand A yogurt exceeds 1.225 kilograms. [2]

(ii) Find the probability that the mean mass of three cups of Brand B yogurt is more than twice the mass of one cup of Brand A yogurt by at least 10 grams. [2]

(iii) Ten cups of Brand A yogurt are chosen at random. Find the probability that at least three of them weigh more than 250 grams. [3]

The cost price of a cup of yogurt is determined based on its mass. The cost price of a cup of Brand A yogurt is 50 cents per 100 grams and the cost price of a cup of Brand B yogurt is 60 cents per 100 grams. Assume that the mass of an empty cup is negligible.

(iv) Find the probability that the total cost price of five cups of Brand A yogurt is at most 20 cents more than two times the cost price of a cup of Brand B yogurt. [3]

11(a) It is given that $\mathrm{P}(A \cap B') = 0.3$ and $\mathrm{P}(A \mid B') = 0.7$.

Find exactly

(i) $\mathrm{P}(B)$ [2]

(ii) $\mathrm{P}(A' \cap B')$ [1]

(b) As part of a Charity Run, Ben runs along the Punggol Waterway running track from Serangoon Reservoir end to Punggol Reservoir end. Following table illustrates the probabilities that he completes his run within a time frame on a wet day and on a dry day.

Time Taken (x min)	$x < 20$	$20 \leq x \leq 25$	$x > 25$
Probability on a wet day	a	$\frac{1}{5}$	$\frac{7}{10}$
Probability on a dry day	$\frac{2}{3}$	$\frac{1}{6}$	b

The probability that a day will be a wet day is given as $\frac{1}{4}$.

(i) Draw a tree diagram to illustrate the above information stating the exact values of a and b. [2]

(ii) Find the probability that he takes at most 25 minutes to complete his run on a particular day. [1]

(iii) Find the probability that he runs on a wet day. given that he takes more than 25 minutes to complete his run. [3]

(iv) The Charity Run is held on three different days and Ben participates on all the three days. Find the probability that he takes at most 25 minutes to complete his run on two days and more than 25 minutes on one day. [2]

12(a) The amount of mineral water consumed by a worker in a factory has mean 2.5 litres and standard deviation 0.4 litres. Given that the probability that the total amount of mineral water consumed by n workers ($n > 50$) in that factory between $2.5n$ litres and $2.55n$ litres is at least 0.40, find the least value of n. [3]

(b) A new car company "Telstar" claims that their latest car model is more economical in terms of its fuel consumption as compared to its previous models. The company claims that the car consumes at most 9.5 litres for a 100 km drive. A test is conducted on 60 such cars and their average fuel consumptions are recorded as shown in the following table.

Average fuel consumption in litres, for 100 km.	8.5	8.7	8.8	9.0	9.4	9.7	10.0	10.3	10.5
Number of cars	1	1	3	6	13	12	10	9	5

(i) Calculate the mean of this sample and the unbiased estimate of the population variance. [2]

(ii) Test at 5% level of significance whether the company's claim is justified. [3]

(iii) Is it necessary to assume that the fuel consumption of a car follows normal distribution? [1]

(iv) Explain the meaning of p-value in this context. [1]

Another test is conducted on another 50 such cars. It is given that the population variance is 2.5 litres^2, find the range of values of the sample mean if the company's claim was rejected at 5% level of significance. [2]

END OF PAPER

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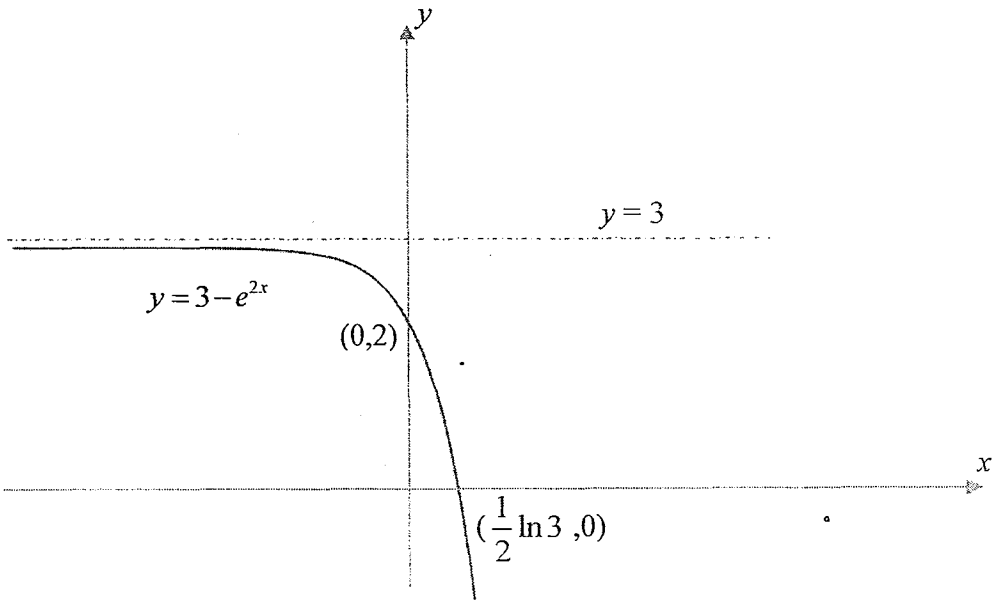
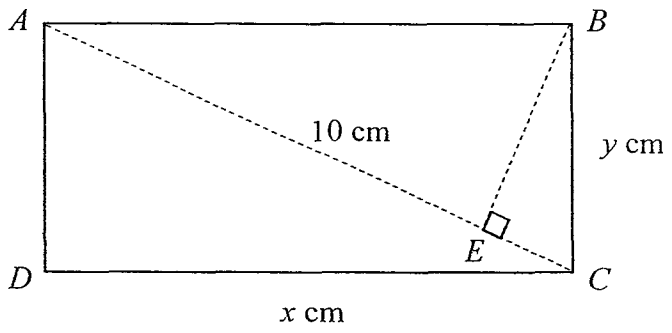
	Section A: Pure Mathematics (35 Marks)	
1	Find the exact value of $\int_1^4 \left(\frac{x^2+1}{\sqrt{x}} - e^{-2x} \right) dx$	[3]
	Solution: $\int_1^4 \left(\frac{x^2+1}{\sqrt{x}} - e^{-2x} \right) dx = \left[\frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{1}{2}} + \frac{1}{2}e^{-2x} \right]_1^4 = \left(\frac{64}{5} + 4 + \frac{1}{2}e^{-8} \right) - \left(\frac{2}{5} + 2 + \frac{1}{2}e^{-2} \right)$ $= \frac{72}{5} + \frac{1}{2}\left(e^{-8} - e^{-2} \right)$	
2	**(i)** Differentiate the expression $\ln\sqrt{\frac{2x-1}{1-x}}$ with respect to x.	[2]
	(ii) The curve $y = \ln\sqrt{\frac{2x-1}{1-x}}$ has gradient $\frac{1}{2k}$ at a point (x, y). **(a)** Show that $2x^2 - 3x + k + 1 = 0$. **(b)** Find the range of values of k if there are two points on the curve where the gradient is $\frac{1}{2k}$.	[1] [3]
(i)	**Solution:** $\ln\sqrt{\frac{2x-1}{1-x}} = \frac{1}{2}\left[\ln(2x-1) - \ln(1-x) \right]$ $\frac{d}{dx}\ln\sqrt{\frac{2x-1}{1-x}} = \frac{1}{2}\left[\frac{2}{2x-1} + \frac{1}{1-x} \right]$ $= \frac{1}{2(2x-1)(1-x)}$	
(ii)(a)	$\frac{1}{2(2x-1)(1-x)} = \frac{1}{2k}$ $(2x-1)(1-x) = k,\ k \neq 0$ $2x^2 - 3x + k + 1 = 0$	
(b)	There are two distinct values for x. Therefore the discriminant is > 0. $(-3)^2 - 4(2)(k+1) > 0$, $1 - 8k > 0$ $k < \frac{1}{8}$, $k \neq 0$ That is either $k < 0$ or $0 < k < \frac{1}{8}$ for the answer $k < \frac{1}{8}$	

3	Sketch the graph of the function $y=3-e^{2x}$ indicating clearly the equation(s) of asymptote(s), if any, and the exact coordinates of points of intersection with the axes. Without the use of a graphic calculator, find the exact equation of the tangent to the curve $y=3-e^{2x}$ at the point where $x = 1$.	[3] [4]
	Solution:  $\frac{dy}{dx}=-2e^{2x}$ Gradient of the tangent at $x = 1$ is $-2e^2$. When $x = 1$, $y=3-e^2$. Equation of the tangent, $y-(3-e^2)=-2e^2(x-1)$ $y=-2e^2x+e^2+3$	

4



The diagram shows a piece of paper $ABCD$ in the shape of a rectangle of length x cm and width y cm, where $x > y$. The perimeter of the rectangle $ABCD$ is 28 cm and the length of the diagonal AC is 10 cm.

(i) Find the values of x and y. [4]

(ii) Hence, calculate the area of the triangle ABC. [1]

The point E is on AC such that BE is perpendicular to AC.

(iii) Find the length of BE. [1]

Suggested Solution

(i)

$2x+2y=28$

$x+y=14$

$y=14-x$ ---------- (1)

$x^2+y^2=10^2$ ---------- (2)

Substitute (1) into (2),

$x^2+(14-x)^2=100$

$x^2+196-28x+x^2=100$

$2x^2-28x+96=0$

$x^2-14x+48=0$

$(x-6)(x-8)=0$

$x=6$ or 8

When $x=6$, $y=8$ (Rejected since $x > y$)

When $x=8$, $y=6$

(ii) Area of triangle ABC

$= \frac{1}{2}(6)(8)$

$= 24 \text{ cm}^2$

(iii) $\frac{1}{2}(10)(BE) = 24$

$BE = 4.8$ cm

5 The curve C has equation $y = (k-1)x^3 + \frac{3}{2}x^2 - 6x + h$, where k and h are constants. The stationary points of C are at A and B.

(i) Given that A has coordinates $\left(1, \frac{1}{2}\right)$, show that $k = 2$ and find the value of h. [3]

(ii) Hence, find the exact value of the coordinates of B. [2]

(iii) Sketch the graph of C, stating the coordinates of A and B. [3]

(iv) Hence, by sketching a suitable graph on the same diagram as **(iii)**, solve the inequality $(k-1)x^3 + \frac{3}{2}x^2 - 6x + h \geq \frac{1}{2}x$. [2]

(v) For $x > 0$ and $y > 0$, use a non-calculator method to find the exact area of the region bounded by the curve C, the line $y = \frac{1}{2}x$ and the $y-$axis. [3]

Suggested Solution

(i)

$y = (k-1)x^3 + \frac{3}{2}x^2 - 6x + h,$

$\frac{dy}{dx} = 3(k-1)x^2 + 3x - 6$

When $x = 1$, $\frac{dy}{dx} = 0$

$3(k-1)(1)^2 + 3(1) - 6 = 0$

$3k - 6 = 0 \quad , k = 2$ (Shown)

When $x = 1$, $y = \frac{1}{2}$

$\frac{1}{2} = (1)^3 + \frac{3}{2}(1)^2 - 6(1) + h$

$h = 4$

(ii)

$\frac{dy}{dx} = 3x^2 + 3x - 6$

$3x^2 + 3x - 6 = 0$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = 1$ (rej, since this is point A) or $x = -2$

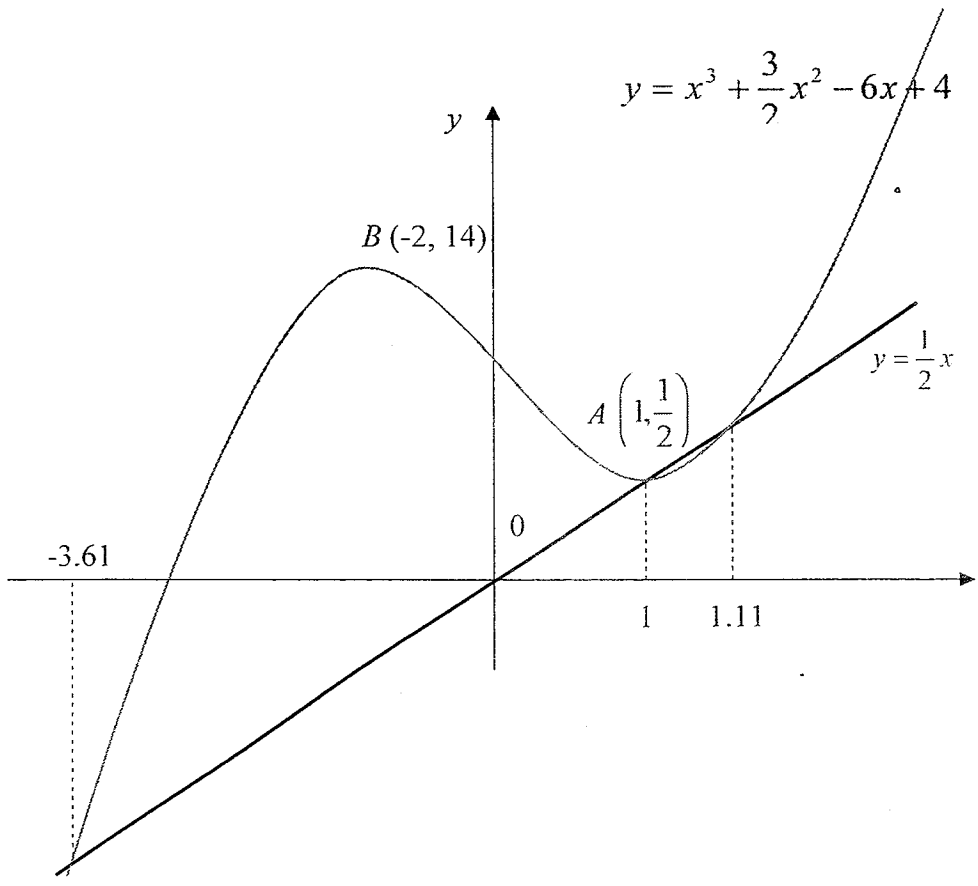
When $x = -2$,

$y = (-2)^3 + \frac{3}{2}(-2)^2 - 6(-2) + 4$

$y = 14$

Hence, B is $(-2, 14)$

(iii)



(iv)

Using GC, the intersection points between the curve C and the line $y=\frac{1}{2}x$ is

$\left(-3.61,-1.80\right)$, $\left(1,\frac{1}{2}\right)$ and $\left(1.11,0.554\right)$.

Hence, the range of values of x is $-3.61\le x\le 1$ or $x\ge 1.11$

(v)

Area of region

$$=\int_0^1\left(x^3+\frac{3}{2}x^2-6x+4\right)-\left(\frac{1}{2}x\right)\,\mathrm{d}x$$

$$=\int_0^1\left(x^3+\frac{3}{2}x^2-\frac{13}{2}x+4\right)\mathrm{d}x$$

$$=\left[\frac{1}{4}x^4+\frac{1}{2}x^3-\frac{13}{4}x^2+4x\right]_0^1$$

$$=\frac{1}{4}(1)^4+\frac{1}{2}(1)^3-\frac{13}{4}(1)^2+4(1)$$

$=1.5$ units^2

Alternatively,

Area of region

$$=\int_0^1\left(x^3+\frac{3}{2}x^2-6x+4\right)\mathrm{d}x-\frac{1}{2}(1)\left(\frac{1}{2}\right)$$

$$=\left[\frac{1}{4}x^4+\frac{1}{2}x^3-3x^2+4x\right]_0^1-\frac{1}{4}$$

$$=\left[\frac{1}{4}(1)^4+\frac{1}{2}(1)^3-3(1)^2+4(1)\right]-\frac{1}{4}$$

$=1.5$ units^2

	Section B: Statistics (60 Marks)	
6	The random variable X follows a normal distribution with mean μ and variance σ^2. Given that $\mathrm{P}(X<24)=\mathrm{P}(X>30)=0.256$, write down the value of μ. Hence, calculate the value of σ, correct to 3 decimal places.	[4]
	Suggested Solution	
	$\mu=27$	
	$X\sim \mathrm{N}(27,\sigma^2)$ $P(X<24)=0.256$ $P\left(Z<\frac{24-27}{\sigma}\right)=0.256$ $-\frac{3}{\sigma}=-0.65573$ $\sigma=4.575$ (to 3 d.p.)	
7	There are 1000 students in a particular educational institution. For each student, the monthly stationery shopping is done either by going to the school bookstore or by ordering online and having the order delivered. The numbers using each method of shopping are recorded, according to the age in years, of the student responsible for the shopping. The data is summarised in the following table. | Age | School Bookstore | Online | | 7 – 12 years | 280 | 20 | | 13 – 18 years | 250 | 450 | A teacher carries out a survey to investigate the amount spent on stationery per month. She decides to use a sample of size 100 from these students.	
	(i) Describe how she might obtain a systematic sample.	[2]
	(ii) Describe how she might obtain a stratified sample, identifying the strata and finding the size of the sample taken from each of the strata.	[2]
	(iii) State, with a reason, whether a systematic sample or a stratified sample would be more appropriate in this context.	[1]

	Suggested Solution	
	(i) Firstly, number all the students from 1 to 1000 according to some order (e.g. in order of birth date). The sample interval $k = \frac{1000}{100} = 10$. Random select a number from 1 to 10, say the 5th student, and select every 10th student thereafter, until a sample of 100 students is obtained.	B2
	(ii) Calculate the sample size to be collected from the population as follows: Within each stratum, a sample is randomly selected to meet the sample size calculated above. **(iii)** Stratified sampling is more appropriate as it will produce a sample that is more representative of the different strata of shopping methods and different age groups.	
8	**(i)** In a large population, the proportion having blood type C is p%. A random sample of 15 people is taken and their blood specimens are to be tested. It is known that 3 people are expected to have blood type C. Show that the value of p is 20.	[2]
	(ii) Another random sample of 25 people is taken. Show that the probability that more than 24% of the people have blood type C is 0.21996, corrected to 5 decimal places.	[2]
	(iii) One hundred samples of 25 people are taken. Estimate the probability that there are at least 30 samples but less than 55 samples with more than 24% of the people having blood type C. You should state the mean and variance of any distribution that you use.	[4]
	Suggested Solution	
	(i) Let X denote "the number of people who have blood type C, out of a sample of 15 people" $X \sim \mathrm{B}\left(15, \frac{p}{100}\right)$	

Age	School Bookstore	Online
7 – 12 years	$\frac{280}{1000} \times 100 = 28$	$\frac{20}{1000} \times 100 = 2$
13 – 18 years	$\frac{250}{1000} \times 100 = 25$	$\frac{450}{1000} \times 100 = 45$

Since $\mathrm{E}(X)=3$,

$\frac{15p}{100}=3$

$p=20$ (Shown)

(ii)

Let Y denote "the number of people who have blood type C, out of a sample of 25 people"

$Y \sim \mathrm{B}(25,0.2)$

$\mathrm{P}(Y>0.24(25)) = \mathrm{P}(Y>6)$

$=1-\mathrm{P}(Y\leq 6)$

$= 0.21996$

(iii)

Let W denote "the number of samples of 25 people with more than 6 people having blood type C, out of 100 samples"

$W \sim \mathrm{B}(100,0.21996)$

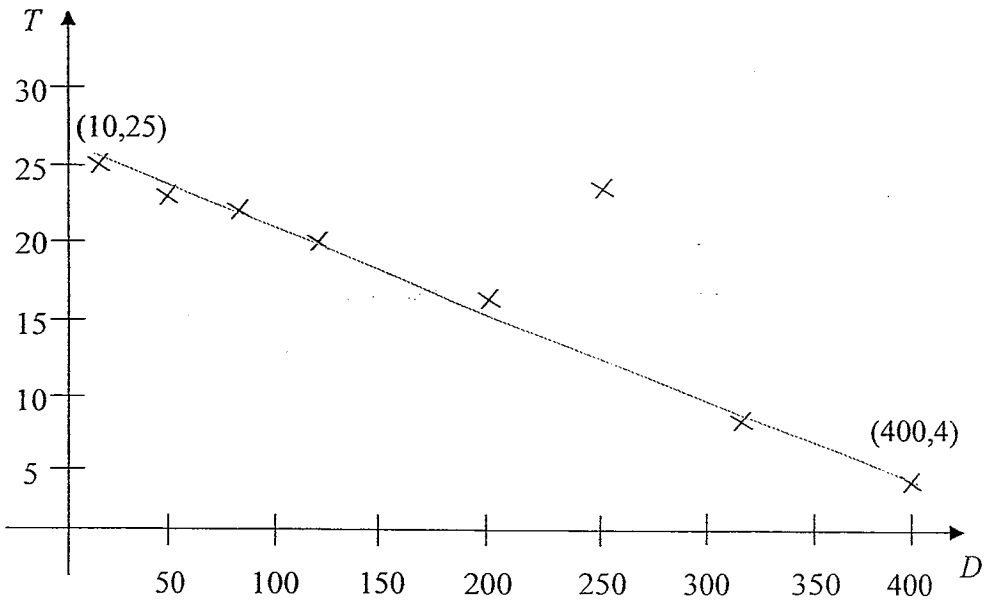
Since $n = 100$ is large, $np=21.996\ (>5)$, $nq=78.004\ (>5)$,

$W \sim \mathrm{N}(21.996,17.158)$ approximately

$\mathrm{P}(30\leq W<55) \xrightarrow{c.c} \mathrm{P}(29.5<W<54.5)$

$= 0.0350$ (to 3 s.f.)

9	A researcher recorded the water temperature T, in °C, and the depth D, in metres, at noon on a certain day at each of the eight locations in a lake. The results are summarized in the table below. D (m): 10, 50, 80, 120, 200, 250, 320, 400 T (°C): 25.0, 23.0, 22.2, 20.0, 16.4, 24.2, 8.0, 4.0	
	(i) Give a sketch of the scatter diagram for the data, as shown on your calculator.	[2]
	(ii) One pair of data was incorrectly recorded. Identify this pair of data by writing down its coordinates.	[1]
	The incorrect pair of data is now removed from the data set.	
	(iii) Calculate the linear product moment correlation coefficient for the revised data and comment on its value in the context of the question.	[2]
	(iv) Find the equation of the regression line T on D and sketch this line on your scatter diagram.	[2]
	(v) Hence, estimate the water temperature when the depth of the water is 600 metres. Comment on the reliability of this estimate.	[2]
	(vi) Another researcher uses this set of revised data for his research. The depth of the water was, however, recorded in kilometres. Without any further calculations, state with a reason, if you would expect a change in the value of the linear product moment correlation coefficient found in part **(iii)**.	[1]
	Suggested Solution	
	(i)	



(ii)

The incorrect data pair is $(250, 24.2)$

(iii)

$r = -0.996$. It indicates a strong negative linear correlation between the depth of water and the temperature of water. As the depth of water increases, the temperature of water decreases.

(iv)

Regression line T on D:

$T = -0.054983D + 26.211$

$T = -0.0550D + 26.2$ (3 s.f.)

(v)

When $D = 600$,

$T = -0.054983(600) + 26.211$

$T = -6.78$

The estimate is not reliable as $D = 600$ falls outside the data range [10, 400]. The data may not follow a linear trend outside the given data range.

(vi)

No, we would not expect a change in the value of the linear product moment correlation coefficient found in part **(iii)**. As r measures the degree of scatter of the data set, its value will not be affected by a change in the unit of calculation.

10 A particular dairy products company produces two types of yogurt namely Brand A and Brand B. The mass of a cup of Brand A yogurt follows a normal distribution with mean 240 grams and standard deviation 5 grams. The mass of a cup of Brand B yogurt follows a normal distribution with mean 500 grams and standard deviation 7 grams.

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(ii) Find the probability that the mean mass of three cups of Brand B yogurt is more than twice the mass of one cup of Brand A yogurt by at least 10 grams. [2]

(iii) Ten cups of Brand A yogurt are chosen at random. Find the probability that at least three of them weigh more than 250 grams.
The cost price of a cup of yogurt is determined based on its mass. The cost price of a cup of Brand A yogurt is 50 cents per 100 grams and the cost price of a cup of Brand B yogurt is 60 cents per 100 grams. Assume that the mass of an empty cup is negligible. [3]

(iv) Find the probability that the total cost price of five cups of Brand A yogurt is at most 20 cents more than two times the cost price of a cup of Brand B yogurt. [3]

Solution:

Let X and Y be the random variables the mass of cup of Brand A and the mass of a cup of Brand B yogurt respectively.

$X \sim \mathrm{N}(240, 25)$

$Y \sim \mathrm{N}(500, 49)$

(i) Let $T = X_1 + X_2 + X_3 + X_4 + X_5 \sim N(1200, 125)$

$P(T > 1225) = 0.012674 \approx 0.0127$

(ii) Let $S = \frac{Y_1 + Y_2 + Y_3}{3} - 2X \sim N\left(500 - 2(240), \frac{1}{9}(49 + 49 + 49) + 4(25)\right)$

$S \sim N(20, 116, 33)$

$P(S \geq 10) = 0.823$

(iii) $P(X > 250) = 0.02275$

Let V be the random variable , number of cups of Brand A yogurt out of 10, which weigh at least 250 grams.

$V \sim \mathrm{B}(10, 0.02275)$

$P(V \geq 3) = 1 - P(V \leq 2)$

$= 0.0012531 \approx 0.00125$

(iv) $0.005(X_1+X_2+X_3+X_4+X_5)\sim N(6,0.003125)$

$0.006(2Y)\sim N(6,0.007056)$

Let $W=0.005(X_1+X_2+X_3+X_4+X_5)-0.006(2Y)\sim N(0,0.010181)$

$P(W\le 0.2)=0.976$

11(a) It is given that $\mathrm{P}(A\cap B')=0.3$ and $\mathrm{P}(A|B')=0.7$.

Find exactly

(i) $\mathrm{P}(B)$ [2]

(ii) $P(A'\cap B')$ [1]

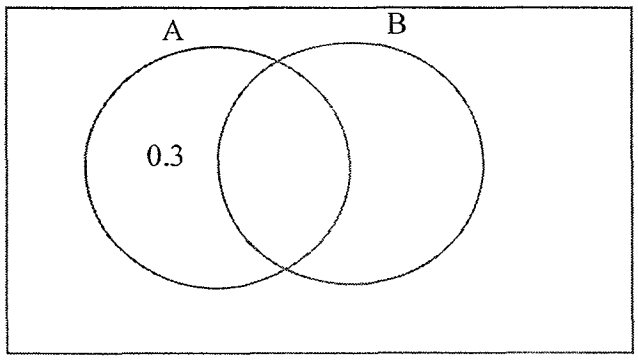
Solution:

(i) $\mathrm{P}(A|B')=\dfrac{\mathrm{P}(A\cap B')}{\mathrm{P}(B')}$

$0.7=\dfrac{0.3}{P(B')}$

$\mathrm{P}(B')=\dfrac{3}{7}\Rightarrow \mathrm{P}(B)=\dfrac{4}{7}$

$\mathrm{P}(A'\cap B')=1-0.3-\dfrac{4}{7}=\dfrac{9}{70}$



(b) As part of a Charity Run, Ben runs along the Punggol Waterway running track from Serangoon Reservoir end to Punggol Reservoir end. Following table illustrates the probabilities that he completes his run within a time frame on a wet day and on a dry day.

Time Taken(x min)	$x<20$	$20\le x\le 25$	$x>25$
Probability on a wet day	a	$\frac{1}{5}$	$\frac{7}{10}$
Probability on a dry day	$\frac{2}{3}$	$\frac{1}{6}$	b

The probability that a day will be a wet day is given as $\dfrac{1}{4}$.

(i) Draw a tree diagram to illustrate the above information stating the values of a and b. [2]

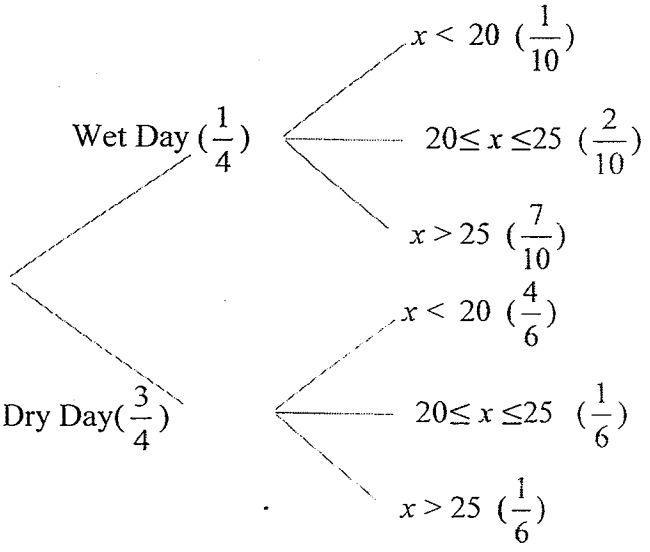
(ii) Find the probability that he takes at most 25 minutes to complete his run on a particular day. [1]

(iii) Find the probability that he runs on a wet day, given that he takes more than 25 minutes to complete his run. [3]

	(iv) The Charity Run is held on three different days and Ben participates on all the three days. Find the probability that he takes at most 25 minutes to complete his run on two days and more than 25 minutes on one day.	[2]

Solution:

(i)



$a = \frac{1}{10}$, $b = \frac{1}{6}$

(ii) Probability $= \left(\frac{1}{4}\right)\left(\frac{3}{10}\right) + \left(\frac{3}{4}\right)\left(\frac{5}{6}\right) = \frac{7}{10}$

(iii) Probability $= \dfrac{\left(\frac{1}{4}\right)\left(\frac{7}{10}\right)}{\frac{3}{10}}$

$= \frac{7}{12}$

(iv) $\left(\frac{7}{10}\right)^2\left(\frac{3}{10}\right) \times 3 = \frac{441}{1000}$

12(a) It is known that the average amount of mineral water consumed by a worker in a factory is 2.5 litres with a standard deviation 0.4 litres. Given that the probability that the total amount of mineral water consumed by n workers ($n > 50$) in that factory between $2.5n$ litres and $2.55n$ litres is at least 0.40, find the least value of n. [3]

Let X be the random variable the amount of mineral water, in litres, consumed by a worker.

Given $E(X) = 2.5$ and $Var(X) = 0.4^2$

Since n is large, by Central Limit Theorem,

$T = X_1 + X_2 + X_3 + ... + X_n \sim N(2.5n, 0.16n)$

Given $P(2.5n < T < 2.55n) \geq 0.40$

$P(T < 2.55n) \geq 0.90$

$P(Z < \frac{2.55n - 2.5n}{0.4\sqrt{n}}) \geq 0.90$

$P(Z < 0.125\sqrt{n}) \geq 0.90$

$0.125\sqrt{n} \geq 1.28155$

$n \geq 105.11$

Least $n = 106$.

OR Using GC,

$n = 105, p = 0.39988$

$n = 106, p = 0.40094$

$n = 107, p = 0.402$

Least $n = 106$

(b) A new car company "Telstar" claims that their latest car model is more economical in terms of its fuel consumption as compared to its previous models. The company claims that the car consumes at most 9.5 litres for a 100 km drive. A test is conducted on 60 such cars and their average fuel consumptions are recorded as in the following table.

Average fuel consumption in litres, for 100 km.	8.5	8.7	8.8	9.0	9.4	9.7	10.0	10.3	10.5
Number of cars	1	1	3	6	13	12	10	9	5

(i) Calculate the mean of this sample and the unbiased estimate of the population variance. [2]

(ii) Test at 5% level of significance whether the company's claim is justified. [3]

(iii) Is it necessary to assume that the fuel consumption of a car follows normal distribution? [1]

(iv) Explain the meaning of p-value in this context. [1]

Another test is conducted on 50 such cars. It is given that the population variance is 2.5. Find the range of values of the sample mean if the company's claim was rejected at 5% level of significance. [2]

Solution:

Let X be the random variable the fuel consumption of a car in litres/100 km.

(i) Using GC

$\bar{x} = 9.69$, $s^2 = 0.27651 \approx 0.277$

(ii) Test $H_0 : \mu = 9.5$ against

$H_1 : \mu > 9.5$ at 5% level of significance.

Under H_0, $\bar{X} \sim N(9.5, \frac{0.27651}{60})$ approximately by Central Limit theorem since n is large.

From GC

p- value is $0.00256 < 0.05$. H_0 is rejected.

Hence there is sufficient evidence to conclude that the manufacturer's claim that the mean fuel consumption is at most 9.5 litres per 100 km is rejected at 5% level of significance.

(iii) It is not necessary to assume that the distribution is normal as we can apply Central Limit theorem since n is large.

(iv) The p-value is the probability that the mean fuel consumption is either equal to 9.69 km or more for 100 litres assuming that H_0 is true.

OR p- value is the lowest significance level at which the evidence is sufficient to conclude that the mean fuel consumption is more than 9.5 litres/100 km

$$\text{Test Statistic} = \frac{\bar{x} - 9.5}{\sqrt{\frac{2.5}{50}}} \geq 1.6449$$

$$\bar{x} \geq 9.8678 \approx 9.87$$