MERIDIAN JUNIOR COLLEGE
JC2 Preliminary Examination
Higher 1

H1 Mathematics 8865/01

Paper 1 **14 September 2017**

3 Hours

Additional Materials: Writing paper
Graph Paper
List of Formulae (MF 26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics group on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

1 Azel, Brenda, Cathy and Dillion went to the wet market to buy three different kinds of fish. As there were no receipts provided, they did not know how much they paid for the individual prices of fish per kilogram. However, Azel, Brenda and Cathy can remember the total amount that they each paid. The weights of the different kinds of fish and the total amounts paid are shown in the following table.

	Azel	Brenda	Cathy	Dillon
Black pomfret (kg)	0.55	0.60	0.40	0.70
Sea Bass (kg)	0.45	0.58	0.75	0.34
Golden snapper (kg)	1.45	1.60	1.70	1.42
Total amounts paid in dollars	38.77	44.18	45.81	[redacted]

Assuming that, for each kind of fish, the price per kilogram paid by them is the same, calculate the total amount that Dillon paid. [5]

2 Differentiate $\ln\left(2x^2+1\right)$ with respect to x. Hence find the exact value of $\int_1^3 \frac{x}{2x^2+1}\,\mathrm{d}x$, leaving your answer in the form $a\ln b$ where a and b are constants to be determined. [5]

3 Find the exact equation of the tangent to the curve $\ln y=\left(2-x\right)^2$ at the point where $x=3$. [5]

4 A curve C has equation $y = kx^2 + k$, where k is a positive constant.

(i) Sketch C. [2]

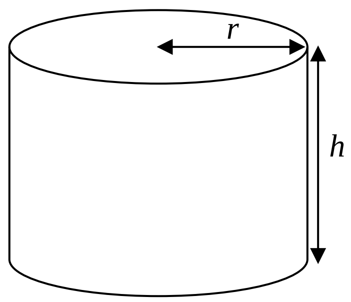
(ii) Find the range of values of k for which the line $y = 4x + 3$ intersects the curve C at 2 distinct points. [4]

It is given that $k = 3$.

(iii) Find the exact x-coordinates of the points of intersection between the line $y = 4x + 3$ and the curve C. [2]

(iv) Hence find the exact area bounded by the line $y = 4x + 3$ and the curve C. [4]

5



A factory decides to design a closed cylindrical water tank of radius r cm and height h cm to hold a maximum of 16000π cm^3 of water. The outer surface area (including the base and lid) of the tank is to be coated with a layer of paint. It is assumed that the thickness of the cylindrical water tank is negligible.

(i) Show that the outer surface area of the tank, S cm^2 is given by $S = \dfrac{32000\pi}{r} + 2\pi r^2$. [3]

(ii) In order to reduce the amount of paint used, the factory wishes to minimize the value of S. Using differentiation, find the value of r for which S is minimized. Hence find the minimum value of S. [5]

(iii) Sketch, in this context, the graph of S against r. [2]

(iv) The tank with minimum value of S is being manufactured. Water is being poured into this cylindrical tank at a constant rate of 1000 cm^3 per minute. Find the rate of change of the depth of water. [3]

Section B: Statistics [60 marks]

6 Five numbers 1, 3, 6, 7 and 8 are used to form a five-digit number. If each number can only be used once, find the number of ways such that the five-digit number is

(i) formed without further restrictions, [1]

(ii) odd and between 30000 and 80000. [3]

It is now given that all five numbers can be used with repetitions. Find the number of ways to form the five-digit number if it must be even. [2]

7 A manufacturer produces a large number of mugs everyday and the mugs are sold in batches of 50. On average, a proportion p of the mugs are defective. The random variable X is the number of defective mugs in a randomly chosen batch of 50. It is assumed that X has the distribution $\mathrm{B}(50, p)$.

(i) Given that $\mathrm{P}(X = 0 \text{ or } 1) = 0.15$, formulate an equation in p and find the value of p. [3]

For the rest of the question, use $p = 0.06$.

In order to ensure the highest quality in their product, the manufacturer decides to carry out a quality control test using one of the following methods:

Method A: Select 10 mugs from the batch at random and accept the batch if there are fewer than 3 defective mugs, otherwise reject the batch.

Method B: Select 5 mugs from the batch at random and accept the batch if there is no defective mug, reject if there are two or more defective mugs, otherwise select another 5 mugs at random from the batch. If the second sample is drawn, accept the batch if there are fewer than 2 defective mugs, otherwise reject the batch.

(ii) By considering the probability of accepting the batch in each method, justify which method should the manufacturer adopt to carry out the quality control test. [6]

8 **(a)** A and B are two events such that $\mathrm{P}\left(A \mid B'\right)=\frac{4}{17}$, $\mathrm{P}(B)=\frac{23}{40}$ and $\mathrm{P}(A \cap B)=\frac{3}{8}$.

By using a Venn diagram or otherwise, find $\mathrm{P}\left(A' \cap B'\right)$. [3]

Determine if A and B are independent events. [2]

(b) A basket contains 35 durians, of which 15 are MSW durians and 20 are D24 durians. Of the MSW durians, 4 are infested with maggots and of the D24 durians, 3 are infested with maggots. Two durians are chosen at random from the basket.

(i) Show that the probability that both are MSW durians and at least one durian is infested with maggots is $\frac{10}{119}$. [2]

(ii) Given that at least one durian is infested with maggots, find the probability that both are MSW durians. [3]

9 Peter is interested to find out how the sale of his books varies with the selling price. Over a period of eight weeks, he sells the books at a different selling price, x (in dollars), which is fixed for each week. He also records the number of books, y, sold in that week. The results are summarised in the following table.

Week	1	2	3	4	5	6	7	8
Selling Price (x dollars)	70	60	50	35	26	17	12	10
Number of Books Sold (y)	3	80	8	20	30	50	75	100

(i) Draw a scatter diagram to illustrate the above information, labelling the axes clearly. [2]

(ii) On the scatter diagram, circle the data point which is an outlier and label it P. [1]

(iii) Omitting the point P, find the product moment correlation coefficient and the least square regression line $y = a + bx$. Sketch this line on your scatter diagram. You are not required to find the axial intercepts. [3]

(iv) Use the least square regression line to estimate the number of books sold when the selling price of the book for that week is \$75. Comment on the reliability of this estimate. [2]

(v) Comment on whether a linear model would be appropriate, referring to both the scatter diagram (omitting the point P) and the context of the question. [2]

10 **(a)** A chocolate company claims that, on average, the consumption of their dark chocolate over time can decrease one's cholesterol level by at least 20 mg/dL. Over a period of time, a random sample of 80 volunteers who consume this dark chocolate daily have their cholesterol level measured. The table below shows the decrease in the cholesterol level, measured in mg/dL for the 80 volunteers.

Decrease in the cholesterol level (in mg/dL)	17	18	19	20	21	22	23	24
Number of volunteers	8	15	13	18	14	6	4	2

Test the chocolate company's claim at 5% significance level. [6]

(b) The principal of Meridian Childcare Centre claims that a child at the centre spends an average of μ_0 hours on afternoon nap. A survey was conducted and the time, x hours, spent by 60 randomly chosen children on afternoon naps is as follows:

$$\sum x = 185, \quad \sum x^2 = 626$$

(i) Find unbiased estimates of the population mean and variance. [2]

(ii) Find the range of values of μ_0 such that the principal is confident, at 4% level of significance, that he did not indicate wrongly the mean time that the children spend on afternoon naps. [4]

[Question 11 is printed on the next page.]

[Turn Over

11 In a particular supermarket, fishes are priced according to their weight. The weight of a randomly chosen Black Tilapia fish has a normal distribution with mean μ grams and standard deviation 32 grams. It is found that 10% of the Black Tilapia fish weigh heavier than 541 grams.

(i) Show that the mean weight of a Black Tilapia fish is 500 grams, correct to the nearest grams. [3]

(ii) Find the probability that out of three randomly chosen Black Tilapia fish, two weigh between 440 grams and 550 grams and one weighs more than 550 grams. [3]

(iii) The Black Tilapia fish is priced at \$ a per kg. It is given that the probability of the total price of 2 randomly chosen Black Tilapia fish cost less than \$6.90 is less than 0.84241, correct to 5 significant figures. Find the range of values of a. [4]

The supermarket also sells Grey Mullet fish. The weight of a randomly chosen Grey Mullet fish follows an independent normal distribution with mean weight 800 grams and standard deviation 50 grams.

(iv) Find the probability that the average weight of two randomly chosen Black Tilapia fish and three randomly chosen Grey Mullet fish exceeds 690 grams. [3]

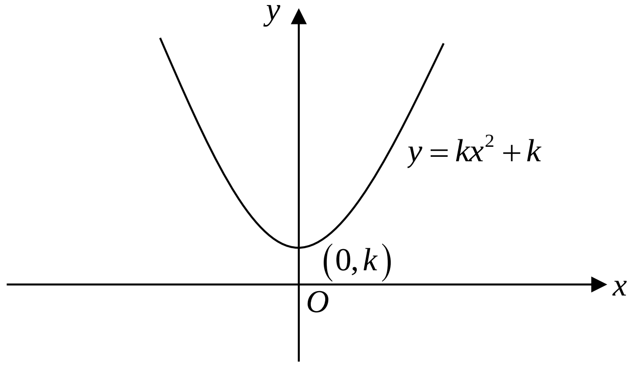
End of Paper

2017 H1 MATH (8865/01) JC 2 PRELIM – SOLUTIONS

Qn	Solution
1	**System of Linear Equations**
	Let x = price of black pomfret per kilogram (in dollars) y = price of sea bass per kilogram (in dollars) z = price of golden snapper per kilogram (in dollars) For Azel: $0.55x+0.45y+1.45z=38.77$ For Brenda: $0.60x+0.58y+1.6z=44.18$ For Cathy: $0.4x+0.75y+1.7z=45.81$ Using GC, $x=14.90$, $y=18.00$, $z=15.50$ Total price paid by Dillon = \$($(0.7\times 14.90)+(0.34\times 18.00)+(1.42\times 15.50)$) = \$ 38.56

Qn	Solution
2	**Techniques of Differentiation and Integration**
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(2x^2+1\right)\right)=\frac{4x}{2x^2+1}$ $\int_1^3 \frac{x}{2x^2+1}\,\mathrm{d}x=\frac{1}{4}\int_1^3 \frac{4x}{2x^2+1}\,\mathrm{d}x$ $=\frac{1}{4}\left[\ln\left(2x^2+1\right)\right]_1^3$ $=\frac{1}{4}\left(\ln 19-\ln 3\right)$ $=\frac{1}{4}\ln\frac{19}{3}$ $\therefore a=\frac{1}{4},\ b=\frac{19}{3}$

Qn	Solution
3	**Exponential and Logarithm (Equation of Tangent)**
	$\ln y = (2-x)^2$ $y = \mathrm{e}^{(2-x)^2}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -2(2-x)\mathrm{e}^{(2-x)^2}$ When $x = 3$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2\mathrm{e}$, $y = \mathrm{e}$. Equation of tangent is $y - \mathrm{e} = 2\mathrm{e}(x-3)$ $\Rightarrow y = 2\mathrm{e}x - 6\mathrm{e} + \mathrm{e}$ $\Rightarrow y = 2\mathrm{e}x - 5\mathrm{e}$

Qn	Solution
4	**Curve Sketching and Application of Integration**
(i)	
(ii)	$kx^2 + k = 4x + 3$ $kx^2 - 4x + k - 3 = 0$ Discriminant > 0 $(-4)^2 - 4(k)(k-3) > 0$ $16 - 4k^2 + 12k > 0$ $k^2 - 3k - 4 < 0$ $(k-4)(k+1) < 0$ $-1 < k < 4$ Since $k > 0$, $0 < k < 4$
(iii)	When $k = 3$ $3x^2 - 4x + 3 - 3 = 0$ $x(3x-4) = 0$ $x = 0$ or $x = \frac{4}{3}$
(iv)	$\int_0^{\frac{4}{3}} 4x + 3 - (3x^2 + 3)\,\mathrm{d}x$ $= \int_0^{\frac{4}{3}} 4x - 3x^2\,\mathrm{d}x$ $= \left[2x^2 - x^3\right]_0^{\frac{4}{3}}$ $= \left[2\left(\frac{4}{3}\right)^2 - \left(\frac{4}{3}\right)^3\right]$ $= \frac{32}{27}$

Qn	Solution
5	**Maxima/Minima and Connected Rate of Change**

(i)

$16000\pi = \pi r^2 h$

$h = \dfrac{16000}{r^2}$ --- (1)

$S = 2\pi rh + 2\pi r^2$ --- (2)

Substitute (1) into (2),

$S = 2\pi r\left(\dfrac{16000}{r^2}\right) + 2\pi r^2$

$S = \dfrac{32000\pi}{r} + 2\pi r^2$ (shown)

(ii)

For minimum S, $\dfrac{dS}{dr} = 0$.

$$\therefore \frac{dS}{dr} = -\frac{32000\pi}{r^2} + 4\pi r = 0$$

$$4\pi r = \frac{32000\pi}{r^2}$$

$$r^3 = 8000$$

$$r = \sqrt[3]{8000}$$

$$r = 20$$

For $r = 20$,

r	20^-	20	20^+
$\dfrac{dS}{dr}$	$-$	0	$+$
Slope of curve			

$\therefore S$ is minimum when $r = 20$.

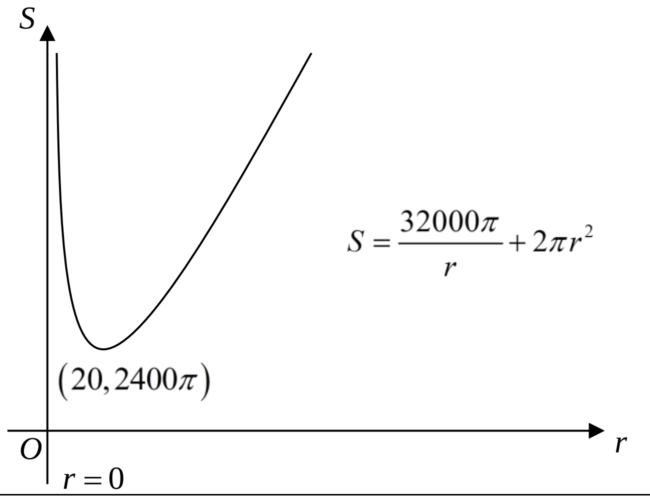
When $r = 20$,

$S = \dfrac{32000\pi}{20} + 2\pi(20)^2$

$= 1600\pi + 800\pi$

$= 2400\pi \text{ cm}^2$ or 7540 cm^2 (3s.f)

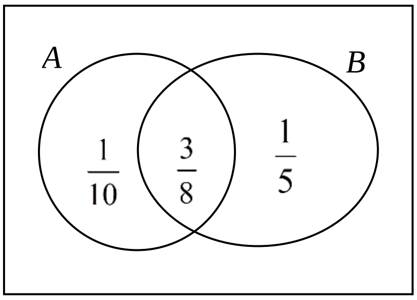
(iii)



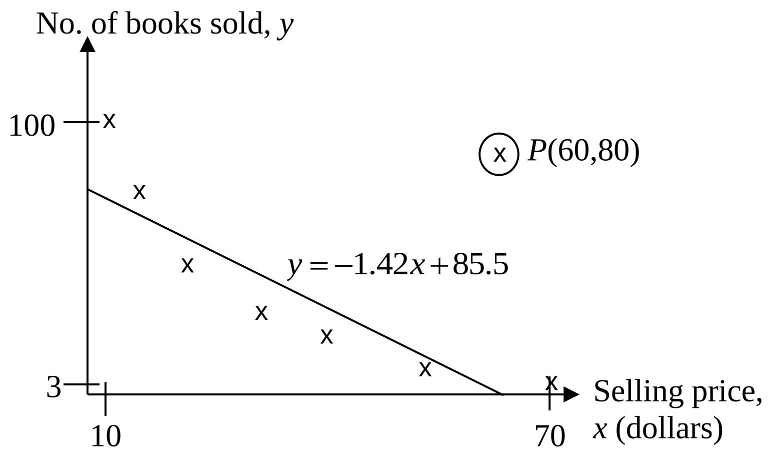
(iv)	Given: $\frac{\mathrm{d}V}{\mathrm{d}t} = 1000 \text{ cm}^3/\text{min}$ $V = \pi r^2 h$ $V = \pi (20)^2 h$ $V = 400\pi h$ $\frac{\mathrm{d}V}{\mathrm{d}h} = 400\pi$ Chain rule: $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \times \frac{\mathrm{d}h}{\mathrm{d}V}$ $= (1000) \times \frac{1}{400\pi}$ $= 0.796 \text{ cm / min } (3 \text{ s.f})$

Qn	Solution
6	**Permutations and Combinations**
(i)	Number of ways = $5! = 120$
(ii)	Case 1 (first digit 3 or 7) Number of ways $= 2 \times 2 \times 3! = 24$ Case 2 (first digit 6) Number of ways $= 3 \times 3! = 18$ Total number of ways $= 24 + 18 = 42$
	Number of ways = $5^4 \times 2 = 1250$

Qn	Solution
7	**Binomial Distribution**
(i)	Let X be the number of mugs, out of 50, that are defective. $X \sim \mathrm{B}(50, p)$ $\mathrm{P}(X = 0 \text{ or } 1) = 0.15$ $\mathrm{P}(X = 0) + \mathrm{P}(X = 1) = 0.15$ $\binom{50}{0} p^0 (1-p)^{50} + \binom{50}{1} p^1 (1-p)^{49} = 0.15$ $(1-p)^{50} + 50p(1-p)^{49} = 0.15$ Using GC, $p = 0.0659$ (3 s.f.)
(ii)	**Method A:** Let Y be the number of mugs, out of 10, that are defective. $Y \sim \mathrm{B}(10, 0.06)$ $\mathrm{P}(\text{accept the batch})$ $= \mathrm{P}(Y < 3)$ $= \mathrm{P}(Y \le 2)$ $= 0.981$ (3 s.f) **Method B:** Let W be the number of mugs, out of 5, that are defective. $W \sim \mathrm{B}(5, 0.06)$ $\mathrm{P}(\text{accept the batch})$ $= \mathrm{P}(W = 0) + \mathrm{P}(W = 1)\mathrm{P}(W < 2)$ $= \mathrm{P}(W = 0) + \mathrm{P}(W = 1)\mathrm{P}(W \le 1)$ $= 0.73390 + (0.23422)(0.96813)$ $= 0.961$ (3 s.f) Since the **probability** of accepting the batch is **higher for Method A**, the manufacturer should adopt **<u>Method A</u>** to carry out the quality control test as it will shorten the process of the quality control test. Since the **probability** of accepting the batch is **lower for Method B**, the manufacturer should adopt **<u>Method B</u>** to carry out the quality control test as it ensures a more stringent quality control test.

Qn	Solution
8	**Probability**
(a)	$\text{P}(A\|B')=\frac{4}{17}$ $\frac{\text{P}(A\cap B')}{\text{P}(B')}=\frac{4}{17}$ $\text{P}(A\cap B')=\frac{4}{17}\left(1-\frac{23}{40}\right)$ $=\frac{1}{10}$  $\text{P}(A'\cap B')=1-\text{P}(A\cup B)$ $=1-\left(\frac{1}{10}+\frac{23}{40}\right)$ $=\frac{13}{40}$
	$\text{P}(A\cap B)=\frac{3}{8}$ $\text{P}(A)=\frac{1}{10}+\frac{3}{8}=\frac{19}{40}$ $\text{P}(A)\text{P}(B)=\left(\frac{19}{40}\right)\left(\frac{23}{40}\right)=\frac{437}{1600}$ Since $\text{P}(A\cap B)\neq\text{P}(A)\text{P}(B)$, A and B are not independent. (Alternative method) $\text{P}(A)=\frac{1}{10}+\frac{3}{8}=\frac{19}{40}$ $\text{P}(A\|B')=\frac{4}{17}$ Since $\text{P}(A\|B')\neq\text{P}(A)$, A and B' are not independent events, therefore A and B are not independent events.

(b) (i)	Let X be the event that a randomly chosen durian is MSW and it is infested with maggots. Let Y be the event that a randomly chosen durian is MSW and it is not infested with maggots. P(both are MSW durians and at least one durian is infested with maggots) $=\mathrm{P}(XY)+\mathrm{P}(YX)+\mathrm{P}(XX)$ $=\left(\frac{4}{35}\right)\left(\frac{11}{34}\right)\times 2+\left(\frac{4}{35}\right)\left(\frac{3}{34}\right)$ $=\frac{10}{119}$
(ii)	$\mathrm{P}(\text{at least one durian is infested with maggots})$ $=1-\mathrm{P}(\text{both durians are not infested with maggots})$ $=1-\left(\frac{28}{35}\right)\left(\frac{27}{34}\right)$ $=\frac{31}{85}$ Required probability $=\frac{\frac{10}{119}}{\frac{31}{85}}$ $=\frac{50}{217}$

Qn	Solution
9	**Correlation and Regression**
(i), (ii), (iii)	
(iii)	$r = -0.869$ $y = -1.4209x + 85.513$ $= -1.42x + 85.5$ (3 s.f.)
(iv)	When $x = 75$, $y = -1.4209(75) + 85.513$ $= -21.1$(3 s.f.) Since $x = 75$ lies outside the data range of *x*, the estimated value of *y* is not reliable since the linear relationship between *x* and *y* may not longer holds. Or Since the value of *y* (the number of books sold) cannot be negative, the estimated value of *y* is not reliable.
(v)	For a linear model, the number of books sold might fall below zero, hence a linear model might not be appropriate. From the scatter diagram, as the selling price of each book (*x*) increases, the number of books sold (*y*) decreases at a decreasing rate. Thus, a linear model might not be appropriate.

Qn	Solution
10	**Hypothesis Testing**
(a)	Let X be the decrease in cholesterol level for a randomly chosen volunteer (in mg/dL) Let μ denote the population mean decrease in cholesterol level in volunteers (in mg/dL) Since n = 80 is large, by Central Limit Theorem, $\bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$ approximately. Using GC, $\bar{x} = 19.7375$ (exact), $s^2 = 1.7628^2$ (5s.f) H_0: $\mu = 20$ H_1: $\mu < 20$ Test statistic: $Z = \frac{\bar{X} - \mu}{S/\sqrt{n}}$ Level of significance: 5% Reject H_0 if $p-\text{value} < 0.05$ Under H_0, using GC, $p-\text{value} = 0.0914$ (3 s.f) Conclusion: Since $p-\text{value} = 0.0914 > 0.05$, we **do not reject H_0** and conclude that there is **insufficient evidence**, at the 5% significance level, that the mean decrease in cholesterol level is less than 20 mg/dL. Thus, the chocolate company's claim is valid at 5% level of significance.
(b)(i)	Unbiased estimate of μ is $\bar{x} = \frac{185}{60} = 3.08$ (3 s.f) Unbiased estimate of σ^2 is $s^2 = \frac{1}{59}\left[626 - \frac{185^2}{60}\right] = \frac{667}{708} = 0.942$ (3 s.f)
(ii)	Since n = 60 is large, by Central Limit Theorem, $\bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$ approximately H_0: $\mu = \mu_0$ H_1: $\mu \neq \mu_0$ Test statistic: $Z = \frac{\bar{X} - \mu}{S/\sqrt{n}}$ Level of significance: 4% Reject H_0 if $z-\text{value} < -2.0537$ or $z-\text{value} > 2.0537$

	Under H_0, $z-\text{value}=\dfrac{3.0833-\mu_0}{\sqrt{0.94209/60}}$ Since the principal is confident that he did not indicate wrongly the mean time that the children took for afternoon naps at 4% level of significance, H_0 is not rejected. $-2.0537<z-\text{value}<2.0537$ $-2.0537<\dfrac{3.0833-\mu_0}{\sqrt{0.94209/60}}<2.0537$. $2.83<\mu_0<3.34 \quad (3\text{s.f})$

Qn	Solution
11	**Normal and Sampling Distribution**
(i)	Let X be the weight of a randomly chosen Black Tilapia fish, in grams. $X \sim \mathrm{N}\left(\mu, 32^2\right)$ Given $\mathrm{P}\left(X > 541\right) = 0.10$ Standardizing, $Z \sim \mathrm{N}(0,1)$ $\mathrm{P}\left(X > 541\right) = 0.10$ $1 - \mathrm{P}\left(X \le 541\right) = 0.10$ $\mathrm{P}\left(X \le 541\right) = 0.9$ $\frac{541-\mu}{32} = 1.28155$ $\therefore \mu = 500$ (nearest gram)
(ii)	$X \sim \mathrm{N}\left(500, 32^2\right)$ $\mathrm{P}(440 < X < 550) \times \mathrm{P}(440 < X < 550) \times \mathrm{P}(X > 550) \times \frac{3!}{2!}$ $= \left(0.91052\right)^2 \times \left(0.059085\right) \times 3$ $= 0.147$ (3 s.f)
(iii)	$X \sim \mathrm{N}\left(500, 32^2\right)$ $\frac{a}{1000}\left(X_1 + X_2\right) \sim \mathrm{N}\left(\frac{a}{1000}(2)\left(500\right), \left(\frac{a}{1000}\right)^2 (2)(32^2)\right)$ $\frac{a}{1000}\left(X_1 + X_2\right) \sim \mathrm{N}\left(\mathrm{a}, 0.002048a^2\right)$ $\mathrm{P}\left(\frac{a}{1000}\left(X_1 + X_2\right) < 6.9\right) < 0.84241$ $\mathrm{P}\left(Z < \frac{6.9-a}{\sqrt{0.002048a^2}}\right) < 0.84241$ $\frac{6.9-a}{\sqrt{0.002048a^2}} < 1.00441$ $6.9 - a < 0.045455a$ $1.045455a > 6.9$ $a > 6.60$ (3 s.f) Alternatively: $\mathrm{P}\left(\frac{a}{1000}\left(X_1 + X_2\right) < 6.9\right) < 0.84241$ Using GC, $\therefore a > 6.60\,(3\,\text{s.f})$
(iv)	Let Y be the weight of a randomly chosen Grey Mullet fish, in grams. $Y \sim \mathrm{N}\left(800, 50^2\right)$ Let $T = \frac{X_1 + X_2 + Y_1 + Y_2 + Y_3}{5} \sim \mathrm{N}\left(\frac{1}{5}\left(2(500) + 3(800)\right), \ \frac{1}{5^2}\left(2(32^2) + 3(50^2)\right)\right)$

$T \sim \mathrm{N}\left(680,\ 381.92\right)$

$\mathrm{P}(T > 690) = 0.304$ (3 s.f)

Alternatively:

$T = X_1 + X_2 + Y_1 + Y_2 + Y_3 \sim \mathrm{N}\left(2(500) + 3(800),\ 2(32^2) + 3(50^2)\right)$

$T \sim \mathrm{N}\left(3400,\ 9548\right)$

$\mathrm{P}(T > 690 \times 5) = \mathrm{P}(T > 3450) = 0.304$ (3 s.f)