15

CATHOLIC JUNIOR COLLEGE
General Certificate of Education Advanced Level
Higher 2
JC2 Preliminary Examination

CANDIDATE NAME

CLASS

INDEX NUMBER

MATHEMATICS

Paper 1

9758/01
31 August 2021
3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Question	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**
Marks											
Total	4	7	9	10	10	11	12	13	12	12	**100**

This document consists of 28 printed pages, including this cover page.

1 A curve has equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \begin{cases} \sqrt{8x+1} & \text{for } 0 \le x \le 3, \\ 11-2x & \text{for } 3 < x \le 5, \end{cases}$$

and $\mathrm{f}(x) = \mathrm{f}(x-5)$ for all real values of x.

(i) Sketch the curve for $-2 \le x \le 8$. [2]

(ii) On a separate diagram, sketch the curve with equation $y = \mathrm{f}\left(\frac{1}{2}x - 1\right)$, for $-2 \le x \le 8$. [2]

2 Referred to the origin O, points A and B have position vectors **a** and **b** respectively, where **a** and **b** are non-zero and non-parallel vectors. Point C lies on AB, such that $AC : CB = 3:1$.

(i) Express $\overrightarrow{OC}$ in terms of **a** and **b**. [1]

Point D lies on OB produced such that $OD : BD = \lambda : \mu$, where $\lambda > \mu > 0$.

(ii) By finding $\overrightarrow{OD}$, show that the area of triangle OCD can be written as $k|\mathbf{a} \times \mathbf{b}|$, where k is a constant in terms of λ and μ. [3]

(iii) It is given that **b** is a unit vector and area of triangle OCD is 2. Find the shortest distance between C and the line OB, giving your answer in terms of λ and μ. [3]

3 The curve $y = \frac{1}{x}$ is transformed onto the curve with equation $y = \mathrm{f}(x)$. The vertical and horizontal asymptotes of $y = \mathrm{f}(x)$ are $x = a$ and $y = b$ respectively. The curve $y = \mathrm{f}(x)$ passes through the point with coordinates $(0, c)$. It is given that $\mathrm{f}(x)$ has the form $p + \frac{k}{x-q}$ where k, p and q are real constants and that a, b and c are positive real constants.

(i) Express k, p and q in terms of a, b and/or c. [2]

The curve $y = \mathrm{f}(x)$ also passes through the point with coordinates $(d, 0)$ where d is a positive real constant with $a > d$ and $b > c$. By sketching the curve $y = \mathrm{f}(x)$,

(ii) sketch the curve $y = \frac{1}{\mathrm{f}(x)}$, stating, in terms of a, b, c and d, the coordinates of any points where $y = \frac{1}{\mathrm{f}(x)}$ crosses the axes and the equations of any asymptotes clearly. [4]

(iii) write down, in terms of a and d, the range of values of x for $\mathrm{f}(x) < 0$. Hence or otherwise, solve the inequality $\mathrm{f}(|x| + d) < 0$. [3]

4 A sequence $u_1, u_2, u_3, \ldots$ is such that $u_n = \dfrac{1}{an^2+bn+c}$, where a, b and c are real constants and $n \geq 1$.

(i) Given that $u_1 = \dfrac{1}{3}$, $u_2 = \dfrac{1}{15}$ and $u_3 = \dfrac{1}{35}$, find u_{10}. [4]

It is now given that $a=4$, $b=0$ and $c=-1$.

(ii) By expressing u_n in the form $\dfrac{P}{2n-1}+\dfrac{Q}{2n+1}$, where P and Q are real constants, find $\sum_{r=1}^{n} u_r$ in terms of n. [4]

(iii) Hence find $\sum_{r=2}^{\infty} u_r$. [2]

5 **(i)** Describe a series of transformations which transforms the graph of

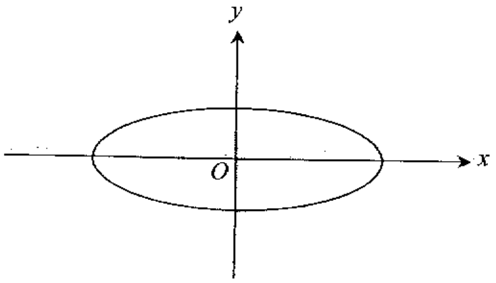
$$(x-1)^2+y^2=1$$

onto the graph of

$$\frac{x^2}{3^2}+\frac{y^2}{2^2}=1.$$ [3]

The diagram below shows the graph of an ellipse with equation

$$\frac{x^2}{3^2}+\frac{y^2}{2^2}=1.$$



(ii) Show that the area of the ellipse can be expressed as

$$k\int_0^3 \sqrt{1-\frac{x^2}{9}}\,\mathrm{d}x\ ,$$

where k is a constant to be determined. [2]

(iii) Hence, using the substitution $x = 3\sin\theta$, find the exact area of the ellipse. [5]

[Turn Over

6 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{1+x^2}=\frac{x}{\sqrt{1+x^2}}$.

Hence, find $\int \frac{3x^3}{\sqrt{1+x^2}}\,\mathrm{d}x$. [4]

(b) It is given that $\mathrm{f}(x)=\cos 2mx+\cos 2nx$, where m and n are positive integers and $m \neq n$.

(i) Find $\int \cos 2mx \cos 2nx\,\mathrm{d}x$. [3]

(ii) Find $\int_0^{\pi}\left(\mathrm{f}(x)\right)^2\,\mathrm{d}x$. [4]

7 A curve C has parametric equations

$$x=t^3-4t+1 \quad \text{and} \quad y=\mathrm{e}^t-1 \quad \text{where } 0 \leq t \leq \lambda.$$

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t and hence show that the equation of normal at the point $t=1$ is

$$y=\frac{1}{\mathrm{e}}x+\frac{2+\mathrm{e}^2-\mathrm{e}}{\mathrm{e}}.$$ [3]

(ii) The tangent at the point $\left(\lambda^3-4\lambda+1,\ \mathrm{e}^{\lambda}-1\right)$ on C is parallel to the y-axis. Find the exact value of λ. [2]

(iii) Using the value of λ found in part **(ii)**, sketch C, indicating clearly the coordinates of the end-point(s). [2]

(iv) Show that

$$\int_{-2}^{1} y^2\,\mathrm{d}x=\int_1^k\left(\mathrm{e}^t-1\right)^2\left(3t^2+b\right)\mathrm{d}t,$$

where b and k are constants to be determined. [2]

(v) Hence, find the volume of the solid obtained when the region bounded by the curve C, the x-axis and the normal at $t=1$ is rotated 2π radians about the x-axis. [3]

8 The line l_1 has equation $\frac{x-2}{4}=\frac{3-y}{2}=\frac{z+1}{3}$ and the line l_2 has equation $\mathbf{r}=(5+\mu)\mathbf{i}+(4-\mu)\mathbf{j}-(2+2\mu)\mathbf{k}$ where $\mu \in \mathbb{R}$.

(i) Determine whether the lines l_1 and l_2 are parallel, intersecting or skew. [3]

It is given that point B lies on line l_2. The position vector of the point B is such that $\mu=2$.

(ii) Find a cartesian equation of the plane p that includes the point B and the line l_1. [3]

(iii) Find the acute angle between p and l_2. [3]

(iv) The plane Π is parallel to plane p and contains the point $W(3,5,k)$. Given that the distance between Π and p is $\frac{34}{\sqrt{1430}}$, find the possible values of k. [4]

9 Alex and Gopal embark on a journey, travelling the same route separately. Alex travels 1000 km on Day 1 and for each subsequent day, he travels 5 km less than the previous day. Gopal starts his journey on Day 5, travelling 1500 km on that day and for each subsequent day, he travels 2% less than the previous day.

(i) Find the distance travelled by Alex on Day 10. [1]

(ii) Show that the distance travelled by Gopal on Day 10 is 1356 km, correct to the nearest whole number. [2]

(iii) Given that Alex completes the journey by the end of Day 100, determine if Gopal is able to complete the journey no matter how many days he takes. [4]

It is given that the total distance travelled by Gopal at the end of Day n exceeds the total distance travelled by Alex at the end of Day $n-1$.

(iv) Find the least value of n and the total distance travelled by Gopal at the end of this day, giving your answer correct to the nearest whole number. [5]

10 The game designer for a video game is running a simulation of a particular stage where the Hero is trapped underground and is trying to make his way back up to the ground level. The Hero can only move vertically upwards. At time t minutes, his distance from the starting point is x units.

The Hero starts to move vertically upwards from the starting point at a rate inversely proportional to x. However, he encounters traps and monsters along the way, which will push him vertically downwards at a rate proportional to x.

(i) Show that the Hero's movement can be modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{a-kx^2}{x},$$

where a and k are positive constants. [1]

During the simulation, the game designer sees k as the "difficulty factor" and sets $a=8$.

(ii) Show that

$$x=\sqrt{\frac{8-8\mathrm{e}^{-2kt}}{k}}.$$ [5]

(iii) If the simulation is left to run for a long time, deduce the eventual position of the Hero. [2]

(iv) Hence, sketch the graph of x against t. [2]

[2]

[2]

(v) If the ground level is supposed to be 4 units above the Hero's starting point, what is the range of values that the game designer should set for the "difficulty factor" such that the Hero is able to make it to the ground level and complete the stage? [2]

- END -

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General Certificate of Education Advanced Level
Higher 2
JC2 Preliminary Examination

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MATHEMATICS

Paper 2

9758/02
15 September 2021
3 hours

Candidates answer on the Question Paper.

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[Turn Over

Section A: Pure Mathematics [40 marks]

1 A particular brand of power bank charges up to its rated voltage of 5 V (volts) according to the equation

$$V = \frac{1}{0.2 + \mathrm{e}^{-2t}} \quad \text{for } t \geq 0,$$

where t is measured in hours.

(i) Find an expression for the rate of charging. [1]

(ii) Without the use of a calculator, find the time when the rate of charging is most rapid. [4]

(iii) With a sketch of a suitable graph, explain if the power bank can be fully charged to 5 V . [2]

2 By successively differentiating $x\ln(2+x)$, find the first three non-zero terms in the Maclaurin series for $x\ln(2+x)$. [4]

Hence, or otherwise, find the first three non-zero terms in the Maclaurin expansion of $x\ln(2-x)$. [2]

Deduce the first two non-zero terms in the Maclaurin expansion of $x\ln(4-x^2)$. [2]

3 **Do not use a calculator in answering this question.**

It is given that z_1 and z_2 are the roots of the equation $z^2 - 2\mathrm{i}z - 2 = 0$, where $\arg(z_1) < \arg(z_2)$.

(i) Show that $z_1 = 1 + \mathrm{i}$. [3]

(ii) Given that $x = z_1$, find x^2, x^3 and x^4 in cartesian form. Given also that $x = z_1$ is a root of the equation $x^4 - 6x^3 + sx^2 - 18x + 10 = 0$, where s is real, find the value of s and the other roots of the equation. [6]

(iii) The complex conjugate of z_1 is denoted by $z_1{}^*$. Given that $\frac{z_1{}^n}{z_1{}^*}$ is purely imaginary, find the two smallest positive integers of n. [3]

4 The functions f and g are defined by

$$\mathrm{f}: x \mapsto 15-2x-x^2, \quad x \in \mathbb{R}, x < a$$

$$\mathrm{g}: x \mapsto \mathrm{e}^{-x}-4, \quad x \in \mathbb{R}, x \geq 0$$

where a is a real constant.

(i) State the largest value of a such that the inverse function f^{-1} exists. [1]

In the rest of the question, use $a=-2$.

(ii) Find $\mathrm{f}^{-1}(x)$ and state the domain for f^{-1}. [4]

(iii) On the same diagram, sketch the graphs of $y=\mathrm{f}(x)$ and $y=\mathrm{f}^{-1}(x)$, stating the coordinates of the end points and axial intercepts. [3]

(iv) Explain why the x-coordinate of the point of intersection of the curves in part **(iii)** satisfies the equation

$$x^2+3x-15=0.$$ [1]

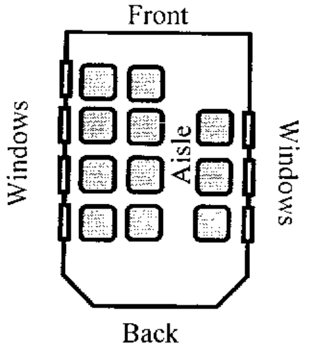
(v) Find an expression for $\mathrm{fg}(x)$ and state the domain for fg. [2]

(vi) Find the range of fg. [2]

[Turn Over

Section B: Statistics [60 marks]

5



A large private plane has 11 seats for passengers. The seats are arranged in 3 rows of 3 seats and 2 front seats as seen in the diagram above. A family of 10 boards the plane.

(i) Find the number of possible seating arrangements for the family of 10. [1]

The family of 10 consists of 2 married couples, Mr and Mrs Lim and Mr and Mrs Tan, 4 children and 2 grandparents.

(ii) Find the number of possible seating arrangements if the

- 2 grandparents sit in the front row,
- 4 children each sits at a window seat,
- Mr and Mrs Lim sit in the same row on the same side of the aisle,
- Mr and Mrs Tan sit in another row on the same side of the aisle. [3]

(iii) Find the probability that Mr Lim sits directly behind a child, Mr Tan sits in the front row and the others do not mind where they sit. [3]

6 As part of its customer loyalty program, Sugureta Sushi allows customers to spin the wheel once for every \$100 they spend. There are four different equally likely outcomes, namely \$10 discount, \$5 discount, \$2 discount and "Better luck next time!"

Each discount has three equally likely multiplier sub-segments, namely $\times 1$, $\times 2$ and $\times 3$. For example, if a customer spins a \$5 discount with $\times 2$ sub-segment, he will get \$10 off his bill.

Let Y be the random variable denoting the total discount a customer who spends \$100 can obtain.

(i) Tabulate the probability distribution of Y. [2]

(ii) Find $\mathrm{E}(Y)$ and $\mathrm{Var}(Y)$. [3]

(iii) Find the probability that a customer who spends \$200 pays \$155 or less after the discount. [3]

7 - Common Last Topic -

8 **In this question you should state clearly all the distributions that you use, together with the values of the appropriate parameters.**

A supermarket stocks two different brands of toilet paper, namely Cleanex and Woofsoft. The masses, in grams, of rolls of Cleanex and Woofsoft toilet paper are modelled as having independent normal distributions with means and standard deviations as shown in the table.

Toilet Paper	Mean Mass	Standard Deviation
Rolls of Cleanex	200	10
Rolls of Woofsoft	220	15

(i) Find the probability that the mass of a randomly chosen roll of Cleanex toilet paper is between 202 and 208 grams. [1]

The rolls of toilet paper are sold in packs of 10.

(ii) Find the probability that the mass of a randomly chosen pack of Woofsoft toilet paper would exceed the mass of a randomly chosen pack of Cleanex toilet paper by 5%. [3]

(iii) The probability that a randomly chosen pack of Cleanex and 3 randomly chosen packs of Wooftsoft have total mass less than k grams is 0.4. Find the value of k. [3]

To meet the increasing demand for toilet paper due to the Covid-19 pandemic, the supermarket decides to stock a third brand of toilet paper, Upwards.

(iv) The masses of rolls of Upwards toilet paper are normally distributed such that 4% of them have a mass less than 220 grams and 80% of them have a mass greater than 230 grams. Find the mean and standard deviation of the masses of rolls of Upwards toilet paper. [4]

9 A company employs experienced and novice telesales executives. Their responsibilities include calling existing and potential customers to persuade them to purchase their company products and services. A call that results in a sale is considered a successful call.

On average, 15% of the calls made by an experienced telesales executive are successful. An experienced telesales executive handles 15 calls a day.

(i) State, in context, two assumptions needed for the number of successful calls made by an experienced telesales executive to be well modelled by a binomial distribution. [2]

Assume now that the number of successful calls made by an experienced telesales executive follows a binomial distribution.

(ii) Find the probability that Tom, an experienced telesales executive, makes more than 4 successful calls a day. [2]

(iii) The number of successful calls made by an experienced telesales executive in a day is independent of other days. Find the probability that, in a randomly chosen working week of 5 days, Tom makes more than 4 successful calls on no more than 2 days. [2]

The number of successful calls made by a novice telesales executive also follows a binomial distribution. The probability of a successful call made by a novice telesales executive is 0.09. A novice telesales executive handles 9 calls a day. Jerry is a novice telesales executive in the company.

(iv) Find the probability that Tom and Jerry make 2 sales each in a day. [2]

(v) Find the probability that Tom and Jerry make a total of 3 sales in a day given that Jerry makes at least two sales in a day. [3]

(vi) State an assumption for your calculations in parts **(iv)** and **(v)** to be valid. [1]

10 The Simozer vaccine developed by a pharmaceutical company is marketed to have a 91% efficacy rate against Covid-19, measured from seven days to six months after the completion of doses. The government of the country Elbonia is interested in purchasing this vaccine for its people. It decides to test a random sample of 100 citizens and measure the efficacy rate, x%. The results obtained are summarised as follows.

$$\sum x = 9052 \qquad \sum x^2 = 962373$$

(i) Explain the meaning of 'a random sample' in the context of the question. [1]

(ii) Calculate unbiased estimates of the population mean and variance of the efficacy rate. [2]

(iii) Test, at the 5% level of significance, if the company has overstated the vaccine's efficacy rate. You should state your hypotheses and define any symbols you use. [5]

Three months later, the government decides to check the efficacy rate for the same 100 citizens. It found that the mean efficacy rate is 90.2% and the standard deviation was m%. A test is carried out at the 1% level of significance.

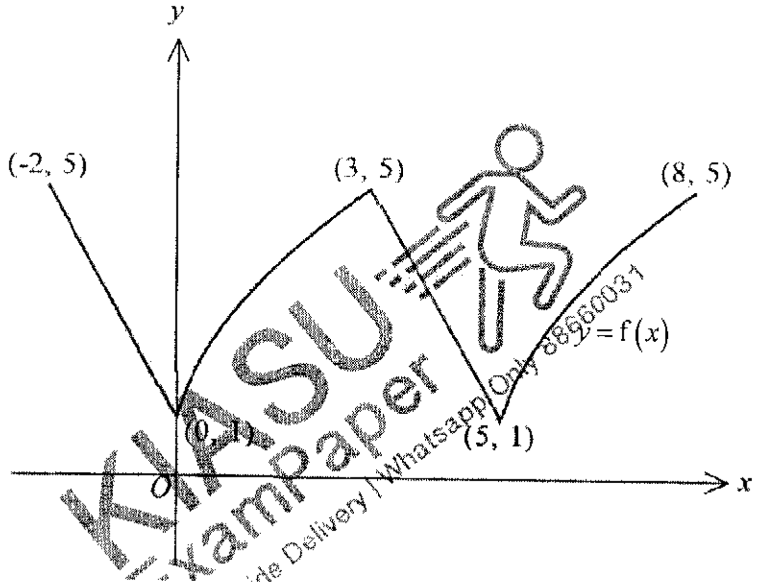
(iv) Find the range of values of m if there is sufficient evidence supporting the claim that the efficacy rate was overstated by the pharmaceutical company. [4]

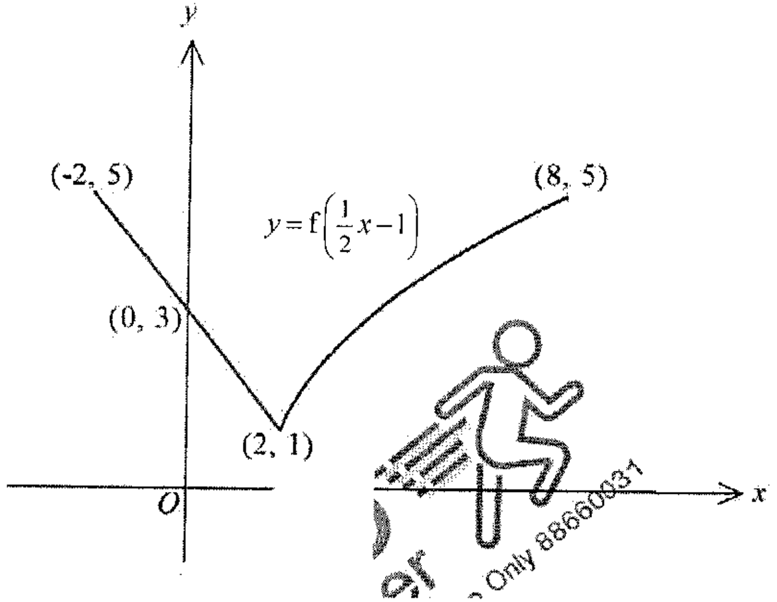
(v) Explain why there is no need for the government of Elbonia to know anything about the population distribution of the vaccine's efficacy rate. [2]

- END -

CATHOLIC JUNIOR COLLEGE
H2 MATHEMATICS
2021 JC2 PRELIMINARY EXAMINATION (PAPER I)

Q1. Functions + Transformations

Assessment Objectives	Solution		Examiner Feedback
Sketch a periodic piecewise function	(i)		Quite a number of students did not label the coordinates of the points on the graph. Some students just draw dotted lines from the x and y axes and expect the examiners to interpret the coordinates of the points! Please do not expect the examiners to look for the answers. Some of the graphs with points $(-2,5),(3,5)$ and $(8,5)$ not levelled which should not be the case, similarly with the points $(0,1)$ and $(5,1)$. Some students put ●○ one closed and opened circle at the points $(0,1),(3,5)$ and $(5,1)$ which should not happen since a point can only be included or excluded, **not both**.

Sketch periodic, piecewise graph after a sequence of 2 transformations for a specific interval [HOT]	(ii)	 $y = \mathrm{f}(x)$ ↓ translate 1 unit in the positive x-direction $y = \mathrm{f}(x-1)$ ↓ scale parallel to x-axis by a scale factor of 2 $y = \mathrm{f}\left(\frac{1}{2}x-1\right)$	Quite a lot of students either leave blank or do not know how to proceed. The question requested for the graph to be drawn in the range of $-2 \le x \le 8$, however, quite a number of students drawn for $-2 \le x \le 18$. Quite a number of students have the point $(-2,3)$ instead of $(-2,5)$.

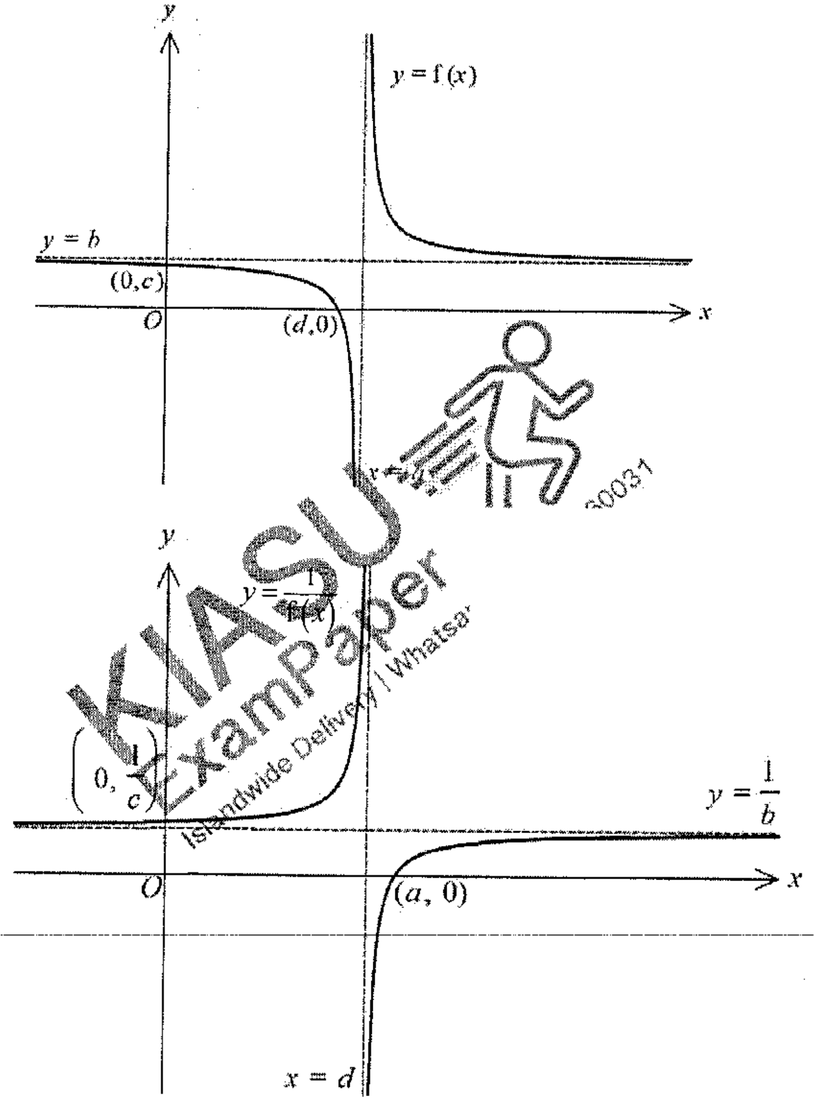
Q2. Vectors (Basic)

Assessment Objectives	Solution		Examiner Feedback
Interpret the question and apply Ratio Theorem	(i)	Using Ratio Theorem, $\overrightarrow{OC}=\frac{\underset{\sim}{a}+3\underset{\sim}{b}}{3+1}$ $=\frac{1}{4}(\underset{\sim}{a}+3\underset{\sim}{b})$	Generally well done except for a few careless mistakes.
Application of Ratio Theorem for ratios involving unknowns **[HOT]** Use 2 appropriate vectors for Cross Product to find area of a triangle. Apply properties of Cross Product in the simplification process to show the given result.	(ii)	$\frac{\overrightarrow{OD}}{\overrightarrow{OB}}=\frac{\lambda}{\lambda-\mu}\Rightarrow\overrightarrow{OD}=\frac{\lambda}{\lambda-\mu}\overrightarrow{OB}$ $=\left(\frac{\lambda}{\lambda-\mu}\right)\underset{\sim}{b}$  Area of triangle $OCD=\frac{1}{2}\left\|\overrightarrow{OC}\times\overrightarrow{OD}\right\|$ $=\frac{1}{2}\left\|\frac{1}{4}(\underset{\sim}{a}+3\underset{\sim}{b})\times\left(\frac{\lambda}{\lambda-\mu}\right)\underset{\sim}{b}\right\|$ $=\frac{1}{2}\left(\frac{1}{4}\right)\left(\frac{\lambda}{\lambda-\mu}\right)\left\|(\underset{\sim}{a}\times\underset{\sim}{b})+(3\underset{\sim}{b}\times\underset{\sim}{b})\right\|$ $=\frac{1}{8}\left(\frac{\lambda}{\lambda-\mu}\right)\left\|\underset{\sim}{a}\times\underset{\sim}{b}\right\|$, since $\underset{\sim}{b}\times\underset{\sim}{b}=0$ (shown)	Common mistakes are: • $\overrightarrow{OD}=\frac{\lambda}{\lambda+\mu}\overrightarrow{OB}$ • $\overrightarrow{OD}=k\,\overrightarrow{OB}$ • $\overrightarrow{OD}=\lambda\,\overrightarrow{OB}$ • $\underset{\sim}{b}=\lambda-\mu$ • $\underset{\sim}{d}=\underset{\sim}{b}+\mu$ • $\underset{\sim}{b}\times\underset{\sim}{b}=\left\|\underset{\sim}{b}\right\|^2=0$ Most students know that Area of triangle OCD $=\frac{1}{2}\left\|\overrightarrow{OC}\times\overrightarrow{OD}\right\|$, however, due to the above mistakes, they could not get the correct answers.

Find the shortest distance (perpendicular distance) between a point and a given vector involving unknown values Interpret that the magnitude of a unit vector is 1	**(iii)**	**Method ①:** Let the shortest distance between C and line OB be x. Area of $\Delta OCD = 2$ $\frac{1}{2}x\left\|\overline{OD}\right\| = 2$ $\frac{1}{2}x\left\|\frac{\lambda}{\lambda-\mu}b\right\| = 2$ $\frac{1}{2}x\left(\frac{\lambda}{\lambda-\mu}\right)\|b\| = 2$ $\left(\because \|b\| = 1 \text{ and } \frac{\lambda}{\lambda-\mu} > 0\right)$ $x = 4\left(\frac{\lambda-\mu}{\lambda}\right)$ **Method ②:** Shortest distance between C and **b** = $\left\|\overrightarrow{OC}\times\hat{b}\right\|$ $= \left\|\frac{1}{4}(a+3b)\times\frac{b}{\|b\|}\right\|$ $= \frac{1}{4}\left(\frac{1}{\|b\|}\right)\|(a+3b)\times b\|$ $= \frac{1}{4}\left(\frac{1}{\|b\|}\right)\|(a\times b)+(3b\times b)\|$ $= \frac{1}{4}\left(\frac{1}{\|b\|}\right)\|a\times b\|$ Given: **b** is a unit vector and $\frac{1}{8}\left(\frac{\lambda}{\lambda-\mu}\right)\|a\times b\| = 2$	Quite a number of students assumed that the height of the triangle OCD is BC, which is wrong. Quite of number of students wrote $\hat{b} = 1$, which is wrong since $\hat{b}$ is a vector. Another common mistake is $\|a\times b\| = -\|b\times a\|$. In fact, $\|a\times b\| = \|b\times a\|$. Most students knew that shortest distance between C and $b = \left\|\overline{OC}\times\hat{b}\right\|$, however, due to the carelessness in algebraic manipulations and misconceptions as mentioned above, they did not get the full marks.
Perform simple algebraic manipulation to obtain $\|a\times b\|$		So $\|b\| = 1$ and $\|a\times b\| = 16\left(\frac{\lambda-\mu}{\lambda}\right)$ Shortest distance between C and **b** $= \frac{1}{4}\left(\frac{1}{1}\right)\left[16\left(\frac{\lambda-\mu}{\lambda}\right)\right] = 4\left(\frac{\lambda-\mu}{\lambda}\right)$	

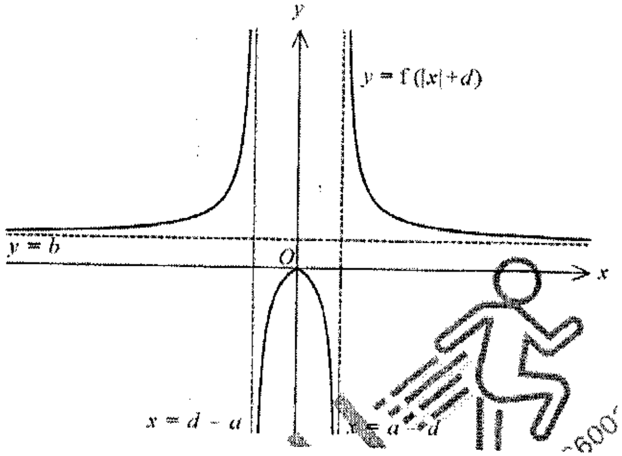
Q3. Graphing Techniques + Transformations + Inequalities

Assessment Objectives	Solution		Examiner Feedback
Use the essential features of a rectangular hyperbola to form equations	**(i)**	For vertical asymptote, $x=a$ and $x=q$ $\therefore q=a$ For horizontal asymptote, $y=b$ and $y=p$ $\therefore p=b$ At $(0,c)$, $c=p-\frac{k}{q}$ $k=q(p-c)$ $\quad =a(b-c)$	For students who attempted this question, they were generally able to find p and q. Common mistakes: • Some students confused the vertical and horizontal asymptotes. • Some students simply assumed $k=c$ when k does not represent the y-intercept in the question. • Omission of negative sign when substituting the point $(0,c)$ into the equation of the curve, resulting in wrong expression of k. • Did not know how to manipulate to find k correctly: $\frac{k}{a}=b-c \not\Rightarrow k=\frac{b-c}{a}$.

Sketch the reciprocal graph of a given function	**(ii)**		Some students did not attempt **(ii)** and **(iii)**, possible due to the presence of many unknowns in **(i)**. Common mistakes: • Omission of the sketch of the graph of $y = \mathrm{f}(x)$, resulting in a loss of marks. • Expressed the axial intercepts and/or equation of asymptotes of $y = \dfrac{1}{\mathrm{f}(x)}$ in terms of p, q and k instead of a, b, c and d. • Sketched the asymptotes $x = d$ and/or $y = \dfrac{1}{b}$ in the negative *x*- and *y*- regions respectively. • Did not write down the equation(s) of asymptotes (simply labelled the *x*/*y*-coordinate). • The graph $y = \dfrac{1}{\mathrm{f}(x)}$ for $x < d$ approached the *y*-axis instead of the horizontal asymptote $y = \dfrac{1}{b}$. • Did not take into account that

			$a > d$ and $b > c$, resulting in wrong shape of the graphs $y = \mathrm{f}(x)$ and $y = \dfrac{1}{\mathrm{f}(x)}$. • Presence of stationary points in the graph of $y = \dfrac{1}{\mathrm{f}(x)}$ when there are no stationary points in the graph of $y = \mathrm{f}(x)$. *There was no penalty in this paper when students did not write down the axial intercepts in coordinates form. However, students should bear in mind to do so during A-Levels to ensure that they will get the credits.*
Solve an inequality using graphical method and substitution	**(iii)**	For $\mathrm{f}(x) < 0$, $d < x < a$.	This first part is well-attempted. Some students included the equality sign despite the strict inequality in $\mathrm{f}(x) < 0$.

		Method ①: Hence For $\mathrm{f}(\lvert x\rvert+d)<0$. $d<\lvert x\rvert+d<a$ $0<\lvert x\rvert<a-d$ $\lvert x\rvert>0$ and $\lvert x\rvert<a-d$ $x>0$ or $x<0$ and $-(a-d)<x<a-d$ $x>0$ or $x<0$ and $-a+d<x<a-d$ $\therefore$ $-a+d<x<0$ or $0<x<a-d$ **Alternatively,** $y=\mathrm{f}(x)$ $\downarrow$ translate d units in the negative x-direction [replace x by $x+d$] $y=\mathrm{f}(x+d)$ $\downarrow$ retain the graph for $x\geq 0$ only and reflect this about y-axis [replace x by $\lvert x\rvert$] $y=\mathrm{f}(\lvert x\rvert+d)$ After the first transformation (translation), $d<x+d<a$ $d-d<x<a-d$ $0<x<a-d$ After the second transformation, $0<\lvert x\rvert<a-d$ $\therefore$ $-a+d<x<0$ or $0<x<a-d$ (by the reflection in y-axis)	Most students either identified the correct replacement (replaced x by $\lvert x\rvert+d$) or used the knowledge on the transformations (by performing translation first) to arrive at $0<\lvert x\rvert<a-d$. A significant percentage of students failed to exclude the value $x=0$.

		Method ②: Otherwise  $d-a<x<0$ or $0<x<a-d$ "OR" $d-a<x<a-d,\ x\neq 0$	There was another group of students who attempted to sketch the graph of $y=\mathrm{f}\left(\|x\|+d\right)$ to solve $\mathrm{f}\left(\|x\|+d\right)<0$. The common mistake is the wrong order of the sequences of transformations (did not perform translation first).

Q4. Sequences and Series/Sigma Notation

Assessment Objectives	Solution		Examiner Feedback
Formulate and solve a system of linear equation.	(i)	$u_1 = \frac{1}{a(1)^2 + b(1) + c} = \frac{1}{3} \quad \Rightarrow \quad a + b + c = 3$ $u_2 = \frac{1}{a(2)^2 + b(2) + c} = \frac{1}{15} \quad \Rightarrow \quad 4a + 2b + c = 15$ $u_3 = \frac{1}{a(3)^2 + b(3) + c} = \frac{1}{35} \quad \Rightarrow \quad 9a + 3b + c = 35$ Using GC, $a = 4,\ b = 0,\ c = -1$. $u_{10} = \frac{1}{4(10)^2 + 0(10) - 1} = \frac{1}{399}$	This part was generally well done. Most students were able to set up 3 equations correctly. However, there was a small percentage of students who assumed the sequence was either A.P. or G.P.. Most students used G.C. to solve the 3 equations. • However, some students did not know how to interpret the value of b found from the G.C. ($-1.73\text{E}-12 = -1.73 \times 10^{-12}$ arising from the algorithm used in solving the equations by GC). Students must know that $b = 0$ in this case, otherwise it will result in a loss of marks. • Some students solved the system of linear equations algebraically. However, there were some students who ended up with complicated equations through substitution rather than by elimination of c from the 3 equations.

Find the summation of a series using partial fraction and the method of differences.	**(ii)**	$u_n = \frac{1}{4n^2-1} = \frac{1}{2}\left[\frac{1}{2n-1} - \frac{1}{2n+1}\right]$ where $P = \frac{1}{2}$ and $Q = -\frac{1}{2}$ $\sum_{r=1}^{n} u_r = \sum_{r=1}^{n} \frac{1}{4r^2-1}$ $= \sum_{r=1}^{n}\left[\frac{1}{2}\left(\frac{1}{2r-1} - \frac{1}{2r+1}\right)\right]$ $= \frac{1}{2}\left[\begin{array}{ccccc} & \frac{1}{1} & - & \cancel{\frac{1}{3}} \\ + & \cancel{\frac{1}{3}} & - & \cancel{\frac{1}{5}} \\ & & \vdots & \\ + & \cancel{\frac{1}{2n-3}} & - & \cancel{\frac{1}{2n-1}} \\ + & \cancel{\frac{1}{2n-1}} & - & \frac{1}{2n+1} \end{array}\right]$ $= \frac{1}{2}\left(1 - \frac{1}{2n+1}\right)$	This was relatively well done though many students chose their own letters when applying partial fractions. Students who encountered issues were those who used P and Q which is the opposite of that stated in the question. It lead to confusion and subsequently they got the values of both letters mixed up. Students must understand that method of difference involves the negative sign. There were students who did not have the negative sign and went on to do cancellation which is impossible.

Apply the change of limits and find the sum to infinity of a series. **[HOT]**	**(iii)**	As $n \to \infty$, $\frac{1}{2n+1} \to 0$, $\sum_{r=1}^{n} u_r \to \frac{1}{2}$ $\sum_{r=2}^{\infty} u_r = \sum_{r=1}^{\infty} u_r - u_1$ $= \frac{1}{2} - \frac{1}{3}$ $= \frac{1}{6}$	A common mistake seen was students applying change of limits in this manner: $\sum_{r=2}^{\infty} u_r = \sum_{r=1}^{\infty} u_r - \sum_{r=1}^{2} u_r$ when it should have been $\sum_{r=2}^{\infty} u_r = \sum_{r=1}^{\infty} u_r - \sum_{r=1}^{1} u_r$. This part of the question clearly state HENCE. However, a handful of students went on to redo method of difference (OTHERWISE), similar to (ii). They were not awarded any credit even though they achieved the final answer.

Q5. Integration/Application

Assessment Objectives	Solution		Examiner Feedback
Describe a series of transformations	**(a)**	$(x-1)^2+y^2=1$ $\downarrow$ Replace y by $\frac{y}{2}$ Scale parallel to y-axis by a scale factor of 2 $(x-1)^2+\left(\frac{y}{2}\right)^2=1$ $\downarrow$ Replace x by $x+1$ Translate 1 unit in the negative x-direction $x^2+\left(\frac{y}{2}\right)^2=1$ $\downarrow$ Replace x by $\frac{x}{3}$ Scale parallel to x-axis by a scale factor of 3 $\left(\frac{x}{3}\right)^2+\left(\frac{y}{2}\right)^2=1$	Many students understood that they should do translation first in the x-direction then following by scaling parallel in the x-axis to minimize complications. Students should put in the effort to learn the proper description for transformations. They should not attempt to use their own description. Many of them then failed to use the necessary key words or got mixed up scaling description with translation description. Common mistakes include • Scale x by 2 units • Transform 1 unit in negative x-direction There is also a habit for students to use shortform which is not allowed in examination such as • u (units?) • sf (scale factor?)

Recall properties of ellipse Setting up integrand to find area	(b)(i)	Since shape is symmetrical about both axes, just consider area in 1st quadrant and multiply by 4 $\frac{x^2}{3^2}+\frac{y^2}{2^2}=1 \Rightarrow y=2\sqrt{1-\frac{x^2}{9}}$ Area of shape $=4\int_0^3 2\sqrt{1-\frac{x^2}{9}}\,\mathrm{d}x$ $=8\int_0^3 \sqrt{1-\frac{x^2}{9}}\,\mathrm{d}x$ (shown)	Students were not careful in reading this part of the question. Almost all students were successful in making y the subject. Students who failed more commonly made algebraic slips in their working. Many student failed to see that since shape is symmetrical about both axes, just consider area in 1st quadrant and multiply by 4.
Procedure for integrating by substitution	(ii)	Area of Ellipse $=8\int_0^3 \sqrt{1-\frac{x^2}{9}}\,\mathrm{d}x$ $=8\int_0^{\frac{\pi}{2}} \sqrt{1-\frac{(3\sin\theta)^2}{9}}\,(3\cos\theta)\,\mathrm{d}\theta$ $=8\int_0^{\frac{\pi}{2}} \sqrt{1-\sin^2\theta}\,(3\cos\theta)\,\mathrm{d}\theta$ $=8\int_0^{\frac{\pi}{2}} 3\cos^2\theta\,\mathrm{d}\theta$ $=8\int_0^{\frac{\pi}{2}} 3\left(\frac{\cos 2\theta+1}{2}\right)\mathrm{d}\theta$ $=12\left[\frac{\sin 2\theta}{2}+\theta\right]_0^{\frac{\pi}{2}}$ $=12\left[0+\frac{\pi}{2}-0\right]$ $=6\pi$ **Substitution:** $x=3\sin\theta$ $\frac{\mathrm{d}x}{\mathrm{d}\theta}=3\cos\theta$ when $x=0,\ \theta=0$ When $x=3,\ \theta=\frac{\pi}{2}$	Students who were able to handle this part of the question did so effortlessly with minimal working. There were some who failed to notice that the question was asking for EXACT area. Students who struggled in this part clearly were not able to perform integration by substation. They were unable to change limits and apply chain rule by introducing $\frac{\mathrm{d}x}{\mathrm{d}\theta}$. A number of students were unable to integrate $\cos^2\theta$. They should apply double angle formula in MF26 then perform the integration.

Q6. Integration/Application

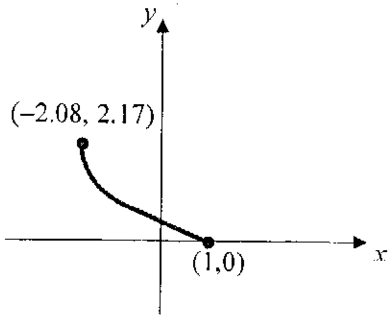
Assessment Objectives	Solution		Examiner Feedback
Recognize the need for Integrating by parts. Ability to identify the "u" and the "$\frac{dv}{dx}$"	**(a)**	$\frac{d}{dx}\sqrt{1+x^2}=\frac{1}{2}\cdot\frac{2x}{\sqrt{1+x^2}}=\frac{x}{\sqrt{1+x^2}}$ (shown) $\int\frac{3x^3}{\sqrt{1+x^2}}\,dx=\int(3x^2)\cdot\frac{x}{\sqrt{1+x^2}}\,dx$ $=(3x^2)\sqrt{1+x^2}-\int(6x)\sqrt{1+x^2}\,dx$ $=(3x^2)\sqrt{1+x^2}-3\int(2x)\sqrt{1+x^2}\,dx$ $=(3x^2)\sqrt{1+x^2}-3\cdot\frac{(1+x^2)^{3/2}}{3/2}+c$ $=(3x^2)\sqrt{1+x^2}-2(1+x^2)^{3/2}$ a By parts: $u=3x^2$ $\frac{dv}{dx}=\frac{x}{\sqrt{1+x^2}}$ $\frac{du}{dx}=6x$ $v=\sqrt{1+x^2}$ (from previous part) Power formula: $\int f'(x)(f(x))^n\,dx=\frac{(f(x))^{n+1}}{n+1}$ $f(x)=1+x^2$ $f'(x)=2x$	First part on differentiation was well done. Many students could not perform integration by parts with the hint from first part. LIATE will not be useful here. For those who have successfully done the by parts, many could not do $\int x\sqrt{1+x^2}\,dx$ and went to do by parts again. They should use power formula: $\int f'(x)(f(x))^n\,dx$ $=\frac{(f(x))^{n+1}}{n+1}$ Some students missed out the constant 3 in their final answer.

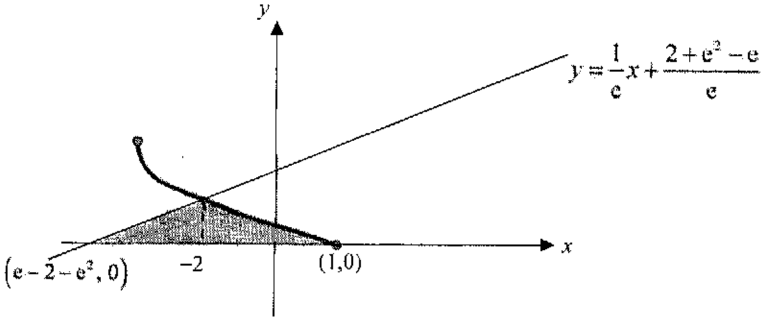
Apply factor formula in integration	(b)(i)	$\int \cos 2mx \cos 2nx \, dx$ $= \frac{1}{2}\int \cos(2mx+2nx)+\cos(2mx-2nx) \, dx$ $= \frac{1}{2}\int \cos(2m+2n)x+\cos(2m-2n)x \, dx$ $= \frac{1}{2}\frac{\sin(2m+2n)x}{2m+2n}+\frac{1}{2}\frac{\sin(2m-2n)x}{2m-2n}+C$ $= \frac{\sin(2m+2n)x}{4m+4n}+\frac{\sin(2m-2n)x}{4m-4n}+C$ From MF26: $\cos P+\cos Q = 2\cos\frac{1}{2}(P+Q)\cos\frac{1}{2}(P-Q)$ $\cos\underbrace{\frac{1}{2}(P+Q)}_{2mx}\cos\underbrace{\frac{1}{2}(P-Q)}_{2nx} = \frac{1}{2}(\cos P+\cos Q)$ $\frac{1}{2}(P+Q)=2mx-(1) \quad \frac{1}{2}(P-Q)=2nx-(2)$ $(1)+(2)$: $P=2mx+2nx$ $(1)-(2)$: $Q=2mx-2nx$ $\therefore \cos 2mx \cos 2nx$ $= \frac{1}{2}(\cos(2mx+2nx)+\cos(2mx-2nx))$	Many did not know that they should use factor formula here. They applied factor formula wrongly. Some missed out the factor $\frac{1}{2}$. Many did not know how to integrate $\int \cos(2m+2n)x \, dx$ as they failed to recognize that $(2m+2n)$ is just a constant say k. $\int \cos kx \, dx = \frac{\sin kx}{k}$

Integration using solution obtained earlier **[HOT]**	**(ii)**	$\int_0^\pi (\mathrm{f}(x))^2 \, \mathrm{d}x$ $= \int_0^\pi (\cos 2mx + \cos 2nx)^2 \, \mathrm{d}x$ $= \int_0^\pi \cos^2(2mx) + 2\cos(2mx)\cos(2nx) + \cos^2(2nx) \, \mathrm{d}x$ $= \int_0^\pi \left[\frac{1}{2}(\cos(2)(2mx)+1)\right] + 2\left[\cos(2mx)\cos(2nx)\right] + \left[\frac{1}{2}(\cos(2)(2nx)+1)\right] \mathrm{d}x$ $= \int_0^\pi \left[\frac{1}{2}\cos(4mx)+\frac{1}{2}\right] + 2\left[\cos(2mx)\cos(2nx)\right] + \left[\frac{1}{2}\cos(4nx)+\frac{1}{2}\right] \mathrm{d}x$ $= \left[\frac{\sin(4mx)}{8m}+\frac{x}{2}\right]_0^\pi + 2\left[\frac{\sin(2m+2n)x}{4m+4n}+\frac{\sin(2m-2n)x}{4m-4n}\right]_0^\pi + \left[\frac{\sin(4nx)}{8n}+\frac{x}{2}\right]_0^\pi$ $= \frac{\pi}{2}+0+\frac{\pi}{2}$ $= \pi$	Some candidates copy the question wrongly as $\mathrm{f}(x)$ is already given as $\cos 2mx + \cos 2nx$. Candidates should expand the expression first using $(a+b)^2 = a^2 + 2ab + b^2$ - some missed out the term $2ab$. Followed by applying double angle formula to rewrite $\cos^2(2mx) = \frac{1}{2}(\cos 2(2mx)) + 1$. Note the +1 does not belong to factor $\frac{1}{2}$. Note that reexpressing is NOT integration. Candidates will need to integrate $\frac{1}{2}(\cos 2(2mx)) + 1$. Many candidates are also not able to recognize that $\sin(\text{even integer})\pi$ gives zero, thereby did not give the final answer.

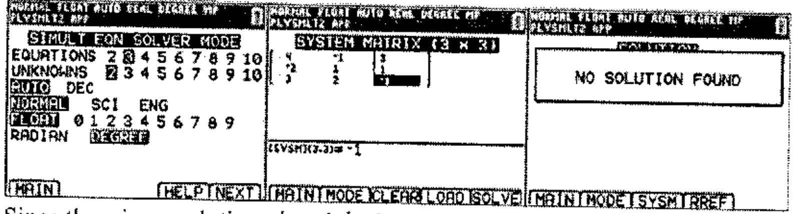
Q7. Parametric Equations

Assessment Objectives	Solution		Examiner Feedback
Procedure to find equation of normal from a curve defined parametrically	(i)	$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{e^t}{3t^2-4}$ When $t=1$, $\frac{dy}{dx}=-e$, $x_1=-2$, $y_1=e-1$ Equation of normal: $y-(e-1)=\frac{1}{e}[x-(-2)]$ $y=\frac{1}{e}[x-(-2)]+(e-1)$ $y=\frac{1}{e}x+\frac{2}{e}+e-1$ $y=\frac{1}{e}x+\frac{2+e^2-e}{e}$ (shown)	Most students are able to obtain $\frac{dy}{dx}$. Those who didn't will need to revisit their differentiation techniques. Many students used equation of straight line $y=mx+c$ which is also an acceptable method. Students should put $t=1$ first before substituting into the equation of normal, to avoid lengthy complicated expressions.
Understand that curve is parallel to the y-axis, $\frac{1}{dy/dx}=0$. [HOT]	(ii)	$\frac{dy}{dx}=\frac{e^t}{3t^2-4}$ When tangent to the curve is parallel to the y-axis, $3t^2-4=0$ $t=\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$ or $t=-\frac{2}{\sqrt{3}}$ (reject as $t\geq 0$) $\therefore \lambda=\frac{2\sqrt{3}}{3}$	Students should simplify their answer. Many students did not reject the negative root or write the reason for rejection. Common mistake: setting the numerator to zero instead of denominator.

Sketch parametric curves	**(iii)**		Students need to ensure end point (-2.08, 2.17) is parallel to y-axis. End points given in exact form should be simplified to $\left(-\frac{16\sqrt{3}}{9}+1, e^{\frac{2\sqrt{3}}{3}}-1\right)$
	(iv)	$\int_{-2}^{1} y^2 \, \mathrm{d}x = \int_{1}^{0} y^2 \cdot \frac{\mathrm{d}x}{\mathrm{d}t} \, \mathrm{d}t$ $= \int_{1}^{0} \left(e^t - 1\right)^2 \left(3t^2 - 4\right) \mathrm{d}t$ Sub $x = 1$ into $x = t^3 - 4t + 1$ $1 = t^3 - 4t + 1$ $0 = t^3 - 4t$ $t = 0$ or $t = \pm 2$ $k = 0, \;\; b = -4$	Most students are able to obtain $b = -4$ Some students gave the value of $k = \pm 2$ where they should have rejected those values based on the domain of t.

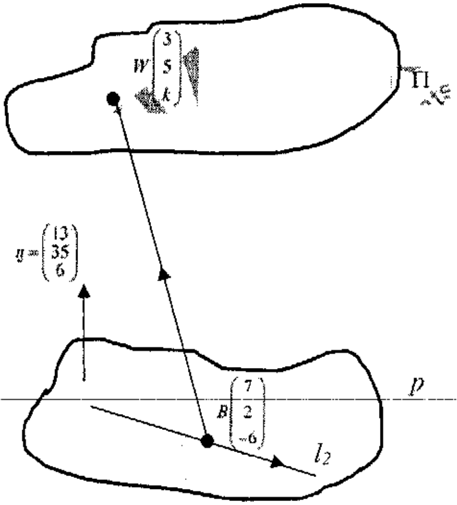
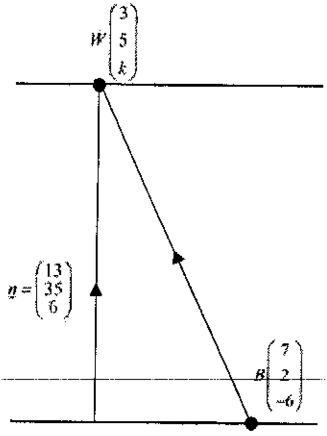
Find volume using G.C.	(v)	 **Method ①:** When $x=-2$, $y=\frac{1}{e}(-2)+\frac{2+e^2-e}{e}$ $=e-1$ Volume generated $=\frac{1}{3}\pi(e-1)^2(-e+e^2)+\pi\int_{-2}^{1}(y_c)^2\,dx$ $=4.5968\pi+\pi\int_{1}^{0}(e^t-1)^2(3t^2-4)\,dt$ $=4.5968\pi+1.5498\pi$ $=19.3$ unit^3 **Method ②:** Volume generated $=\pi\int_{e-2-e^2}^{-2}\left(\frac{1}{e}x+\frac{2+e^2-e}{e}\right)^2 dx+\pi\int_{-2}^{1}(y_c)^2\,dx$ $=\pi\int_{e-2-e^2}^{-2}\left(\frac{1}{e}x+\frac{2+e^2-e}{e}\right)^2 dx+\pi\int_{1}^{0}(e^t-1)^2(3t^2-4)\,dt$ $=4.5968\pi+1.5498\pi$ $=19.3$ unit^3	Students should draw the graph to see the region when they are finding volume. Some students tried to integrate algebraically, wasting a lot of time.

Q8. Vectors (Lines & Planes)

Assessment Objectives	Solution		Examiner Feedback
The ability to convert equation of a line to and from the 3 forms: - Parametric Form - Cartesian Form - Vector Form	(i)	Parametric Equation of line l_1: $\lambda=\frac{x-2}{4}\Rightarrow x=2+4\lambda$ $\lambda=\frac{3-y}{2}\Rightarrow y=3-2\lambda$ $\lambda=\frac{z+1}{3}\Rightarrow z=-1+3\lambda$ Vector equation of line l_1: $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}2+4\lambda\\3-2\lambda\\-1+3\lambda\end{pmatrix}\Rightarrow r=\begin{pmatrix}2\\3\\-1\end{pmatrix}+\lambda\begin{pmatrix}4\\-2\\3\end{pmatrix}$ Vector equation of line l_2: $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}5+\mu\\4-\mu\\-2-2\mu\end{pmatrix}\Rightarrow r=\begin{pmatrix}5\\4\\-2\end{pmatrix}+\mu\begin{pmatrix}1\\-1\\-2\end{pmatrix}$	This part (i) of the question is similar to Vectors Lecture (Lines) Example 25. Most candidates were able to convert the parametric equation of line l_1 into the vector equation form. There were some candidates who converted wrongly, which affected subsequent parts.
Perform test for parallel lines		Since $\begin{pmatrix}4\\-2\\3\end{pmatrix}\neq k\begin{pmatrix}1\\-1\\-2\end{pmatrix}$, l_1 and l_2 are not parallel.	Some candidates remembered and completed the series steps but did not make any logical conclusion after each check. For example, after finding the vector equations of l_1 and l_2, there was no conclusion or mention of the line being not parallel, and proceeded to find intersection points.
Perform test for intersecting lines		$\begin{pmatrix}2+4\lambda\\3-2\lambda\\-1+3\lambda\end{pmatrix}=\begin{pmatrix}5+\mu\\4-\mu\\-2-2\mu\end{pmatrix}\Rightarrow\begin{cases}2+4\lambda=5+\mu\\3-2\lambda=4-\mu\\-1+3\lambda=-2-2\mu\end{cases}$ $\begin{matrix}\Rightarrow 4\lambda-\mu=3\cdots\cdots(1)\\\Rightarrow -2\lambda+\mu=1\cdots\cdots(2)\\\Rightarrow 3\lambda+2\mu=-1\cdots\cdots(3)\end{matrix}$	
Make a logical conclusion. The relationship between 2 lines in a 3D space are either: - Parallel - Intersecting - Skew (i.e. non-parallel and non-intersectig)		Solving, SIMULT EQN SOLVER MODE EQUATIONS 2 3 4 5 6 7 8 9 10 UNKNOWNS 2 3 4 5 6 7 8 9 10 AUTO DEC NORMAL SCI ENG FLOAT 0 1 2 3 4 5 6 7 8 9 RADIAN DEGREE SYSTEM MATRIX (3 × 3) NO SOLUTION FOUND Since there is no solution, l_1 and l_2 do not intersect.	This signals a lack of understanding behind the procedures to systematically and logically determine the relationship between two lines.

		Therefore l_1 and l_2 are skew lines.	
Find the vector equation of a plane 2 non-parallel vectors that lie in the same plane are required Use Cross Product to find normal vector of the plane	(ii)	Let $\mu=2$, $\overrightarrow{OB}=\begin{pmatrix}5\\4\\-2\end{pmatrix}+(2)\begin{pmatrix}1\\-1\\-2\end{pmatrix}=\begin{pmatrix}5+2\\4-2\\-2-4\end{pmatrix}=\begin{pmatrix}7\\2\\-6\end{pmatrix}$ Second vector that lies in p : $\begin{pmatrix}7\\2\\-6\end{pmatrix}-\begin{pmatrix}2\\3\\-1\end{pmatrix}=\begin{pmatrix}5\\-1\\-5\end{pmatrix}$ Normal vector of p : $\underset{\sim}{n}=\begin{pmatrix}5\\-1\\-5\end{pmatrix}\times\begin{pmatrix}4\\-2\\3\end{pmatrix}$ $=\begin{pmatrix}(-1\times 3)-(-5\times -2)\\-[(5\times 3)-(-5\times 4)]\\(5\times -2)-(-1\times 4)\end{pmatrix}$ $=\begin{pmatrix}-13\\-35\\-6\end{pmatrix}=-\begin{pmatrix}13\\35\\6\end{pmatrix}$ $\left[\textit{Note}: \text{Both} \begin{pmatrix}-13\\-35\\-6\end{pmatrix} \text{or} \begin{pmatrix}13\\35\\6\end{pmatrix} \text{can be used for calculations}\right]$ $\underset{\sim}{r}\cdot\begin{pmatrix}13\\35\\6\end{pmatrix}=\begin{pmatrix}2\\3\\-1\end{pmatrix}\cdot\begin{pmatrix}13\\35\\6\end{pmatrix}=125$ (using fixed point $\begin{pmatrix}2\\3\\-1\end{pmatrix}$ on l_1) OR	A number of candidates made errors in (i) when converting to vector equation form and hence were only credited for method. Many candidates did not manage to identify the correct second vector that lies in the plane p. As a result, the normal vector for p was incorrect. There were some candidates who made careless mistakes while performing cross product.
		$\underset{\sim}{r}\cdot\begin{pmatrix}13\\35\\6\end{pmatrix}=\begin{pmatrix}7\\2\\-6\end{pmatrix}\cdot\begin{pmatrix}13\\35\\6\end{pmatrix}=125$ (using fixed point $B\begin{pmatrix}7\\2\\-6\end{pmatrix}$) $\therefore 13x+35y+6z=125$	There were some candidates who did not give the equation of the plane in the required Cartesian form.

Find acute angle between a line and a plane	(iii)	**Method ①:** Acute angle α between normal of p and l_2 : $\cos\alpha = \frac{\left\lvert\begin{pmatrix}13\\35\\6\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\-2\end{pmatrix}\right\rvert}{\left\lvert\begin{pmatrix}13\\35\\6\end{pmatrix}\right\rvert\left\lvert\begin{pmatrix}1\\-1\\-2\end{pmatrix}\right\rvert} = \frac{\lvert 13-35-12\rvert}{\sqrt{13^2+35^2+6^2}\sqrt{1^2+1^2+2^2}}$ $= \frac{34}{\sqrt{1430}\sqrt{6}}$ $\alpha = \cos^{-1}\left(\frac{34}{\sqrt{1430}\sqrt{6}}\right) = 68.4657°$	Some candidates who used Method 1 did not realise that the angle found using cosine is not the correct angle. Drawing a diagram may have helped visualize the required angle.
Identify correct angle between line and plane		Angle between p and l_2, $\theta = 90° - 68.4657° = 21.5°$ (0.376 rad) **Method ②:** $\sin\theta = \frac{\left\lvert\begin{pmatrix}13\\35\\6\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\-2\end{pmatrix}\right\rvert}{\left\lvert\begin{pmatrix}13\\35\\6\end{pmatrix}\right\rvert\left\lvert\begin{pmatrix}1\\-1\\-2\end{pmatrix}\right\rvert} = \frac{\lvert 13-35-12\rvert}{\sqrt{13^2+35^2+6^2}\sqrt{1^2+1^2+2^2}}$ $\theta = 21.5°$ (0.376 rad)	

Using given distance between 2 planes to find an unknown point. **[HOT]** Formulate an equation to represent the distance between two parallel planes with information provided	**(iv)**	Since the plane Π is parallel to plane p, $\underset{\sim}{n}=\begin{pmatrix}13\\35\\6\end{pmatrix}$. **<u>Method ①:</u>** Since $B\begin{pmatrix}7\\2\\-6\end{pmatrix}$ lies on plane p, distance between plane Π and plane p is the length of projection of $\overrightarrow{BW}$ onto $\underset{\sim}{n}$. $\overrightarrow{BW}=\overrightarrow{OW}-\overrightarrow{OB}$ $=\begin{pmatrix}3\\5\\k\end{pmatrix}-\begin{pmatrix}7\\2\\-6\end{pmatrix}=\begin{pmatrix}-4\\3\\k+6\end{pmatrix}$  	Not many candidates were able to get (iv) correct due to mistakes from previous parts. They were credited for correct method. By drawing a diagram to represent the conditions given for plane p, and plane Π, the length of projection can be identified. Not many candidates were able to identify the correct points and directional vectors.

Use the correct vectors to find length of projection		Distance between plane Π and plane p $= \text{Length of projection of } \overrightarrow{BW} \text{ onto } \underset{\sim}{n}$ $= \left\| \dfrac{\overrightarrow{BW} \cdot \underset{\sim}{n}}{\|\underset{\sim}{n}\|} \right\|$ $= \left\| \dfrac{\begin{pmatrix} -4 \\ 3 \\ k+6 \end{pmatrix} \cdot \begin{pmatrix} 13 \\ 35 \\ 6 \end{pmatrix}}{\left\| \begin{pmatrix} 13 \\ 35 \\ 6 \end{pmatrix} \right\|} \right\|$ $= \left\| \dfrac{-52+105+6k+36}{\sqrt{13^2+35^2+6^2}} \right\|$ $= \left\| \dfrac{6k+89}{\sqrt{1430}} \right\|$ $= \dfrac{\|6k+89\|}{\sqrt{1430}}$	Some candidates who arrived at this step left out the modulus when finding the length of projection. This will result in obtaining one answer.
Apply modulus results for simplification		Given $\dfrac{\|89+6k\|}{\sqrt{1430}} = \dfrac{34}{\sqrt{1430}}$, $\|89+6k\| = 34 \Rightarrow \begin{cases} 89+6k=34 \\ 89+6k=-34 \\ (\text{or} -(89+6k)=34) \end{cases}$ $89+6k=34$ or $89+6k=-34$ $k=-\dfrac{55}{6}$ or $k=-\dfrac{123}{6}$	There were some candidates who squared both sides to simplify the modulus. This is also a correct method but may complicate the expression.

Method ②:

Since plane Π contains the point $W(3,5,k)$,

$$\underset{\sim}{r}\cdot\begin{pmatrix}-13\\-35\\-6\end{pmatrix}=\begin{pmatrix}3\\5\\k\end{pmatrix}\cdot\begin{pmatrix}-13\\-35\\-6\end{pmatrix}=-39-175-6k$$

$$\text{i.e. } \underset{\sim}{r}\cdot\begin{pmatrix}-13\\-35\\-6\end{pmatrix}=-214-6k$$

Distance between plane Π and plane p $=\left|\underset{\sim}{a}_1\cdot\hat{\underset{\sim}{n}}-\underset{\sim}{a}_2\cdot\hat{\underset{\sim}{n}}\right|$

$$=\left|\frac{-125-(-214-6k)}{\left|\begin{pmatrix}-13\\-35\\-6\end{pmatrix}\right|}\right|$$

$$=\left|\frac{89+6k}{\sqrt{13^2+35^2+6^2}}\right|$$

$$=\frac{|89+6k|}{\sqrt{1430}}$$

Simplify equations involving modulus

Given $\frac{|89+6k|}{\sqrt{1430}}=\frac{34}{\sqrt{1430}}$, $|89+6k|=34\Rightarrow\begin{cases}89+6k=34\\89+6k=-34\\(\text{or}-(89+6k)=34)\end{cases}$

$89+6k=34$ or $89+6k=-34$

$k=-\frac{55}{6}$ or $k=-\frac{123}{6}$

Q9. Arithmetic & Geometric Progression

Assessment Objectives	Solution		Examiner Feedback
Find the nth term an arithmetic series.	(i)	Let A_n be the distance travelled by Alex on Day n. $A_{10} = 1000 + (10-1)(-5)$ $= 955$ km	Majority of the students were able to recognise that the question was asking for the *n*th term of an arithmetic series and successfully solved the problem. However, there were students who used 5 as the common difference instead of -5. There were also students who misinterpreted the question and found the sum of the first *n* terms of an arithmetic series instead. **It is thus important to read the question carefully and understand the requirement of the question.**
Find the nth term a geometric series.	(ii)	Let G_n be the distance travelled by Gopal on Day n. $G_5 = 1500,\ G_6 = 1500(0.98),\ G_7 = 1500(0.98)^2,\ \ldots G_n = 1500(0.98)^{n-5}$ $G_{10} = 1500(0.98)^{10-5}$ $= 1355.88$ km ≈ 1356 km (to nearest whole number) (shown)	Majority of the students were able to recognise that the question was asking for the *n*th term of a geometric series and successfully showed the distance travelled by Gopal on Day 10, either by using the formula for the *n*th term of a geometric series or by listing down the subsequent terms. However, there were students who wrote the incorrect power, e.g. $1500(0.98)^4$, or used the incorrect common ratio, e.g. $1500(0.02)^5$. There were also students who worked with the *n*th term instead of the sum of the first *n* terms. In addition, for students who listed down the terms, it was evident that they did not realise

			that the distance travelled on Day 5 was the first term of the geometric series and not the fifth term of the series. **It is thus important to know the formula for the geometric series and use it well should the question ask for the distance travelled on Day 50, which may be tedious to list down one by one.**
Find the sum of an arithmetic series and the sum to infinity of a geometric series, and draw conclusion based on the context of the question.	(iii)	Total distance travelled by Alex by the end of Day 100 $=\frac{100}{2}\left[2(1000)+(100-1)(-5)\right]$ $=75250$ km $1500+1500(0.98)+\ldots+1500(0.98)^{n-5}+\ldots$ $=\frac{1500}{1-0.98}$ $=75000$ km <75250 km Hence, Gopal is unable to complete the journey.	Many students were able to apply the sum of the first *n* terms of an arithmetic series to find the total distance of the journey. However, a significant number of students equated this value to the sum of the first *n* terms of a geometric series and concluded that there was no value of *n* and thus Gopal could not complete the journey. This method was not accepted because the concept of a sum to infinity was omitted although the question hinted on the sum to infinity. There were also students who were unable to recall the formula accordingly. **It is thus important to know the formula well and to recognise hints provided in the question to approach the problem.**

<table>
<tr>
<td>Find the sum of an arithmetic series and the sum of a geometric series, and to formulate and solve an inequality based on the context of the question.
[HOT]</td>
<td>(iv)</td>
<td>Total distance travelled by Gopal at the end of Day n
$=1500+1500(0.98)+\ldots+1500(0.98)^{n-5}$
$=\dfrac{1500\left(1-0.98^{n-4}\right)}{1-0.98}$

Total distance travelled by Alex at the end of Day $n-1$
$=1000+995+\ldots+\left[1000+(n-1-1)(-5)\right]$
$=\dfrac{n-1}{2}\left[2(1000)+(n-2)(-5)\right]$

$\dfrac{n-1}{2}\left[2(1000)+(n-2)(-5)\right]<\dfrac{1500\left(1-0.98^{n-4}\right)}{1-0.98}$
$\dfrac{n-1}{2}\left[2(1000)+(n-2)(-5)\right]-\dfrac{1500\left(1-0.98^{n-4}\right)}{1-0.98}<0$

Using G.C.,
<table>
<tr><td>n</td><td>$\dfrac{n-1}{2}\left[2(1000)+(n-2)(-5)\right]-\dfrac{1500\left(1-0.98^{n-4}\right)}{1-0.98}$</td></tr>
<tr><td>10</td><td>258.18</td></tr>
<tr><td>11</td><td>-115.59</td></tr>
<tr><td>12</td><td>-467.77</td></tr>
</table>

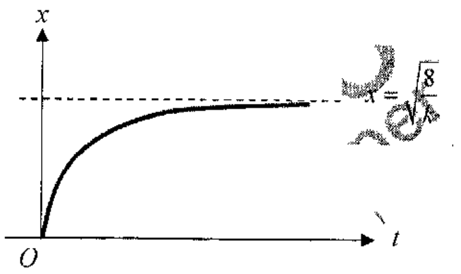
least $n=11$.

Total distance travelled by Gopal by the end of Day 11
$=\dfrac{1500\left(1-0.98^{11-4}\right)}{1-0.98}$
$=9890.585$ km
$=9891$ (to nearest whole no.)</td>
<td>Majority of the students could not answer this question well. Many students used $\dfrac{1500\left(1-0.98^{n}\right)}{1-0.98}$ as the total distance travelled by Gopal at the end of Day n, forgetting that Gopal only started his journey on Day 5. There were also other incorrect values being substituted into the formula, although most students were able to recall the formula for the sum of the first n terms of an arithmetic and geometric series correctly, and formed the inequality accordingly. For those who successfully formulated the correct inequality, many went on to solve it algebraically which was not possible. Instead, students were expected to use the GC to solve the inequality.

It is thus important to know the approaches to solving an inequality and to check back on the question to see if the use of a calculator is allowed in the question.</td>
</tr>
</table>

Q10. Differential Equations

Assessment Objectives	Solution		Examiner Feedback
Interpretation of context and representing it mathematically	(i)	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{a}{x}-bx=\frac{a-bx^2}{x}$ $\therefore\ \frac{\mathrm{d}x}{\mathrm{d}t}=\frac{a-kx^2}{x}$ where $k=b$ (shown)	A significant number of candidates are able to get this. Those who did not get it should realize that a and b are dummy constants so they can use different letters to represent the constants of proportionality at first, then convert to a and b towards the end. However, they must be careful NOT to use the same constant for the upward and the downward motion.
Ability to solve differential equation to get particular solution using the variable separable method	(ii)	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{8-kx^2}{x}$ $\int\frac{x}{8-kx^2}\,\mathrm{d}x=\int 1\,\mathrm{d}t$ $-\frac{1}{2k}\ln\lvert 8-kx^2\rvert=t+C$ $\ln\lvert 8-kx^2\rvert=-2kt-2kC$ $\lvert 8-kx^2\rvert=\mathrm{e}^{-2kt-2kC}$ $8-kx^2=A\mathrm{e}^{-2kt},\ A=\pm\mathrm{e}^{-2kC}$ When $t=0,\ x=0,$ $8-0=A\mathrm{e}^0$ $A=8$ $\therefore 8-kx^2=8\mathrm{e}^{-2kt}$ $kx^2=8-8\mathrm{e}^{-2kt}$	Some candidates did not realize that the differential equation can be solved using the variable separable method, or even when they realized so, were not able to "separate" the variables to their respective sides. They thus ended up integrating an expression in x with respect to t Some candidates forgot to include the arbitrary constant C after integrating. Some candidates forgot to apply the boundary (initial conditions) and were not able to figure out what the value of A is.
		$x=\sqrt{\frac{8-8\mathrm{e}^{-2kt}}{k}}$ or $-\sqrt{\frac{8-8\mathrm{e}^{-2kt}}{k}}$ (reject $\because x\ge 0$) (shown)	Some candidates forgot include (and then reject) the negative solution for x.

Interpretation and finding of long-term effect in a modelling	**(iii)**	When $t \to \infty$, $e^{-2kt} \to 0$, hence $x \to \sqrt{\frac{8-8(0)}{k}} = \sqrt{\frac{8}{k}}$ units The Hero will be stranded at a point $\sqrt{\frac{8}{k}}$ units from the starting point.	There are presentation issues. Many candidates have written something along the lines of: " When $t \to \infty$, $x = \sqrt{\frac{8}{k}}$ " which is technically not correct. Even with correct presentation: " When $t \to \infty$, $x \to \sqrt{\frac{8}{k}}$ " , candidates must realize they have only deduced a numerical behaviour. It is good to have a statement to interpret this behaviour in context.
Ability to produce a graphical representation of model	**(iv)**		Generally well attempted. Those who did not get the full credit are those who either drew the curve to touch the asymptote, or that the curve did not pass the origin. Candidates should also realize that, in sketches, they must label all special features (especially asymptotes) unless instructed otherwise
Manipulation of parameters in model to achieve prescribed condition **[HOT]**	**(v)**	Completion point must occur before point of being stranded, therefore $\sqrt{\frac{8}{k}} > 4$ $\frac{8}{k} > 16$ Hence $0 < k < 0.5$	Not well attempted. Common mistakes are setting an equality, and then suddenly turning the solution into an inequality without explanation. Some left the solution as a specific value, indicating a lack of awareness of what is going on.

Q1. Applications of Differentiation

Assessment Objectives	Solution		Examiner Feedback
Perform differentiation using either Quotient Rule or Product Rule	(i)	For rate of charging, Using Product Rule $V=\frac{1}{0.2+\mathrm{e}^{-2t}}=\left(0.2+\mathrm{e}^{-2t}\right)^{-1}$ $\frac{\mathrm{d}V}{\mathrm{d}t}=-\left(0.2+\mathrm{e}^{-2t}\right)^{-2}\left(-2\mathrm{e}^{-2t}\right)$ $=\left(2\mathrm{e}^{-2t}\right)\left(0.2+\mathrm{e}^{-2t}\right)^{-2}$ $=\frac{2\mathrm{e}^{-2t}}{\left(0.2+\mathrm{e}^{-2t}\right)^{2}}$ Using Quotient Rule $V=\frac{1}{0.2+\mathrm{e}^{-2t}}$ $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\left(0.2+\mathrm{e}^{-2t}\right)(0)-(1)\left(-2\mathrm{e}^{-2t}\right)}{\left(0.2+\mathrm{e}^{-2t}\right)^{2}}$ $=\frac{2\mathrm{e}^{-2t}}{\left(0.2+\mathrm{e}^{-2t}\right)^{2}}$	Almost all candidates were able to perform differentiation correctly. There were a few candidates who tried to use implicit differentiation but were unsuccessful.
Interpret the question correctly, where the rate of charging must be maximum, i.e. find maximum of $\frac{\mathrm{d}V}{\mathrm{d}t}$	(ii)	For maximum rate of charging, let $\frac{\mathrm{d}^2V}{\mathrm{d}t^2}=0$ Using Product Rule $\frac{\mathrm{d}^2V}{\mathrm{d}t^2}=\left[(-2)\left(0.2+\mathrm{e}^{-2t}\right)^{-3}\left(-2\mathrm{e}^{-2t}\right)\left(2\mathrm{e}^{-2t}\right)\right]+\left[\left(0.2+\mathrm{e}^{-2t}\right)^{-2}\left(-4\mathrm{e}^{-2t}\right)\right]$ $=\left[\frac{8\mathrm{e}^{-4t}}{\left(0.2+\mathrm{e}^{-2t}\right)^{3}}\right]+\left[\frac{-4\mathrm{e}^{-2t}}{\left(0.2+\mathrm{e}^{-2t}\right)^{2}}\right]$ $=\frac{8\mathrm{e}^{-4t}-\left(4\mathrm{e}^{-2t}\right)\left(0.2+\mathrm{e}^{-2t}\right)}{\left(0.2+\mathrm{e}^{-2t}\right)^{3}}$	Many candidates did not manage to interpret the question correctly. If $\frac{\mathrm{d}V}{\mathrm{d}t}$ is the "rate of charging", then for the "rate of charging" to be maximum, $\frac{\mathrm{d}^2V}{\mathrm{d}t^2}=0$. Some candidates who interpreted the question correctly proceeded to find $\frac{\mathrm{d}^2V}{\mathrm{d}t^2}$ but had slips when
			simplify the expression. There was no need to simplify the expression of $\frac{\mathrm{d}^2V}{\mathrm{d}t^2}$ here.

<table>
<tr>
<td>Apply First Derivative Test to test for maximum.</td>
<td></td>
<td>
OR using Quotient Rule

$$\frac{d^2V}{dt^2}=\frac{\left(0.2+e^{-2t}\right)^2\left(-4e^{-2t}\right)-\left(2e^{-2t}\right)(2)\left(0.2+e^{-2t}\right)\left(-2e^{-2t}\right)}{\left[\left(0.2+e^{-2t}\right)^2\right]^2}$$

$$=\frac{\left(-4e^{-2t}\right)\left(0.2+e^{-2t}\right)^2+\left(8e^{-4t}\right)\left(0.2+e^{-2t}\right)}{\left(0.2+e^{-2t}\right)^4}$$

$$=\frac{\left(-4e^{-2t}\right)\left(0.2+e^{-2t}\right)+\left(8e^{-4t}\right)}{\left(0.2+e^{-2t}\right)^3}$$

Let $\frac{d^2V}{dt^2}=0$

$$\frac{8e^{-4t}-\left(4e^{-2t}\right)\left(0.2+e^{-2t}\right)}{\left(0.2+e^{-2t}\right)^3}=0$$

$$8e^{-4t}-\left(4e^{-2t}\right)\left(0.2+e^{-2t}\right)=0$$

$$8e^{-4t}=4e^{-2t}\left(0.2+e^{-2t}\right)$$

$$2e^{-2t}=0.2+e^{-2t}$$

$$e^{-2t}=0.2$$

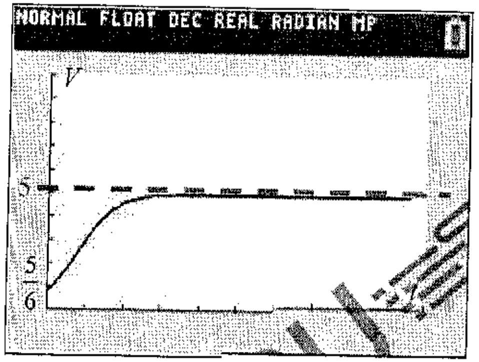
$$-2t=\ln 0.2$$

$$t=-\frac{1}{2}\ln 0.2=\frac{1}{2}\ln 5$$

First Derivative Test:

<table>
<tr><td></td><td>$t=\left(\frac{1}{2}\ln 5\right)^-$</td><td>$t=\frac{1}{2}\ln 5$</td><td>$t=\left(\frac{1}{2}\ln 5\right)^+$</td></tr>
<tr><td>Sign of $\frac{d^2V}{dt^2}$</td><td>+ve</td><td>0</td><td>-ve</td></tr>
<tr><td>Shape of $\frac{d^2V}{dt^2}$ graph</td><td>/</td><td>–</td><td>\</td></tr>
</table>

Therefore $\frac{dV}{dt}$ is maximum when $t=-\frac{1}{2}\ln 0.2$
</td>
<td>Most candidates did not perform the check for maximum/minimum.
Both First Derivative test and Second Derivative test are acceptable methods, but here the First Derivative test is the easier method to show maximum value.</td>
</tr>
</table>

Consider the behavior of $V = \frac{1}{0.2+e^{-2t}}$ as $t \to +\infty$, **[HOT]** Apply the behavior of the graph as $t \to +\infty$, to the context to the question and make a judgement.	**(iii)**	$V = \frac{1}{0.2+e^{-2t}}$ As $t \to +\infty$, $e^{-2t} \to 0$ $\therefore V \to \frac{1}{0.2} = 5\text{ V}$  Graphically, $V = 5$ is a horizontal asymptote. In the context of the question, it means that the voltage will never reach 5V. Hence the power bank can never be fully charged to 5 V.	The main feature of the graph is the horizontal asymptote at $V = 5$ and must be clearly shown. Candidates who included the graph for $t < 0$, omitted or gave the wrong y intercept (i.e. when $t = 0, V = \frac{5}{6}$) were not penalized for this question. Final conclusion should make reference to the graph.

Q2. Maclaurin Series

Assessment Objectives	Solution	Examiner Feedback
Perform repeated differentiation involving use of product rule Determine Maclaurin expansion of a function	Let $y = x\ln(2+x)$. $\frac{dy}{dx} = x \cdot \frac{1}{2+x} + \ln(2+x)$ $= x \cdot (2+x)^{-1} + \ln(2+x)$ $\frac{d^2y}{dx^2} = x \cdot (-1)(2+x)^{-2} + (2+x)^{-1} + \frac{1}{2+x}$ $= -x \cdot (2+x)^{-2} + 2(2+x)^{-1}$ $\frac{d^3y}{dx^3} = -x \cdot (-2)(2+x)^{-3} + (2+x)^{-2} \cdot (-1) + 2(-1)(2+x)^{-2}$ $= 2x \cdot (2+x)^{-3} - 3(2+x)^{-2}$ When $x = 0$, $f(0) = 0$ $f'(0) = \ln 2$ $f''(0) = 1$ $f'''(0) = -\frac{3}{4}$ $x\ln(2+x) = 0 + \ln 2 \cdot x + 1 \cdot \frac{x^2}{2!} - \frac{3}{4} \cdot \frac{x^3}{3!} + \ldots$ $= (\ln 2)x + \frac{x^2}{2} - \frac{x^3}{8} + \ldots$	Students must learn to read the question carefully. Question requires students to perform successive differentiation. There were students who used the formula in MF26. For students who performed differentiation, many applied product rule and quotient rule wrongly. These are assumed knowledge from secondary school and they are expected to be able to do it fluently. There were students who were unable to achieve full credit for this part of the question due to algebraic slips made in the evaluation of $f'(0)$, $f''(0)$ and $f'''(0)$.

Deduce a Maclaurin expansion of a function using replacement	**<u>Method ①:</u> Hence** Replace x with $-x$, $(-x)\ln(2-x)=(\ln 2)(-x)+\frac{(-x)^2}{2}-\frac{(-x)^3}{8}+\ldots$ $=-(\ln 2)x+\frac{x^2}{2}+\frac{x^3}{8}+\ldots$ $x\ln(2-x)=(\ln 2)x-\frac{x^2}{2}-\frac{x^3}{8}+\ldots$ **<u>Method ②:</u> Otherwise** Let $y=x\ln(2-x)$. $\frac{\mathrm{d}y}{\mathrm{d}x}=x\cdot\frac{-1}{2-x}+\ln(2-x)$ $=-x\cdot(2-x)^{-1}+\ln(2-x)$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-x\cdot(-1)(2-x)^{-2}(-1)+(2-x)^{-1}(-1)+\frac{-1}{2-x}$ $=-x\cdot(2-x)^{-2}-2(2-x)^{-1}$ $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=-x\cdot(-2)(2-x)^{-3}(-1)+(2-x)^{-2}\cdot(-1)-2(-1)(2-x)^{-2}(-1)$ $=-2x\cdot(2-x)^{-3}-3(2-x)^{-2}$ When $x=0$, $\mathrm{f}(0)=0$ $\mathrm{f}'(0)=\ln 2$ $\mathrm{f}''(0)=-1$ $\mathrm{f}'''(0)=-\frac{3}{4}$ $x\ln(2-x)=0+\ln 2\cdot x-1\cdot\frac{x^2}{2!}-\frac{3}{4}\cdot\frac{x^3}{3!}+\ldots$ $=(\ln 2)x-\frac{x^2}{2}-\frac{x^3}{8}+\ldots$	Given the marks allocated for this part of the question, Method ② is the least efficient because repeated differentiation has to be carried out all over again. Many algebraic slips were made during differentiation. Most commonly seen was the exclusion of negative signs. Method ① require students to make an observation that x is replaced by $-x$ in the earlier part of the question. Method ③ make use of standard series expansion in finding the Maclaurin expansion of $x\ln(2-x)$.

	Method ③: Otherwise $x\ln(2-x)=x\ln\left[2\left(1-\frac{x}{2}\right)\right]$ $=x\left[\ln 2+\ln\left(1-\frac{x}{2}\right)\right]$ $=(\ln 2)x+x\ln\left(1-\frac{x}{2}\right)$ $=(\ln 2)x+x\left[\left(-\frac{x}{2}\right)-\frac{1}{2}\left(-\frac{x}{2}\right)^2+\ldots\right]$ $=(\ln 2)x-\frac{x^2}{2}-\frac{x^3}{8}+\ldots$	
Deduce a Maclaurin expansion of a function based on previous results **[HOT]**	$x\ln(4-x^2)=x\ln[(2-x)(2+x)]$ $=x\ln(2-x)+x\ln(2+x)$ $=\left[(\ln 2)x+\frac{x^2}{2}-\frac{x^3}{8}+\ldots\right]+\left[(\ln 2)x-\frac{x^2}{2}-\frac{x^3}{8}+\ldots\right]$ $=(2\ln 2)x-\frac{x^3}{4}+\ldots$	This part of the question require students to first apply ln properties, i.e. $\ln ab=\ln a+\ln b$, and subsequently making use of the results obtained earlier. It was poorly attempted. Students who attempted commonly used $\ln ab=\ln a\times\ln b$ which is wrong.

Q3. Complex Numbers

Assessment Objectives	Solution		Examiner Feedback
Ability to solve complex roots of a quadratic equation, and to find argument of a complex number.	(i)	$z=\frac{-(-2\mathrm{i})\pm\sqrt{(-2\mathrm{i})^2-4(1)(-2)}}{2(1)}$ $=\frac{2\mathrm{i}\pm\sqrt{4\mathrm{i}^2+8}}{2}$ $=\frac{2\mathrm{i}\pm 2}{2}$ $=\mathrm{i}\pm 1$ Note that $\arg(\mathrm{i}+1)=\frac{\pi}{4}$ and $\arg(\mathrm{i}-1)=\frac{3\pi}{4}$ Since $\arg(z_1)<\arg(z_2)$, $\therefore z_1 = 1 + \mathrm{i}$ (shown)	Majority of the students were unable to answer this part of the question correctly. Many students let the roots be $a+b\mathrm{i}$ and $c+d\mathrm{i}$, and attempt to solve for the unknown constants, which is often tedious and leads to mistakes. Students who used the formula for quadratic equation were mostly successful. Common mistakes observed for this part of the question include: • Misread question and showed that $1+\mathrm{i}$ is a root of the equation instead of solving for the root. • Letting the roots be $a+b\mathrm{i}$ and $a-b\mathrm{i}$ when the roots are not conjugate pairs. • For students who attempt to solve for the unknown constants, many left out at least one of the cases or were unable to solve it fully. • Having the assumption that $\mathrm{i}\pm 1$ is the same as $1\pm\mathrm{i}$. • For students who successfully found the two roots, explanation (based on argument) were not provided on why $z_1=1+\mathrm{i}$.

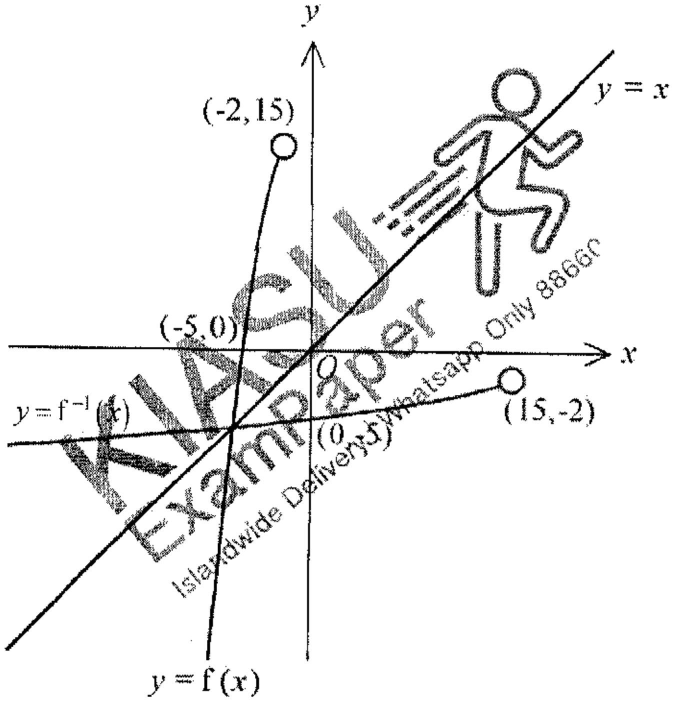
			It is thus important to acquire the technique of solving a quadratic equation involving complex numbers and be mindful of the coefficients to decide which approach to use to solve the problem. It is also important to know the rigour of a "show" question where sufficient explanation or justification has to be provided to be awarded the marks.
Ability to solve a polynomial equation with real coefficients.	**(ii)**	$x^2=(1+i)^2=1+2i+i^2=2i$ $x^3=(2i)(1+i)=2i+2i^2=-2+2i$ $x^4=(2i)^2=4i^2=-4$ $(1+i)^4-6(1+i)^3+s(1+i)^2-18(1+i)+10=0$ $-4-6(-2+2i)+s(2i)-18-18i+10=0$ By comparing imaginary parts, $-12+2s-18=0$ $\therefore s=15$ Since the coefficients of the equation are all real, and $1+i$ is a root of the equation, $1-i$ is also a root of the equation. $[x-(1+i)][x-(1-i)]=(x-1)^2-i^2$ $=x^2-2x+2$ By long division, $x^4-6x^3+15x^2-18x+10=(x^2-2x+2)(x^2-4x+5)$	Majority of the students were able to answer this part of the question to some extent. Common mistakes observed in this part of the question include: • Not answering the part on finding x^2, x^3 and x^4. For the majority who attempted, there were a few who were unsure of a Cartesian form. There were also some students who did not show sufficient working to be awarded the marks since this question does not permit the use of a calculator. • Not mentioning that since all the coefficients are real, then the conjugate root is also a root of the equation. • Algebraic slips in the process of long division or comparing coefficients to find the other quadratic factor.

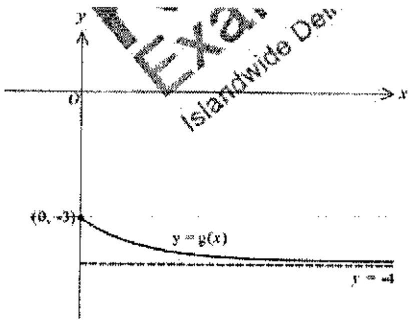
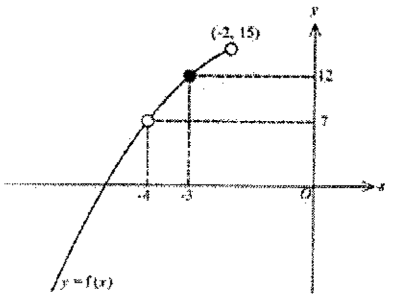
		Solving $x^2-4x+5=0$, $x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(5)}}{2(1)}$ $=\frac{4\pm\sqrt{-4}}{2}$ $=2\pm\mathrm{i}$ The other roots are $1-\mathrm{i}$, $2+\mathrm{i}$ and $2-\mathrm{i}$.	- Using the GC to solve for the roots after finding the value of *s*. **It is thus important to read the question properly to avoid missing out of certain part of the question and to be mindful on the restriction on the use of the calculator, although the GC can be used to check the answer. In addition, since this part of the question is a routine question, students need to be careful and proficient in the process of long division and comparing coefficients to avoid making mistakes.**
Ability to apply multiplication properties of two complex numbers, and to solve for *n* given that the complex number is purely imaginary. [HOT] (Seen)	(iii)	$\arg\left(\frac{z_1^{\ n}}{z_1^{\ *}}\right)=n\arg(z_1)-\arg(z_1^{\ *})$ $=n\arg(z_1)+\arg(z_1)$ $=(n+1)\frac{\pi}{4}$ Since $\frac{z_1^{\ n}}{z_1^{\ *}}$ is purely imaginary, $(n+1)\frac{\pi}{4}=\frac{\pi}{2}+k\pi$, where $k\in\mathbb{Z}$ $\frac{1}{4}(n+1)=\frac{1}{2}+k$ $n+1=2+4k$ $n=1+4k$ The two smallest positive integers of *n* are 1 and 5.	This part of the question was poorly attempted. Some students who did by guess and check were often not able to justify their solution properly. Common mistakes include: - Not able to use the properties of argument to find $\arg\left(\frac{z_1^{\ n}}{z_1^{\ *}}\right)$ - Not able to draw connections of purely imaginary to $\arg\left(\frac{z_1^{\ n}}{z_1^{\ *}}\right)$. - For students who were successful in formulating an equation and arriving at

			$n = 1 + 4k$, or equivalent, a significant number gave their answer as $n = 5$ and 9, failing to consider the value of n when $k = 0$. • For students who attempted via the Guess and Check method (not recommended), only $n = 1$ and $n = 5$ were considered and omitting showing why $n = 2, 3, 4$ did not satisfy the condition. **It is thus important to know the properties of complex numbers (both modulus and argument) and to be able to draw connections to terms such as "pure imaginary", "real and positive" etc.**

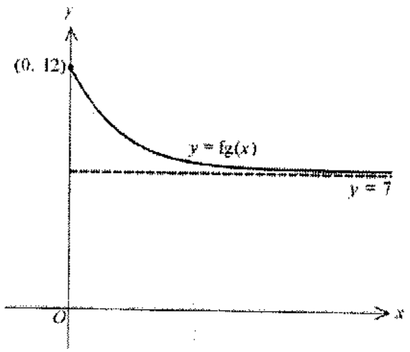
Q4. Functions

Assessment Objectives	Solution		Examiner Feedback
Understand and apply the condition (one-to-one function) for inverse function to exist	**(i)**	For the inverse function to exist, the function f must be one-to-one. Largest value of $a = -1$.	The question asked for the **largest value of *a***, however quite a number of students only stated $x < -1$ without answering the question. No mark will be awarded in this case. **It is important to read the question carefully and take note of what is required.**
Find the inverse function and its domain	**(ii)**	Let $y = 15 - 2x - x^2$ $y = 15 - 2x - x^2$ $= -\left(x^2 + 2x - 15\right)$ $= -\left[\left(x+1\right)^2 - 1^2 - 15\right]$ $= 16 - \left(x+1\right)^2$ $\left(x+1\right)^2 = 16 - y$ $x + 1 = \sqrt{16 - y}$ or $x+1 = -\sqrt{16-y}$ $x = -1 + \sqrt{16 - y}$ or $x = -1 - \sqrt{16-y}$ (reject $\because$ $R_{f^{-1}} = D_f = (-\infty, -2)$) $\therefore f^{-1}(x) = -1 - \sqrt{16 - x}$ $D_{f^{-1}} = R_f = \left(-\infty, 15\right)$	Some students did not know that they need to complete the squares for this question. Those who did completed the squares made some common mistakes below: • $y = 15 - 2x - x^2$ $\Rightarrow y = x^2 + 2x - 15$ Missing out the negative sign in the LHS. • $\left(x+1\right)^2 = 16 - y$ $\Rightarrow x + 1 = \sqrt{16 - y}$ Leaving out the "±" when taking square root. • Some students did not reject $x + 1 = \sqrt{16 - y}$ and include both $x = -1 \pm \sqrt{16 - y}$ as the answer **without checking the domain of function f.** • Some students even thought that the inverse function must take positive square root **without checking the**

			domain of function f. • Quite a number of students put $D_{f^{-1}} = R_f = (-\infty, 15]$ as the answer of the domain for inverse function of f. Again **without checking the range and/or domain of function f.**
Sketch the graphs of function and its inverse function, in particular, the end points, asymptotes, point of intersection and symmetry.	**(iii)**		In order to see the reflection of the graphs of f and f^{-1} about the line $y = x$ on the same diagram, you should draw the ***same scales*** for both the x and y axis. Some common mistakes: • The line $y = x$ is missing in the diagram. • The end points and the axial intercepts are not labelled. • The points $(-2,15)$ and $(15,-2)$ were not excluded with an open circle. • The graphs f and f^{-1} were not labelled.

Identify that the intersection of the graphs of $y=\mathrm{f}(x)$ and $y=\mathrm{f}^{-1}(x)$ is simply the intersection between graphs of $y=\mathrm{f}(x)$ and $y=x$	(iv)	$\mathrm{f}(x)=\mathrm{f}^{-1}(x)$ $\mathrm{f}(x)=x$ $15-2x-x^2=x$ $x^2+3x-15=0$	Badly done. Students did not understand the question. Some tried to find the roots to the quadratic equation: $x^2+3x-15=0$ when they are supposed to show this result using the fact that the curves of f and f^{-1} intersect.
Find the rule of a composite function and its domain	(v)	$\mathrm{fg}(x)$ $=\mathrm{f}(\mathrm{e}^{-x}-4)$ $=15-2(\mathrm{e}^{-x}-4)-(\mathrm{e}^{-x}-4)^2$ or $7+6\mathrm{e}^{-x}-\mathrm{e}^{-2x}$ or $\frac{7\mathrm{e}^{2x}+6\mathrm{e}^{x}-1}{\mathrm{e}^{2x}}$ $\mathrm{D}_{\mathrm{fg}}=[0,\infty)$	Quite well done **except for some careless mistakes in algebraic manipulation.**
Determine the range of a composite function	(vi)	**Method ① (Mapping Approach):** $[0,\infty)\xrightarrow{g}(-4,-3]\xrightarrow{f}(7,12]$ $\mathrm{R}_{\mathrm{fg}}=(7,12]$  	A common mistake that students made is the way the set notation should be written. Some students wrote $\mathrm{R}_{\mathrm{fg}}=[12,7)$ as the answer instead of $\mathrm{R}_{\mathrm{fg}}=(7,12]$.

Method ②:



$R_{fg} = (7,12]$

Q5. Permutations and Combinations &/or Probability			
Assessment Objectives	**Solution**		**Examiner Feedback**
Find number of ways with no restrictions	(i)	Number of ways $= \underbrace{{}^{11}C_{10}}_{\text{choose 10 seats out of 11 for the 10 people}} \times \underbrace{10!}_{\text{arrange these 10 people among the 10 chosen seats}}$ OR $\underbrace{11!}_{\text{treat empty seat as an object}}$ $=39916800$	This part was not well done. A significant percentage of students did not know how to consider the empty seat in the number of arrangements, often giving 10! as their answers. Some students applied the formulae $\frac{n!}{n}$ when there was no mention of round table. A small percentage of students worked out the number of arrangement using a long method by considering the seats for each group of passengers: ${}^{11}C_2 \times 2! \times {}^{9}C_4 \times 4! \times {}^{5}C_2 \times 2! \times {}^{3}C_2 \times 2!$ which is not recommended.
Find number of ways with restrictions	(ii)	Number of ways $= \underbrace{2!}_{\text{arrange 2 grandparents in the front row}} \times \overbrace{{}^{3}C_2}^{\text{2 possible positions of 3 for Mr and Mrs Lim and Mr and Mrs Tan}} \times \overbrace{2!}^{\text{arrange Mr and Mrs Lim and Mr and Mrs Tan}} \times \underbrace{2!}_{\text{arrange Mr and Mrs Lim}} \times \underbrace{2!}_{\text{arrange Mr and Mrs Tan}} \times \underbrace{4!}_{\text{arrange 4 children among the 4 window seats left}}$ $=1152$	This part was not well done. Some students misinterpreted the question. They found 4 separate answers for each condition. Credit was not given in this case. Some students did not interpret the third and fourth conditions correctly. The condition "Mr and Mrs Lim sit in the **same row** on the **same side** of the aisle" means that Mr and Mrs Lim are next to each other. Similarly, Mr and Mrs

		Alternatively, Number of ways $= \underbrace{2!}_{\text{arrange 2 grandparents in the front row}} \times \overbrace{{}^{3}C_{1}}^{\text{1 possible position out of 3 window seats for the 4th child}} \times \overbrace{4!}^{\text{arrange 4 children among the 4 windows seats}} \times \overbrace{{}^{2}C_{1}}^{\text{1 possible row out of 2 rows for the first couple}} \times \underbrace{2!}_{\text{arrange Mr and Mrs Lim}} \times \overbrace{{}^{1}C_{1}}^{\text{1 possible row out of 1 row for the second couple}} \times \underbrace{2!}_{\text{arrange Mr and Mrs Tan}}$ $= 1152$	Tan are also next to each other. A significant percentage of students did not know clearly when to apply Addition and Multiplication Principles. The number of ways for each condition were added together instead of being multiplied together. Common mistakes: • ${}^{6}C_{4} \times 4!$ - The children do not have 6 window seats to choose from since the two couples will occupy 2 rows. • ${}^{4}C_{2} \times 2!$ - The first couple does not have 4 seats to choose from since this couple must be seated next to each other. • Some students missed out a factor $2!$ as they forgot that the couples can swap their seats with each other (swapping the 2 rows). • Some students failed to consider all the cases.

Find probability	**(iii)**	Required probability $= \dfrac{\overbrace{{}^{2}C_1}^{\text{2 possible front row seats for Mr Tan}} \times \overbrace{{}^{7}C_1}^{\text{7 possible seats for Mr Lim such that there is a seat behind for a child}} \times \overbrace{{}^{4}C_1}^{\text{choose 1 child out of 4 to seat behind Mr Lim}} \times \overbrace{{}^{8}C_7}^{\text{choose 7 seats out of 8 for the remaining 7 people}} \times \overbrace{7!}^{\text{arrange these 7 people among the 7 chosen seats}}}{\underbrace{39916800}_{\text{arrange 10 people without restriction}}}$ $= \dfrac{28}{495}$ or 0.0566 (to 3 s.f.) **Alternatively,** **<u>Method ①</u>:** **Case 1: Child is at the front row (equivalent to Mr Lim is in 2nd row)** Number of ways $= \overbrace{{}^{2}C_1}^{\text{1 possible seat ut of 2 front seats for Mr Tan}} \times \overbrace{{}^{4}C_1}^{\text{select 1 child out of 4 children to be seated in front of Mr Lim}} \times \overbrace{{}^{8}C_7}^{\text{choose 7 seats out of 8 for the remaining 7 people}} \times \overbrace{7!}^{\text{arrange these 7 people among the 7 chosen seats}}$ $= 322560$ **Case 2: Child is not at the front row (equivalent to Mr Lim is in 3rd or 4th row)** Number of ways $= \overbrace{{}^{2}C_1}^{\text{1 possible seat ut of 2 front seats for Mr Tan}} \times \overbrace{{}^{4}C_1}^{\text{select 1 child out of 4 children to be seated in front of Mr Lim}} \times \overbrace{{}^{6}C_1}^{\text{1 possible seat out of 6 seats for Mr Lim}} \times \overbrace{{}^{8}C_7}^{\text{choose 7 seats out of 8 for the remaining 7 people}} \times \overbrace{7!}^{\text{arrange these 7 people among the 7 chosen seats}}$ $= 1935360$ Required probability $= \dfrac{322560 + 1935360}{39916800}$ $= \dfrac{28}{495}$	This part is not well done. Some students forgot to find probability and left their answers as the number of ways. Partial credit was given for students who used their answer in **(i)** as the total number of ways to arrange without restriction (even though their answer in (i) was wrong). Students often omitted the selection of child in their working. Question says 'a child', not 'a *particular* child'. A small percentage of students attempted to find the required probability by using individual probabilities for the events (Alternative Method 2). However, most failed to account for the non-replacable seats. Some students also assumed the 2 events 'Mr Lim sits directly behind a child' and 'Mr Tan sits in the front row" were independent events which was not correct.

Method ②:

Required probability

$$= \left(\overbrace{\frac{2}{11}}^{\text{2 possible seats out of 11 seats for Mr Tan}} \times \overbrace{\frac{7}{10}}^{\text{7 possible seats out of 10 remaining seats for the chosen child in front of Mr Lim}} \times \overbrace{\frac{1}{9}}^{\text{1 possible seat out of 9 remaining seats for Mr Lim}} \right) \times \overbrace{4}^{\text{4 possible children to be chosen}}$$

$$= \frac{28}{495}$$

Q6. Discrete Random Variables

<table>
<tr><th>Assessment Objectives</th><th colspan="2">Solution</th><th>Examiner Feedback</th></tr>
<tr><td>Finding probability distribution of DRV</td><td>(i)</td><td>
<table>
<tr><td>y</td><td>0</td><td>2</td><td>4</td><td>5</td><td>6</td><td>10</td><td>15</td><td>20</td><td>30</td></tr>
<tr><td>$\mathrm{P}(Y=y)$</td><td>$\frac{3}{12}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td><td>$\frac{2}{12}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td></tr>
</table>
</td><td>Question was well done.

Some mistakes were missing out the outcome 0, or stating it as 'Better luck next time' which is not a discount.</td></tr>
<tr><td>Calculating mean and variance of DRV</td><td>(ii)</td><td>$\mathrm{E}(Y)=0\left(\frac{3}{12}\right)+2\left(\frac{1}{12}\right)+4\left(\frac{1}{12}\right)+5\left(\frac{1}{12}\right)+6\left(\frac{1}{12}\right)+10\left(\frac{2}{12}\right)$
$+15\left(\frac{1}{12}\right)+20\left(\frac{1}{12}\right)+30\left(\frac{1}{12}\right)$
$=8.5$
$\mathrm{E}(Y^2)=0^2\left(\frac{3}{12}\right)+2^2\left(\frac{1}{12}\right)+4^2\left(\frac{1}{12}\right)+5^2\left(\frac{1}{12}\right)+6^2\left(\frac{1}{12}\right)+10^2\left(\frac{2}{12}\right)$
$+15^2\left(\frac{1}{12}\right)+20^2\left(\frac{1}{12}\right)+30^2\left(\frac{1}{12}\right)$
$=150.5$
$\mathrm{Var}(Y)=\mathrm{E}(Y^2)-[\mathrm{E}(Y)]^2$
$=150.5-8.5^2$
$=78.25$</td><td>Question was well done. Students should use G.C. (1-Var Stats) to work out this part quickly or to check answers.

Common mistake were
• Mistake 150.5 as $\mathrm{Var}(Y)$, when it is actually $\mathrm{E}(Y^2)$
• Using $\frac{1}{12}$ as the corresponding probability for the outcome of 10, even though the table had $\frac{2}{12}$
• Using the unbiased estimate of population variance formula</td></tr>
<tr><td>Application of DRV, listing cases, finding probability, addition and multiplication principles for counting [HOT]</td><td>(iii)</td><td>$\mathrm{P}(Y_1+Y_2\geq 45)$
$=\mathrm{P}(Y_1=30,Y_2=30)+2\mathrm{P}(Y_1=30,Y_2=20)+2\mathrm{P}(Y_1=30,Y_2=15)$
$=\left(\frac{1}{12}\right)^2+2\left(\frac{1}{12}\right)^2+2\left(\frac{1}{12}\right)^2$
$=\frac{5}{144}$</td><td>Question was poorly done.

Common mistakes were
• Writing $\mathrm{P}(Y\geq 45)$: Y is associated with \$100, not \$200 as required in (iii)
• Writing $\mathrm{P}(2Y\geq 45)$: This is 2 times of one outcome, not</td></tr>
</table>

			two different outcomes. • Using $\geq$ instead of $>$: $\mathrm{P}(Y_1+Y_2=45)$ matters in DRV and binomial, the two inequalities are not equivalent. • Missing the case of $Y_1=30, Y_2=30$: This is not a "without replacement" type of question. It is possible to get identical outcomes. • Including the cases (20, 20) or (10, 30): A discount of at least \$45 is required so these are not a favourable outcome • Not multiplying by 2 at all: We need to account for 5 distinct cases are (15,30), (30,15), (20,30), (30,20) and (30,30). • Multiplying by 2 to $\mathrm{P}(Y_1=30, Y_2=30)$: The distinct cases are (15,30), (30,15), (20,30), (30,20) and (30,30). So not every case requires us to times 2. • Use of normal distribution: Finding mean/variance in (ii) does not imply that the subsequent part is a normal distribution. All distributions have mean and variance. •

Q8. Normal Distribution

Assessment Objectives	Solution		Examiner Feedback
Finding the value of $\mathrm{P}(x_1 < X < x_2)$, given the values of x_1 , x_2 , μ , σ	(i)	Let C and W be the random variables denoting the mass of a roll of Cleanex and WoolSoft toilet paper in grams, respectively. $C \sim \mathrm{N}(200,10^2)$ $\qquad$ $W \sim \mathrm{N}(220,15^2)$ $\mathrm{P}(202 < C < 208) = 0.20888 = 0.209$	Well attempted. There are some scripts with rounding off errors.
Solving problems involving the use of $\mathrm{E}(aX+bY)$ and $\mathrm{Var}(aX+bY)$, where X and Y are independent, $\mathrm{E}(X_1+X_2+...+X_n)$ and $\mathrm{Var}(X_1+X_2+...+X_n)$.	(iii)	Let $A = C_1 + C_2 + ... + C_{10}$ and $B = W_1 + W_2 + ... + W_{10}$ $B - 1.05A \sim \mathrm{N}(10(220) - 1.05(10)(200), 10(15^2) + 1.05^2(10)(10^2))$ $\sim \mathrm{N}(100, 3352.5)$ $\mathrm{P}(B > 1.05A) = \mathrm{P}(B - 1.05A > 0)$ $= 0.95792$ $= 0.958$	Not well attempted. Common mistakes are: • candidates did not take into consideration that they are dealing with a pack of 10 rolls, so their parameters used are that of 1 roll • some who realized it is 10 rolls set up the random variable as 10C or 10W, which is incorrect • There are a variety of incorrect representations for the required event, like $\mathrm{P}(B - 0.95A > 0)$ or $\mathrm{P}(1.05B - A > 0)$ or $\mathrm{P}(0.95B - A > 0)$ etc • Some candidates inexplicably use the equal sign in their setup, $\mathrm{P}(B - 1.05A = 0)$ and used the normpdf command in the G.C.
Solving problems involving the use of $\mathrm{E}(aX+bY)$ and $\mathrm{Var}(aX+bY)$, where X and Y are independent. Application	(iv)	$A + B_1 + B_2 + B_3 \sim N(10(200) + 3(10)(220),\ 10(10)^2 + 3(10)(15)^2)$ $\sim N(8600,\ 7750)$ $\mathrm{P}(A + B_1 + B_2 + B_3 < k) = 0.4$	Mistakes are similar to that in (iii): • candidates did not take into consideration that they are dealing with a pack of 10 rolls, so their parameters used are

		$k = 8577.7$ $= 8580$ (to 3 s.f.)	that of 1 roll • some who realized it is 10 rolls set up the random variable as A+ 3B, or still used 10C + 30W, which is incorrect A number of candidates used the table of values to find the approximate value of k. They must realize the tables are more suitable to find discrete values.

Standard normal distribution Finding a relationship between x_1, x_2, μ, σ given the value of $\mathrm{P}(X < x_1)$ and $\mathrm{P}(X > x_2)$ Solving system of linear equations using GC.	(v)	Let U be the random variable denoting the mass of a roll of 4-ply toilet paper in grams, respectively. $U \sim \mathrm{N}(\mu, \sigma^2)$ $\mathrm{P}(U < 220) = 0.04$ $\mathrm{P}\left(Z < \frac{220-\mu}{\sigma}\right) = 0.04$ $\frac{220-\mu}{\sigma} = -1.7507$ $\mu - 1.7507\sigma = 220$ ------(1) $\mathrm{P}(U > 230) = 0.80$ $\mathrm{P}\left(Z > \frac{230-\mu}{\sigma}\right) = 0.80$ $\frac{230-\mu}{\sigma} = -0.84162$ $\mu - 0.84162\sigma = 230$ ------(2) $\mu = 239.26 = 239$ $\sigma = 11.000 = 11.0$	Generally well-attempted. Some did not get the full credit because of careless errors. There are some who exhibited conceptual errors though, like: • carrying over the probability: $\frac{220-\mu}{\sigma} = 0.04$ • did not standardize correctly: $\mathrm{P}\left(Z < \frac{\mu-220}{\sigma}\right) = 0.04$ or $\mathrm{P}\left(Z < \frac{220-\mu}{\sigma^2}\right) = 0.04$ • did not know they need to standardize at all

Q9. Binomial Distribution

Assessment Objectives	Solution		Examiner Feedback
Assumptions required for a situation to be well modelled by a Binomial distribution	**(i)**	• Whether a call made by an experienced telesales executive is successful or not is independent of any other calls. • The probability that a call made by an experienced telesales executive is successful is a constant at 0.15.	Candidates need to improve on such explanation questions. • Probability of 'success' is **constant at 0.15**. → To state the value of probability i.e. 0.15 → 'same' is accepted. → 'equal' is not accepted unless 0.15 is stated explicitly because probability of success is equal may mean equal to other values not 0.15 • Outcome of trial needs to be stated in context i.e. **success** of the call. • **Probability cannot be independent!** Candidates need to stop producing this answer at this stage.
Use of binomcdf	**(ii)**	Let T be the random variable denoting the number of successful calls, out of 15, made by Tom, an experienced telesales executive. $T \sim \mathrm{B}(15,\ 0.15)$ $\mathrm{P}(T > 4) = 1 - \mathrm{P}(T \le 4)$ $= 0.0617053867$ $= 0.0617$ (to 3 s.f.)	Candidates need to work on presentation such as defining the random variable and recognising this as a Binomial Distribution question. Writing B instead of N (which represents normal distribution) Strongly encouraged for candidates to use the initials T and J for easier reference to the random variables defined.

			Many candidates gave $\mathrm{P}(T>4)=1-\mathrm{P}(T\le 3)$ which is INCORRECT! Candidates are strongly encouraged to write out the answer in more than 5 s.f. to ensure accuracy for subsequent parts.
Use of probability found earlier to aid in solving Binomial distribution	**(iii)**	<u>**Method ①:**</u> **Define a new binomial random variable** Let D be the random variable denoting the number of days, out of 5, where an experienced telesales executive made more than 4 successful calls. $D\sim \mathrm{B}(5,\ 0.0617053867)$ $\mathrm{P}(D\le 2)=0.998$ (to 3 s.f.) <u>**Method ②:**</u> **Consider cases** $\binom{n}{x}p^x(1-p)^{n-x}$ Required probability $={}^5\mathrm{C}_0\left[\mathrm{P}(T>4)\right]^0\left[1-\mathrm{P}(T>4)\right]^5$ $+{}^5\mathrm{C}_1\left[\mathrm{P}(T>4)\right]^1\left[1-\mathrm{P}(T>4)\right]^4$ $+{}^5\mathrm{C}_2\left[\mathrm{P}(T>4)\right]^2\left[1-\mathrm{P}(T>4)\right]^3$ $=(1-0.0617053867)^5$ $+5(0.0617053867)(1-0.0617053867)^4$ $+10(0.0617053867)^2(1-0.0617053867)^3$ $=0.998$ (3 s.f.)	This is a second tier binomial question. Candidates who used the alternative method may be unsuccessful if the wrong formula is used (see Method 2). They tend to miss out the coefficients ${}^5C_x$.

Find probability	**(iv)**	Let J be the random variable denoting the number of successful calls, out of 9, made by Jerry, a novice telesales executive. $J \sim \mathrm{B}(9,\ 0.09)$ $\mathrm{P}(\text{Tom \& Jerry made 2 sales each}) = \mathrm{P}(T=2)\cdot\mathrm{P}(J=2)$ $= (0.2856392285)(0.1506875132)$ $= 0.0430\ \text{(to 3s.f.)}$	Many candidates gave $\mathrm{P}(T=2)+\mathrm{P}(J=2)$ instead which is incorrect. Addition principle is used for mutually exclusive cases. However, the events occurred simultaneously so multiplication principle is used. Some candidates did not understand "Tom & Jerry made 2 sales each" and went to consider $\mathrm{P}(T=0)\cdot\mathrm{P}(J=2)$ etc which is unnecessary.
Use of conditional probability **[HOT]**	(v)	$\mathrm{P}(\text{Tom \& Jerry made 3 sales in total} \mid \text{Jerry makes at least two sales})$ $= \dfrac{\mathrm{P}(\text{Tom \& Jerry made 3 sales in total and Jerry makes at least two sales})}{\mathrm{P}(\text{Jerry makes at least two sales})}$ $= \dfrac{\mathrm{P}(J=2)\cdot\mathrm{P}(T=1)+\mathrm{P}(J=3)\cdot\mathrm{P}(T=0)}{\mathrm{P}(J\geq 2)}$ $= \dfrac{\mathrm{P}(J=2)\cdot\mathrm{P}(T=1)+\mathrm{P}(J=3)\cdot\mathrm{P}(T=0)}{1-\mathrm{P}(J\leq 1)}$ $= \dfrac{(0.1506875132)(0.2312317564)+(0.0347740415)(0.0873542191)}{1-0.8088343476}$ $= 0.198\ \text{(to 3s.f.)}$	Many candidates did not realise that this is a conditional probability question based on the keyword **"given that"**. For those that recognizes that conditional probability is used, they failed to recognise the numerator (see solution), or they tend to miss out the $\cdot\mathrm{P}(T=0)$.

State assumption	(vi)	Successful calls made by Tom, a novice telesales executive are independent of successful calls made by Jerry, an experienced telesales executive. OR All successful calls made are independent.	Not well-attempted. Again, the outcome of all calls must be stated (successful calls). Note: **Number** of calls cannot be independent (hence, this is unaccepted). It is the **outcome of the calls** that can be independent. Once again, probability cannot be independent.

Q10. Hypothesis Testing

Assessment Objectives	Solution		Examiner Feedback
Concept of simple random sample.	(i)	A random sample means that each of the citizens is selected **independently** from each other and each citizen has an **equal** chance of being selected. Equal and independent chance of being selected (penalize).	Question was poorly done. Students need to revise their definitions. Common mistakes: Probability of selecting citizens is independent.
Calculation of unbiased estimates of the population mean and variance from a sample where data is given in summarized form.	(ii)	$\hat{\mu}=\frac{\sum x}{n}=\frac{9052}{100}=90.52$ (exact) $s^2=\frac{1}{99}\left(962373-\frac{9052^2}{100}\right)=1444.3=1440$ (3s.f.)	Generally well done. Many students did not give exact value for $\hat{\mu}$ or round s^2 to 3 s.f.
Concept of null and alternative hypotheses, level of significance and p-value. Formulation of hypotheses and testing for a population mean based on a large sample from any population. Use of CLT Interpretation of the results of a hypothesis test in the context of the problem.	(iii)	Let X be the random variable denoting the efficacy rate of the Simozer vaccine. $H_0: \mu = 91$ $H_1: \mu < 91$, where μ is the population mean efficacy rate of the Simozer vaccine. Under H_0, Since $n = 100$ is large, $\bar{X} \sim N\left(91, \frac{1444.3}{100}\right)$ approx by Central Limit Theorem. $Z = \frac{\bar{X}-91}{\sqrt{\frac{1444.3}{100}}} \sim N(0,1)$ Reject H_0 if p-value ≤ 0.05 p-value $= 0.44975 > 0.05$ Do not reject H_0 and conclude that there is insufficient evidence at 5% level of significance to claim that the efficacy rate of the Simozer vaccine has been overstated.	Many presentation errors. Not defining X or defining X wrongly. Not writing the hypotheses correctly Not writing n is large Not writing approx. Writing 90.52 instead of 91 in the distribution. **Do not write the wrong statement** $X \sim N(91,\ 1444.3)$ Careless mistake $0.44975 < 0.05$ Some had correct inequality but wrong conclusion, or miss out key phrases in the conclusion such as the level of significance, insufficient evidence, do not reject H_0, writing H_1 in the context of the question.

Interpretation of the results of a hypothesis test in the context of the problem. Use of CLT Concept of null and alternative hypotheses, test statistic, critical value Forming inequality **[HOT]**	(iv)	$\bar{x}=90.2$ $s^2=\frac{100}{99}m^2$ $\mathrm{H}_0:\mu=91$ $\mathrm{H}_1:\mu<91$ Under H_0, Since $n=100$ is large, $\bar{X}\sim\mathrm{N}\left(91,\ \frac{\frac{100}{99}m^2}{100}\right)$ approx.. by Central Limit Theorem $Z=\frac{\bar{X}-91}{\sqrt{\frac{\frac{100}{99}m^2}{100}}}\sim\mathrm{N}(0,1)$ At 1% significance level, reject H_0 if $z_{test}\le-2.3263$ $\frac{90.2-91}{\sqrt{\frac{\frac{100}{99}m^2}{100}}}\le-2.3263$ $-0.8\le-2.3263\sqrt{\frac{1}{99}}m$ $m\le3.4217$ $m\le3.42$ $0\le m\le3.42$ (since variance is never negative)	Poorly done. Many students did not realise that *m* is sample standard deviation. Many students left out the approximation by CLT, jumping straight to the inequality Note lower bound for answer.
Interpretation of question	(v)	Since $\underline{n}$ is large ($n\ge30$), the distribution of the mean efficacy rate will approximately follow a normal distribution by Central Limit Theorem. Hence, the hypothesis test could still be carried out without knowledge of the marketed efficacy rate's population distribution.	Common mistake: assume normal distribution Did not write the key words such as $\underline{n}$ is large, approximately, normal distribution, Central Limit Theorem