Anglo-Chinese School (Barker Road)

END-OF-YEAR EXAMINATION 2018

SECONDARY THREE

EXPRESS

ADDITIONAL MATHEMATICS 4047

2 HOURS 30 MINS

Additional Materials: 8 Writing Papers
1 Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
Attach the cover page on top of your answer script.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

This question paper consists of 7 printed pages. **[Turn over**

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 A frozen piece of steak being kept in the freezer at $-18°C$ is removed from the freezer. Its temperature at time t minutes after it is removed from the freezer is given by $T = T_0 - 38e^{-\frac{3}{5}t}$,

where T_0 is a constant.

(i) Find the value of T_0. [1]

(ii) Find the temperature of the steak at 2 minutes. [2]

(ii) Can the steak ever reach 20°C? Justify your answer. [1]

2 The equation of a curve is $y = x^2 + 4x - 10$, find the set of values of x for which the curve lies above the line $y = 2$. [4]

3 The volume of a square pyramid is $(2+5\sqrt{5})$ cm^3. If the length of the perpendicular height of the pyramid is $\left(3+2\sqrt{5}\right)$cm, find, without using a calculator, the area of the base of the pyramid in the form of $\left(a+b\sqrt{5}\right)$cm^2, where a and b are integers. [4]

4 Given that $x+1$ is a factor for the expression $x^9 - x^6 - 3x^3 - p$,

(i) Find the value of p, [2]

(ii) Hence, by substituting $y = x^3$ or otherwise, solve the equation $x^9 - x^6 - 3x^3 - p = 0$, leaving your answer in the exact form. [4]

5 **(i)** Express $\alpha^2 + \beta^2$ in terms of $\alpha + \beta$ and $\alpha\beta$. [1]

The roots of the quadratic equation $x^2 + mx + 8 = 0$, where m is an integer, are α and β.

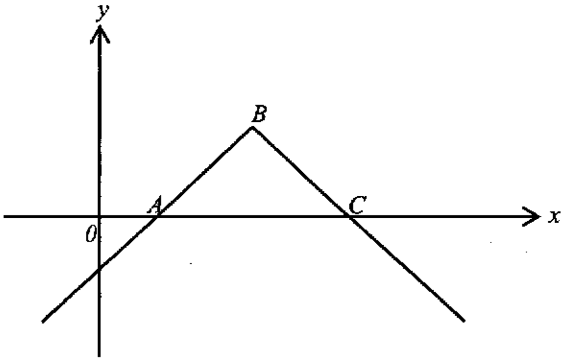
(ii) Express m in terms of α and/or β. [1]

Another quadratic equation $x^2 - 48x + n = 0$, where n is an integer, has roots α^2 and β^2.

(iii) Express n in terms of α and/or β. [1]

(iii) Hence, solve for m and n. [3]

6 The diagram shows part of a graph of $y = 3 - |6 - x|$.



(i) Find the coordinates of the points A, B and C. [3]

(ii) Solve the equation $3 - |6 - x| = 2x$. [3]

7 **(i)** Find the term independent of x in the binomial expansion of $\left(x - \frac{2}{x^2}\right)^9$. [3]

(ii) Hence, find the value of k given that there is no constant term in the expansion of

$(1 + kx^6)\left(x - \frac{2}{x^2}\right)^9$. [4]

8 Given that $\log_x 2 = p$ and $\log_4 y = q$, express the following in terms of p and/or q.

(i) $\log_4 \frac{4x}{y}$, [4]

(ii) xy. [3]

9 **(i)** Write down the equation of the circle with centre A(8,2) and radius $\sqrt{80}$. [1]

This circle intersects the y-axis at points P and Q.

(ii) Find the length PQ. [3]

A second circle, centre B, also passes through P and Q.

(iii) State the y-coordinate of B. [1]

Given that the x-coordinate of B is negative and that the radius of the second circle is 5, find

(iv) the x-coordinate of B. [2]

10 Answer the whole of this question on a piece of graph paper.

The table shows experimental values of two variables, x and y, which are connected by an equation of the form $y - b\sqrt{x} = \dfrac{a}{\sqrt{x}}$, where a and b are constants.

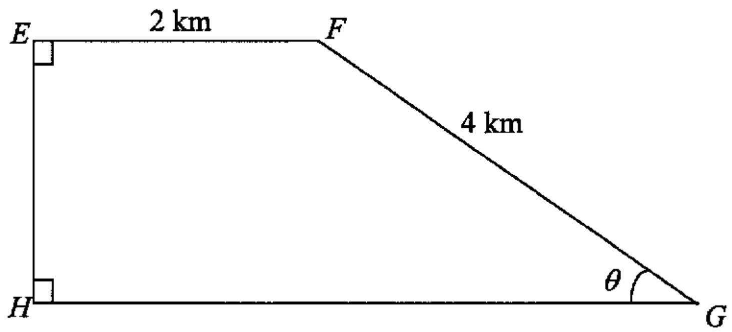
x	1	2	3	4	5	6
y	5	4.95	5.20	5.50	5.81	6.12

(i) Using a scale of 1 cm to represent 1 unit on the $(y\sqrt{x})$-axis and 2 cm to represent 1 unit on the x-axis, plot $y\sqrt{x}$ against x and draw a straight line graph. [3]

(ii) Using your graph, estimate the value of a and of b. [3]

(iii) Using your graph, estimate the value of y when $x = 4.75$. [2]

11 The diagram shows a straight road GH. A runner leaves the road at G and runs 4km in a straight line to a point F. She then turns around at an angle and runs 2km to the point E. At point E, she turns left and runs a certain distance to reach a point H, before making another left turn to run straight back to the point G.

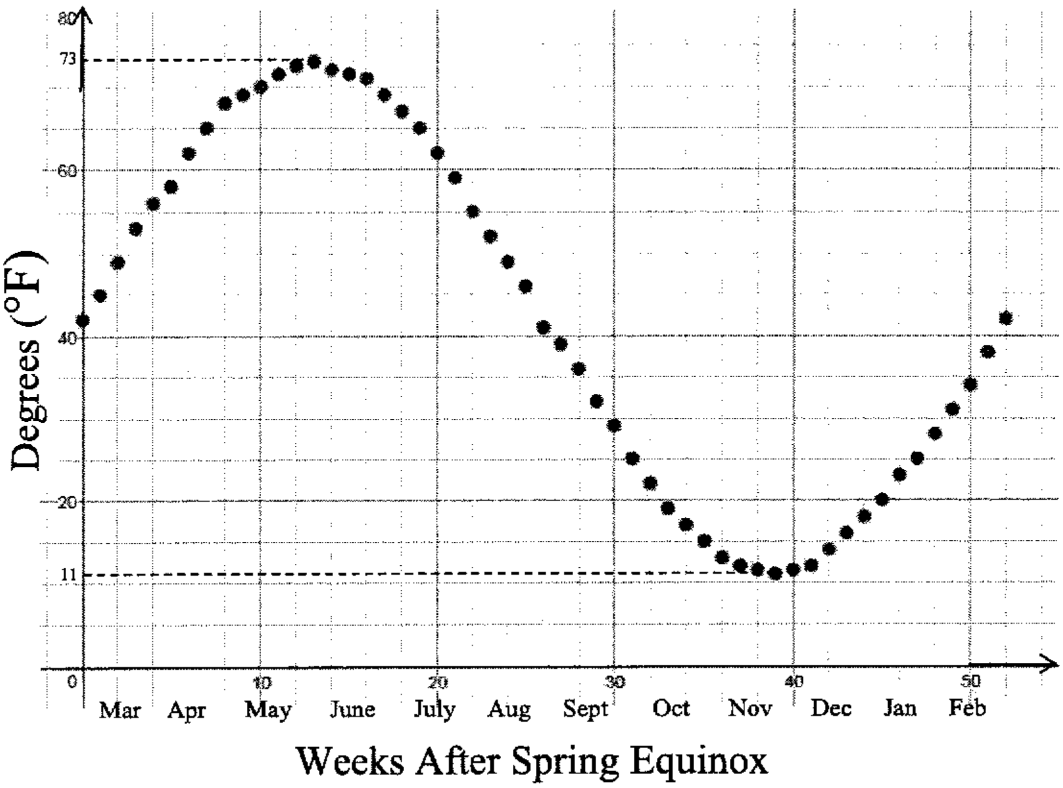


(i) Express the distance, D km which the woman ran, in the form $D = a + b\sin\theta + c\cos\theta$, where a, b and c are integers to be determined. [3]

(ii) Express D in the form $S + R\cos(\theta - \alpha)$, where S, R and α are constants and $0° < \alpha < 90°$. [4]

(iii) Find the maximum value of D and the corresponding value of θ. [2]

12 In New York City, the average weekly temperature, y, in °F, throughout the year can range from a high of 73°F and low of 11°F depending on the season. The average weekly temperatures of New York City from the Spring Equinox (first week of March) were taken for a full year (52 weeks). The data collected resulted in a graph plot as seen below.



From the data points plotted, the average weekly temperature, during both the initial Spring Equinox and the subsequent Spring Equinox 1 year later was 42°F. Using the data provided, for a trigonometric graph that can model the average weekly temperature of New York City, state

(i) the amplitude of this graph, [1]

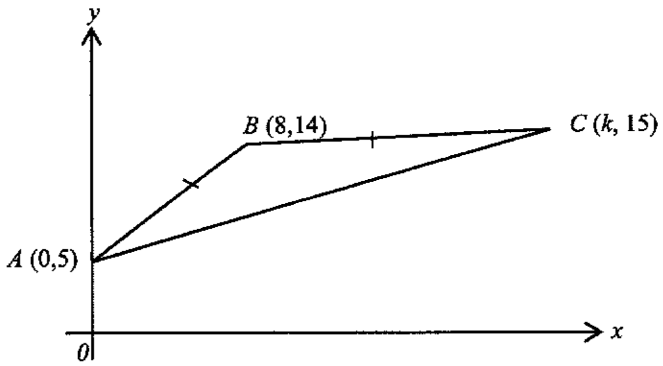
(ii) the period of this graph, [1]

(iii) the trigonometric equation in the form of $y = a\sin\frac{\pi}{b}x + c$, where a, b and c are integers to be determined, that best models the graph. [3]

Mr P wants to visit New York City for a month, but would only like to go when the temperature is at least 60°F or higher, so that he need not bring winter clothes for his trip.

(iv) From the equation of the graph you have found in **(iii)**, which full month can Mr P make his trip to satisfy this requirement? Justify your answer. [4]

13



The diagram shows an isosceles triangle ABC with vertices A (0,5), B (8,14) and C (k,15).

(i) Find the value of k. [4]

D is a point on the x-axis such that $AD = CD$.

(ii) Find the equation of BD. [3]

(iii) Find the coordinates of D. [1]

(iv) Find the areas of triangle ABC and of quadrilateral $ABCD$. [2]

14 **(a)** Prove that $4\sin x\cos^3 x - 4\sin^3 x\cos x = \sin 4x$. [3]

(b) The acute angles A and B are such that tan $(A + B) = 4$ and tan $B = 2$. Without using a calculator, find the exact value of cos A. [5]

(c) Solve the equation $3\cot^2\theta + 10\,\text{cosec}\,\theta = 5$ for $0° \le \theta \le 360°$. [5]

End of Paper

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1 (i)	When $t=0$, $-18=T_0-38e^{-\frac{3}{4}(0)}$ $T_0=38-18$ $=20$			
(ii)	When $t=2$, $T=20-38e^{-\frac{3}{5}(2)}$ $=8.55°C(3s.f)$			
(iii)	No. since $e^{-\frac{3}{5}(t)}>0$, $-38e^{-\frac{3}{5}(t)}<0$ $20-38e^{-\frac{3}{5}(t)}<20(shown)$			
2	$x^2+4x-10>2$ $x^2+4x-12>0$ Roots are 2 or -6 $x<-6$ or $x>2$			
3	$\frac{2+5\sqrt{5}}{3+2\sqrt{5}}\times\frac{3-2\sqrt{5}}{3-2\sqrt{5}}\times 3$ $=\frac{-44+11\sqrt{5}}{-11}\times 3$ $=3(4-\sqrt{5})cm^2$			

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4 (i)	Since $x+1$ is a factor, when $x=-1$, $x^9 - x^6 - 3x^3 - p = 0$ $(-1)^9 - (-1)^6 - 3(-1)^3 - p = 0$ $p = 1$			
(ii)	$x^9 - x^6 - 3x^3 - 1 = 0$ Let $y = x^3$ $y^3 - y^2 - 3y - 1 = 0$ $(y+1)(y^2 - 2y - 1) = 0$ $y = -1$ or $y = \frac{2 \pm \sqrt{8}}{2}$ $= 1 \pm \sqrt{2}$ $x = -1$ or $x = \sqrt[3]{1 \pm \sqrt{2}}$			
5 (i)	$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$			
(ii)	$\alpha + \beta = -m$ $\alpha\beta = 8$			
(iii)	$\alpha^2 + \beta^2 = 48$ $\alpha^2\beta^2 = n$			
(iv)	$(\alpha\beta)^2 = n$ $\therefore 8^2 = n$ $n = 64$ $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ $48 = (-m)^2 - 2(8)$ $m^2 = 64$ $\therefore m = 8$ or -8			
6 (i)	$A\ (3,0)$ $B\ (6, 3)$ $C\ (9, 0)$			
(ii)	$3 - \lvert 6 - x \rvert = 2x$			

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	$\lvert 6-x\rvert = 3-2x$ $6-x=3-2x$ or $6-x=-3+2x$ $x=-3$ $\quad$ $x=3$ (rej)			
7 (i)	T_{r+1} $=\binom{9}{r}(x)^{9-r}\left(-2x^{-2}\right)^r$ $=\binom{9}{r}(-2)^r x^{9-r-2r}$ $=\binom{9}{r}(-2)^r x^{9-3r}$ $9-3r=0$ $r=3$ $\therefore T_4$ $=\binom{9}{3}(-2)^3$ $=-672$			
(ii)	$9-3r=-6$ $r=5$ $\therefore T_6$ $=\binom{9}{5}(-2)^5$ $=-4032$ $\left(1+kx^6\right)\left(x-\frac{2}{x^2}\right)^9=\left(1+kx^6\right)\left(-672-4032x^{-6}+\ldots\right)$ $\therefore (1)(-672)+k(-4032)=0$ $k=-\frac{1}{6}$			

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8 (i)	$\log_4 \frac{4x}{y}$ $= \log_4 4 + \log_4 x - \log_4 y$ $= 1 + \frac{\log_x x}{\log_x 4} - q$ $= 1 + \frac{1}{\log_x 2^2} - q$ $= 1 + \frac{1}{2\log_x 2} - q$ $= 1 + \frac{1}{2p} - q$			
(ii)	$x = 2^{\frac{1}{p}}$ $y = 2^{2q}$ $xy = 2^{\frac{1}{p}+2q}$			

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9 (i)

Eqn of circle: $(x-8)^2+(y-2)^2=80$

(ii)

$x=0$, $64+y^2-4y+4=80$
$y^2-4y-12=0$
$(y-6)(y+2)=0$
$y=6$ or -2

Length $PQ = 6-(-2)$
$= 8$ units

(iii)

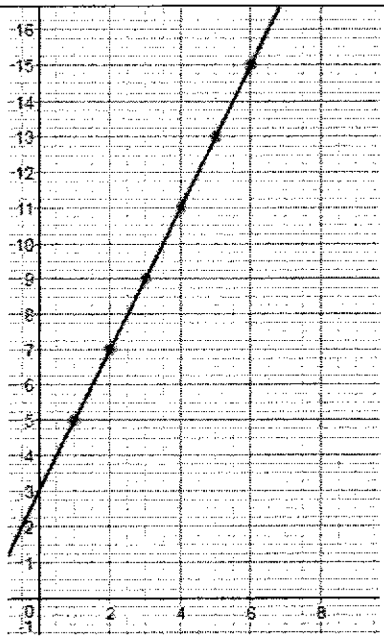
y-coordinate of $B = 2$

(iv)

Let B be $(k,2)$
Length $BP = 5$
$\sqrt{(k-0)^2+(2-6)^2}=5$
$k^2+16=25$
$k^2=9$
$k = 3$(reject) or -3

10 (i)

x	1	2	3	4	5	6
$y\sqrt{x}$	5	7	9	11	13	15

(ii)				
(iii)	$y-b\sqrt{x}=\frac{a}{\sqrt{x}}$ $y\sqrt{x}=bx+a$ $a=3$ $b=\frac{15-13}{6-5}=2$ $y\sqrt{x}=12.5$ (allow for deviation of ±0.1) From graph, the point of intersection is $(4.75,12.5)$ (allow for deviation of ±0.1) $y\sqrt{4.75}=12.5$ $y=5.74$			

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11	(i)	$EH = 4\sin\theta$ $HG - 2 = 4\cos\theta$ $D = 4+2+4\cos\theta+2+4\sin\theta$ $D = 8+4\cos\theta+4\sin\theta$ So $a=8, b=4, c=4$			
	(ii)	$4\sin\theta+4\cos\theta = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$ $4 = R\cos\alpha$ $4 = R\sin\alpha$ $R = \sqrt{4^2+4^2} = \sqrt{32} = 4\sqrt{2}$ $\tan\alpha = \frac{4}{4}$ $\alpha = 45°$ $\therefore D = 8+4\sqrt{2}\cos(\theta - 45°)$			
	(iii)	Max D $= (8+4\sqrt{2})$ km $\theta - 45° = 0°$ $\theta = 45°$			
12	(i)	Amplitude = 31°F			
	(ii)	Period = 52 weeks			
	(iii)	$y = 31\sin\frac{\pi}{26}x + 42$ Award 2 marks if answer is not simplified and left as $y = 31\sin\frac{2\pi}{52}x + 42$			
	(iv)	$60 = 31\sin\frac{\pi}{26}x + 42$			

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		$18 = 31\sin\frac{\pi}{26}x$ $\frac{18}{31} = \sin\frac{\pi}{26}x$ (1st, 2nd quadrants) $\alpha = 0.61952$ $\frac{\pi}{26}x = 0.61952, 2.522071$ $x = 5.127, 20.87$ So, the temperature in NYC will be higher than 60°F from after 6 weeks from the first week of march, and before 20 weeks from the first week of march. Months completely fall within temperature range are May and June. Hence, Mr P should go for the month of May or June.			
13	(i)	$AB = BC$ $\sqrt{(8-0)^2+(14-5)^2} = \sqrt{(k-8)^2+(15-14)^2}$ $145 = k^2 - 16k + 65$ $k^2 - 16k - 80 = 0$ $(k+4)(k-20) = 0$ $k = -4$ (reject) or 20			
	(ii)	*BD* is the perpendicular bisector of *AC*. $MdptAC = (\frac{0+20}{2}, \frac{5+15}{2})$ $= (10,10)$ $GradientAC = \frac{15-5}{20-0}$ $= \frac{1}{2}$ Therefore Gradient*BD* = -2 Equation of *BD*: $y - 10 = -2(x-10)$ $y = -2x + 30$			
	(iii)	At *D*, sub $y = 0$,			

	$0=-2x+30$ $x=15$ $D=(15,0)$			
(iv)	Area $ABC=\frac{1}{2}\begin{vmatrix}0 & 20 & 8 & 0\\ 5 & 15 & 14 & 5\end{vmatrix}$ $=\frac{1}{2}[0+280+40-0-120-100]$ $=50\text{ units}^2$ Area $ABCD=\frac{1}{2}\begin{vmatrix}0 & 15 & 20 & 8 & 0\\ 5 & 0 & 15 & 14 & 5\end{vmatrix}$ $=\frac{1}{2}[0++225+280+40-0-120-0-75]$ $=175\text{ units}^2$			
14 (a)	LHS $=4\sin x\cos x(\cos^2 x-\sin^2 x)$ $=4\sin x\cos x\cos 2x$ $=2\sin 2x\cos 2x$ $=\sin 4x$ = RHS (Proven)			
(b)	$\tan(A+B)=4$ $\frac{\tan A+\tan B}{1-\tan A\tan B}=4$ $\frac{\tan A+2}{1-2\tan A}=4$ $\tan A+2=4-8\tan A$ $9\tan A=2$ $\tan A=\frac{2}{9}$ If A acute, A is in the first quadrant, then $\cos A=\frac{9}{\sqrt{85}}$ $=\frac{9\sqrt{85}}{85}$			
(c)	$\frac{3}{\tan^2\theta}+\frac{10}{\sin\theta}=5$ $3\cot^2\theta+10\operatorname{cosec}\theta=5$ $3(\operatorname{cosec}^2\theta-1)+10\operatorname{cosec}\theta=5$ $3\operatorname{cosec}^2\theta+10\operatorname{cosec}\theta-8=0$			

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	$(\operatorname{cosec}\theta + 4)(3\operatorname{cosec}\theta - 2) = 0$ $\operatorname{cosec}\theta = -4$ or $\frac{2}{3}$ $\sin\theta = -\frac{1}{4}$ (3rd, 4th quad) or $\frac{3}{2}$ (reject) $\alpha = \sin^{-1}(\frac{1}{4})$ $= 14.478°$ (3.d.p) $\theta = 194.5°, 345.5°$			