BEATTY SECONDARY SCHOOL

END OF YEAR EXAMINATION 2018

SUBJECT : Additional Mathematics **LEVEL : Secondary 3 Express**

PAPER : 4047/01 **DURATION : 2 hours**

SETTER : Mr Teo Chye Keong **DATE : 4 October 2018**

CLASS :	NAME :	REG NO :

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This question paper consists of 5 printed pages.

[Turn Over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Express $\dfrac{x^2+4x+7}{(x-1)(x^2+2)}$ in partial fractions. [5]

2 (a) (i) Write down and simplify the first 4 terms in the expansion of $\left(\frac{1}{2}+2x\right)^8$ in ascending powers of x. [2]

(ii) Given that the expansion of $(a-x)\left(\frac{1}{2}+2x\right)^8$ in ascending powers of x is $\frac{1}{128}+\frac{63}{256}x+bx^2+\ldots$, find the value of a and of b. [2]

(b) (i) Write down the general term in the expansion of $\left(x+\frac{1}{2x^2}\right)^{12}$. [1]

(ii) Hence, or otherwise, find the term independent of x in the expansion of $\left(x+\frac{1}{2x^2}\right)^{12}$. [3]

3 (a) Solve the equation

(i) $\lg(x-3)+3\lg 2=1+\lg\left(\frac{1}{5}x\right)$, [3]

(ii) $4\log_6 x-2\log_x 6=7$. [4]

(b) A cup of hot coffee is put into a refrigerator and it cools from its original temperature of 95°C to T°C in x minutes. Given that the relationship between T and x is $T=95(0.92)^x$,

(i) find the temperature of the coffee after putting it in the refrigerator for 10 minutes, [1]

(ii) find the time taken for the temperature of the coffee to reach 25°C. [2]

(iii) Is it possible for the coffee to reach 0°C ? Explain your answer. [1]

4 (i) Find the value of a and of b for which $2x^2+5x-3$ is a factor of $2x^4-x^3+ax^2+bx-15$. [5]

(ii) Hence, show that the equation $2x^4-x^3+ax^2+bx-15=0$ has exactly 2 solutions. [4]

[Turn Over

5 The roots of the equation $3x^2-5x+9=0$ are $\alpha+1$ and $\beta+1$. Find the quadratic equation whose roots are α^2 and β^2. [5]

6 **(a)** Simplify $\dfrac{20\left(4^{2x}\right)}{\left(2^{x-3}\right)\left(8^{x+2}\right)}$. [3]

(b) The area of a triangle is $\left(6\sqrt{3}-8\right)$ cm^2. If the base length of the triangle is $\left(1+2\sqrt{3}\right)$ cm, find the height, in cm, of the triangle in the form $a-b\sqrt{3}$, where a and b are integers. [3]

7 A circle C_1 has the equation $(x-4)^2+(y-6)^2=100$ and another circle C_2 has the equation $x^2+y^2+2x-16y+49=0$.

(i) Find the coordinates of the centre of the circle C_2 and its radius. [2]

(ii) Show that C_2 lies completely inside of C_1. [3]

8 **(a)** Sketch the graph of $y=|3\cos 2x|-1$ for $0\le x\le 2\pi$. [2]

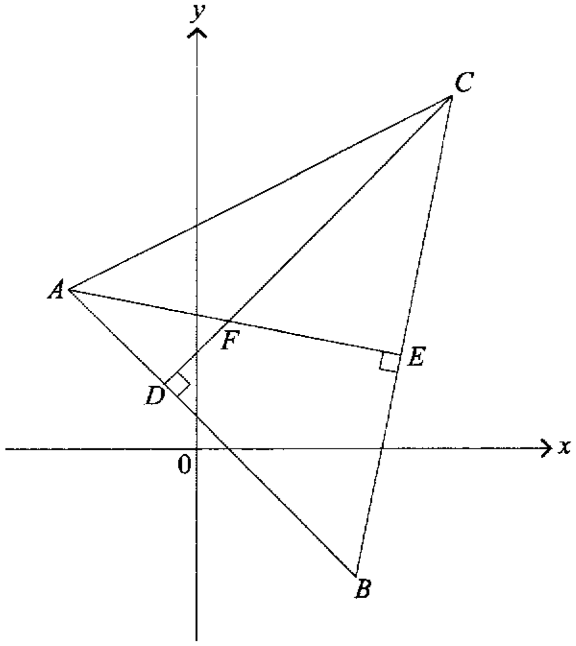
(b) Solve the equation $\sin(2x-34°)=-0.45$ for $0°\le x\le 360°$. [3]

(c) Solve the equation $3\cos x-\cot x=0$ for $-\pi\le x\le\pi$. [3]

9 **(a)** Find the range of values of p for which $2x^2+5x=px-8$ is always positive. [3]

(b) A curve has the equation $y=-x^2+3x-1$. Find the equation of the tangent to the curve that is parallel to the x-axis. [3]

10 Solutions to this question by accurate drawing will not be accepted.



The vertices of the triangle ABC have coordinates $(-4,\ 5)$, $(5,\ -4)$, and $(8,\ 11)$ respectively. AE is perpendicular to BC, CD is perpendicular to AB, and AE and CD meet at F.

(i) Find the coordinates of D and of F. [7]

(ii) Find the area of triangle ABC. [2]

11 Answer the whole of this question on a sheet of graph paper.

The growth rate of the bacteria *Streptococcus lactis* is known to be governed by the equation $N = N_0 a^t$, where N is the number of bacteria after t minutes, and N_0 and a are constants.
In an experiment, some bacteria are grown in a culture. The table below shows the results of the experiment.

t/minutes	10	20	30	40	50	60
N	530	690	850	1080	1400	1880

(a) Using a scale of 2 cm to represent 10 minutes on the horizontal axis and 1 cm to represent 0.1 unit on the vertical axis, plot $\lg N$ against t for $2 \le \lg N \le 4$ and draw a straight line graph. [2]

(b) Use your graph to estimate the value of N_0 and of a. [4]

(c) Use your graph to estimate the time taken for the bacteria population to be doubled. [2]

End of Paper

ANSWER KEY

1	$\frac{4}{x-1}-\frac{3x-1}{x^2+2}$
2(a)(i)	$\frac{1}{256}+\frac{1}{8}x+\frac{7}{4}x^2+14x^3+\ldots$
2(a)(ii)	$a=2, b=\frac{27}{8}$
2(b)(i)	$T_{r+1}=\binom{12}{r}(x)^{12-r}\left(\frac{1}{2x^2}\right)^r$
2(b)(ii)	$\frac{495}{16}$
3(a)(i)	$x=4$
3(a)(ii)	$x=36$
3(b)(i)	41.3°C
3(b)(ii)	16.0 minutes
3(b)(iii)	No, since $0.92^x \neq 0$.
4(i)	$a=-8, b=34$
4(ii)	(show)
5	$x^2+\frac{41}{9}x+\frac{49}{9}=0$
6(a)	$\frac{5}{2}$
6(b)	$8-4\sqrt{3}$
7(i)	centre $=(-1,8)$ radius $=4$ units
7(ii)	(explain)
8(a)	
8(b)	$x=3.6°$ or $120.4°$ or $183.6°$ or $300.4°$
8(c)	$x=-\frac{\pi}{2}$ or 0.340 or $\frac{\pi}{2}$ or 2.80
9(a)	$-3<p<13$
9(b)	$y=\frac{5}{4}$
10(i)	$D(-1,2)$, $F(1,4)$
10(ii)	81 units^2
11(a)	(graph)
11(b)	$a\approx 1.025$, $N_0\approx 407$
11(c)	$t\approx 28$ minutes

3 EXPRESS SA2 2018
ADDITIONAL MATHEMATICS
ANSWER SCHEME

1	$\frac{x^2+4x+7}{(x-1)(x^2+2)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+2}$	
	$x^2+4x+7=A(x^2+2)+(Bx+C)(x-1)$	M1
	by cover up rule: $A=\frac{(1)^2+4(1)+7}{(1^2+2)}$	
	$=4$	B1
	comparing x^2: $A+B=1$	
	$B=-3$	B1
	comparing x^0: $2(4)-C=7$	
	$C=1$	B1
	$\therefore \frac{x^2+4x+7}{(x-1)(x^2+2)}=\frac{4}{x-1}-\frac{3x-1}{x^2+2}$	A1
	TOTAL MARKS: 5	

2(a)(i)	$\left(\frac{1}{2}+2x\right)^8=\binom{8}{0}\left(\frac{1}{2}\right)^8+\binom{8}{1}\left(\frac{1}{2}\right)^7(2x)+\binom{8}{2}\left(\frac{1}{2}\right)^6(2x)^2+\binom{8}{3}\left(\frac{1}{2}\right)^5(2x)^3+\ldots$ $=\frac{1}{256}+\frac{1}{8}x+\frac{7}{4}x^2+14x^3+\ldots$	M1 A1
2(a)(ii)	$(a-x)\left(\frac{1}{2}+2x\right)^8=\frac{1}{128}+\frac{63}{256}x+bx^2+\ldots$ $(a-x)\left(\frac{1}{256}+\frac{1}{8}x+\frac{7}{4}x^2+14x^3\right)=\frac{1}{128}+\frac{63}{256}x+bx^2+\ldots$ comparing x^0: $\frac{a}{256}=\frac{1}{128}$ $a=2$ comparing x^2: $\frac{7}{4}a-\frac{1}{8}=b$ $b=\frac{27}{8}$	 B1 B1
2(b)(i)	$T_{r+1}=\binom{12}{r}(x)^{12-r}\left(\frac{1}{2x^2}\right)^r$	B1
2(b)(ii)	$T_{r+1}=\binom{12}{r}(x)^{12-r}\left(\frac{1}{2x^2}\right)^r$ $=\binom{12}{r}(x)^{12-r}\left(\frac{1}{2}\right)^r\left(\frac{1}{x^2}\right)^r$ $=\binom{12}{r}(x)^{12-3r}\left(\frac{1}{2}\right)^r$ $x^0=x^{12-3r}$ $12-3r=0$ $r=4$ $\therefore$ term independent of $x=\binom{12}{4}(x)^8\left(\frac{1}{2x^2}\right)^4$ $=\frac{495}{16}$	 M1 M1 A1
	TOTAL MARKS: 8	

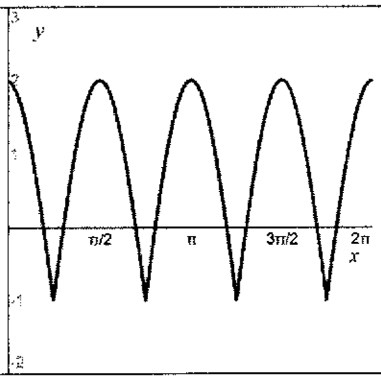
3(a)(i)	$\lg(x-3)+3\lg 2=1+\lg\left(\frac{1}{5}x\right)$ $\lg(x-3)+\lg 2^3-\lg\left(\frac{1}{5}x\right)=\lg 10$ $\frac{(x-3)(2^3)}{\frac{1}{5}x}=10$ $8x-24=2x$ $6x=24$ $x=4$	M1 M1 A1
3(a)(ii)	$4\log_6 x-2\log_x 6=7$ $4\log_6 x-\frac{2\log_6 6}{\log_6 x}=7$ let $u=\log_6 x$ $4u-\frac{2}{u}=7$ $4u^2-7u-2=0$ $(4u+1)(u-2)=0$ $u=-\frac{1}{4}$ or 2 $\log_6 x=-\frac{1}{4}$ (reject since $\log_6 x>0$) or $\log_6 x=2$ $x=36$	M1 M1 M1 A1
3(b)(i)	when $x=10$, $T=95(0.92)^{10}$ $=41.3°\text{C}$	B1
3(b)(ii)	when $T=25$, $25=95(0.92)^x$ $\frac{25}{95}=0.92^x$ $\lg\frac{25}{95}=x\lg 0.92$ $x=16.0$ minutes	M1 A1
3(b)(iii)	No, since $0.92^x\neq 0$.	B1
	TOTAL MARKS: 11	

4(i)	let $f(x)=2x^4-x^3+ax^2+bx-15$ $2x^2+5x-3=(2x-1)(x+3)$ $f\left(\frac{1}{2}\right)=0$ $2\left(\frac{1}{2}\right)^4-\left(\frac{1}{2}\right)^3+a\left(\frac{1}{2}\right)^2+b\left(\frac{1}{2}\right)-15=0$ $a+2b=60$ --- (1) $f(-3)=0$ $2(-3)^4-(-3)^3+a(-3)^2+b(-3)-15=0$ $9a-3b=-174$ --- (2) $(1)\times 9-(2)$: $18b-(-3b)=540-(-174)$ $21b=714$ $b=34$ $a=60-2(34)$ $=-8$	M1 M1 M1 A1 A1
4(ii)	$2x^4-x^3-8x^2+34x-15=0$ $(2x-1)(x+3)(ax^2+bx+c)=0$ comparing x^4: $2a=2$ $a=1$ comparing x^0: $-3c=-15$ $c=5$ comparing x: $-3b-c+6c=34$ $-3b+25=34$ $b=-3$ $(2x-1)(x+3)(x^2-3x+5)=0$ For $(x^2-3x+5)=0$, since $b^2-4ac=(-3)^2-4(1)(5)$ $=9-20$ $=-11<0$ Hence, the equation has only 2 solutions.	M2 M1 A1
	TOTAL MARKS: 9	

5	$\alpha+\beta+2=\frac{5}{3}$ $\alpha\beta+\alpha+\beta+1=3$ $\alpha+\beta=-\frac{1}{3}$ $\alpha\beta=\frac{7}{3}$ $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=\left(-\frac{1}{3}\right)^2-2\left(\frac{7}{3}\right)$ $=-\frac{41}{9}$ $\alpha^2\beta^2=\left(\frac{7}{3}\right)^2$ $=\frac{49}{9}$ $\therefore x^2+\frac{41}{9}x+\frac{49}{9}=0$	M1 (sum and product of roots) M1 (soe) M1 M1 A1
	TOTAL MARKS: 5	

6(a)	$\frac{20(4^{2x})}{(2^{x-3})(8^{x+2})}=\frac{20(2^{4x})}{(2^{x-3})(2^{3x+6})}$ $=\frac{20(2^{4x})}{2^{4x+3}}$ $=\frac{20(2^{4x})}{(2^{4x})(2^3)}$ $=\frac{5}{2}$	M1 M1 A1
6(b)	height $=\frac{6\sqrt{3}-8}{\left(\frac{1}{2}\right)(1+2\sqrt{3})}$ $=\frac{12\sqrt{3}-16}{1+2\sqrt{3}}\times\frac{1-2\sqrt{3}}{1-2\sqrt{3}}$ $=\frac{12\sqrt{3}-72-16+32\sqrt{3}}{1-12}$ $=\frac{44\sqrt{3}-88}{-11}$ $=8-4\sqrt{3}$	M1 M1 A1
	TOTAL MARKS: 6	

7(i)	$x^2+y^2+2x-16y+49=0$ $(x+1)^2+(y-8)^2-1-64+49=0$ $(x+1)^2+(y-8)^2=16$ centre $=(-1,8)$ radius $=4$ units	 B1 B1
7(ii)	let the centres of C_1 and C_2 be O_1 and O_2 respectively $O_1O_2=\sqrt{(8-6)^2+(-1-4)^2}$ $=\sqrt{29}$ since $\sqrt{29}+4<10$, therefore C_2 lies completely inside C_1.	 M1 M1 A1
	TOTAL MARKS: 5	

8(a)		B1 – correct shape B1 – correct labels
8(b)	$\sin(2x-34°)=-0.45$ $\alpha=\sin^{-1}0.45$ $=26.743°$ $0°\le x\le 360°$ $-34°\le 2x-34°\le 686°$ $2x-34°=-26.743°$ or $180°+26.743°$ or $360°-26.743°$ or $540°+26.743°$ $x=3.6°$ or $120.4°$ or $183.6°$ or $300.4°$	 M1 M1 A1
8(c)	$3\cos x-\cot x=0$ $3\cos x-\frac{\cos x}{\sin x}=0$ $\cos x\left(3-\frac{1}{\sin x}\right)=0$ $\cos x=0$ or $\sin x=\frac{1}{3}$ $x=-\frac{\pi}{2}$ or $\frac{\pi}{2}$ or $x=0.340$ or 2.80 $x=-\frac{\pi}{2}$ or 0.340 or $\frac{\pi}{2}$ or 2.80	 M1 M1 A1
	TOTAL MARKS: 8	

9(a)	$2x^2+5x=px-8$ $2x^2+5x-px+8=0$ $b^2-4ac<0$ $(5-p)^2-4(2)(8)<0$ $25-10p+p^2-64<0$ $p^2-10p-39<0$ $(p-13)(p+3)<0$ $-3<p<13$	M1 M1 A1
9(b)	let the equation of the tangent be $y=a$ $a=-x^2+3x-1$ $-x^2+3x-1-a=0$ $b^2-4ac=0$ $(3)^2-4(-1)(-1-a)=0$ $9-4-4a=0$ $a=\frac{5}{4}$ $\therefore y=\frac{5}{4}$ Alternate solution: $y=-x^2+3x-1$ $=-(x^2-3x+1)$ $=-\left(x-\frac{3}{2}\right)^2+\frac{5}{4}$ maximum point $=\left(\frac{3}{2},\frac{5}{4}\right)$ equation of tangent: $y=\frac{5}{4}$	M1 M1 A1 M1 M1 A1
		TOTAL MARKS: 6

10(i)	$m_{AB}=\frac{-4-5}{5-(-4)}$ $=-1$ $m_{CD}=1$ equation of CD: $y-11=1(x-8)$ $y=x+3$ equation of AB: $y-5=-1(x-(-4))$ $y=-x+1$ $x+3=-x+1$ $x=-1$ $y=2$ $\therefore D(-1,2)$ $m_{BC}=\frac{11-(-4)}{8-5}$ $=5$ $m_{AE}=-\frac{1}{5}$ equation of AE: $y-5=-\frac{1}{5}(x-(-4))$ $y=-\frac{1}{5}x+\frac{21}{5}$ $x+3=-\frac{1}{5}x+\frac{21}{5}$ $5x+15=-x+21$ $6x=6$ $x=1$ $y=4$ $\therefore F(1,4)$	M1 - m_{CD} M1 – equation of CD and AB M1 A1 M1 – equation of AE M1 A1
10(ii)	area $=\frac{1}{2}\begin{vmatrix}-4 & 5 & 8 & -4\\ 5 & -4 & 11 & 5\end{vmatrix}$ $=81$ units^2	M1 A1
	TOTAL MARKS: 9	

11	(see attached graph)	
	TOTAL MARKS: 8	