Name:______________________________ ( )	Class: 3E______

SERVE·LEAD·EXCEL

GREENDALE SECONDARY SCHOOL
End-of-Year Examination 2018

ADDITIONAL MATHEMATICS

08 Oct 2018

Secondary Three Express **2 hour 30 minutes**

Candidates answer on the Writing Paper.

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark or blue pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.
Omission of essential working may result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

This document consists of 5 printed pages, including this cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Answer **all** questions and start each question on a **fresh page**.

1 Given that the points $(d,10d)$, $(0.75,0)$ and $(1.5,5+4d)$ are collinear, find the possible values of d. [4]

2 Without using a calculator, find the constants a and b, for which $\frac{2\sqrt{10}+\sqrt{3}}{\sqrt{6}-\sqrt{5}}$ can be expressed as $a\sqrt{2}+b\sqrt{15}$. [5]

3 The function f is defined by $f(x)=-3\cos(0.5x)$.

(i) Write down the period and amplitude of $f(x)$. [2]

(ii) Sketch the graph of $f(x)$ for $0^\circ \leq x \leq 360^\circ$. [3]

4 A curve has the equation $y=2x^2+x+a$, where a is a constant.

(i) When $a=-10$, find the range of values of x for which $y \leq 2x+5$ [3]

(ii) Find the value of a for which the line $y=5x+1$ is tangent to the curve. [4]

5 Solve, for x and y, the simultaneous equations

$$\frac{5^{2x}}{5^{3y}}=\frac{5}{5^3(5^y)}$$

$$\log_3(x-4)=\log_3(y-1)-\log_3 x.$$ [7]

6 The curve $y=3\log_2 kx$ passes through the points with coordinates $(0.25,0)$ and $(8,l)$.

(i) Determine the values of k and l. [2]

(ii) Sketch the graph of $y=3\log_2 kx$ [2]

(iii) By drawing a suitable graph on the same set of axes, find the number of solutions to the equation $3\log_2 kx-\frac{5}{x^2}=0,\ x\neq 0$. [2]

7 **(a)** Fine the exact value of $\dfrac{\tan 45^\circ}{\sin 60^\circ + \sec 45^\circ}$. [3]

(b) Given that $\sin A = \dfrac{3}{5}$ and $\tan B = -\dfrac{12}{5}$, and that A and B are in the same quadrant, find **without using a calculator** the value of

(i) $\cos(90^\circ - A)$ [1]

(ii) $\sin B$ [1]

(iii) $\dfrac{\cot A}{\sin B}$ [2]

8 The roots of the quadratic equation $2x^2 + 5x - 3 = 0$ are α and β.

(i) Show that $\alpha^2 + \beta^2 = 9.25$. [3]

(ii) Find a quadratic equation whose roots are α^3 and β^3. [5]

9 **(a)** **(i)** Write down and simplify the first 3 terms in the expansion of $(2+x)^5$, in ascending powers of x. [2]

(ii) Find the coefficient of a^2 in the expansion of $(2 + a^2 + 3a)^5$. [3]

(b) Find the term independent of x in the binomial expansion of $\left(3x + \dfrac{1}{2x^2}\right)^9$. [4]

10 **(i)** Find the coordinates of all the points at which the graph $y = -|x+5| + 3$ meets the coordinate axes. [4]

(ii) Sketch the graph of $y = -|x+5| + 3$. [2]

(iii) Solve the equation $-|x+5| + 3 = 0.5x$. [3]

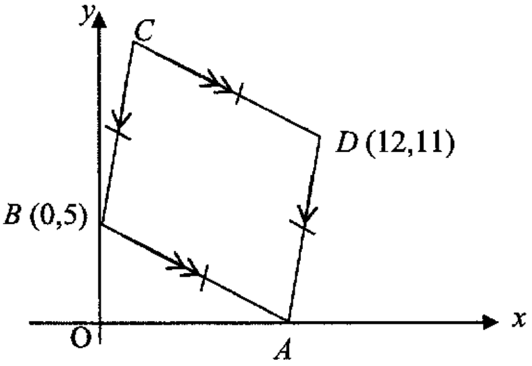
(iv) Hence, state the range of values of x such that $-|x+5| + 3 > 0.5x$. [1]

11 **(i)** Find the remainder when $2x^3 + 3x^2 - 11x - 6$ is divided by $2x + 3$. [2]

(ii) Factorise completely the cubic polynomial $2x^3 + 3x^2 - 11x - 6$. [4]

(iii) Hence, express $\dfrac{9x^2 + 14x + 11}{2x^3 + 3x^2 - 11x - 6}$ in partial fractions. [4]

12 The diagram shows a rhombus $ABCD$. B and D are $(0,5)$ and $(12,11)$ respectively and A lies on the x-axis.



(i) Show that the gradient of AC is -2. [2]

Find,

(ii) the midpoint of BD, [1]

(iii) the coordinates of A and C, [5]

(iv) the area of $ABCD$. [2]

13 **(a)** Solve the equation $2\tan^2\theta = 1 + 5\sec\theta$ for $0 \le \theta \le 2\pi$. [4]

(b) **(i)** Show that

$$\sec\theta + \cot\theta\operatorname{cosec}\theta = \sec\theta\operatorname{cosec}^2\theta.$$ [4]

(ii) **Hence**, solve the equation

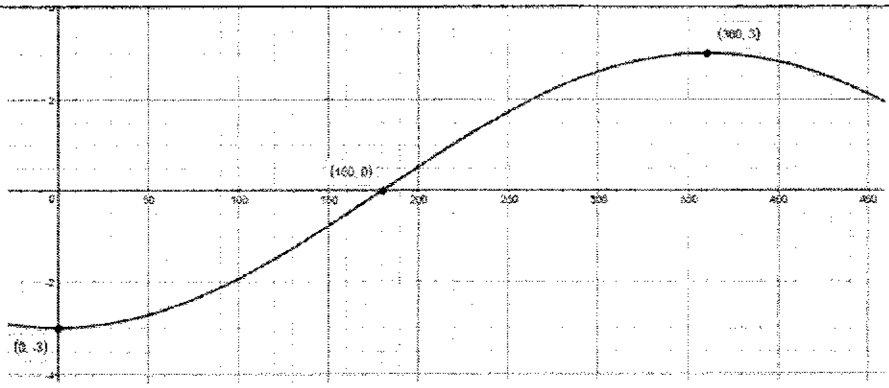
$$\sec\theta + \cot\theta\operatorname{cosec}\theta = \sec\theta\cot^2\theta + 2,$$

for $0^\circ \le x \le 360^\circ$ [4]

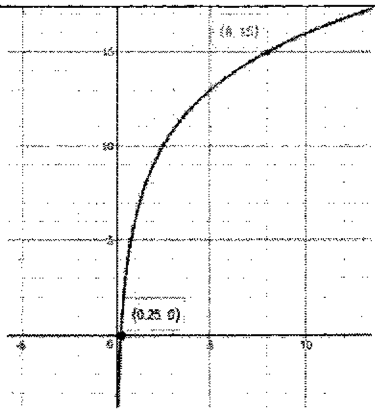
End - of – Paper

2018 END-YEAR EXAMINATION
SEC 3EX AMATH PAPER

Qn	Solutions	Marks
1	Collinear implies that the gradient would be the same for any two lines formed by the 3 points.	
	$\frac{10d-0}{d-(0.75)}=\frac{5+4d-0}{1.5-0.75}$	M1
	$\frac{10d}{d-0.75}=\frac{5+4d}{0.75}$	
	$7.5d=5d-3.75+4d^2-3d$	M1
	$4d^2-5.5d-3.75=0$	
	$d=\frac{5.5\pm\sqrt{(-5.5)^2-4(4)(-3.75)}}{2(4)}$	M1
	$=-0.5 \quad \text{or} \quad =-1.875$	A1
2	$\frac{2\sqrt{10}+\sqrt{3}}{\sqrt{6}-\sqrt{5}}=\frac{2\sqrt{10}+\sqrt{3}}{\sqrt{6}-\sqrt{5}}\times\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}$	M1
	$=\frac{2\sqrt{60}+2\sqrt{50}+\sqrt{18}+\sqrt{15}}{\sqrt{6}^2-\sqrt{5}^2}$	M1
	$=\frac{4\sqrt{15}+10\sqrt{2}+3\sqrt{2}+\sqrt{15}}{6-5}$	M1
	$=\frac{5\sqrt{15}+13\sqrt{2}}{1}$	M1
	$=13\sqrt{2}+5\sqrt{15}$	
	$\therefore a=13,\ b=5$	A1

3(i)	Period $= \dfrac{2\pi}{0.5}$ $= 4\pi$ or 720° Amplitude $= 3$	B1 B1
3(ii)		B1 – general shape B1 – intercepts B1 – max & min pt
4(i)	$2x^2 + x - 10 \le 2x + 5$ $2x^2 - x - 15 \le 0$ $(2x+5)(x-3) \le 0$ $-2.5 \le x \le 3$	M1 M1 A1
4(ii)	$2x^2 + x + a = 5x + 1$ $2x^2 - 4x + a - 1 = 0$ For tangent: $b^2 - 4ac = 0$ $(-4)^2 - 4(2)(a-1) = 0$ $16 - 8a + 8 = 0$ $8a = 24$ $a = 3$	M1 M1 (b^2 - $4ac$) M1 (=0) A1

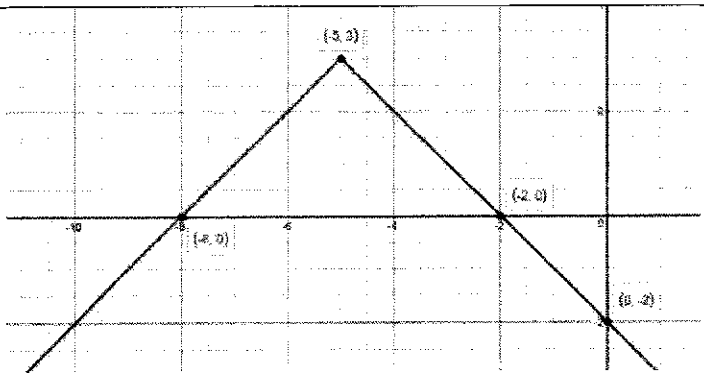
5	$\frac{5^{2x}}{5^{3y}}=\frac{5^1}{5^3(5^y)}$ $5^{2x-3y}=5^{1-3-y}$ $2x-3y=-2-y$ $2x-2y=-2$ $y=x+1------(1)$ $\log_3(x-4)=\log_3(y-1)-\log_3 x$ $\log_3(x-4)=\log_3\frac{(y-1)}{x}$ $x-4=\frac{y-1}{x}$ $x^2-4x=y-1$ $y=x^2-4x+1------(2)$ Sub (1) into (2), $x^2-4x+1=x+1$ $x^2-5x=0$ $x(x-5)=0$ $x=0$ (rej) or $x=5$ $y=6$	 M1 M1 M1 M1 M1 A1 A1

6(i)	$y = 3\log_2 kx$ Sub $x = 0.25$ and $y = 0$, $0 = 3\log_2 k(0.25)$ $\log_2 k(0.25) = 0$, $k(0.25) = 1$ $k = 4$ Sub $x = 8$ and $y = l$, $l = 3\log_2 4(8)$ $l = 3(5)$ $= 15$	B1 B1
6(ii)		B1 – general shape B1 – intercept
6(iii)	$3\log_2 kx - \frac{5}{x^2} = 0$ $3\log_2 kx = \frac{5}{x^2}$ Draw $y = \frac{5}{x^2}$ Therefore there is only 1 solution to the equation.	M1 A1

7(a)	$\frac{\tan 45^\circ}{\sin 60^\circ + \sec 45^\circ} = \frac{1}{\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{1}}$ $= \frac{1}{\frac{\sqrt{3}+2\sqrt{2}}{2}}$ $= \frac{2}{\sqrt{3}+2\sqrt{2}} \times \frac{\sqrt{3}-2\sqrt{2}}{\sqrt{3}-2\sqrt{2}}$ $= \frac{2\sqrt{3}-4\sqrt{2}}{3-8}$ $= \frac{4\sqrt{2}-2\sqrt{3}}{5}$	M1 M1 A1
7(b)(i)	$\cos(90 \; - A) = \sin A$ $= \frac{3}{5}$ But since A is in the 2nd quadrant $\cos(90 \; - A) = -\frac{3}{5}$	B1
7(b)(ii)	$\tan B = -\frac{12}{5}$ $\Rightarrow \text{opp} = 12$ $\Rightarrow \text{adj} = 5$ $\therefore \text{hyp} = 13$(Pythagoras') $\sin B = \frac{12}{13}$	B1
7(b)(iii)	$\sin A = \frac{3}{5}$ $\Rightarrow \text{opp} = 3$ $\Rightarrow \text{hyp} = 5$ $\therefore \text{adj} = 4$(Pythagoras') $\cot A = -\frac{4}{3}$ $\frac{\cot A}{\sin B} = \frac{-\frac{4}{3}}{\frac{12}{13}}$ $= -\frac{13}{9}$	M1 A1

8(i)	$2x^2+5x-3=0$ $\alpha+\beta=-\frac{5}{2}$ $=-2.5$ $\alpha\beta=\frac{-3}{2}$ $=-1.5$ $\alpha^2+\beta^2=\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta$ $=(\alpha+\beta)^2-2\alpha\beta$ $=(-2.5)^2-2(-1.5)$ $=9.25$ (shown)	M1 – for either SOR or POR M1 A1
8(ii)	$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=(\alpha+\beta)\left[(\alpha^2+\beta^2)-\alpha\beta\right]$ $=(-2.5)\left[9.25-(-1.5)\right]$ $=-26\frac{7}{8}$ $\alpha^3\beta^3=(\alpha\beta)^3$ $=(-1.5)^3$ $=-3\frac{3}{8}$ $x^2-\left(-26\frac{7}{8}\right)x+\left(-3\frac{3}{8}\right)=0$ $8x^2+215x-27=0$	M1 M1 M1 M1 A1

9(a)	$(2+x)^5 = 2^5 + {}^5C_1(2)^5(x) + {}^5C_2(2)^4(x)^2 + \dots$ $\approx 32 + 80x + 80x^2$	M1 A1
	$(2+x)^5 = (2+a^2+3a)^5$ $= 32 + 80(a^2+3a) + 80(a^2+3a)^2$ Coefficient of a^2 in $(2+a^2+3a)^5 = (80)[1] + (80)(3^2)$ $= 80 + 720$ $= 800$	M1 M1 A1
9(b)(i)	$T_{r+1} = {}^9C_r(3x)^{9-r}\left(\frac{1}{2x^2}\right)^r$ $= {}^9C_r(3^{9-r}x^{9-r})(2^{-r}x^{-2r})$ $= {}^9C_r 3^{9-r}2^{-r}x^{9-3r}$ Term independent of x: $9-3r=0$ $r=3$ $\therefore$ The 4th term is the independent term. $T_4 = {}^9C_3 3^{9-3}2^{-3}x^{9-9}$ $= 7654\frac{1}{2}$	M1 M1 M1 A1

10(i)	$y = -\|x+5\|+3$ When $x = 0$, $y = -\|5\|+3$ $= -2$ When y = 0, $0 = -\|x+5\|+3$ $\|x+5\| = 3$ $-x-5 = 3$ or $x+5 = 3$ $x = -8$ or $x = -2$ $(0,-2)$, $(-8,0)$, $(-2,0)$	M1 A3
10(ii)		B1 – General shape B1 ecf – labelled intercepts
10(iii)	$-\|x+5\|+3 = 0.5x$ $\|x+5\| = 3-0.5x$ $-x-5 = 3-0.5x$ or $x+5 = 3-0.5x$ $x = -16$ or $x = -\frac{4}{3}$	M1 A1, A1
10(iv)	$-16 < x < -\frac{4}{3}$	B1

11(i)	$P(x)=2x^3+3x^2-11x-6$ $P\left(-\frac{3}{2}\right)=2\left(-\frac{3}{2}\right)^3+3\left(-\frac{3}{2}\right)^2-11\left(-\frac{3}{2}\right)-6$ $=10.5$	M1 A1
11(ii)	$P(x)=2x^3+3x^2-11x-6$ Sub $x=-3$ into $P(x)$ $P(-3)=2(-3)^3+3(-3)^2-11(-3)-6$ $=0$ $\Rightarrow x+3$ is a factor of $P(x)$ $P(x)=(x+3)(2x^2-3x-2)$ $=(x+3)(2x+1)(x-2)$	M1 M1 M1 A1
11(iii)	$\frac{9x^2+14x+11}{2x^3+3x^2-11x-6}=\frac{A}{(x+3)}+\frac{B}{(x-2)}+\frac{C}{(2x+1)}$ $9x^2+14x+11=A(x-2)(2x+1)+B(x+3)(2x+1)+C(x+3)(x-2)$ Sub $x=-3$ $50=25A$ $A=2$ Sub $x=2$ $75=25B$ $B=3$ Sub $x=-0.5$ $6.25=-6.25C$ $C=-1$ $\frac{9x^2+14x+11}{2x^3+3x^2-11x-6}=\frac{2}{(x+3)}+\frac{3}{(x-2)}-\frac{1}{(2x+1)}$	M1 ecf M1 ecf M1 ecf A1

12(i)	$m_{BD}=\frac{11-5}{12-0}$ $=\frac{1}{2}$ Since BD is perpendicular to AC, $m_{BD}\times m_{AC}=-1$ $m_{AC}=-1\div\frac{1}{2}$ $=-2$ (shown)	M1 A1
12(ii)	Midpoint of BD $=\left(\frac{0+12}{2},\frac{5+11}{2}\right)$ $=(6,8)$	B1
12(iii)	Equation of AC: Since midpoint of BD is also the midpoint of AC, $y-8=-2(x-6)$ $y=-2x+12+8$ $y=-2x+20$ At A, $y=0$ $-2x+20=0$ $x=10$ $\therefore A(10,0)$ Let the coordinates of C be (x_1,y_1) $(6,8)=\left(\frac{10+x_1}{2},\frac{0+y_1}{2}\right)$ $(x_1,y_1)=(2,16)$ $\therefore C(2,16)$ OR By observation, $A(10,0)\rightarrow D(12,11)$ $B(0,5)\rightarrow C(2,16)$	M1 M1 A1 B1 (x value) B1 (y value)
12(iv)	Area $=\frac{1}{2}\begin{vmatrix}10 & 0 & 2 & 12 & 10\\ 0 & 5 & 16 & 11 & 0\end{vmatrix}$ $=\frac{1}{2}\left\|(50+22)-(10+192+110)\right\|$ $=\frac{1}{2}\left\|-240\right\|$ $=120$ units^2	M1 ecf A1

13(a)	$2\tan^2\theta = 1+5\sec\theta$ $2(\sec^2\theta-1) = 1+5\sec\theta$ $2\sec^2\theta-2-1-5\sec\theta=0$ $2\sec^2\theta-5\sec\theta-3=0$ $(2\sec\theta+1)(\sec\theta-3)=0$ $2\sec\theta=-1$ or $\sec\theta=3$ $\sec\theta=-0.5$ or $\cos\theta=\frac{1}{3}$ $\cos\theta=-2(\text{NA})$ or basic angle $\alpha=1.2309$ $\theta=1.2309,\ 5.0522$ $=1.23,\ 5.05$ (3sf)	M1 M1 M1 A1
13(b)(i)	$\text{LHS}=\sec\theta+\cot\theta\text{cosec}\,\theta$ $=\frac{1}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\frac{1}{\sin\theta}$ $=\frac{1}{\cos\theta}+\frac{\cos\theta}{\sin^2\theta}$ $=\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin^2\theta}$ $=\frac{1}{\cos\theta\sin^2\theta}$ $=\sec\theta\text{cosec}^2\theta$ $=\text{RHS}$	M1 M1 M1 A1
13(b)(ii)	$\sec\theta+\cot\theta\text{cosec}\,\theta=\sec\theta\cot^2\theta+2$ $\sec\theta\text{cosec}^2\theta=\sec\theta\cot^2\theta+2$ $\sec\theta\text{cosec}^2\theta-\sec\theta\cot^2\theta=2$ $\sec\theta(\text{cosec}^2\theta-\cot^2\theta)=2$ $\sec\theta(1)=2$ $\sec\theta=2$ $\cos\theta=0.5$ basic angle $\alpha=60$ $\theta=60\ ,300$	M1 M1 M1 A1