Register No.	Class

Name : ____________________

BENDEMEER SECONDARY SCHOOL
2021 Preliminary Examination
Secondary Four Express
Additional Mathematics Paper 1
4049/ 01

DATE : **25 August 2021**
DURATION : **2 hours 15 minutes**
TOTAL : **90 Marks**

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a 2B pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** questions.
Write your answers in the spaces provided on the question paper.
All the diagrams in this paper are **not** drawn to scale.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

FOR EXAMINER'S USE
90

This question paper consists of 18 printed pages including this cover page.

[Turn over

MATHEMATICAL FORMULAE

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}.$$

$$a^2=b^2+c^2-2bc\cos A.$$

$$\Delta=\frac{1}{2}bc\sin A.$$

1 **(i)** Find the value of 10^x, given that $4^{x+1} \times 5^{2x-3} = 10^{3x}$. [3]

(ii) Hence, solve $4^{x+1} \times 5^{2x-3} = 10^{3x}$. [1]

2 **(a)** The equation of a curve is $y = 3x^2 + 5x + 1$. Find the set of values of x for which the curve lies completely above the line $y - 3x = 2$. [3]

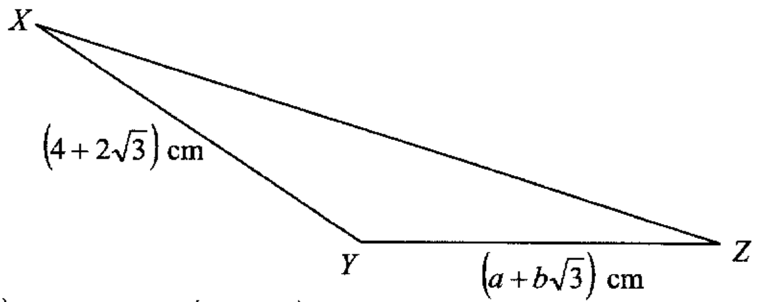
(b) Find the range of values of m for which the equation $mx^2 + 2m = 3x(4 - x)$ has real roots. [3]

3 **(i)** Express $\dfrac{8x^2 - x}{4x^2 - 1}$ in partial fractions. [4]

(ii) Use your results in part (i) to find $\int \dfrac{8x^2 - x}{4x^2 - 1}\, dx$. [2]

4 **(i)** Express $\frac{30+18\sqrt{3}}{2+\sqrt{3}}$ in the form $r+s\sqrt{3}$, where r and s are integers. [3]

(ii) The diagram shows a triangle XYZ.



XY is $(4+2\sqrt{3})$ cm and YZ is $(a+b\sqrt{3})$ cm, where a and b are integers.
The included angle XYZ is 150°.
Given that the area of the triangle is $(15+9\sqrt{3})$ cm^2, find the value of a and of b. [5]

5 **(a)** The equation $\log_2 x + \log_8 x = \log_7 49$ has the solution $x = 2^a$. [4]
Find the value of a.

(b) Show that the equation $\log_3(4x - 11) - \log_3(x - 3) = 1$ has no solution. [4]

6 The equation of a curve is $y = 3 - 4sin2x$.

(i) State the minimum and maximum value of y. [2]

(ii) Sketch the curve $y = 3 - 4sin2x$ for $0 \leq x \leq 2\pi$ [3]

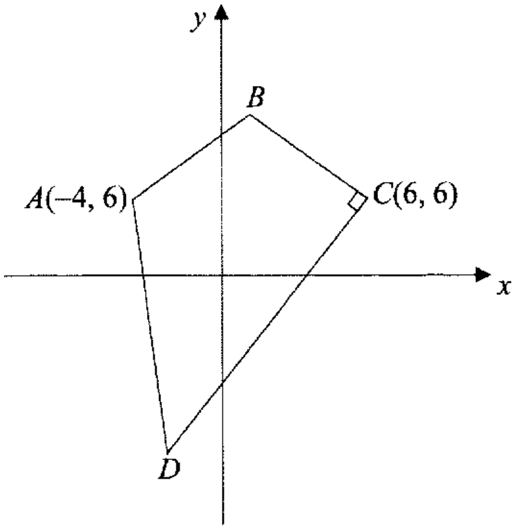
7 The function is defined by $(x) = \frac{x^2-4}{x^2+6}$, $x > 0$.

(i) Explain, with working, whether $g(x)$ is an increasing or decreasing function. [4]

(ii) A point P moves along the curve g(x), such that the y – coordinate of P is increasing at a rate of 0.04 unit per second. Find the rate of increase of the x – coordinate of P when $x = 2$. [2]

8 The gradient function of a curve is given by $\frac{dy}{dx}=\frac{16}{x^3}-4$. Given that the line $2y-x-6=0$ is a normal to the curve, find the equation of the curve. [6]

9 The diagram (not drawn to scale) shows a quadrilateral $ABCD$ such that $AB = BC$ and angle $BCD = 90°$. The point A is $(-4, 6)$ and the point C is $(6, 6)$. Given that the area of triangle ABC is 15 square units and the point D lies on the line $3y + 5x + 42 = 0$.



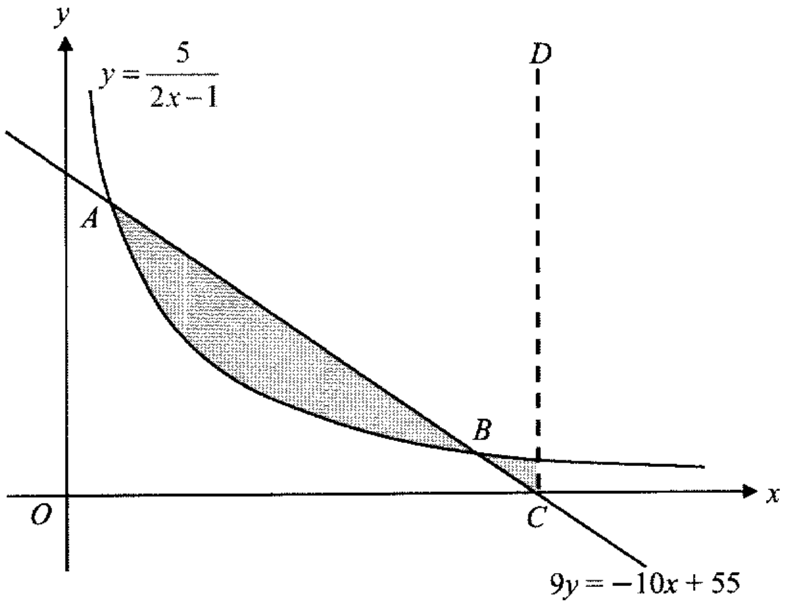
(i) Find the coordinates of B. [2]

(ii) Find the coordinates of D. [4]

9 **(iii)** Find the area of the quadrilateral $ABCD$. [3]

(iv) The point X lies on the line AB produced such that $AB : BX$ is 2 : 3. Find the coordinates of X. [2]

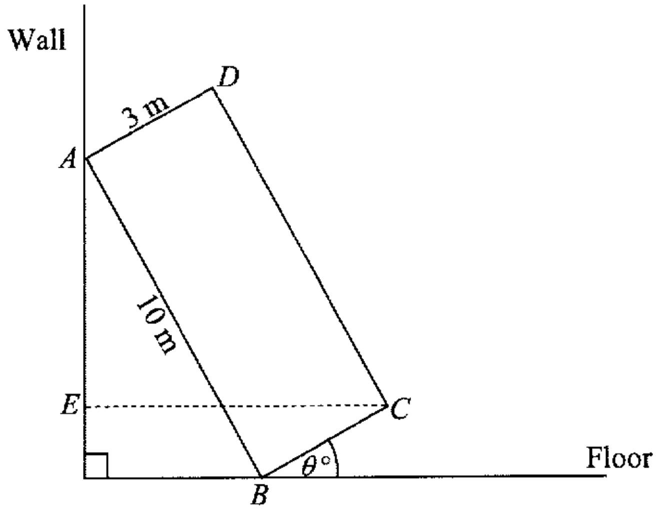
10 The diagram shows part of the curve $y = \frac{5}{2x-1}$. The line $9y = -10x + 55$ intersects the curve at points A and B and meets the x-axis at point C. CD is a straight line parallel to the y – axis.



(i) Find the coordinates of A, B and C. [4]

10 **(ii)** Find the area of the shaded region. [4]

11 The diagram shows a rectangle $ABCD$ that is leaning against a vertical wall.



AB is 10 metres, AD is 3 metres and the side BC makes an angle $\theta°$ with the floor.
EC is horizontal distance of C from the wall.

(i) Show that EC can be expressed in the form $a \sin \theta + b \cos \theta$, where a and b are constants to be found. [2]

(ii) Express EC in the form $R \sin (\theta + \alpha)$ where $R > 0$ and α is an acute angle. [4]

11 The rectangle $ABCD$ will remain leaning against the wall provided EC lies between 7.5 metres and 9.5 metres.

(iii) Find the range of values of θ for the rectangle $ABCD$ to remain leaning against the wall. [4]

12 The points P and Q both lie on a circle and have coordinates (-10, 15) and (10, 0) respectively. The centre of the circle lies on the line $4y = 3x + 37$.

(i) Find the equation of the perpendicular bisector of PQ. [5]

(ii) Find the equation of the circle in the general form. [5]

12 The point R is such that PR is a diameter of the circle.

(iii) Find the coordinates of R. [2]

End of Paper

Register No.	Class

Name :

BENDEMEER SECONDARY SCHOOL
2021 Preliminary Examination
Secondary Four Express
Additional Mathematics Paper 2
4049/ 02

DATE : 31 August 2021
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TOTAL : 90 Marks

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$$\Delta = \frac{1}{2}bc\sin A.$$

1 Given that θ is acute and $\sin\theta = s$, express, in terms of s,

(i) $\tan\theta$ [2]

(ii) $\sec\theta$. [1]

2 Solve the equation $3\cos A + 3\sec A + 7\tan A = 0$ for $0° < A < 360°$. [5]

3 **(i)** Differentiate $3x \sin 2x$ with respect to x. [3]

(ii) Hence, evaluate $\int_0^{\frac{\pi}{4}} x \cos 2x \, dx$. [5]

4 **(i)** Write down the first 3 terms in the expansion of $\left(2-\frac{x}{3}\right)^6$, in ascending power of x. All terms must be expressed in the simplest form. [3]

(ii) Hence, find the term independent of x in the expansion of $\left(2-\frac{x}{3}\right)^6\left(\frac{4}{x}-x\right)^2$. [3]

5 It is given that $f(x) = ax^3 + 3x^2 + bx + 1$ where a and b are constants.

(i) Given that $f(x)$ is divisible by $x+1$ and leaves a remainder of 3 when divided by $2x-1$, find the value of a and of b. [4]

(ii) Hence explain why the equation $f(x) = 0$ has only one real root and state this root. [4]

(iii) Find the remainder when $f(x)$ is divided by $x - 4$. [1]

6 The equation of a curve is $y = e^{x^2-5x}$.

(i) Find the expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [5]

(ii) Find the exact values of the coordinates of the stationary point. [4]

7 A drug is administered to a patient through an injection. The concentration of drug, x *mg/l* in a patient's bloodstream after time, t hours, is believed to be related by the equation

$$x = x_o e^{-kt},$$

where x_o and k are constants. Some values of x and t are recorded in the following table:

t (hours)	2	4	6	8	10
x (*mg/l*)	48.3	41.4	35.6	30.8	26.4

(i) By using a suitable scale, plot a suitable straight line graph for ln x against t. [3]

(ii) Use your graph to estimate the value of k and of x_o. [3]

(iii) Estimate the number of hours after which the concentration of drug in a patient's bloodstream is halved. [2]

(iv) By drawing a suitable straight line on your graph, solve the equation $x_o e^{-kt} = e^{0.04t+3.2}$. [2]

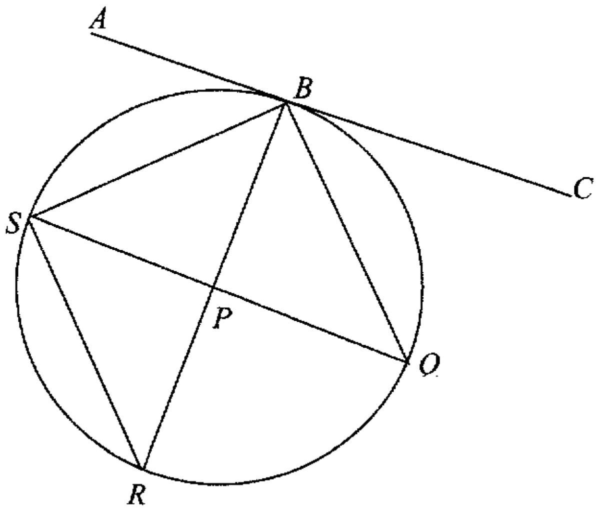
7 (i)

8 **(i)** By writing 105° as a sum of two angles, without using a calculator, show that

$\sin 105\Upsilon = \frac{\sqrt{2}}{4}\left(\sqrt{3}+1\right)$. [2]

(ii) Hence, express 105° in terms of its basic angle and find the exact value of $\cos 165°$. [2]

9 In the diagram (not to scale), points B, Q, R and S all lie on the circumference of the circle. ABC is a tangent to the circle at point B. The chords BR and QS intersect at point P.



(i) Given that QS is the angle bisector of angle BSR, show that $BQ \times PS = RP \times BS$. [3]

It is further given that $BS = SR$, show that

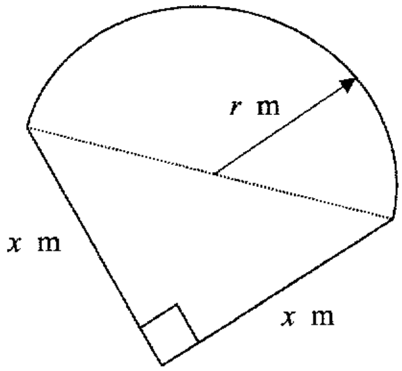
(ii) BS bisects angle ABR, [3]

9 **(iii)** SQ is the diameter of the circle. [3]

10 **(i)** Given that $\frac{\sin(A+B)}{\sin(A-B)} = \frac{1}{5}$, prove that $2\tan A + 3\tan B = 0$. [4]

(ii) Hence, solve the equation $\sin(x-45°) = 5\sin(x+45°)$ for $0° < x < 360°$. [4]

11 The figure shows the outline of a field, which consists of a semi-circular arc with radius r m and sides of a right-angled triangle each of length x m. The perimeter of the field is 10 m.



(i) Show that the area of the field, A m^2, is given by

$$A = \frac{1}{2}\pi r^2 + \frac{1}{8}(10 - \pi r)^2 \ .$$

[3]

11 **(ii)** Given that r can vary, find the value of r which gives a stationary value of A. [3]

(iii) Find the stationary value of A and explain whether this value of A is the smallest or greatest possible area. [3]

12 A cyclist moves along a straight road from home O. His velocity, v m/min, is given by $v = 25 + 10t - 3t^2$, where t is the time in minutes after leaving O. Calculate

(i) the acceleration when the cyclist is instantaneously at rest, [3]

(ii) the displacement of the cyclist when his velocity is a maximum, [3]

12 **(iii)** the average speed of the cyclist in the first 8 minutes. [4]

End of Paper

BENDEMEER SECONDARY SCHOOL
2021 Preliminary Examination
Secondary Four Express
Additional Mathematics Answer Key

1(i)	$\frac{4}{125}$	**8**	$y = -\frac{8}{x^2} - 4x + 14$
1(ii)	-1.49	**9(i)**	B (1, 9)
2(a)	$x < -1$ or $x > \frac{1}{3}$	**9(ii)**	D (-3, -9)
2(b)	$-6 \le m \le 3$	**9(iii)**	Area = 90 units^2
3(i)	$\frac{8x^2 - x}{4x^2 - 1} = 2 + \frac{3}{4(2x-1)} - \frac{5}{4(2x+1)}$	**9(iv)**	X (8.5, 13.5)
3(ii)	$2x + \frac{3}{8}\ln(2x-1) - \frac{5}{8}\ln(2x+1) + c$	**10(i)**	A (1, 5), B $(5, \frac{5}{9})$, C (5.5, 0)
4(i)	$6 + 6\sqrt{3}$	**10(ii)**	5.74 units^2
4(ii)	a = b = 6	**11(i)**	EC = $10\sin\theta + 3\cos\theta$
5(a)	a = 1.5	**11(ii)**	EC = $\sqrt{109}\sin(\theta + 16.7^o)$
5(b)	*No solution*	**11(iii)**	$29.2^0 < \theta < 48.8^o$
6(i)	Min = -1, max = 7	**12(i)**	$y = \frac{4}{3}x + \frac{15}{2}$
6(ii)	graph	**12(ii)**	$x^2 - 6x + y^2 - 23y - 40 = 0$
7(i)	Increasing function	**12(iii)**	R (16, 8)
7(ii)	0.1unit/s		

BENDEMEER SECONDARY SCHOOL
2021 Preliminary Examination
Secondary Four Express
Additional Mathematics Paper 2 Answer Key

1(i)	$\frac{s}{\sqrt{1-s^2}}$	**8(i)**	proof
1(ii)	$\frac{1}{\sqrt{1-s^2}}$	**8(ii)**	$-\frac{\sqrt{2}}{4}(\sqrt{3}+1)$
2	221.8°, 318.2°	**9**	Proof
3(i)	$6x\cos 2x + 3\sin 2x$	**10(i)**	Proof
3(ii)	0.143	**10(ii)**	123.7°, 303.7°
4(i)	$64 - 64x + \frac{80}{3}x^2 + \cdots$	**11(i)**	Proof
4(ii)	$-85\frac{1}{3}$	**11(ii)**	r = 1.40
5(i)	$a = b = 2$	**11(iii)**	A = 7.00 (minimum value)
5(ii)	$x = -1$	**12(i)**	-20m/min^2
5(iii)	185	**12(ii)**	50.9
6(i)	$\frac{dy}{dx} = (2x-5)e^{x^2-5x}$	**12(iii)**	30.25m/min
	$\frac{d^2y}{dx^2} = e^{x^2-5x}(4x^2-20x+27)$		
6(ii)	$(2.5, e^{-6.25})$		
7(ii)	$k = 0.0760$, $x_o = 56.3$		
7(iii)	t = 9h (accept 9 to 9.2)		
7(iv)	t = 7.2h		