1 Given that $\cos x = -\frac{4}{5}$ and $\tan x > 0$, find

(i) $\sin 2x$, [2]

(ii) $\cos \frac{x}{2}$. [3]

2 The graph of $y = 5\sin 3x + c$, where c is a constant, passes through the point $(50°, \frac{1}{2})$.

(i) Show that $c = -2$. [2]

(ii) Hence, sketch the graph of $y = 5\sin 3x + c$ for $0° \leq x \leq 180°$. [3]

3 (a) Express $\frac{9x^2+17x-16}{3x^2+5x-2}$ in partial fractions. [5]

(b) Hence, find $\int \frac{9x^2+17x-16}{3x^2+5x-2}\,dx$. [2]

4 A particle starts from rest at a fixed point O and moves in a straight line towards a point A. Its velocity, v m/s, t seconds after leaving O, is given by $v = 6 - 4e^{-2t}$. The particle reaches A when $t = \ln 2$,

(i) Find the velocity of the particle at O. [1]

(ii) Find the acceleration of the particle at A. [2]

(iii) Explain why the particle never comes to rest. [1]

4 (iv) Find the distance between O and A. [3]

[Turn over

5 (a) Prove that $\dfrac{1+\cos\theta}{\sin\theta}+\dfrac{\sin\theta}{1+\cos\theta}=2\operatorname{cosec}\theta$. [4]

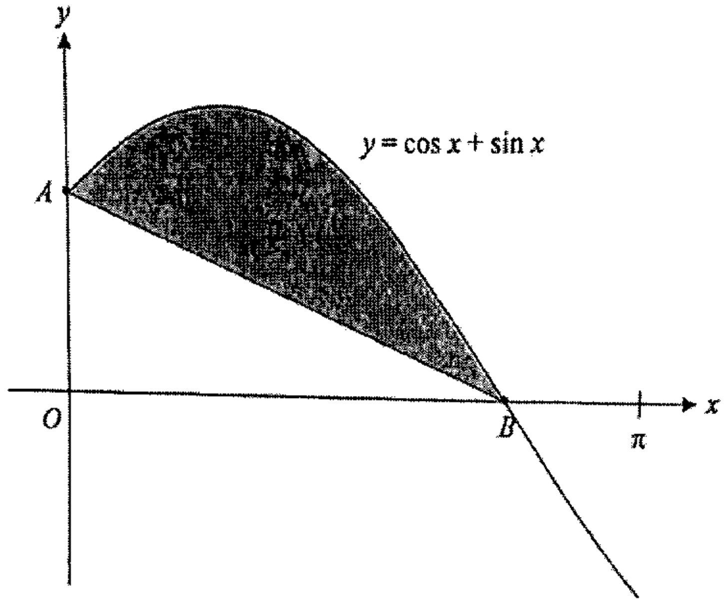
5 (b) A and B are acute angles such that $\sin(A + B) = \dfrac{56}{65}$ and $\sin A \cos B = \dfrac{4}{13}$.

Without using a calculator, find the value of

(i) $\cos A \sin B$, [1]

(ii) $\dfrac{\tan B}{\tan A}$. [2]

6



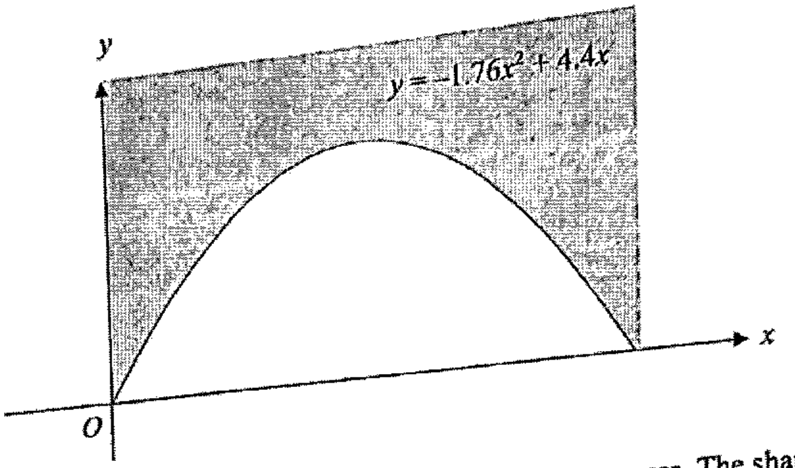
The diagram shows the curve $y = \cos x + \sin x$ for $0 \le x \le \pi$. The curve meets the y-axis at the point A and the x-axis at the point B.

(i) Show that the x-coordinate of B is $\frac{3\pi}{4}$. [2]

6 **(ii)** Find the area of the shaded region, correct to 3 significant figures. [6]

[Turn over

7



The diagram shows the cross section of a tunnel for vehicles to pass. The shape of the arched tunnel can be modelled by the equation $y = -1.76x^2 + 4.4x$, where y m represents the height above the ground and x m represents the horizontal distance from O.

(i) Express y in the form $a(x-b)^2 + c$, where a, b and c are constants. [3]

(ii) Hence or otherwise, state the coordinates of the highest point of the tunnel. [1]

7 (iii) Is it possible for a lorry that is 1.8 m wide and 2 m tall to pass through the tunnel? Justify your answer with calculations. [4]

[Turn over

8 The remainder when $x^3 + 17x^2 + 7x - 9$ is divided by $x - a$ is twice the remainder when it is divided by $x + a$.

(i) Show that $3a^3 - 17a^2 + 21a + 9 = 0$. [3]

(ii) Solve the equation $3a^3 - 17a^2 + 21a + 9 = 0$. [5]

9 The equation of a circle is $x^2 + y^2 + 8x - 20y - 53 = 0$.

(i) Find the radius of the circle and the coordinates of its centre. [3]

(ii) Find the equation of the tangent to the circle at the point $P(1, -2)$. [3]

(iii) The point $Q(9, 10)$ lies on the circle. Explain why the tangent to the circle at Q is parallel to the y-axis. [2]

[Turn over

10 (a) Given that $y = \frac{2x^2}{x-3}$, determine the range of values of x for which y is a decreasing function. [3]

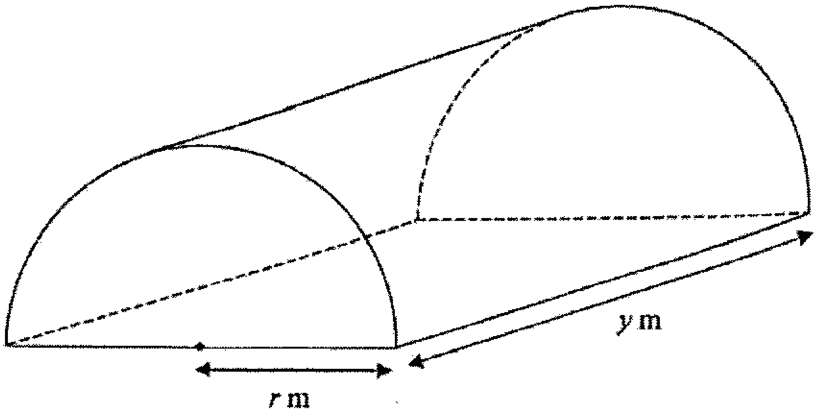
10 (b) Given that $y = x^2 \ln x$ for $x > 0$ and P is a stationary point on the curve,

(i) show that the x-coordinate of P is $\frac{1}{\sqrt{e}}$, [4]

(ii) determine the nature of this stationary point. [2]

[Turn over

11



The diagram shows a greenhouse built on a rectangular base. 140 m^2 of polyethene is used to build the curved roof and the two identical semicircular sides. The radius of each semicircle is r m and the length of the greenhouse is y m.

(i) Show that the volume, V m^3, of the greenhouse is given by $V = 70r - \frac{\pi r^3}{2}$. [4]

11 (ii) Given that r can vary, find the value of r for which V has a stationary value. [3]

(iii) Explain why this value of r gives the largest possible volume of the greenhouse. [2]

[Turn over

12 (a) State the principal value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$. Give your answer in radians as a multiple of π. [1]

(b) (i) Solve the equation $9\sin(2y + 10°) = 4$ for $0° \le y \le 360°$. [3]

(ii) Solve the equation $3\tan^2 x + 5\sec x + 1 = 0$ for $0 \le x \le 2\pi$. Leave your answers in terms of π. [5]

END OF PAPER

1 (i) By considering the general term in the binomial expansion of $\left(x+\frac{1}{ax^2}\right)^7$, where a is a constant, explain why every term is dependent on x. [3]

(ii) The coefficient of x in the expansion of $\left(x+\frac{1}{ax^2}\right)^7$ is $\frac{7}{3}$. Find the possible values of a. [3]

[Turn over

2 (i) A radioactive substance Q decays at a constant rate of 6% of its mass every minute. If M is the initial mass of Q, explain why the mass of Q after t minute is $(0.94)^t M$. [2]

(ii) x% of Q decayed after 10 minutes. Calculate, to 2 significant figures, the value of x. [2]

(iii) Given that the mass of Q left after t minutes can be expressed as Me^{kt}, find the value of the constant k. [2]

3 The table below shows experimental values of two variables x and y. It is known that x and y are related by an equation of the form $y = h(x+1)^k$, where h and k are constants.

x	1	2	3	4	5	6
y	5.2	7.2	9.1	10.9	12.6	14.2

(i) Draw the graph of $\lg y$ plotted against $\lg(1+x)$, using a scale of 2 cm to 0.1 unit on both axes. [3]

3 (ii) Use your graph to estimate the value of each of the constants h and k. [3]

(iii) Explain how the value of y when $x = 1.5$ can be estimated using the straight line graph. [1]

[Turn over

4 (a) (i) Clearly labelling each graph, sketch, on the same axes, the graphs of $y = 2e^{-2x}$ and $y = -\ln x$. [2]

(ii) Hence, explain why the solution of $e^{-2x} + \ln\sqrt{x} = 0$ must lie between 0 and 1. [2]

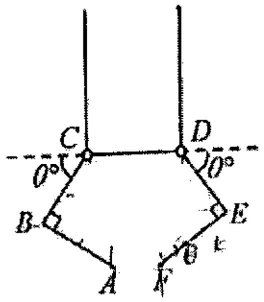
(b) In order to obtain a graphical solution of the equation $x = \ln\sqrt{\frac{4-3xe^{2x}}{4}}$, a suitable straight line can be drawn on the same set of axes as the graph of $y = 2e^{-2x} + 3$. Make $4e^{2x}$ the subject of $x = \ln\sqrt{\frac{4-3xe^{2x}}{4}}$ and hence find the equation of this line. [4]

5 (a) (i) Find the range of values of p for which $8x - p < x^2 + (p-1)x + 6$ for all values of x. [4]

(ii) Represent the solution set of p on a number line. [1]

(b) Find the values of p for which the line $y = 5x - p$ is a tangent to the curve $y = x^2 + px + 3$. [4]

6 The claw of a claw machine is designed such that $AB = BC = CD = DE = EF = k$ cm with angle ABC = angle $FED = 90°$. The two sides of the claw, ABC and FED, hinge at C and D respectively at angle $\theta°$ with $\theta > 45$. The claw closes when A and F touches one another.



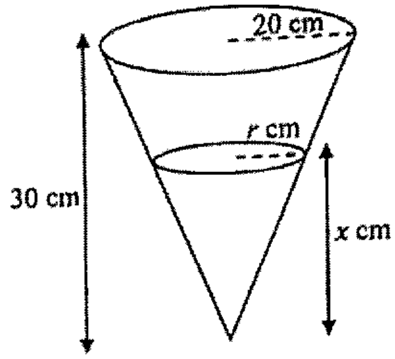
(i) Show that $AF = k - 2k(\sin\theta - \cos\theta)$. [3]

(ii) Express $\sin\theta - \cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

6 (iii) Hence, find the value of θ when the claw closes. [3]

[Turn over

7 A piping bag is in the shape of a cone with radius 20 cm and height 30 cm. A baker filled the piping bag with frosting to a depth of x cm and radius r cm. When piping, the frosting is squeezed out from the tip of the piping bag at a constant rate of 80 cm^3/s.



(i) Show that the volume of the frosting, V cm^3, in the piping bag is given by $V = \frac{4}{27}\pi x^3$. [3]

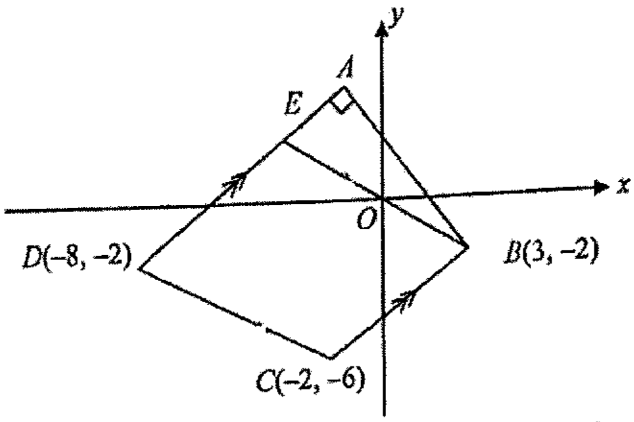
(ii) Find the rate of change of the depth of the frosting when $r = 12$. [4]

7 (iii) Hence, find the rate of change of the radius of the frosting when $r = 12$. [3]

[Turn over

Solutions to this question by accurate drawing will not be accepted.

8



$ABCD$ is a trapezium where AD is parallel to BC and perpendicular to AB. E is a point on AD and BE passes through the origin. Points B, C and D are $(3, -2)$, $(-2, -6)$ and $(-8, -2)$ respectively.

(i) Show that $BCDE$ is a parallelogram. [2]

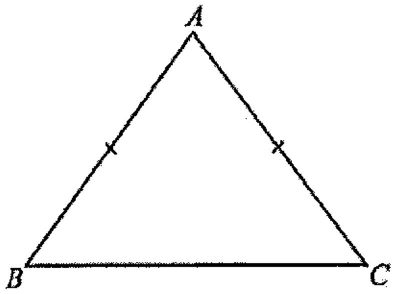
(ii) Find the coordinates of E. [2]

8 (iii) Show that the coordinates of A is $\left(-1\frac{12}{41}, 3\frac{15}{41}\right)$. [4]

(iv) Calculate the area of parallelogram $BCDE$. [2]

[Turn over

9 (a) ABC is an isosceles triangle where $AB = \left(3x - 4\sqrt{5}\right)$ cm, $AC = \left(x\sqrt{5} - 4\right)$ cm, $BC = \left(6\sqrt{5} - 2\right)$ cm and $AB = AC$.



Find, leaving your answer in the form $a\sqrt{5} + b$, where a and b are constants,

(i) the value of x, [3]

(ii) the perimeter of triangle ABC. [2]

9 (b) Given that $p = \log_3 y$, express $\log_y 27$ in terms of p. [2]

(c) Solve the equation $\log_{27} x^3 = \log_3 6 - \dfrac{1}{\log_{5-x} 3}$. [4]

[Turn over

[2]

10 (a) (i) Differentiate $\frac{1}{2}x\sin 2x$ with respect to x.

[4]

(ii) Hence find $\int_0^{\frac{\pi}{4}} 3x\cos 2x \, dx$.

(b) The gradient of a curve is given as $\frac{dy}{dx} = a + \frac{b}{x^2}$. The gradient of the normal at $P(1,3)$ on the curve is $-\frac{1}{5}$ and Q is a stationary point on the curve at $x = \frac{2}{3}$.

(i) Find the value of a and of b. [4]

10 (ii) Find the equation of the curve. [2]

(iii) Explain why the equation of the normal at Q must be of the form $x = k$, where k is a constant. State the value of k. [2]

END OF PAPER

Date: No.

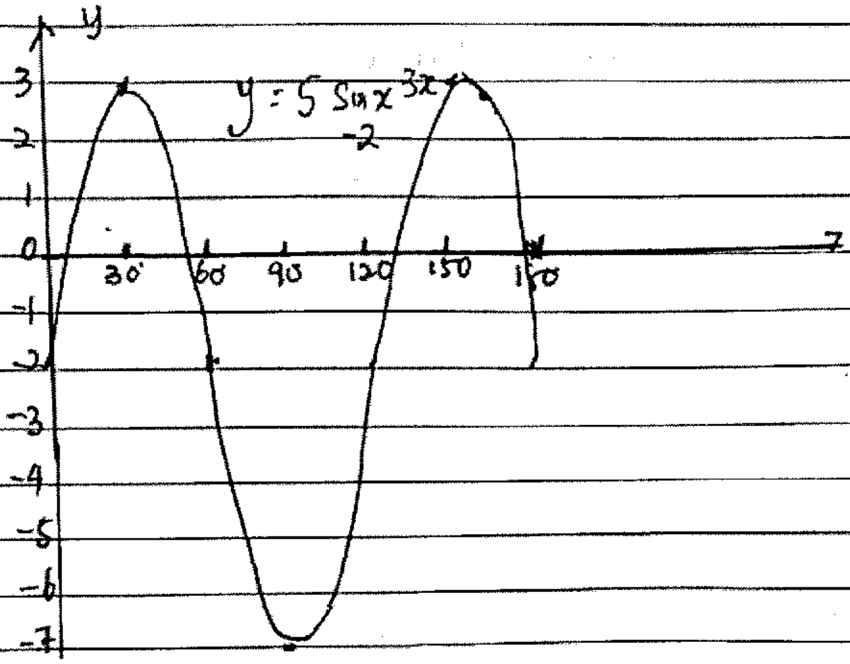
Paper 1 Answer Key

1 i) $\frac{24}{25}$

ii) $-\frac{\sqrt{10}}{10}$

2 i) Shown $c = -2$

ii)



3a) $3 + \frac{2}{x+2} - \frac{4}{3x-1}$

b) $3x + 2\ln(x+2) - \frac{4}{3}\ln(3x-1) + c$

4 i) 2 m/s

ii) 2 m/s²

iii) Since $v = 0$ is not possible, the particle never comes to rest. (Note $t \geq 0$, $t = -0.207$ invalid)

iv) 2.66 m

5 a) Shown

b i) $\frac{36}{65}$

ii) $\frac{9}{5}$

6 i) Shown

ii) 1.24 units^2

7 i) $-\frac{44}{25}\left(x-\frac{5}{4}\right)^2+\frac{11}{4}$

ii) $\left(\frac{5}{4}, \frac{11}{4}\right)$

iii) $y=2$, $x=0.597208$

$y=1.9027$

max width allowed

$1.9027-0.597208$

$=1.31$m

∴ It is possible.

8 i) Shown.

ii) $a=3$, $a=-\frac{1}{3}$

9 i) $R=13$ units,

$c\ (-4, 10)$

ii) $y=\frac{5}{12}x-\frac{29}{12}$

iii) $x=13-4$, $x=9$

(9,10) is a vertical pt and

tangent to the circle ∴ It is parallel to y axis.

10 a) $0<x<6$,

$x\neq 3$

b) i) Shown

ii) minimum point

11 i) Shown

ii) $r=3.85$ m

iii) $\frac{d^2V}{dr^2}<0$

12 a) $-\frac{\pi}{6}$

b i) $8.2°$, $71.8°$, $188.2°$, $251.6°$,

ii) $\frac{2\pi}{3}$, $\frac{4\pi}{3}$

Date: No.

Paper 2 Answer Key

1i) $r = \frac{7}{3}$

Since it is not possible for the coefficient of x to be 0, every term is dependent on x.

ii) $a = 3$, $a = -3$

2i) mass of α after 1 min = $0.94M$

mass of α after 2 min = $0.94 \times 0.94M$

$= 0.94^2 M$

$\therefore$ mass of α after t minutes $= 0.94^t M$.

ii) 46

iii) $k = -0.0619$

3i) Plot lg y vs lg (1+x)

ii) $k = 0.803$

$n = 3.02$

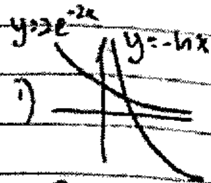
iii) $x = 1.5$

$\lg(1.5+1) = 0.398$

Read $\lg y = 8$

$y = 10^8$

4a) i)



b) Since $y = -\ln x$ has an asymptote of $x = 0$ and $2e^{-2x} > 0$

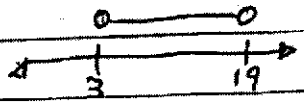
and $-\ln x = 0$ when $x = 1$

$\therefore$ the solution lies between 0 and 1, when two curve intersect.

c) $y = \frac{3}{2}x + 5$

5a) i) $3 < p < 19$

ii)



b) $p = 13$, $p = 1$

6 i) Shown

ii) $\sqrt{2}\sin(\theta - 45^\circ)$

iii) $\theta = 65.7^\circ$

7) i) Shown

ii) -0.177 cm/s

iii) -0.118 cm/s

8 i) Shown

ii) $E(-3, 2)$

iii) Shown

iv) 44 unit²

9 a i) $2\sqrt{5}+2$

ii) $(10\sqrt{5}+10)$ cm

b) $\frac{3}{p}$

c) $x=3,\ x=2$

10 a i) $\frac{1}{2}\sin 2x + x\cos 2x$

ii) 0.428

b) i) $a=9,\ b=-4$

ii) $y = 9x + \frac{4}{x} - 10$

iii) The equation of tangent is $y = 2$ and $\frac{dy}{dx}=0,\ x=\frac{2}{3}$.

∴ Q is at stationary pt

∴ $x = \frac{2}{3}$. (equation of normal)

∴ $k = \frac{2}{3}$