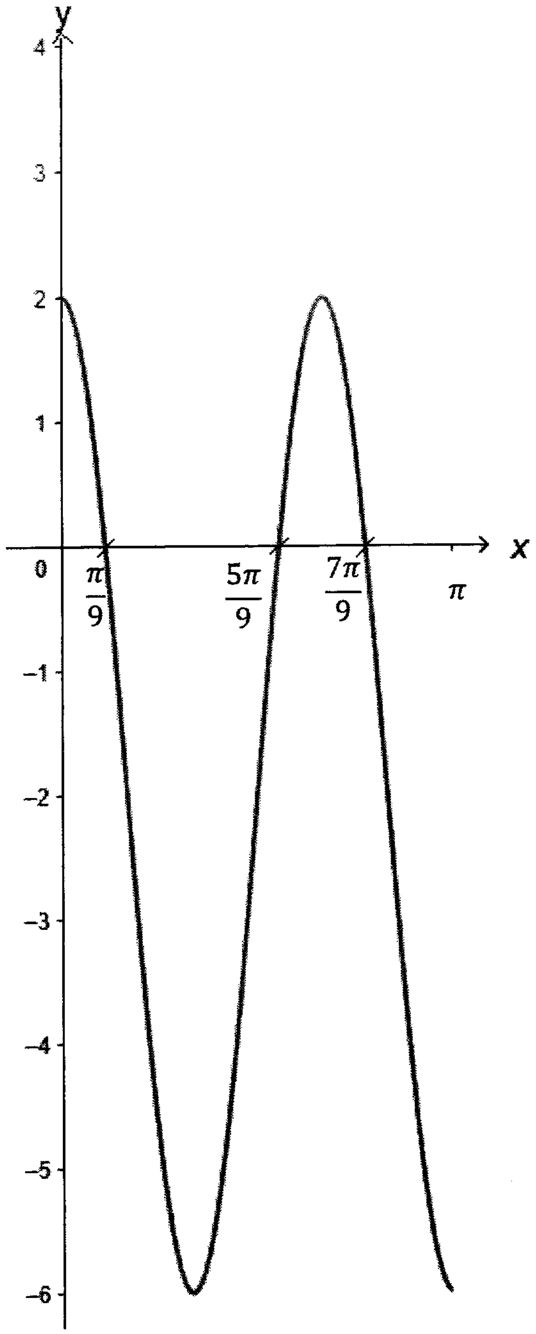
FHSS Prelim AM P1 2022 Marking Scheme

Qn	Solution	Mark allocation	Remarks
1	Volume of a cylinder, $V = \pi r^2 h$ $\left(12+3\sqrt{2}\right)\pi = \pi\left(\sqrt{2}-1\right)^2 h$ $h = \dfrac{12+3\sqrt{2}}{\left(\sqrt{2}-1\right)^2}$	M1	
	$h = \dfrac{12+3\sqrt{2}}{3-2\sqrt{2}}$ $h = \dfrac{\left(12+3\sqrt{2}\right)\left(3+2\sqrt{2}\right)}{\left(3-2\sqrt{2}\right)\left(3+2\sqrt{2}\right)}$	M1	
	$h = \dfrac{\left(48+33\sqrt{2}\right)}{9-8}$ $h = 48+33\sqrt{2}$	A1	
2	$\dfrac{x}{3} = y^2$ —(1) $\dfrac{x}{9} = 9y$ ---(2) From (2), $x = 81y$ From (1), $x = 3y^2$ $3y^2 = 81y$	M1	
	$3y^2 - 81y = 0$ $3y(y-27) = 0$ $y = 0$ (reject) or $y = 27$	M1	
	$x = 2187$ $x - y = 2187 - 27$ $= 2160$	A1	

3	$y=-2x^2-12x+1$ $=-2\left(x^2+6x-\frac{1}{2}\right)$ $=-2\left[x^2+6x+\left(\frac{6}{2}\right)^2-\left(\frac{6}{2}\right)^2-\frac{1}{2}\right]$ $=-2\left[(x+3)^2-9-\frac{1}{2}\right]$ $=-2\left[(x+3)^2-9\frac{1}{2}\right]$ $=19-2(x+3)^2$ Maximum value of y is 19. The maximum value occurs when $x=-3$.	M1 M1 A1 A1	
4	$y=\int \frac{2}{x^2}+\frac{3}{(7-2x)}dx$ $y=-\frac{2}{x}+\frac{3}{-2}\ln(7-2x)+c$ Substitute $P(3, 1)$, $1=-\frac{2}{3}+\frac{3}{-2}\ln(1)+c$ $c=\frac{5}{3}$ Equation of curve is $y=-\frac{2}{x}-\frac{3}{2}\ln(7-2x)+\frac{5}{3}$	M2 M1 A1	
5	$\frac{2x-1}{x^2(x+1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{(x+1)}$ $=\frac{Ax(x+1)+B(x+1)+Cx^2}{x^2(x+1)}$ $=\frac{Ax^2+Cx^2+Ax+Bx+B}{x^2(x+1)}$	M1 M1 M1	

	Comparing coefficient of: $x^0 : -1 = B$ $x^1 :$ $2 = A + B$ $2 + 1 = A$ $3 = A$ $x^2 :$ $0 = A + C$ $0 = 3 + C$ $-3 = C$ $\therefore \frac{2x-1}{x^2(x+1)} = \frac{3}{x} - \frac{1}{x^2} - \frac{3}{(x+1)}$	M1 M1 A1	
	<u>Alternative method,</u> $\frac{2x-1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x+1)}$ $= \frac{Ax(x+1) + B(x+1) + Cx^2}{x^2(x+1)}$ $2x-1 = Ax(x+1) + B(x+1) + Cx^2$ Let $x = -1$, $2(-1) - 1 = C(-1)^2$ $C = -3$ Let $x = 0$, $-1 = B(1)$ $B = -1$ Let $x = 1$, $2 - 1 = A(1)(2) + (-1)(2) + (-3)(1)^2$ $A = 3$	M1 M1 M1 M1 M1	

	$\therefore \frac{2x-1}{x^2(x+1)} = \frac{3}{x} - \frac{1}{x^2} - \frac{3}{(x+1)}$	A1	
6a	$f(-2c) = -32c^3$ $3(-2c)^3 - 5c(-2c)^2 + kc^2(-2c) + 4c^3 = -32c^3$ $-24c^3 - 20c^3 - 2kc^3 + 4c^3 = -32c^3$ $-2kc^3 = 8c^3$ $k = -4$	M1 A1	Allow long division method
6b	$f(x) = 3x^3 - 5cx^2 - 4c^2x + 4c^3$ $x^2 - cx - 2c^2 \overline{)3x^3 - 5cx^2 - 4c^2x + 4c^3}$ quotient $3x - 2c$ $\underline{3x^3 - 3cx^2 - 6c^2x}$ $-2cx^2 + 2c^2x + 4c^3$ $\underline{-2cx^2 + 2c^2x + 4c^3}$ Since remainder is 0, $x^2 - cx - 2c^2$ is a factor of $f(x)$. <u>Alternative method,</u> $x^2 - cx - 2c^2 = (x+c)(x-2c)$ $f(-1) = 0$ $f(2c) = 0$	M1 M1 A1 M1 M1	
7i	Amplitude = 4	B1	
7ii	$\frac{2\pi}{3} = \frac{2\pi}{a}$ $a = 3$ $b = -2$	 B1 B1	

7iii		Shape -1 Intercept - 1 Min, max -1 Period -1	

8a	In right-angled ΔABC, by Pythagoras' Theorem, $AC^2 = AB^2 + BC^2$ $13^2 = 12^2 + BC^2$ $BC^2 = 13^2 - 12^2$ $= 25$ $BC = \pm 5$ Since $BC>0$, $BC=5$. By similar triangles ΔABC and ΔQPC, $\frac{y}{12} = \frac{5-x}{5}$ $y = \frac{12(5-x)}{5}$ $= \frac{60-12x}{5}$ (shown)	M1 A1	
8b	$A = xy$ $= x\left(\frac{60-12x}{5}\right)$ $= x\left(12 - \frac{12x}{5}\right)$ $= 12x - \frac{12x^2}{5}$ (shown)	B1	
8c	$A = 12x - \frac{12x^2}{5}$ $\frac{dA}{dx} = 12 - \frac{24x}{5}$ For stationary values, $\frac{dA}{dx} = 0$. $12 - \frac{24x}{5} = 0$ $60 - 24x = 0$ $24x = 60$ $x = 2.5$	M1 M1	

	When $x=2.5$, $A=12(2.5)-\frac{12(2.5)^2}{5}$ $=15$ $\frac{dA}{dx}=12-\frac{24x}{5}$ $\frac{d^2A}{dx^2}=-\frac{24}{5}<0$ By the second derivative test, A is a maximum. Hence, the stationary value of A is 15 cm^2 and it is a maximum.	A1 A1	
9ai	$m_{AC}=-2$ Equation of AC is $y=-2x+6$	M1 A1	
9aii	$y=-2x+6$ ---- (1) $x+5y=-6$ --- (2) Sub (1) into (2) $x+5(-2x+6)=-6$ $-9x=-36$ $x=4$ $y=-2$ $C(4,-2)$	M1 A1	

9b	$D=(-6,0)$ Area of $\Delta ACD=\frac{1}{2}\begin{vmatrix}0 & -6 & 4 & 0\\6 & 0 & -2 & 6\end{vmatrix}$ $=\frac{1}{2}\left(12+24-\left(-36\right)\right)$ $=36\text{units}^2$ Area of $\Delta ABC=\frac{36}{1.5}$ $=24\text{units}^2$ Let $B=(k,6)$ $\frac{1}{2}\begin{vmatrix}0 & 4 & k & 0\\6 & -2 & 6 & 6\end{vmatrix}=24$ $\frac{1}{2}(24+6k-24+2k)=24$ $k=6$ $B=(6,6)$ <u>Alternative method.</u> $\frac{1}{2}(AB)(8)=24$ $AB=6$ $B(6,6)$	M1 M1 A1 M1 M1 A1	
9c	$\lvert AB\rvert=6\text{units}$ $\lvert AD\rvert=\sqrt{72}\text{units}$ $\lvert CD\rvert=\sqrt{(4+6)^2+(-2)^2}$ $=\sqrt{104}\text{units}$ Since $\lvert AB\rvert\neq\lvert AD\rvert\neq\lvert CD\rvert$, $ABCD$ is not a kite.	 M1 A1	

10a	$\dfrac{1-\sin^4 x}{\sin^2 x}=\dfrac{1}{\sin^2 x}-\sin^2 x$ $=\text{cosec}^2 x-(1-\cos^2 x)$ $=\cot^2 x+1-1+\cos^2 x$ $=\cot^2 x+\cos^2 x$ (Proven)	M1 M1 M1 A1	
10b	$\cot^2 2x+\cos^2 2x=0$ $\dfrac{1-\sin^4 2x}{\sin^2 2x}=0$ $1-\sin^4 2x=0$ $\sin^4 2x=1$ $\sin 2x=1$ or $\sin 2x=-1$ basic angle = 90° $2x=90°, 270°$ $x=45°, 135°$	M1 M1 M1 A1	
11a	$\angle CBA=180°-\angle ADC$ (opp. $\angle$s of cyclic *quadrilateral*) $\angle ADE=180°-\angle ADC$ (adj. $\angle$s on a str. line) $\therefore \angle CBA=\angle ADE$	M1 A1	
11b	In triangles DAE and BAC, $\angle CBA=\angle ADE$ (shown in (a)) $\angle DAE=\angle ACD$ (tangent-chord theorem) $=\angle CAB$ (alt. $\angle$) Hence, triangles BAE and DAC are similar.	M1 A1	
11c	$AD=DE$, implying that triangles DAE and BAC are similar isosceles triangles. $\angle ACB=\angle CAB$ (alt. $\angle$s) $=\angle DCA$ Hence, the line AC bisects the angle BCD.	M1 M1 A1	

12a	$2^{x-2}\times 3^{x+2}=6^{2x}$ $\frac{2^x\times 3^x\times 3^2}{2^2}=6^{2x}$ $\frac{6^x\times 9}{4}=6^{2x}$ $6^x=\frac{9}{4}$ $x=\frac{\lg\left(\frac{9}{4}\right)}{\lg 6}$ $=\frac{\lg 9-\lg 4}{\lg 6}$ (shown)	M1 M1 M1 A1	
12b	$\frac{\log_3 y}{\log_3 9}-2=\log_3 2y$ $\frac{1}{2}\log_3 y-2=\log_3 2y$ $\frac{1}{2}\log_3 y-\log_3 2y=2$ $\log_3 y-2\log_3 2y=4$ $\log_3 y-\log_3(2y)^2=4$ $\log_3\frac{y}{4y^2}=4$ $\frac{1}{4y}=3^4$ (since y>0) $y=\frac{1}{324}$	M1 M1 M1 A1	
13ai	When $V=334\pi$ cm^3, $10\pi+\frac{4\pi h^3}{9}=334\pi$ $\frac{4\pi h^3}{9}=324\pi$ $h^3=729$ $h=9$ Hence, the value of h is 9.	M1 A1	

13aii	$V = 10\pi + \frac{4\pi h^3}{9}$ $\frac{dV}{dh} = 3\left(\frac{4\pi h^2}{9}\right)$ $= \frac{4\pi h^2}{3}$ Since $\frac{dV}{dt} = -150$ cm^3/s, when h=9, $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ $-150 = \frac{4\pi(9)^2}{3} \times \frac{dh}{dt}$ $-150 = 108\pi \times \frac{dh}{dt}$ $\frac{dh}{dt} = -\frac{25}{18\pi}$ $= -0.442$ (correct to 3 sig. fig.) Hence, the rate of change of h is -0.442 cm/s.	M1 M1 A1	
13bi	$\frac{s}{v} = \frac{1}{6} + \frac{v}{50}$ $s = \frac{1}{6}v + \frac{v^2}{50}$ $\frac{ds}{dv} = \frac{1}{6} + \frac{2v}{50}$ $\left.\frac{ds}{dv}\right\|_{v=60} = \frac{1}{6} + \frac{2(60)}{50}$ $= \frac{77}{30} \frac{\text{m}}{\text{km/h}}$	M1 M1 A1	
13bii	$\frac{ds}{dv}$ is the rate of change of the stopping distance with respect to the speed of the car. When the car is travelling at 60 km/h, for every 1km/h increase in its speed, its stopping distance increases by approximately 2.57 m.	B1	
14a	To find the coordinates of A,		

	When $y = 0$, $0 = \sqrt{3x-2}$ $x = \frac{2}{3}$ Therefore, $A(\frac{2}{3}, 0)$. Coordinates of $B(9, 5)$ To find the coordinates of C, $\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{3x-2}\right) = \frac{3}{2\sqrt{3x-2}}$ At $x = 9$, Grad. of tangent $= \frac{3}{10}$ Gradient of normal $= -\frac{10}{3}$ Let $C = (c, 0)$, $\frac{5-0}{9-c} = -\frac{10}{3}$ $15 = -90 + 10c$ Therefore $C = (10.5, 0)$.	M1 A1 M1 M1 A1	

14b	Area of shaded region $= \int_{\frac{2}{3}}^{9} \sqrt{3x-2}\ \mathrm{d}x + \text{Area of } \Delta$	M1	
	$= \left[\dfrac{(3x-2)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)(3)} \right]_{\frac{2}{3}}^{9} + \left(\dfrac{1}{2} \times \dfrac{3}{2} \times 5\right)$	M2	
	$= \left[\dfrac{25^{\frac{3}{2}}}{\frac{9}{2}} - \dfrac{0^{\frac{3}{2}}}{\frac{9}{2}} \right] + \dfrac{15}{4}$	M1	
	$\approx 31.5 \text{ units}^2$	A1	

1. $e^{\frac{1}{2}x} = 2 + 24e^{-\frac{1}{2}x}$

$e^{\frac{1}{2}x} = 2 + \frac{24}{e^{\frac{1}{2}x}}$

let $u = e^{\frac{1}{2}x}$

$u = 2 + \frac{24}{u}$ M1

$u^2 = 2u + 24$

$u^2 - 2u - 24 = 0$

$(u-6)(u+4) = 0$ M1

$u - 6 = 0$ or $u + 4 = 0$

$u = 6$ or $u = -4$ M1

$e^{\frac{1}{2}x} = 6$ or $e^{\frac{1}{2}x} = -4$

$\frac{1}{2}x = \ln 6$ A1 $\qquad \frac{1}{2}x = \ln -4$ (rej.)

$x = 2\ln 6$

A1

2a $\int \left(\frac{6}{e^{3-x}}\right)^2 dx$

$= \int \frac{36}{e^{6-2x}}$

$= 36\int e^{2x-6} \, dx$

$= \frac{36e^{2x-6}}{2} + c$

$= 18e^{2x-6} +$

2b let $p(x) = 2x^3 + 3x^2 + 2x + 8$

$p(-2) = 2(-2)^3 + 3(-2)^2 + 2(-2) + 8$ M1

$= 0$

$x+2$ is a factor

$$\begin{array}{r|l} & 2x^2 - x + 4 \\ \hline x+2 & 2x^3 + 3x^2 + 2x + 8 \\ & -(2x^3 + 4x^2) \\ \hline & -x^2 + 2x \\ & -(-x^2 - 2x) \\ \hline & 4x + 8 \\ & -(4x + 8) \\ \hline & 0 \end{array}$$

M1

$P(x) = (x+2)(2x^2 - x + 4)$

$P(x) = 0$

$(x+2)(2x^2 - x + 4) = 0$

$x+2 = 0$

$x = -2$

A1

$2x^2 - x + 4 = 0$

$(-1)^2 - 4(2)(4) = -31 < 0$ M1

$b^2 - 4ac < 0$

Since $b^2 - 4ac < 0$, $2x^2 - x + 4 =$ has no real solutions

$\therefore P(x) = 0$ has only one real root A1

3a $\quad y = x^2\sqrt{2x-1}$

$$u = x^2 \qquad v = (2x-1)^{\frac{1}{2}}$$

$$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = \frac{1}{2}(2x-1)^{-\frac{1}{2}}(2)$$

$$= (2x-1)^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{x^2}{\sqrt{2x-1}} + 2x\sqrt{2x-1}$$

$$= \frac{x^2 + 2x\sqrt{2x-1}\,(\sqrt{2x-1})}{\sqrt{2x-1}}$$

$$= \frac{x^2 + 2x(2x-1)}{\sqrt{2x-1}}$$

$$= \frac{x^2 + 4x^2 - 2x}{\sqrt{2x-1}}$$

$$= \frac{5x^2 - 2x}{\sqrt{2x-1}}$$

$$= \frac{x(5x-2)}{\sqrt{2x-1}}$$

$2x-1 > 0$

$x > \frac{1}{2}$ B1

$5x-2 > \frac{1}{2}$

$\therefore \quad 5x-2 > 0$

$x(5x-2) > 0$

$\frac{dy}{dx} > 0$

It is an increasing function.

3b

$$\int_1^5 \frac{5x^2-2x+1}{\sqrt{2x-1}}\,dx$$

$$= \int_1^5 \frac{5x^2-2x}{\sqrt{2x-1}}\,dx + \int_1^5 \frac{1}{\sqrt{2x-1}}\,dx \quad m1$$

$$= \left(x^2\sqrt{2x-1}\right)_1^5 + \left[\frac{(2x-1)^{\frac{1}{2}}}{\frac{1}{2}\times 2}\right]_1^5$$

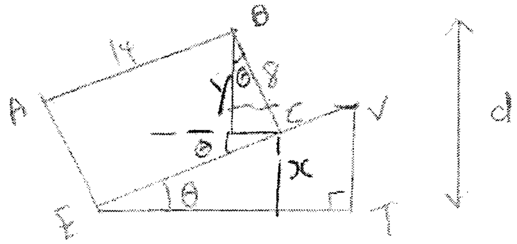
$$= \left[5^2\sqrt{2(5)-1} - 1^2\sqrt{2(1)-1}\right] + \left[(2x-1)^{\frac{1}{2}}\right]_1^5$$

$$= 74 + \left[2(5)-1\right]^{\frac{1}{2}} - \left[2(1)-1\right]^{\frac{1}{2}} \quad m1$$

$$= 74 + 2$$

$$= 76$$

4.a.



$\sin\theta = \frac{x}{14}$

$x = 14\sin\theta$ m1

$\cos\theta =$

180 − (90+90−θ

$y = 8\cos$

$d = x + y$

$d = 14\sin\theta + 8 \quad \theta$. A1

4b.

$= \sqrt{14^2 \quad 8^2}$ m1

$= \sqrt{260}$

$= 2\sqrt{65}$

$\tan\alpha = \frac{8}{14}$ m1

$=$

$d = \quad + 29.7°)$ A1

4c maximum ~~of~~ $d = 2\sqrt{65}$ B1

$= 16.1$

$(\theta + 29.7448°) = 90°$

corresponding $\theta = 60.2552°$

$= 60.3°$ (1dp) B1

4d. $13 = 2\sqrt{65} \sin(\theta + 29.7°)$

$\sin(\theta + 29.7448°) = \frac{13}{2\sqrt{65}}$ M1

$\theta + 29.7448° = 53.7288$

smallest value of $\theta = 23.9800$ (4dp)

$= 24.0$ (1dp) A1

5ai

$$x^2 - 2x + 3$$

$$= x^2 - 2x + \left(\frac{-2}{2}\right)^2 + 3 - \left(\frac{-2}{2}\right)^2 \quad \text{M1}$$

$$= (x-1)^2 + 2$$

Since $(x-1)^2 \geq 0$

$(x-1)^2 + 2 \geq 2$

$\therefore$ $x^2 - 2x + 3$ is always positive for all real values of x. B1

5b

Since $x^2 - 2x + 3$ is always positive,

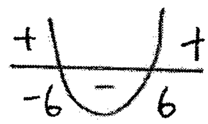
$3x^2 + px + 3 > 0$ M1

$\therefore \quad b^2 - 4ac < 0$

$(p)^2 - 4(3)(3) < 0$ M1

$p^2 - 36 < 0$

$(p-6)(p+6) < 0$ M1



$-6 < p < 6$ A1

6a. $T_{r+1} = \binom{n}{r} a^{n-r} b^{r}$

$= \binom{5}{r} (x)^{5-r} \left(\frac{k}{x}\right)^{r}$ M1

$= \binom{5}{r} x^{5-r} k^{r} x^{-r}$

$= \binom{5}{r} k^{r} x^{5-2r}$

For $\frac{1}{x^3}$, $x^{5-2r} = x^{-3}$ M1

$5-2r = -3$

$2r = 8$

$r = 4$

coefficient of $\frac{1}{x^3} = \binom{5}{4} k^{4}$

$= 5k^{4}$

For $\frac{1}{x}$, $x^{5-2r} = x^{-1}$ M1

$5-2r = -1$

$2r = 6$

$r = 3$

coefficient of $\frac{1}{x} = \binom{5}{3} k^{3}$

$= 10k^{3}$

$5k^4 = 10k^3$

$5k^4 - 10k^3 = 0$ M1

$5k^3(k-2) = 0$

$k = 0$ or $k - 2 = 0$
(rej) $k = 2$ A1

6b. $T_{r+1} = \binom{5}{r} k^r x^{5-2r}$

For constant term, $5 - 2r = 0$

$2r$

$r \quad \frac{5}{2}$

coefficient of $\frac{1}{x^2}$ $5 - 2r = -2$ M1

$2r = 7$

$r = \frac{7}{2}$

Since r is not a positive integer,
there is no constant term in
the expansion of $(1-3x^2)\left(x + \frac{k}{x}\right)^5$

7a $\frac{3y^2}{k} + \frac{2x}{h} = 6$

$\frac{3y^2}{k} = -\frac{2x}{h} + 6$

$3y^2 = \frac{-2k}{h}x + 6k$

$y^2 = \frac{-2k}{3h}x + 2k$ M1

$Y = y^2$, $X = x$, $m = \frac{-2k}{3h}$, $c = 2k$

By Comparison

$2k = 8$

$k = 4$

$-6 = \frac{-2(4)}{3h}$

$3h = \frac{4}{3}$

$h = \frac{4}{9}$

$\therefore k = 4, \; h = \frac{4}{9}$

7bi

$$T = 25 + p \cdot e^{-kn}$$

$$T - 25 = p \cdot e^{-kn}$$

$$\ln(T-25) = \ln p e^{-kn}$$

$$= \ln p + \ln e^{-kn}$$

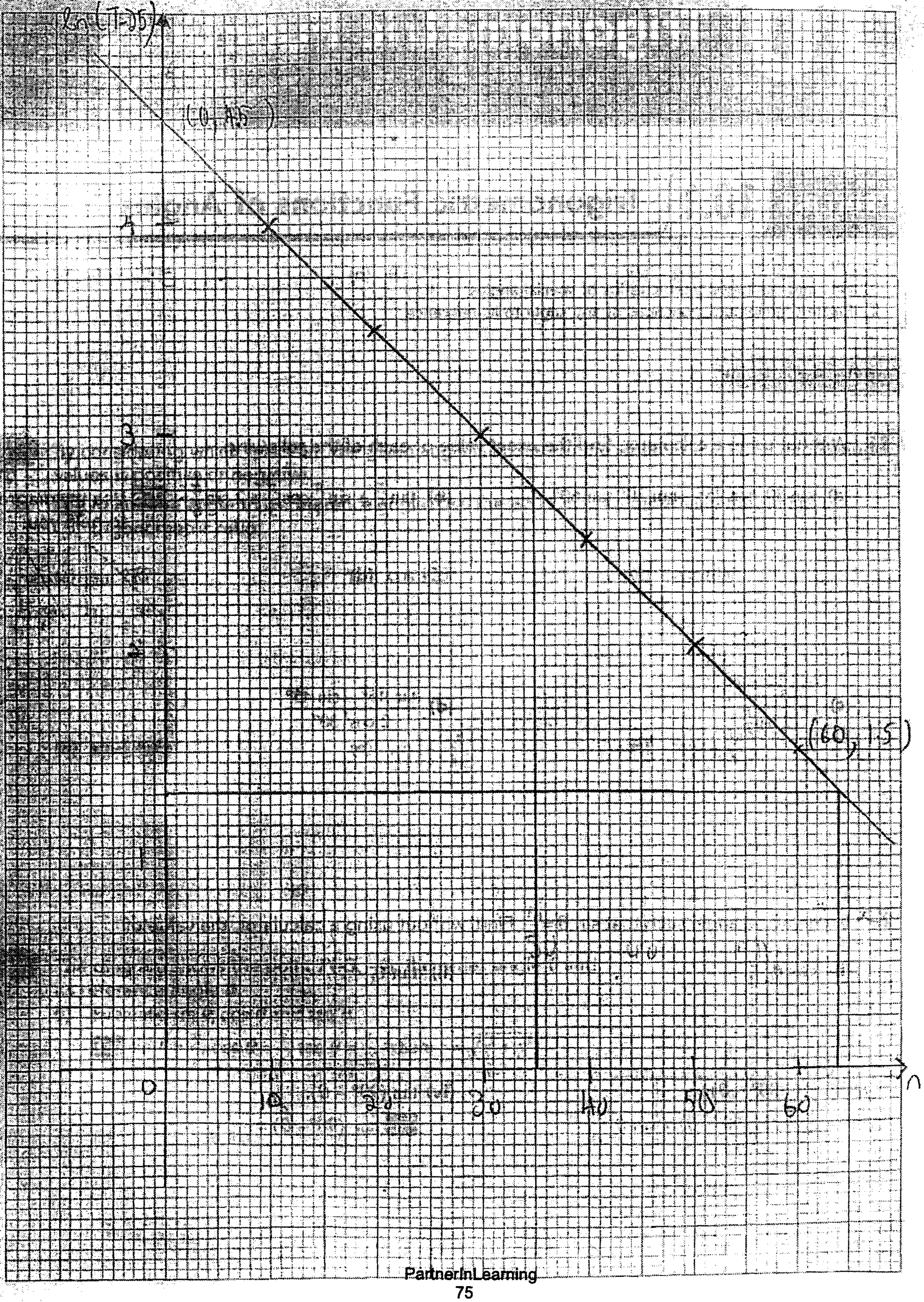
$$\ln(T-25) = -kn + \ln p$$

$Y = \ln(T-25)$, $X = n$, $m = -k$, $c = \ln p$

n	10	20	30	40	50
$\ln(T-25)$	4.0	3.5	3.0	2.5	2.0

Plot $\ln(T-25)$ against n B1

Qn 7bi



7bii

$-k \approx \dfrac{1.5-4.5}{60}$ M1

$k \approx -0.05$ A1

$\ln \approx 4.5$

$p \approx e^{4.5}$ M1

≈ 90.017 (5s.f)

≈ 90.0 (3s.f) A1

7biii $n = 35$

$\ln(T-25) \approx 2.75$ M1

$T \approx 40.642°C$ (5s.f)

$\approx 40.6°C$ (3s.f) A1

The temperarture is 40.6°C after 35 minutes.

7biv $T = 25 + 90e^{-0.05n}$

$n = 0$, $T = 115$

25% of $T = 28.75$ M1

$\ln(28.75 - 25) \approx$

At 1.3 $n \approx$ 64 minutes

It takes 64 minutes for the temperature of the liquid to drop by 75% of its initial value.

9a. Equation of circle, C_1

$$(x+5)^2 + (y-3)^2 = 5^2$$

$$(x+5)^2 + (y-3)^2 =$$

9bi $y = -3x+11$ $A(-5,-2)$

$y = -3x + c$

Sub $A(-5,-2)$ into eqⁿ

$-2 = -3(-5) + c$

$c = -17$

$\therefore$

$y = -3x - 17$ A1

$m_1 \times m_2 = -1$

$-3 \times m_2 = -1$ m_1

$m_2 = \frac{1}{3}$

$y = \frac{1}{3}x + c$

Sub C $(-5, 3)$ into eq$\underline{n}$

$3 = \frac{1}{3}(-5) + c$

$c = \frac{14}{3}$

Equation of perpendicular bisector

$y = \frac{1}{3}x + \frac{14}{3}$

9bii $y = -3x - 17$ — (1)

$y = \frac{1}{3}x + \frac{14}{3}$ — (2)

(1) = (2) $-3x - 17 = \frac{1}{3}x + \frac{14}{3}$ M1

$3\frac{1}{3}x = -21\frac{2}{3}$

$x = -\frac{13}{2}$

Sub $x = -\frac{13}{2}$ (1) : $y = -3\left(-\frac{13}{2}\right) - 17$

$= \frac{5}{2}$

$\therefore$ midpoint of AB $= \left(-\frac{13}{2}, \frac{5}{2}\right)$

let coordinates of B (x_1, y_1)

$\frac{x_1 + (-5)}{2} = -\frac{13}{2}$ $\frac{y_1 + (-2)}{2} = \frac{5}{2}$

$x_1 = -8$ $y_1 = 7$

$\therefore$ coordinates of B $(-8, 7)$

9c Let coordinates of centre of C_2 be (x_2, y_2)

$$\frac{x_2 + (-5)}{2} = -\frac{13}{2} \quad M1 \qquad \frac{y_2 + (3)}{2} = \frac{5}{2} \quad M1$$

$$x_2 = -8 \qquad y_2 = 2$$

coordinates of centre of C_2 $(-8, 2)$

Equation of circle, C_2

$$(x+8)^2 + (y-2)^2 = 5^2$$

$$(x+8)^2 + (y-2)^2 = 25 \quad A1$$

10

$y = 2\sin 2x + 1$

$\frac{dy}{dx} = 4\cos 2x$

$4\cos 2x = 0$

$\cos 2x = 0$

$2x = \frac{\pi}{2}$

$x = \frac{\pi}{4}$

$x = \frac{\pi}{4}$, $y = 2\sin 2\left(\frac{\pi}{4}\right) + 1$

$= 3$

coordinates of P $\left(\frac{\pi}{4}, 3\right)$

$\frac{d^2y}{dx^2} = -8\sin 2x$

At $x = \frac{\pi}{4}$

$\frac{d^2y}{dx^2} = -8\sin 2\left(\frac{\pi}{4}\right)$

$= -8 < 0$

$\therefore$ P is maximum point

$4\cos 2x = -4$

$\cos 2x = -1$

$2x = \pi$

$x =$

$y = 2\sin 2\left(\frac{\pi}{2}\right) + 1$

$= 1$

coordinates of Q $\left(\frac{\pi}{2}, 1\right)$

Area of shaded region

$$= \int_0^{\frac{\pi}{4}} (2\sin 2x + 1)\,dx + \left[\frac{1}{2} \times (3+1) \times \left(\frac{\pi}{2} - \frac{\pi}{4}\right)\right]$$ M1 M1

$$= \left(\frac{-2\cos 2x}{2} + x\right)_0^{\frac{\pi}{4}} + \frac{\pi}{2}$$ M1

$$= \left(-\cos 2x + x\right)_0^{\frac{\pi}{4}} + \frac{\pi}{2}$$

$$= -(-1) + \frac{\pi}{2}$$

$$= 1 + \frac{3\pi}{4}$$ A1