Name | Reg. No | Class

MAYFLOWER SECONDARY SCHOOL

4EX/5NA

ADDITIONAL MATHEMATICS 4049/01

Paper 1 [90 marks]

PRELIMINARY EXAMINATION

23 AUGUST 2022

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

INSTRUCTIONS TO CANDIDATES

Do not open this booklet until you are told to do so.

Write your name, register number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **ALL** questions.

Write your answers in the space provided.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

Write the brand and model of your calculator in the space provided below.

INFORMATION FOR CANDIDATES

You are expected to use an electronic calculator to evaluate explicit numerical expressions.

If the degree of accuracy is not specified in the question, and if the answer is not exact, the answer should be given to **three** significant figures. Answers in degrees should be given to **one** decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

Brand / Model of Calculator

This question paper consists of **17** printed pages and **1** blank page

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\text{Area of } \Delta = \frac{1}{2}ab\sin C$$

1 **(a)** Find the remainder when $9x^3 - 26x^2 + 3x + 14$ is divided by $x + 1$. [1]

(b) Given that $f(x) = bx^3 - 5x^2 + 2x + 4$ and $g(x) = bx^3 + 6x - 8$ have a common factor $x - a$, where a is an integer, find the value of b. [4]

2 A curve has the equation $y = \frac{2-x}{3x-4}$, $x \neq \frac{4}{3}$.

(i) Find an expression for $\frac{dy}{dx}$. [2]

(ii) Find the coordinates of the points on the curve where the normal is parallel to the line $2y - 16x = 3$. [5]

3 **(a)** Find the range of values of a for which the inequality $x^2 + ax - 2 < 2(x-1)^2$ for all real values of x. [4]

(b) The equation of a curve is $y = 2x^3 + (b+1)x^2 - x + b$, where b is a constant.

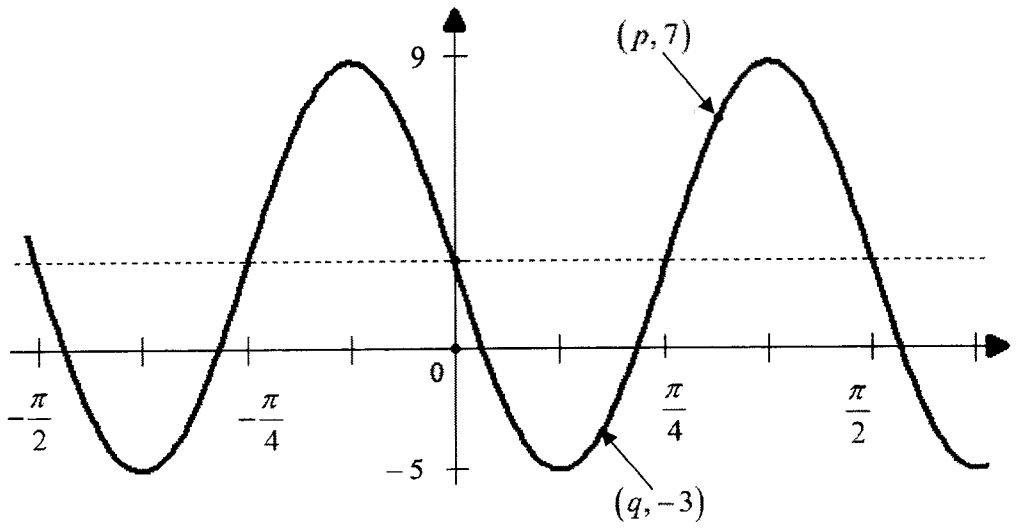
Show that, for all real values of b, the curve will have two distinct stationary points. [4]

4 **(a)** State the range of values

(i) for the principal value of $\cos^{-1} x$, [1]

(ii) of y such that $\sin^{-1} y$ is not defined. [1]

(b)



The diagram shows part of the graph of $y = a\sin bx + c$, passing through the points $(p, 7)$ and $(q, -3)$.

(i) Find the value of the constants a, b and c. [3]

(ii) Form an equation connecting p, q and π. [2]

[Turn over

5 **(a)** The population, N, of a certain virus is given by $N = N_o(3^{kt})$, where N_o and k are constants and t is measured in days. Given that the population of the virus increases by 200% at the end of 15 days, calculate the value of k. [3]

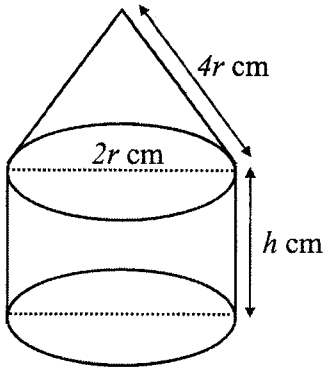
(b) The equation $\log_3 x + 2\log_3 5 = \log_{27} x$ has the solution $x = 5^m$. Find the value of m. [4]

6 Given that $\int_8^n \frac{x-6}{x^2-2x-24}\,dx = \ln\frac{4}{3}$,

(i) state the value(s) of x for the integral to be undefined. [2]

(ii) find the value of n. [4]

7 The diagram shows a **solid** made up by a right circular cone and a cylinder of diameter $2r$ cm. The slant height of the cone is $4r$ cm and height of the cylinder is h cm.



(i) Given that the total surface area of the solid is 300 cm^2, express h in terms of r. [2]

(ii) Show that the volume, V cm^3, of the solid is given by $V = 150r + \left(\frac{\sqrt{15}}{3} - \frac{5}{2}\right)\pi r^3$. [3]

(iii) Given that r can vary, find the stationary value of V and determine whether this value of V is maximum or minimum. [5]

8 **(a)** Prove that $\dfrac{\cos A+\sin A}{\cos A-\sin A} = \sec 2A + \tan 2A$. [3]

(b) The equation of the **gradient** of a curve is $f'(\theta)=\cos 2\theta-3\sin 2\theta$. Given that the curve $y=f(\theta)$ passes through the point $\left(\dfrac{\pi}{2}, -\dfrac{1}{2}\right)$, find

(i) the equation of the curve, [3]

(ii) the coordinates of the turning points of the curve for $0 \leq \theta \leq \pi$. [4]

9 **(a)** **(i)** Write down, and simplify, the first three terms in the expansion of $(2-3x)^5$ in ascending powers of x. [2]

(ii) Hence, find the coefficient of x^2 in the expansion of $\left(1-4x^2\right)(2-3x)^5$. [2]

(b) In the binomial expansion of $\left(\frac{k}{x^3}+x\right)^{12}$, where k is a positive constant, the term independent of x is 27500. Show that $k=5$. [4]

10 An ice cube retains its shape during melting. When its length is x mm, the surface area, A, is decreasing at a rate of $10\ \text{mm}^2/\text{s}$. The volume, V, of the ice cube changes at the rate of $-45\ \text{mm}^3/\text{s}$,

(i) find the value of x. [5]

(ii) find the rate of change of x for the **value found in (i)**. [1]

11 **(i)** $\sqrt{48+24\sqrt{3}}$ can be expressed in the form $a+2\sqrt{3}$, where a is an integer.

Find the value of a. [2]

(ii) If the area of a given square is $48+24\sqrt{3}$ cm^2, find the length of one side of the square. [1]

(iii) The square in part **(ii)** is the base of a right pyramid with volume $\left(50+20\sqrt{3}\right)$ cm^3.

Find the perpendicular height of the pyramid, leaving your answer in the form $\left(b-c\sqrt{3}\right)$ cm, where b and c are constants. [3]

12 A particle moves in a straight line, such that, t seconds after passing a fixed point O, its velocity, v m/s, is given by $v = \frac{2}{5}e^{3t} - 6e^{\frac{1}{2}-t}$. The particle comes to an instantaneous rest at point A.

(i) Show that the particle reaches A when $t = \frac{1}{8}(1+2\ln 15)$. [2]

(ii) Find the acceleration of the particle at A. [2]

(iii) Find the distance OA. [4]

(iv) Explain whether the particle will pass through point O **again** at some instant during the 2nd second. [2]

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Name | Reg. No | Class

MAYFLOWER SECONDARY SCHOOL

4E/5N

ADDITIONAL MATHEMATICS 4049/02

PAPER 2 [90 marks]

PRELIMINARY EXAMINATION

26 August 2022

2 hours 15 minutes

Candidates answer in the Question Paper
No additional material required

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Brand / Model of Calculator

For Examiner's Use

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Mathematical Formulae

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$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

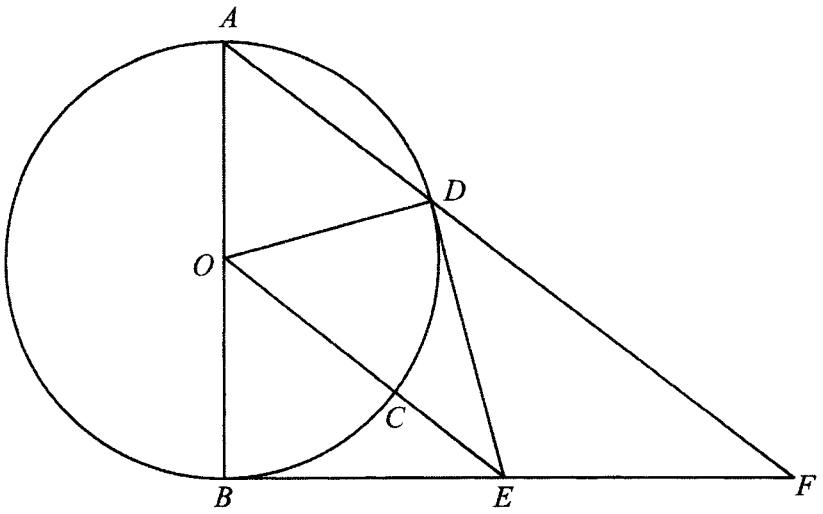
$$\text{Area of } \Delta = \frac{1}{2}ab \sin C$$

1 Express $5+8x-2x^2$ in the form $a(x-b)^2+c$, and hence state the coordinates of the turning point of the curve $y=5+8x-2x^2$. [4]

2 Express $\dfrac{4x^2+5x-11}{(x-2)(x^2+1)}$ in partial fractions. [6]

TURN OVER FOR QUESTION 3

3



A, *B*, *C* and *D* are points on a circle with centre *O*. *AB* is the diameter of the circle, *ADF* and *OCE* are straight lines and lines *BEF* and *DE* are tangents to the circle at *B* and *D* respectively.

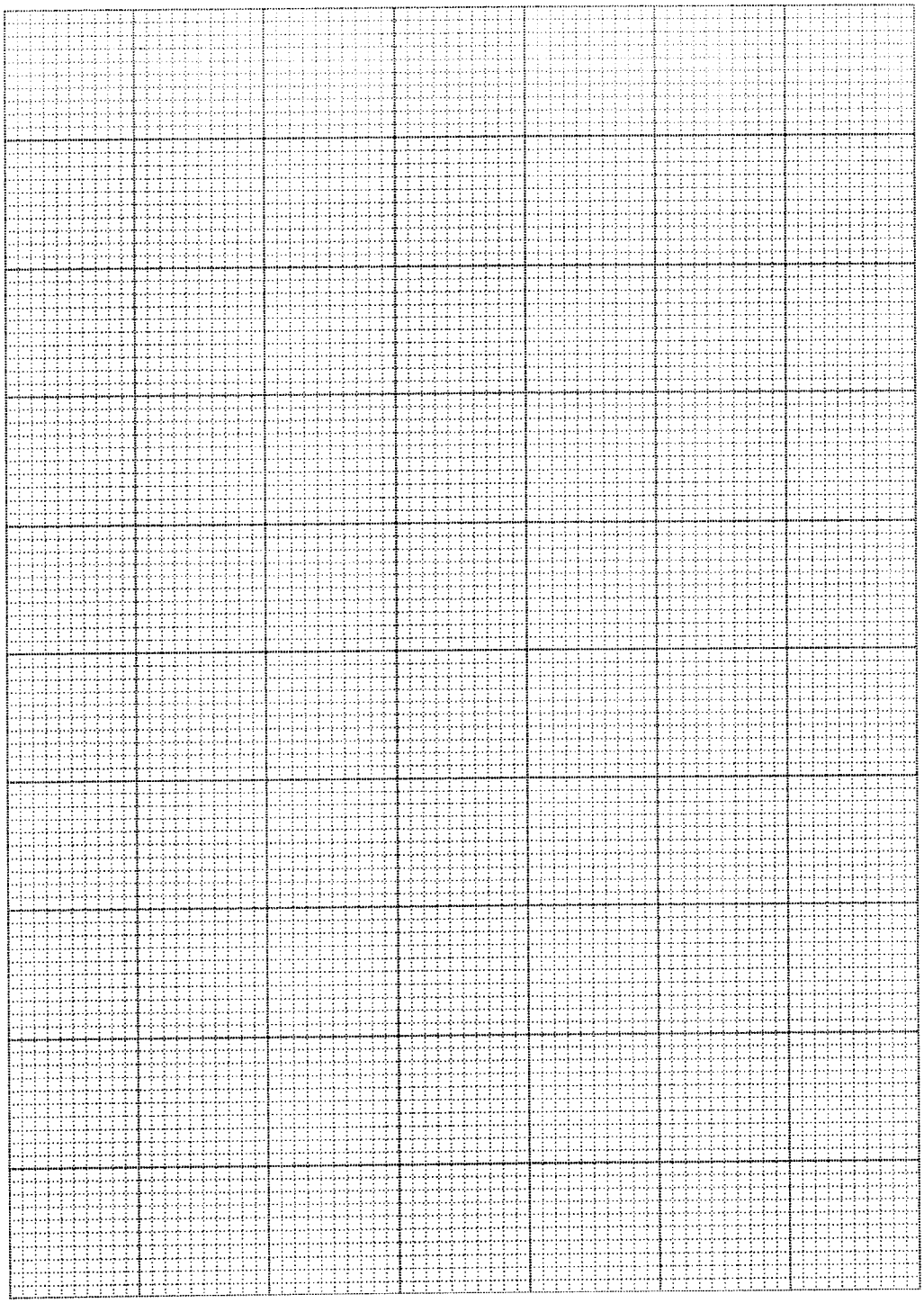
(i) Prove that triangle *OBE* is congruent to triangle *ODE*. [4]

(ii) Show that E is the midpoint of BF. [4]

4 It is known that t and N are related by an equation of the form $N = AB^t$, where A and B are constants. The table shows experimental values of two variables t and N.

t	1	2	3	4	5
N	199	1258	7943	50118	316227

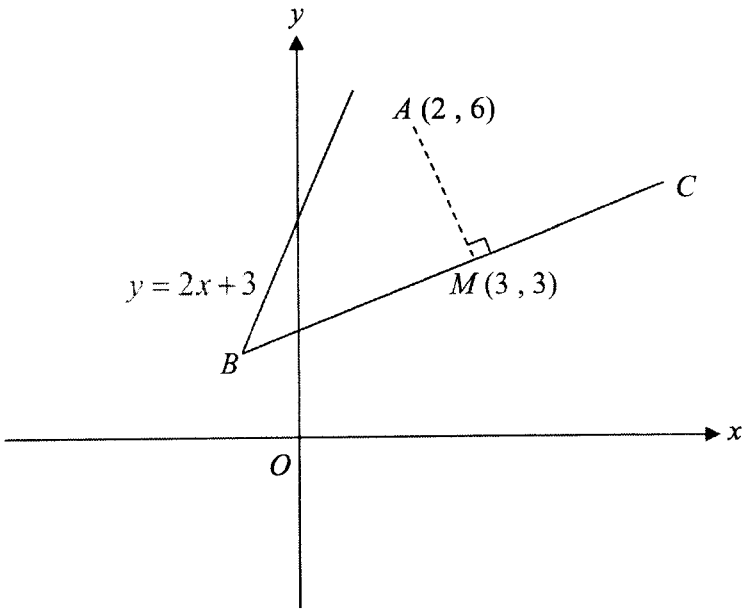
(i) By plotting lg N against t, draw a straight line to represent the above data. [3]



Continuation of working space for question **4(i)**

(ii) Use your graph to estimate the value of A and of B. [4]

5



Point A is (2 , 6), M is (3 , 3) and line AM is perpendicular to line BC.
Point B lies on the line with equation $y = 2x + 3$.

(i) Find the equation of line BC. [3]

(ii) Find the coordinates of B. [2]

M is the midpoint of BC.

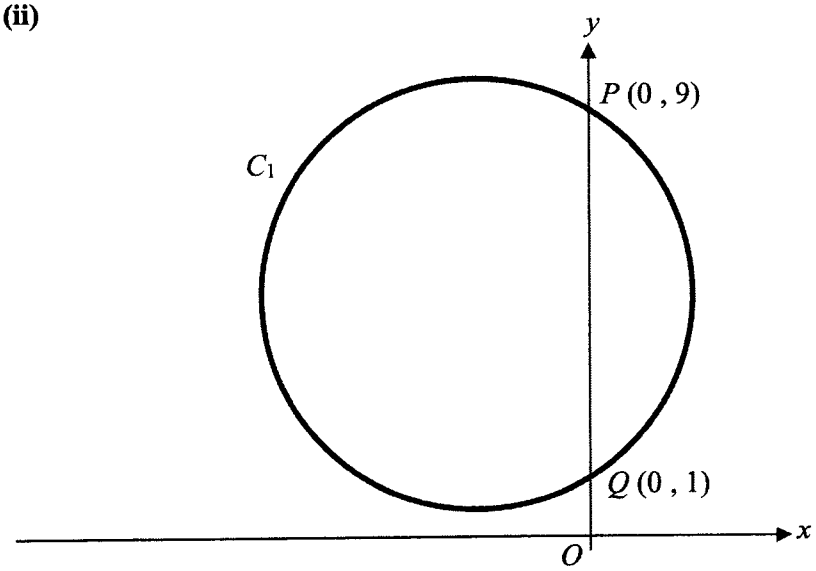
(iii) Find the coordinates of C. [2]

(iv) Calculate the area of triangle OAM. [2]

6 **(a)** A circle, C_1 intersects the y-axis at $P\ (0\ ,\ 9)$ and $Q\ (0\ ,\ 1)$ and its centre is 3 units to the left of the y-axis.

(i) Find the coordinates of the centre and radius of C_1. [3]

(ii)



Hammond drew a diagram of the circle by using the information given.

Explain why his diagram is wrong. [2]

(b) Another circle, C_2 has equation $x^2 + y^2 - 14x - 20y + 113 = 0$. Let L be the centre of C_2.

(i) State the coordinates of L and calculate the radius of C_2. [3]

M (2 , 2) is a point outside C_2 and N is a point on C_2.

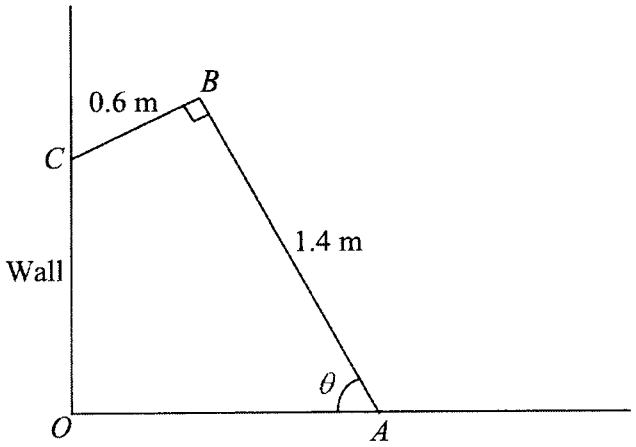
(ii) Find, in degrees, the greatest angle LMN. [3]

7 **(a)** Differentiate $\ln\left[\dfrac{(x-1)^{\frac{1}{2}}}{x+2}\right]$ with respect to x, for $x > 1$. [3]

(b) **(i)** Given that $y = \dfrac{x}{(3x+2)^2}$, show that $\dfrac{dy}{dx} = \dfrac{2-3x}{(3x+2)^3}$. [4]

(ii) Hence, find the value of $\int_0^1 \dfrac{-3x}{(3x+2)^3}\,dx$. [4]

8



A crowbar ABC is leaning against a vertical wall, where angle $ABC = 90°$, $AB = 1.4$ m and $BC = 0.6$ m. OA is horizontal and angle OAB is θ and is measured in degrees.

(i) Show that $OC = 1.4\sin\theta - 0.6\cos\theta$ [2]

(ii) Express OC in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° \leq \alpha \leq 90°$ [4]

(iii) Find the value of θ if $OC = 1$ m. [2]

(iv) Explain the significance when OC is minimum and write down the corresponding value of θ. [2]

9 **(a)** **(i)** Prove the identity $\sin 2\theta - \tan\theta\cos 2\theta = \tan\theta$. [3]

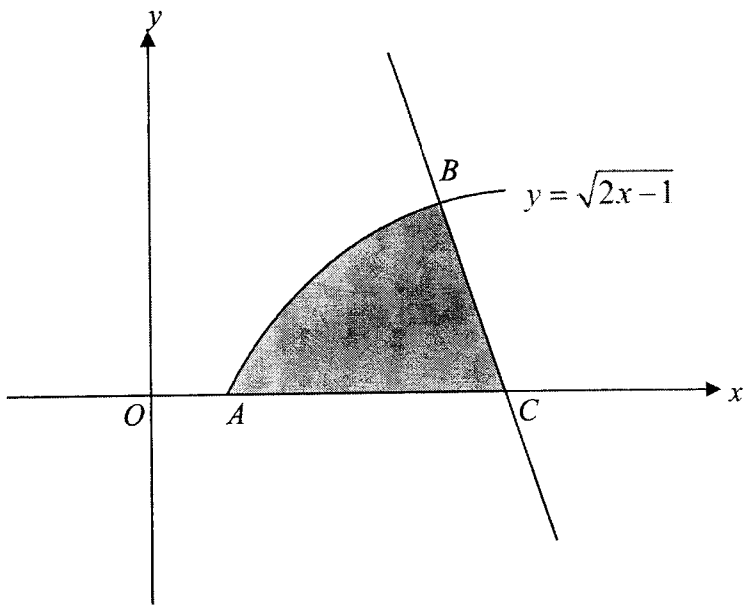
(ii) Hence, solve the equation $\sin 2\theta - \tan\theta\cos 2\theta = \dfrac{3}{\tan\theta}$ for $0 \le \theta \le 2\pi$. [4]

(b) Given that $\sin A = -\frac{3}{5}$, $\cos B = \frac{5}{6}$ and both angle A and angle B are in the same quadrant, find the exact value of

(i) $\cos(A-B)$. [3]

(ii) $\sin\frac{B}{2}$. [2]

10



The diagram shows part of the graph of $y = \sqrt{2x-1}$ intersecting the x-axis at A and has a gradient of $\frac{1}{3}$ at B. The normal of the graph at B intersects the x-axis at C. By first finding the coordinates of A, B and C or otherwise, show that the area of the shaded region is $10\frac{1}{2}$ units^2. [12]

Continuation of working space for question **10**

End of Paper

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