Peirce Secondary School
2022 Preliminary Examination 4E/5A Add Math Paper 1 Solutions

Qn	Suggested solution
1	$\text{Height} = \dfrac{(6\sqrt{7}-2\sqrt{3})}{(\sqrt{7}-\sqrt{3})^2}$ $= \dfrac{(6\sqrt{7}-2\sqrt{3})}{10-2\sqrt{21}} \times \dfrac{10+2\sqrt{21}}{10+2\sqrt{21}}$ $= \dfrac{60\sqrt{7}-20\sqrt{3}+84\sqrt{3}-12\sqrt{7}}{100-4(21)}$ $= \dfrac{48\sqrt{7}+64\sqrt{3}}{16}$ $= (3\sqrt{7}+4\sqrt{3})$ cm
2	$2x = y+3$ $y = 2x-3$ (1) $2x^2 + y + 9 = 10x$ ——— (2) Subst (1) into (2). $2x^2 + 2x - 3 + 9 - 10x = 0$ $2x^2 - 8x + 6 = 0$ $x^2 - 4x + 3 = 0$ $(x-1)(x-3) = 0$ When $x = 1, y = -1$ When $x = 3, y = 3$

Qn	Suggested solution
3 (i)	$R=\frac{1}{2}\left[(x-2)+(11-2x)\right](x+2)$ $=\frac{1}{2}(9-x)(x+2)$ $=\frac{1}{2}(18+7x-x^2)$ $R=-\frac{1}{2}x^2+\frac{7}{2}x+9$ (Shown)
3 (ii)	$-\frac{1}{2}x^2+\frac{7}{2}x+9$ $=-\frac{1}{2}(x^2-7x)+9$ $=-\frac{1}{2}\left[\left(x-\frac{7}{2}\right)^2-\left(\frac{7}{2}\right)^2\right]+9$ $=-\frac{1}{2}\left(x-\frac{7}{2}\right)^2+\frac{121}{8}$ $\therefore$ the maximum value of R = $\frac{121}{8}$ when x = $3\frac{1}{2}$ (or $\frac{7}{2}$)
4	$\int\frac{2}{3}(4x+3)^3\,dx+\int\frac{4}{x^2}\,dx$ $=\frac{2}{3}\left[\frac{(4x+3)^4}{16}\right]-\frac{4}{x}+c$ where c is a constant $=\frac{1}{24}(4x+3)^4-\frac{4}{x}+c$

Qn	Suggested solution
5	Long division: $x^2-2x-3 \overline{)5x^2-11x-4}$, quotient 5; $-(5x^2-10x-15)$; remainder $-x+11$ and $x^2-2x-3=(x-3)(x+1)$ $\therefore \dfrac{5x^2-11x-4}{x^2-2x-3}=5+\dfrac{11-x}{x^2-2x-3}$ Let $\dfrac{11-x}{x^2-2x-3}=\dfrac{A}{x-3}+\dfrac{B}{x+1}$ $\Rightarrow 11-x=A(x+1)+B(x-3)$ Subst $x=3$, $8=4A$ $A=2$ Subst. $x=-1$, $12=-4B$ $B=-3$ $\therefore \dfrac{5x^2-11x-4}{x^2-2x-3}=5+\dfrac{2}{x-3}-\dfrac{3}{x+1}$
6 (a)	$27x^3-8=(3x-2)(9x^2+6x+4)$ For $27x^3-8=0$ $(3x-2)(9x^2+6x+4)=0$ $3x-2=0$, $9x^2+6x+4=0$ $x=\frac{2}{3}$, Discriminant for $9x^2+6x+4=36-4(9)(4)$ $=-108$ D < 0 has no real roots. Hence $27x^3-8=0$ has only one real solution.
6 (b)	Let $2x^3-x^2-13x-6=(2x^2+ax-3)(x+2)$ The term in x: $-13x=2ax-3x$ $2a=-10$, $a=-5$

Qn	Suggested solution
7 (a) (i)	$0 \le \cos^{-1} x \le \pi$
7 (a) (ii)	$-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$
7 (b)(i)	$c = \frac{1}{2}$ (max value of y + min value of y) $\therefore c = \frac{1+(-2)}{2}$ $= -\frac{1}{2}$
7 (b)(ii)	$\text{period} = \frac{2\pi}{b}$ $\frac{4}{3}\pi = \frac{2\pi}{b}$ $b = \frac{3}{2}$
7 (b)(iii)	$y = \frac{3}{2}\cos\frac{3}{2}x - \frac{1}{2}$
7 (b)(iv)	$P = \left(\frac{2\pi}{3}, -2\right)$
7 (b)(v)	$-2 < k < 1$
8 (i)	Height of the isoceles triangle $= \sqrt{(5x)^2 - (3x)^2}$ $= 4x$ $\text{Volume} = \frac{1}{2}(6x)(4x)l$ $240 = 12x^2 l$ $l = \frac{20}{x^2}$ (shown)
8 (ii)	$A = \frac{1}{2}(6x)(4x) \times 2 + 16x\left(\frac{20}{x^2}\right)$ $A = 24x^2 + \frac{320}{x}$ (shown)
8 (iii)	$A = 24x^2 + \frac{320}{x}$ $\frac{dA}{dx} = 48x - \frac{320}{x^2}$ Let $\frac{dA}{dx} = 0$, $48x - \frac{320}{x^2} = 0$ $x^3 = \frac{320}{48}$ $x = 1.88$ (correct to 2 d.p.)
8 (iv)	$A = 255$ (to 3 s.f.)

Qn	Suggested solution
9 (i)	Gradient of $AD = \frac{k+2}{7}$ Gradient of $CD = -\frac{k+2}{7}$ or $\frac{k+1}{-6}$
9 (ii)	$-\frac{k+2}{7} = \frac{k+1}{-6}$ $6(k+2) = 7(k+1)$ $k = 5$ (shown)
9 (iii)	Gradient of DC = Gradient of AB $\frac{5+1}{0-6} = \frac{-2+8}{-7-h}$ $\frac{6}{-6} = \frac{6}{-7-h}$ $h = -1$
9 (iv)	Area of $ABCD = \frac{1}{2}\begin{vmatrix} 0 & -7 & -1 & 6 & 0 \\ 5 & -2 & -8 & -1 & 5 \end{vmatrix}$ $= \frac{1}{2}\left[(0+56+1+30)-(-35+2-48-0)\right]$ $= 84$ sq units.

Qn	Suggested solution
10 (a)	From LHS: $=\dfrac{\cos^2\theta+(1-\sin\theta)^2}{\cos\theta(1-\sin\theta)}$ $=\dfrac{\cos^2\theta+1-2\sin\theta+\sin^2\theta}{\cos\theta(1-\sin\theta)}$ $=\dfrac{2-2\sin\theta}{\cos\theta(1-\sin\theta)}$ $=\dfrac{2(1-\sin\theta)}{\cos\theta(1-\sin\theta)}$ $=2\sec\theta$ (= RHS)
10 (b)	$\dfrac{\cos3\beta}{1-\sin3\beta}+\dfrac{1-\sin3\beta}{\cos3\beta}=4$ ⇨ $\dfrac{2}{\cos3\beta}=4$ $\cos3\beta=\dfrac{1}{2}$ Basic angle = 60° $3\beta=60°, 360°-60°, 360°+60°$ $\beta=20°, 100°, 140°$.
11 (a)	$7^x\left(5^{2x}\right)=x^{3x}$ $7^x\left(25^x\right)=x^{3x}$ $(7\times25)^x=\left(x^3\right)^x$ $175=x^3$ $x=5.59$ (correct to 3 s.f.)
11 (b)	$2\log_a b-\dfrac{5}{\log_b a}=3$ $2\log_a b-\dfrac{5}{\frac{\log_a a}{\log_a b}}=3$ $2\log_a b-5\log_a b=3$ $-3\log_a b=3$ $\log_a b=-1$ $b=\dfrac{1}{a}$ or $b=a^{-1}$

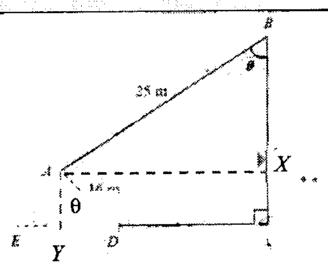
Qn	Suggested solution
12 (i)	$\frac{r}{h}=\frac{6}{12}$ $r=\frac{1}{2}h$ $V=\frac{1}{3}\pi r^2 h$ Where V is the volume of water in the inverted cone $V=\frac{1}{3}\pi\left(\frac{h^3}{4}\right)$ $V=\frac{\pi}{12}h^3$ (Shown)
12 (ii)	$V=\frac{\pi}{12}h^3$ $\frac{dV}{dh}=\frac{\pi}{4}h^2$ $\therefore\ \frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}$ $-2=\frac{\pi}{4}(3)^2\times\frac{dh}{dt}$ $\frac{dh}{dt}=-\frac{8}{9\pi}$ or -0.283 cm/s
12 (iii)	$A=\pi r^2$ $=\pi\left(\frac{h}{2}\right)^2$ $=\frac{\pi}{4}h^2$ $\frac{dA}{dh}=\frac{\pi}{2}h$ $\therefore\frac{dA}{dt}=\frac{\pi}{2}h\times\frac{dh}{dt}$ $\therefore\frac{dA}{dt}=\frac{\pi}{2}(3)\times\left(\frac{-8}{9\pi}\right)$ $\therefore\frac{dA}{dt}=-\frac{4}{3}\text{ cm}^2/s$

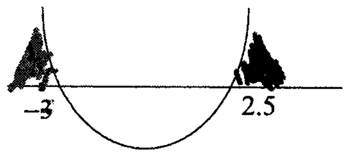
Qn	Suggested solution	
13 (a) (i)	Subst. $x = 0$ into $y=\frac{1}{2x+1}$, => $y=1$ $\therefore A\ (0, 1)$ Subst. $y = 0$ into $5y=x-1$, => $x=1$ $\therefore B\ (1, 0)$	
13 (a) (ii)	$y=\frac{1}{2x+1}$ ———— (1) $5y=x-1$ ———— (2) Subst. equation (1) into (2): $5\left(\frac{1}{2x+1}\right)=x-1$ $5=(x-1)(2x+1)$ $2x^2-x-6=0$ $(2x+3)(x-3)=0$ $x=2$ or $x=-\frac{3}{2}$ Hence the x-coordinate of C is 2 (verified)	
13 (b)	Area of the shaded region = $\int_0^2 \frac{1}{2x+1}\,dx-\frac{1}{5}\int_1^2 (x-1)\,dx$ $=\frac{1}{2}\left[\ln(2x+1)\right]_0^2-\frac{1}{15}\left[\frac{x^2}{2}-x\right]_0^2$, $=\frac{1}{2}\ln 5-\frac{1}{10}$ units^2	
14 (i)	$\angle YXZ=\angle XBY$ (Common Angle) $\angle XYX=\angle AXZ$ (tangent-chord theorem or alternate segment theorem) and $\angle AXZ=\angle XBY$ (Alternate angle) ⇨ $\angle XYX=\angle XBY$ (3 pairs of corresponding angles are equal) Hence $\triangle XYZ$ is similar to $\triangle XBY$.	
14 (ii)	$\frac{XY}{XB}=\frac{YZ}{BY}=\frac{XZ}{XY}$ (Properties of similar triangles) $\therefore XY^2=XB\times XZ$ (Proven)	

Peirce Secondary School

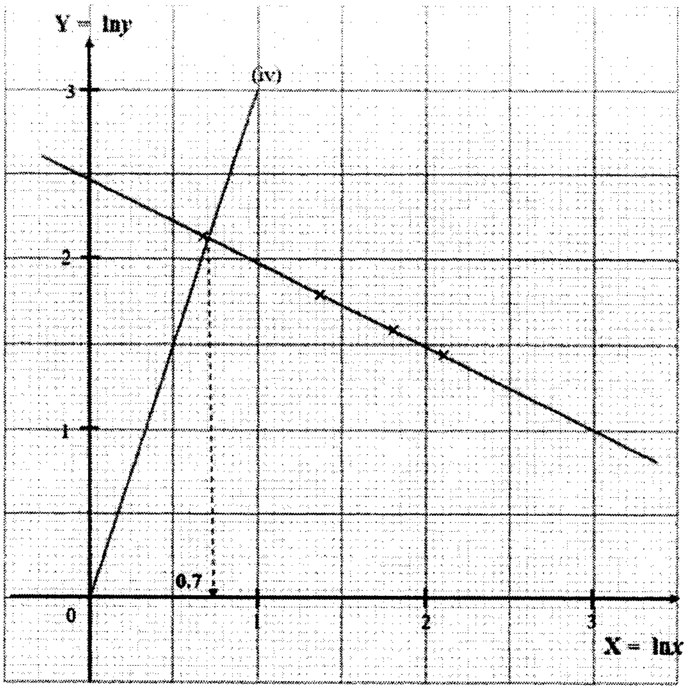
2022 Preliminary Examination 4E/5A Add Math Paper 2 Solutions

Qn	Suggested solution
1	$y=2(7^{2x})-3(7^{x+1})+19$ $y=2(7^{2x})-3(7)(7^{x})+19$ $y=2(7^{2x})-21(7^{x})+19$ Let $u=7^x$ $y=2u^2-21u+19$ Given $y=30$, $2u^2-21u+19=30$ $2u^2-21u-11=0$ $(2u+1)(u-11)=0$ $u=-\frac{1}{2},\ u=11$ $7^x=-\frac{1}{2}$ (NA) , $7^x=11$ $x=\frac{\lg 11}{\lg 7}$ $=1.23$ (to 3 s.f.)
2 (i)	Given p $\left(\frac{1}{3}\right)=0$ $p\left(\frac{1}{3}\right)=m\left(\frac{1}{3}\right)^3-29\left(\frac{1}{3}\right)^2+39\left(\frac{1}{3}\right)+n=0$ $\frac{1}{27}m-\frac{29}{9}+13+n=0$ × 27: $m+27n=-264$ ——— (1) Given $p(1)=6$ $m(1)^3-29(1)^2+39(1)+n=6$ $m+n=-4$ ——— (2) Equation (1) – (2): $26n=-260$ $n=-10$ From (2): $m=6$
2 (ii)	$p(x)=6x^3-29x^2+39x-10$ Let $6x^3-29x^2+39x-10=(3x-1)(2x^2+bx+10)$ The term in x: $30x-bx=39x$ $b=-9$ Let $6x^3-29x^2+39x-10=(3x-1)(2x^2+bx+10)$ Let $p(x)=0$, $(3x-1)(2x^2-9x+10)=0$ $(3x-1)=0$, $2x^2-9x+10=0$ $(2x-5)(x-2)=0$

Qn	Suggested solution
	$x=\frac{1}{3}$, $x=2$, $x=\frac{5}{2}$ (accept $2\frac{1}{2}$, 2.5) ,
3 (a)	$y=x^2\ln x$ $\frac{dy}{dx}=2x(\ln x)+x^2\left(\frac{1}{x}\right)$, $=2x(\ln x)+x$
3 (b)	$\frac{dy}{dx}=2x(\ln x)+x$ $[y]_2^4=\int_2^4 2x(\ln x)\,dx+\int_2^4 x\,dx$ $\left[x^2\ln x\right]_2^4=\int_2^4 2x(\ln x)\,dx+\left[\frac{x^2}{2}\right]_2^4$ $16\ln 4-4\ln 2=\int_2^4 2x(\ln x)\,dx+(8-2)$, $8\ln 4-2\ln 2=\int_2^4 x(\ln x)\,dx+3$ $16\ln 2-2\ln 2-3=\int_2^4 x(\ln x)\,dx$ $\int_2^4 x(\ln x)\,dx=14\ln 2-3$
4 (i)	 Let X be the foot of perpendicular from A to BC. Let Y be the foot of perpendicular from A to DE $BX=25\cos\theta$ and $AY=XC=16\sin\theta$ $AX=25\sin\theta$ and $YD=16\cos\theta$ $\therefore DC=25\sin\theta-16\cos\theta$ and $BC=25\cos\theta+16\sin\theta$ $\therefore L=25+(25\sin\theta-16\cos\theta)+(25\cos\theta+16\sin\theta)$ $L=25+9\cos\theta+41\sin\theta$ (shown)
4 (ii)	Let $R\cos(\theta-\alpha)=9\cos\theta+41\sin\theta$ $R=\sqrt{9^2+41^2}=\sqrt{1762}$

Qn	Suggested solution
	$\alpha = \tan^{-1}\left(\frac{41}{9}\right) = 77.61^o$ $\therefore L = 25 + \sqrt{1762}\ \cos\left(\theta - 77.6^\circ\right)$
4 (iii)	$25 + \sqrt{1762}\ \cos\left(\theta - 77.61^\circ\right) = 50$ $\sqrt{1762}\ \cos\left(\theta - 77.61^\circ\right) = 25$ $\cos\left(\theta - 77.61^\circ\right) = \frac{25}{\sqrt{1762}} = 0.59557$ $\theta - 77.61^\circ = 53.44^o$; $360 - 53.44^o$ (and include $-$ 53.44° angle in clockwise) $\theta = 53.44^o + 77.61^\circ$; $306.55 + 77.61^\circ(-53.44^o + 77.61^\circ = 24.2^o)$ $\theta = 131.05^o$; 384.16 $= 384.16 - 360$ $= 24.2^o$ Answer: $\theta = 24.2^o$ given θ is an acute angle.
4 (iv)	$L = 25 + \sqrt{1762}\ \cos\left(\theta - 77.6^\circ\right)$ Max Length $= 25 + \sqrt{1762}$ $= 66.97$ m Hence it is **not possible** to run the running track of 70 m.
5 (a)	$2x^2 + x - 6 > 9$ $2x^2 + x - 15 > 0$ $(2x-5)(x+3) > 0$ $x < -3,\ x > \frac{5}{2}$  
5 (b)	Gradient of tangent = 3 $\Rightarrow \frac{dy}{dx} = 3$

Qn	Suggested solution
	$4x+1=3$ $x=\frac{1}{2}$ and $y=2\left(\frac{1}{2}\right)^2+\frac{1}{2}-6=-5$ Hence the coordinates of $H=\left(\frac{1}{2}, -5\right)$
5 (c)	Subst $x=\frac{1}{2}$, $y=-5$ into $y=3x+k$ ⇨ $-5=3(\frac{1}{2})+k$ ⇨ $k=-6\frac{1}{2}$
6 (i)	$\left(2-\frac{x}{4}\right)^6=2^6+\binom{6}{1}(2^5)\left(\frac{-x}{4}\right)+\binom{6}{2}(2^4)\left(\frac{-x}{4}\right)^2+...$ $=64-48x+15x^2-...$
6 (ii)	$(4+kx+x^2)(64-48x+15x^2-...)$ The term in $x=-192x+64kx$ $=(-192+64k)x$ The term in $x^2=60x^2-48kx^2+64x^2$ $=(124-48k)x^2$ Given that $-192+64k+124-48k=-4$ $16k=64$ $k=4$
7 (a)	$y=ax(x+b)$ $\frac{y}{x}=ax+ab$ Plot $\frac{y}{x}$ against x, a straight line graph can be drawn. Gradient of the straight line is a and ab is the vertical intercept ***Alternative solution***: $y=ax(x+b)$ $y=ax^2+abx$ $\frac{y}{x^2}=a+\frac{ab}{x}$ Plot $\frac{y}{x^2}$ against $\frac{1}{x}$, a straight line graph can be drawn. Gradient of the straight line is ab and a is the vertical intercept

Qn	Suggested solution
7 (b) (i)	$yx^n = k$ $\ln y + n\ln x = \ln k$ $\ln y = -n\ln x + \ln k$ $X = \ln x$: 0.7, 1.4, 1.8, 2.1 $Y = \ln y$: 2.2, 1.8, 1.6, 1.4 
7 (b)(ii)	From graph, gradient $= \frac{2.45-1}{0-3}$ $-n = -0.4833$ (range ± 0.03) $\ln k = 2.45$ $k = e^{2.45}$ $= 11.6$ (correct to 3 s.f.) (range ± 0.5)
7 (b)(iii)	From Graph, $\ln x = 0.925$ $x = e^{0.925} = 2.52$ (to 3 s.f.) (± 0.03)
7 (b)(iv)	$\ln y = 3\ln x$ $(3+n)\ln x = \ln k$ $3\ln x = -n\ln x + \ln k$ From graph, $\ln x = 0.7$ $x = e^{0.7} = 2.01$ (to 3 s.f.)

Qn	Suggested solution
8 (i)	$v = 15e^{kt} + \frac{3}{4}t$ When $t = 0$, $v = 15$ m/s When $t = 10$, $30 = 15e^{10k} + \frac{3}{4}(10)$ $15e^{10k} = \frac{45}{2}$ $10k = \ln\frac{3}{2}$ $k = \frac{1}{10}\ln\frac{3}{2}$ or $= 0.0405$ (to 3 s.f.)
8 (ii)	$s = \int_0^{10}\left(15e^{\left(\frac{1}{10}\ln\frac{3}{2}\right)t} + \frac{3}{4}t\right)dt$ $s = \left[\frac{15e^{\left(\frac{1}{10}\ln\frac{3}{2}\right)t}}{\frac{1}{10}\ln\frac{3}{2}} + \frac{3}{8}t^2\right]_0^{10}$ $s = \left[\frac{15e^{\ln\frac{3}{2}}}{\frac{1}{10}\ln\frac{3}{2}} + \frac{3}{8}(10)^2\right] - \frac{150}{\ln\frac{3}{2}}$ $= 222$ m (correct to 3 s.f.)
8 (iii)	$v = 15e^{\left(\frac{1}{10}\ln\frac{3}{2}\right)t} + \frac{3}{4}t$ $a = \frac{dv}{dt}$ $\frac{dv}{dx} = 15\left(\frac{1}{10}\ln\frac{3}{2}\right)e^{\left(\frac{1}{10}\ln\frac{3}{2}\right)t} + \frac{3}{4}$ When $t = 2$, $\frac{dv}{dx} = \frac{3}{2}\left(\ln\frac{3}{2}\right)e^{\left(\frac{1}{10}\ln\frac{3}{2}\right)(2)} + \frac{3}{4}$ $= 1.41$ m/s^2 (correct to 3 s.f.)

Qn	Suggested solution
9 (a)	Gradient of AB, $m_{AB} = \frac{0-3}{3-(-2)} = \frac{-3}{5}$ Gradient of BC, $m_{BC} = \frac{0-5}{3-6} = \frac{5}{3}$ $m_{AB} \times m_{BC} = \frac{-3}{5} \times \frac{5}{3} = -1$ $\therefore \angle ABC = 90°$
9 (b)	$D(2, 4)$ AC is the diameter of circle, $\angle$s in semi-circle)
9 (c)	Radius of circle $= \sqrt{(3-2)^2 + (0-4)^2} = \sqrt{17}$ units $\therefore$ the equation of circle is $(x-2)^2 + (y-4)^2 = 17$
9(d)	Let $E = (x, y)$ $\left(\frac{3+x}{2}, \frac{0+y}{2}\right) = (2, 4)$ $\therefore E = (1, 8)$ Gradient of $DE = \frac{8-4}{1-2} = -4$ ⇨ Gradient of tangent at $E = \frac{1}{4}$. Hence the equation of tangent is $y - 8 = \frac{1}{4}(x-1)$ $y = \frac{1}{4}x + \frac{31}{4}$ or $y = \frac{1}{4}x + 7\frac{3}{4}$ or $4y = x + 31$

Qn	Suggested solution
10	$\frac{dy}{dx}=2\cos x-1$ Let $\frac{dy}{dx}=0$, => $2\cos x-1=0$ $\cos x=\frac{1}{2}$ $x=\frac{\pi}{3}$ When $x=\frac{\pi}{3}$, $y=2\sin\frac{\pi}{3}-\frac{\pi}{3}$ $y=2\left(\frac{\sqrt{3}}{2}\right)-\frac{\pi}{3}$ $y=\sqrt{3}-\frac{\pi}{3}$ $\therefore$ Area of the shaded region $=\int_0^{\frac{\pi}{3}}(2\sin x-x)\,dx-\frac{1}{2}\left(\frac{\pi}{3}\right)\left(\sqrt{3}-\frac{\pi}{3}\right)$ $=\left[-2\cos x-\frac{x^2}{2}\right]_0^{\frac{\pi}{3}}-\frac{\pi}{6}\left(\sqrt{3}-\frac{\pi}{3}\right)$ $=\left(-2\cos\frac{\pi}{3}-\frac{\pi^2}{18}\right)-(-2)-\frac{\sqrt{3}}{6}\pi+\frac{\pi^2}{18}$ $=-2\left(\frac{1}{2}\right)+2-\frac{\sqrt{3}}{6}\pi$ $=\left(1-\frac{\sqrt{3}}{6}\pi\right)$ units^2