Name	( )	Class	

南华中学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2022

Subject	:	**Mathematics**
Paper	:	**4048/01**
Level	:	**Secondary Four Express**
Date	:	**17 August 2022**
Duration	:	**2 hours**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue, correction fluid or correction tape.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use

This paper consists of 21 printed pages.

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** questions.

1 The number of people living in a town is given as 60 000, correct to 2 significant figures. Write down values for the smallest and largest possible number of people who could be in the town.

Smallest possible number is 59500
Largest possible number is 60499

2 **(a)** Use prime factors to explain why 135×200 is a perfect cube.

$$135 \times 200 = 5 \times 3^3 \times 10^2 \times 2$$
$$= 5 \times 3^3 \times 5^2 \times 2^2 \times 2$$
$$= 2^3 \times 3^3 \times 5^3$$

Since the index of each prime factor is multiple of 3, 135×200 is a perfect cube.
OR
$2^3 \times 3^3 \times 5^3 = (2 \times 3 \times 5)^3$

(b) The lowest common multiple of x and 135 is $2 \times 3^4 \times 5 \times 7$.
Find the smallest possible value of x.

$135 = 5 \times 3^3$
Smallest possible value of $x = 2 \times 3^4 \times 7$
$= 1134$

3

Write the following numbers in order of size, starting with the **smallest**.

$$\left(\frac{3}{10}\right)^2,\ -\frac{21}{90},\ 90\%,\ -0.23\ ,\ \sqrt{0.3}$$

$-\frac{21}{90}$, -0.23, $\left(\frac{3}{10}\right)^2$, $\sqrt{0.3}$, 90%

4 Factorise $\dfrac{6a-15a^2+20ab-8b}{25a^3-4a}$ completely.

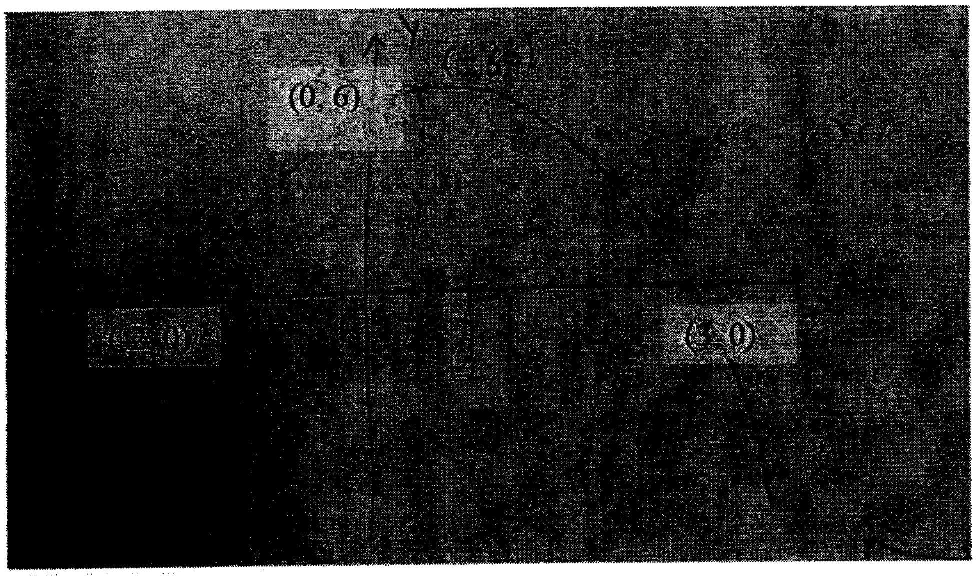
$$\begin{aligned}\frac{6a-15a^2+20ab-8b}{25a^3-4a} &= \frac{3a(2-5a)+4b(5a-2)}{a(25a^2-4)} \\ &= \frac{-3a(5a-2)+4b(5a-2)}{a(5a+2)(5a-2)} \\ &= \frac{(5a-2)(4b-3a)}{a(5a+2)(5a-2)} \\ &= \frac{4b-3a}{a(5a+2)}\end{aligned}$$

5 Rearrange the formula $\frac{1}{a} - \frac{1}{2b} = \frac{1}{3c}$ to make b the subject.

5	$\frac{1}{a} - \frac{1}{2b} = \frac{1}{3c}$ $6bc - 3ac = 2ab$ $6bc - 2ab = 3ac$ $2b(3c - a) = 3ac$ $b = \frac{3ac}{2(3c-a)}$ OR $\frac{3ac}{6c - 2a}$

6 Sketch the graph of $y = (3 - x)(x + 2)$ on the axes below.

Indicate clearly the coordinates of the points where the graph crosses the axes and the turning point of the curve.



7 The scale of a map is 8 cm : 2 km.

(a) Write this scale in the form $1 : n$.

8 cm :2 km

1 cm: 0.25 km

1 cm: 25 000 cm

Scale is 1: 25000

(b) The actual area of a lake is $90\,000\ \text{m}^2$.
Calculate the area, in square centimetres, of the lake on the map.

250 m :1 cm

$1\ \text{m}: \dfrac{1}{250}\ \text{cm}$

$1\ \text{m}^2: \left(\dfrac{1}{250}\right)^2\ \text{cm}^2$

$90000\ \text{m}^2: 90000\times\left(\dfrac{1}{250}\right)^2$

$=1.44\ \text{cm}^2$

8

3.8 is the mean of 5 positive numbers a, b, c, d and e.
The sum of their squares is 360. Each of the numbers is now multiplied by 2.
Find the new standard deviation.

$$\text{Standard deviation} = \sqrt{\frac{2^2(360)}{5}-(2\times3.8)^2}$$
$$=15.2 \text{ (to 3 s.f)}$$

9 One solution of the equation $(k+1)x^2+kx=15$ is $x=-3$.

(a) Find the value of k.

Sub x = -3,

$$(k+1)(-3)^2-3k=15$$
$$9k+9-3k=15$$
$$6k=6$$
$$k=1$$

(b) Find the second possible value of x.

$2x^2+x=15$

$2x^2+x-15=0$

$(2x-5)(x+3)=0$

$x=2.5$ or -3

The second possible value of x is 2.5

10 The cash price of a washing machine is \$840.
If paid by hire purchase scheme, the deposit is 15% of the cash price and the subsequent 24 equal monthly payments is \$33.50.
Calculate the interest rate per annum.

Total hire purchase price = $(0.15\times\$840)+(24\times\$33.50)$

$=\$930$

Interest = \$930 - \$840

= \$90

$$I=\frac{PRT}{100}$$

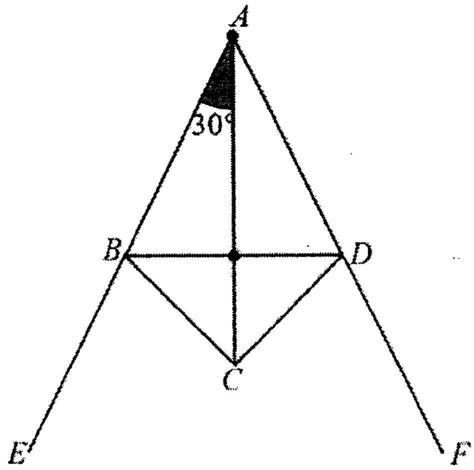
$$90=\frac{(0.85\times840)\times R\times2}{100}$$

$$R=\frac{90\times100}{0.85\times840\times2}$$

$=6.30$(correct to 3 s.f)

Interest rate per annum is 6.30%

11



The diagram above shows a kite $ABCD$ where angle $BAC = 30°$.
EB, BD and DF are three sides of a regular polygon.
ABE and ADF are straight lines.
The ratio of angle CBE to angle CBD is 3: 2.

(a) Calculate the number of sides of the polygon.

Mark the point X on diagram where the diagonals intersect

$\angle AXB = 90°$ (diagonals $\perp$ to each other)

$\angle ABX = 180° - 90° - 30°$ ($\angle$ sum of Δ)

$= 60°$

$n = \dfrac{360°}{60°}$

$= 6$

OR

$\angle CAD = 30°$ (longer diagonal bisects interior angles)

$AB = AD \Rightarrow \Delta ABD$ is an isosceles triangle.

$\angle ABD = \dfrac{180° - 60°}{2}$ ($\angle$ sum of Δ)

$= 60°$

$n = \dfrac{360°}{60°}$

$= 6$

(b) Find angle BCD.

$\angle EBD = 180° - 60° (\angle \text{ sum of } \Delta)$
$= 120°$

$\angle CBD = \frac{2}{5} \times 120°$
$= 48°$

$BC = CD \Rightarrow \Delta BCD$ is an isosceles Δ

$\angle BCD = 180° - 48° - 48° (\angle \text{ sum of } \Delta)$
$= 84°$

12 The first four terms in a sequence of numbers are given below.

$T_1 = 2^2 + 7$

$T_2 = 3^2 + 12$

$T_3 = 4^2 + 17$

$T_4 = 5^2 + 22$

(a) Explain why the value of T_n must be odd for all values of n.

$T_n = (n+1)^2 + (5n+2)$

When n is odd, $(n+1)^2$ is even and $(5n+2)$ is odd.

When n is even, $(n+1)^2$ is odd and $(5n+2)$ is even.

The sum of an even number and an odd number is always odd.

(b) The product of the first n terms of a sequence is given by $2n^2+3n$.
Find the 12^{th} term of this sequence.

$$\text{Product of first 11 terms} = 2(11)^2 + 3(11)$$
$$= 275$$
$$\text{Product of first 12 terms} = 2(12)^2 + 3(12)$$
$$= 324$$
$$\text{12th term} = \frac{324}{275}$$
$$= 1\frac{49}{275}$$

13 The thickness of a layer of ice in a water body is 0.00205 m.

(a) Write 0.00205 in standard form.

$0.00205 = 2.05 \times 10^{-3}$

(b) The ice covers an area of 1.60×10^5 m^2.
Assuming that all the ice melts and ignoring the expansion of volume when water freezes, calculate the volume of water, in litres.

$$\text{Volume of water} = 1.60 \times 10^5 \times 0.00205$$
$$= 328 \text{ m}^3$$
$$= 328000\ l$$

Answer ....l [2]

(c) A thunderstorm occurs and rainwater is falling at an average rate of 5×10^{-1} litres per second over the water body.
Calculate the percentage increase in the volume of water in the water body after two hours.

$$\text{Volume of rain water} = 2 \times 3600 \times 5 \times 10^{-1}$$
$$= 3600\ l$$
$$\text{Percentage increase} = \frac{3600l}{328000l} \times 100\%$$
$$= 1.10\% \text{ (to 3s.f)}$$

14 Given that the coordinates of A is $(2,-3)$, $\overrightarrow{AB}=\begin{pmatrix}-5\\7\end{pmatrix}$ and $\overrightarrow{FE}=k\begin{pmatrix}2.5\\-3.5\end{pmatrix}$.

(a) Find the value of k if $ABFE$ is a parallelogram.

$$\overrightarrow{AB}=\overrightarrow{EF}$$
$$\begin{pmatrix}-5\\7\end{pmatrix}=k\begin{pmatrix}-2.5\\3.5\end{pmatrix}$$
$$k=2$$

(b) Find the coordinates of B.

$$AB=OB-OA$$
$$\begin{pmatrix}-5\\7\end{pmatrix}=OB-\begin{pmatrix}2\\-3\end{pmatrix}$$
$$OB=\begin{pmatrix}-3\\4\end{pmatrix}$$
B is $(-3,4)$

(c) C is the point $(6,-10)$.
Justify if A, B and C are collinear.

Answer [2]

$$\overrightarrow{AB}=\begin{pmatrix}-5\\7\end{pmatrix}$$

$$\text{gradient of } AB=\frac{7}{-5}$$

$$\text{gradient of } AC=\frac{-3+10}{2-6}$$

$$=\frac{7}{-4}$$

Since gradient of $AB\neq$ gradient of AC, A, B and C are not collinear

OR

$$\overrightarrow{AC}=\begin{pmatrix}6\\-10\end{pmatrix}-\begin{pmatrix}2\\-3\end{pmatrix}$$

$$=\begin{pmatrix}4\\-7\end{pmatrix}$$

$$\overrightarrow{AB}=\begin{pmatrix}-5\\7\end{pmatrix}$$

Since $\overrightarrow{AB}\neq k\overrightarrow{AC}$, where k is a constant, hence, A, B and C are not collinear

15 A tour agency records the total number of people buying tour packages to Thailand and Vietnam in the months of November and December.
In November, 144 people bought the Thailand tour package and 100 people bought the Vietnam tour package.
In December, 208 people bought the Thailand tour package and 180 people bought the Vietnam tour package.

This information can be represented by the matrix, $\mathbf{M} = \begin{array}{cc} \text{Thailand} & \text{Vietnam} \\ \left(\begin{array}{cc} 144 & 100 \\ 208 & 180 \end{array}\right) & \begin{array}{l} \text{November} \\ \text{December} \end{array} \end{array}$

(a) The price of the Thailand and Vietnam package is \$890 and \$750 respectively.
Represent the price of the tour package by a 2×1 column matrix **K**.

$$K = \begin{pmatrix} 890 \\ 750 \end{pmatrix}$$

(b) Evaluate the matrix $\mathbf{R} = \mathbf{MK}$.

$$\begin{aligned} R &= MK \\ &= \begin{pmatrix} 144 & 100 \\ 208 & 180 \end{pmatrix}\begin{pmatrix} 890 \\ 750 \end{pmatrix} \\ &= \begin{pmatrix} 203160 \\ 320120 \end{pmatrix} \end{aligned}$$

(c) State what the elements of **R** represent.

Answer
The cash received for total sales of tour packages sold in the months of November and December respectively.

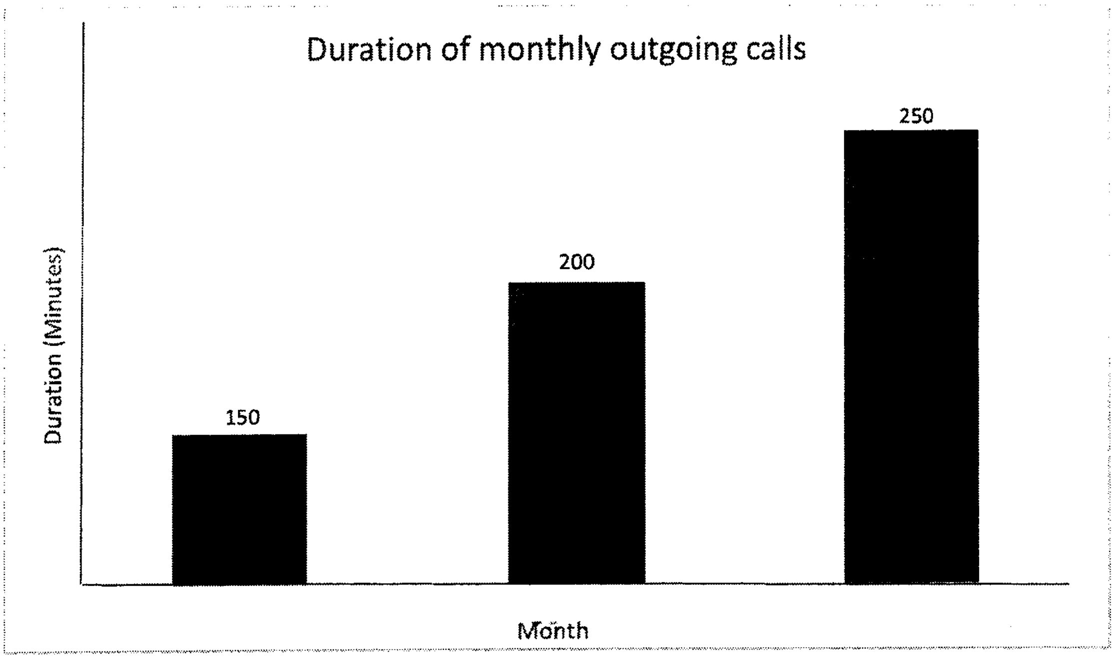
(d) Evaluate $\frac{1}{2}(1 \quad 1)\mathbf{R}$ and explain what the answer represents.

$$\frac{1}{2}(1 \quad 1)\begin{pmatrix} 203160 \\ 320120 \end{pmatrix} = (261640)$$

Answer

The average sales of tour packages sold in the months of November and December.

16 Peter draws this graph to show the duration of his monthly outgoing calls (in minutes) for the last three months.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Vertical axis did not start from zero OR vertical height of the bars are not in proportion. It gives readers the wrong impression that the second month's duration of outgoing calls is twice that of first month's duration OR the third month's duration of outgoing calls is thrice that of first month's duration. OR The horizontal axis did not state the order of months, whether was the latest month the bar furthest on the right. It doesn't allow readers to make any conclusions on its trend whether is duration increasing or decreasing across the months.

17 $\xi = \{\text{integer } x : 0 < x \leq 20\}$
$P = \{\text{perfect square}\}$
$Q = \{\text{even number which solves } 3x > 11\}$
$R = \{\text{ multiple of } 4\}$

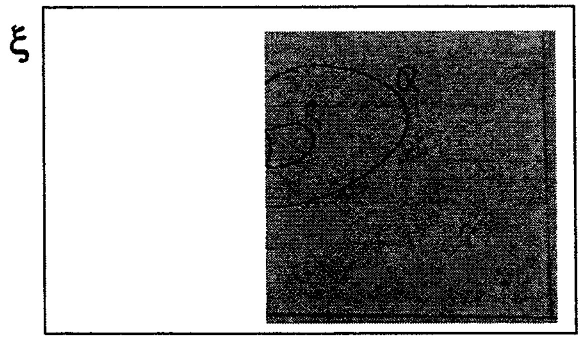
(a) (i) List all the elements in P.

$P = \{1, 4, 9, 16\}$

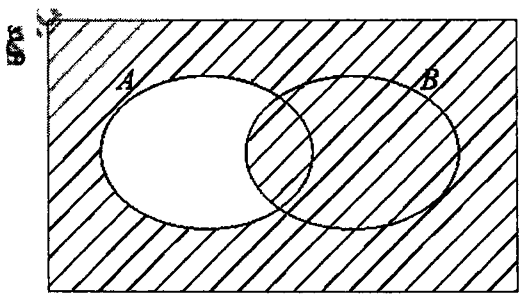
(ii) Find $\mathrm{n}(P' \cap Q)$.

$\mathrm{n}(P' \cap Q) = 7$

(iii) On the answer space provided, draw the Venn diagram to illustrate the relationship between sets P, Q and R. [2]

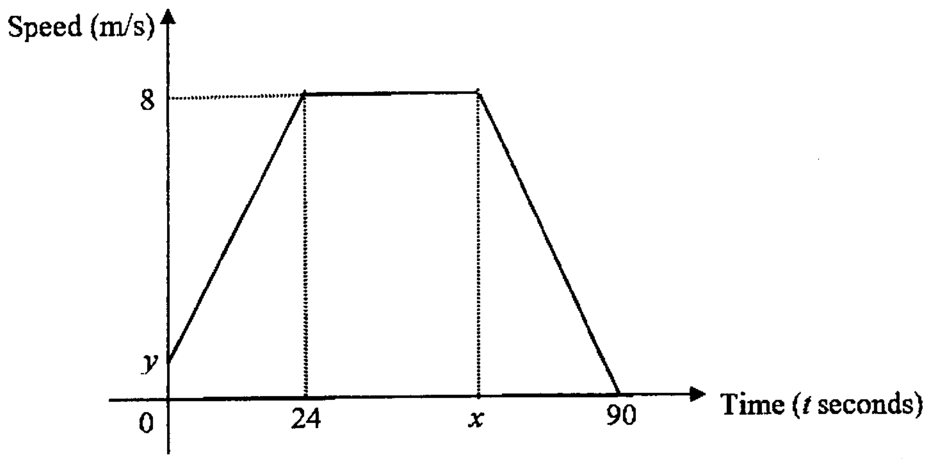


(b) Use set notation to describe the set shaded in the Venn diagram below.



$A' \cup B$

18 The diagram shows the speed-time graph for a cyclist's journey for a period of 90 seconds.
The cyclist accelerates at 0.25 m/s^2 in the first 24 seconds.
He then travels at a constant speed of 8 m/s for a distance of 288 m.



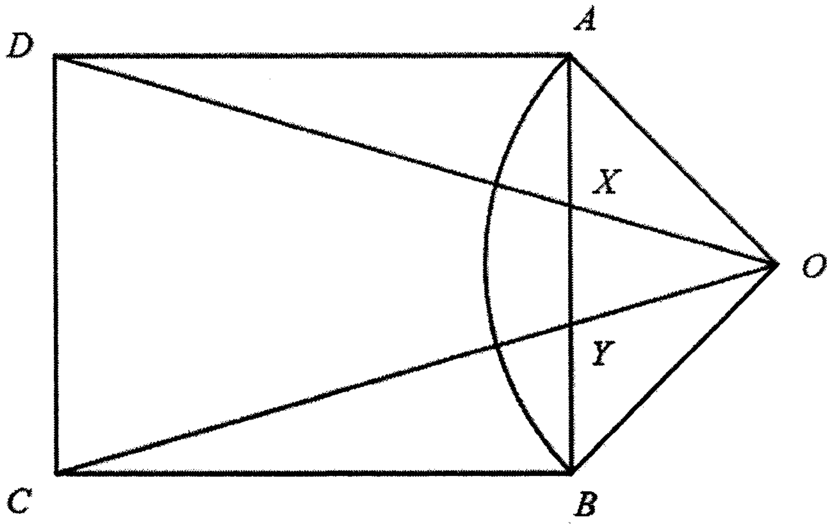
(a) Find the values of x and y.

$$\frac{8-y}{24}=0.25$$
$$8-y=6$$
$$y=2$$
$$288\div 8=36$$
$$x=24+36$$
$$=60$$

(b) On the grid provided, complete the distance-time graph of the journey from $t=0$ to $t=90$ seconds.

[redacted]

19 In the diagram, $ABCD$ is a rectangle.
OAB is a sector of a circle, centre O.
OXD and OYC are straight lines.



(a) Show that triangle OAD is congruent to triangle OBC.
Give a reason for each statement you make. [3]

$OA = OB$ (radius of circle)
$DA = CB$ (length of rectangle)
Since $\angle DAX = \angle CBY$ (right angle of rectangle),
$\angle OAX = \angle OBY$ (base $\angle$s of isosceles Δ),
$\angle DAO = \angle DAX + \angle OAX$
$= \angle CBY + \angle OBY$
$= \angle CBO$
$\therefore \Delta OAD \equiv \Delta OBC$ (SAS congruency)

(b) Show that triangle OXY is similar to triangle ODC.
Give a reason for each statement you make. [2]

$\angle DOC = \angle XOY$ (common $\angle$)
$\angle OXY = \angle ODC$ (corresponding $\angle s$, $AB // DC$)
ΔOXY is similar to ΔODC (AA similarity)

(c) The area of triangle ODC is 36 times that of the area of triangle OXY.
Find the ratio of the area of quadrilateral $DCYX$ to area of triangle OAB.
Ans: 35:6

20 The ages of 18 swimmers and 11 cyclists in a sports carnival race were recorded.
The results are shown in the stem-and-leaf diagram.

Swimmers		Cyclists
4 4 3 2 1 1 0	2	1 1 2 2 3 3
9 x 5 5	2	5 7
4 1	3	
9 8 8	3	6 9
3 2	4	0

Key (Swimmers)
1 | 2 means 21 years old

Key (cyclists)
2 | 3 means 23 years old

(a) Given that the median age of the swimmers is 26 years old, find the value of x.

Median of swimmers' age = 26

$$\frac{25+\text{data}}{2}=26$$

data = 27

$x = 7$

(b) Find the interquartile range of the cyclists' age.

lower quartile = 22

upper quartile = 36

interquartile range = 36 - 22

= 14

(c) Make two comments comparing the ages of the swimmers and the cyclists.

Median of cyclist' age = 23

interquartile range for swimmers' age = 38 – 23

= 15

The cyclists are generally younger than the swimmers as they have a lower median age.

The ages of swimmers have a larger spread of age than that of the cyclists due to its larger interquartile range.

*only award if median of cyclists is correct and IQR of swimmers and cyclists correct

21 The plan of a triangular-shaped garden, PQR, is such that $QR = 9.5\text{cm}$ and $\angle PQR = 30°$. PQ has been drawn for you.

(a) Construct triangle PQR in the space provided. [2]

(b) (i) Construct the perpendicular bisector of R and P. [1]

(ii) Construct the bisector of angle QPR. [1]

(c) A bench S needs to be built inside the garden such that it is nearer to PR than to PQ and $SR \geq 3$ cm. Shade the region where S could be possibly built. [2]

22 Adam invested \$$P$ in a savings account X with interest compounded quarterly at the rate of 1.5% per annum.

Ben invested \$$P$ in a savings account Y, paying simple interest at the rate of x% per year.

At the end of 5 years, Ben made 10% more than Adam.

Mary would like to invest \$$P$ for 20 years.
She believes that savings account Y is better as Ben made more money than Adam.
Justify with clear mathematical working, whether Mary is correct. [4]

$$\text{amount of compound interest} = \left[P\left(1+\frac{1.5}{400}\right)^{20}\right]-P$$

$$= P\left[\left(1+\frac{1.5}{400}\right)^{20}-1\right]$$

$$\text{amount of simple interest} = P \times \frac{x}{100} \times 5$$

$$= \frac{5Px}{100}$$

$$\frac{5Px}{100} = 1.1P\left[\left(1+\frac{1.5}{400}\right)^{20}-1\right]$$

$$x = 1.71013$$

If compounded using X,

$$\text{interest amount} = P\left[\left(1+\frac{1.5}{400}\right)^{80}-1\right]$$

$$= 0.34910P^*$$

If simple interest using Y,

$$\text{interest amount} = P \times \frac{1.71013}{100} \times 20$$

$$= 0.34203P^*$$

Mary is not correct.

– End of Paper –

Name	( )	Class	

南 华 中 学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2022

Subject	:	**Mathematics**
Paper	:	**4048/02**
Level	:	**Secondary Four Express**
Date	:	**18 August 2022**
Duration	:	**2 hours 30 minutes**

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The total of the marks for this paper is 100.

For Examiner's Use

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	**TOTAL**

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Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** questions.

1 (a) Simplify $\frac{3a^2}{5b^3} \div \left(\frac{12c}{25ab}\right)^2$.

$$\frac{3a^2}{5b^3} \div \left(\frac{12c}{25ab}\right)^2$$
$$= \frac{3a^2}{5b^3} \div \frac{144c^2}{625a^2b^2}$$
$$= \frac{3a^2}{5b^3} \times \frac{625a^2b^2}{144c^2}$$
$$= \frac{125a^4}{48bc^2}$$

Answer .... [3]

(b) n is a positive integer.

Show that, for all n, $(3n+2)^2-(3n-2)^2$ is a multiple of 3.

Answer

$$(3n+2)^2-(3n-2)^2$$
$$=(3n+2+3n-2)[3n+2-(3n-2)]$$
$$=(6n)(4)$$
$$=24n$$
$$=3(8n)$$
Since n is a positive integer, $3(8n)$ is a multiple of 3.

[2]

(c) Solve the equation $\dfrac{2}{x-4}+\dfrac{7x}{3x-2}=1$.

$$\frac{2}{x-4}+\frac{7x}{3x-2}=1$$

$$\frac{2(3x-2)+7x(x-4)}{(x-4)(3x-2)}=1$$

$$2(3x-2)+7x(x-4)=(x-4)(3x-2)$$

$$4x^2-8x-12=0$$

$$x^2-2x-3=0$$

$$(x-3)(x+1)=0$$

$$x=3 \text{ or } x=-1$$

Answer $x=$ .... or .... [4]

(d) Use the quadratic formula to solve the equation.

$$3x^2-7x-4=0$$

$$3x^2-7x-4=0$$

$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(3)(-4)}}{2(3)}$$

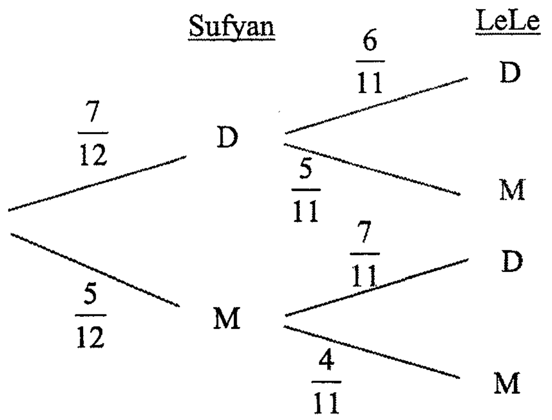
$$=2.81 \text{ or } -0.475$$

Answer $x=$ .... or .... [2]

2 A box of chocolate contains 7 dark chocolates and 5 milk chocolates.
Sufyan takes a chocolate, selected at random, from the box and eats it.
LeLe then takes a chocolate, selected at random, from the box.

(i) Draw a tree diagram to show the probabilities of the possible outcomes.

Answer



Legend
D – Dark Chocolate
M – Milk Chocolate

[2]

(ii) Find, as a fraction in its simplest form, the probability that

(a) Sufyan and LeLe both picked dark chocolates,

$$\text{P(both dark chocolate)} = \frac{7}{12} \times \frac{6}{11} = \frac{7}{22}$$

Answer .... [1]

(b) LeLe picked a milk chocolate,

$$\text{P(LeLe picked milk chocolate)} = \frac{7}{12} \times \frac{5}{11} + \frac{5}{12} \times \frac{4}{11}$$
$$= \frac{5}{12}$$

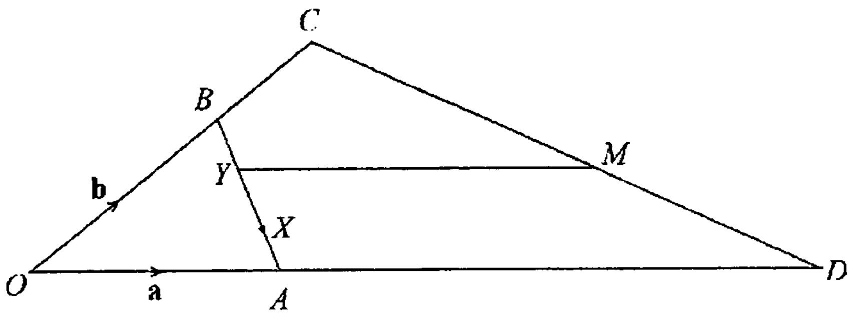
Answer .... [2]

(c) at least one milk chocolate was chosen.

$$\text{P(at least one milk chocolate was chosen)} = 1 - \frac{7}{12} \times \frac{6}{11}$$
$$= \frac{15}{22}$$

Answer .... [2]

3



In the diagram, $OA = \mathbf{a}$ and $OB = \mathbf{b}$.

$2OB = 3BC$, $OD = 3OA$, and $2AX = XB$.

(i) Express, as simply as possible, in terms of **a** and **b**,

(a) BA,

Answer $BA = \ldots\; \mathbf{a}-\mathbf{b} \;\ldots$ [1]

(b) OX,

$OX = OA + AX$

$= OA + \frac{1}{3}AB$

$= \mathbf{a} + \left(-\frac{1}{3}\right)(\mathbf{a}-\mathbf{b})$

$= \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$

Answer $\overrightarrow{OX} = \ldots\ldots\ldots\ldots\ldots\ldots$ [1]

(c) $\overrightarrow{CD}$.

$\overrightarrow{CD} = \overrightarrow{CO} + \overrightarrow{OD}$

$= 3\mathbf{a} - \frac{5}{3}\mathbf{b}$

Answer $\overrightarrow{CD} = \ldots\ldots\ldots\ldots\ldots\ldots$ [1]

(ii) It is given that $CM : MD$ is $10 : 9$.

(a) Express $\overrightarrow{OM}$, as simply as possible, in terms of **a** and **b**.

$\frac{CM}{MD} = \frac{10}{9}$

$\overrightarrow{OM} = \frac{10}{9}\mathbf{b} + \frac{10}{19}\left(3\mathbf{a} - \frac{5}{3}\mathbf{b}\right)$

$= \frac{30}{19}\mathbf{a} + \frac{15}{19}\mathbf{b}$

Answer $\overline{OM} =$ .... [1]

(b) Show that O, X and M are collinear.

Answer

$$\overline{OM} = \frac{15}{19}\mathbf{b} + \frac{30}{19}\mathbf{a}$$
$$= \frac{45}{19}\left(\frac{1}{3}\mathbf{b} + \frac{2}{3}\mathbf{a}\right)$$
$$= \frac{45}{19}OX$$

Hence, $OM \,//\, OX$ and O is a common point.
Thus O, X and M are collinear.

[2]

(c) Given that MY is parallel to OD, express MY in terms of **a** and **b**.

$$\frac{OM}{OX} = \frac{45}{19}$$
$$\Rightarrow \frac{XM}{OX} = \frac{YM}{OA} = \frac{26}{19}$$
$$YM = \frac{26}{19}OA$$
$$MY = -\frac{26}{19}\mathbf{a}$$

Answer $MY =$ .... [1]

(iii) Find the ratio of

(a) $\dfrac{\text{area of } \Delta XMY}{\text{area of } \Delta XOA}$,

$$\frac{\text{area of } \Delta XMY}{\text{area of } \Delta XOA} = \left(\frac{26}{19}\right)^2 = \frac{676}{361}$$

Answer .... [1]

(b) $\dfrac{\text{area of } \Delta OBX}{\text{area of } \Delta OYA}$.

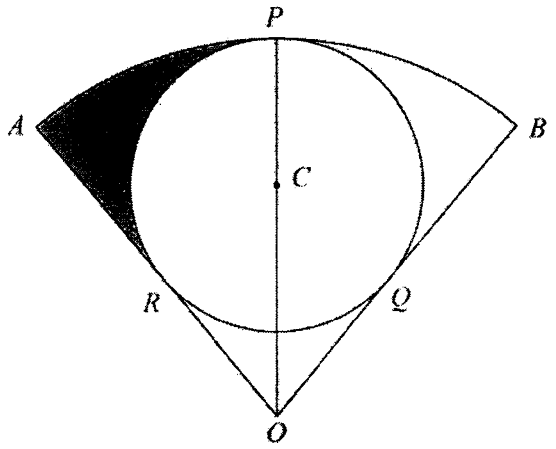
$$\frac{AX}{XB} = \frac{1}{2}$$

BX	:	XA	:	AY
2	:	1		
		19	:	45
38	:	19	:	45

$$\frac{\text{area of } \Delta OBX}{\text{area of } \Delta OYA} = \frac{38}{45}$$

Answer .... [2]

4



In the diagram, $OAPB$ is a sector of circle with centre O and radius 9 cm.
Angle $AOB = 1.4$ radians.
C is the centre of the circle enclosed inside the sector.
OCP is a straight line and the circle touches the sector at points P, Q and R.

(i) Show that the radius of the enclosed circle is 3.526 cm, correct to 3 decimal places.

Answer

Let the radius of the enclosed circle be r cm

$\angle CRO = 90°$ (tangent $\perp$ radius)

$\angle ROC = \dfrac{1.4}{2}$ (tangents from external point)

$= 0.7$ rad

$\sin \angle ROC = \dfrac{RC}{CO}$

$\sin 0.7 = \dfrac{r}{9-r}$

$9\sin 0.7 - r\sin 0.7 = r$

$r = \dfrac{9\sin 0.7}{1+\sin 0.7}$

$= 3.5262$ (4 dp)

$= 3.526$ (3 dp)

[5]

(ii) Calculate the area of the shaded region.

$$\angle RCO = \pi - \frac{\pi}{2} - 0.7 \ (\angle \text{ sum of } \Delta)$$

$$= \frac{\pi}{2} - 0.7$$

$$\angle RCP = \pi - \left(\frac{\pi}{2} - 0.7\right) \ (\text{adjacent } \angle\text{s on a straight line})$$

$$= \frac{\pi}{2} + 0.7$$

$$\text{area of sector } RCP = \frac{1}{2}\left(\frac{\pi}{2} + 0.7\right)(3.526)^2$$

$$= 14.116 \text{ cm}^2 \ (5\text{sf})$$

$$\text{area of } \Delta RCO = \frac{1}{2}(3.526)(9 - 3.526)\sin\left(\frac{\pi}{2} - 0.7\right)$$

$$= 7.3812 \text{ cm}^2 \ (5\text{sf})$$

$$\text{area of sector } OAP = \frac{1}{2}(0.7)(9)^2$$

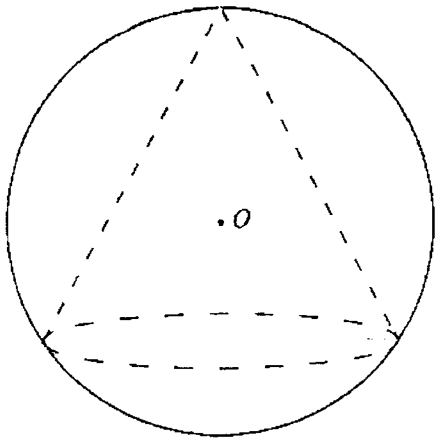
$$= 28.35 \text{ cm}^2$$

$$\text{area of shaded region} = 28.35 - 7.3812 - 14.116$$

$$= 6.85 \text{ cm}^2 \ (3\text{sf})$$

Answer .... cm^2 [4]

5



The diagram shows a right circular cone cut from a solid steel sphere.
Point O is the centre of the sphere with radius 12 cm.

(a) Given the circumference of the base of the cone is 50 cm, show that the height of the cone is 21.0 cm, corrected to 3 significant figures.

Answer

$$\text{radius of cone} = \frac{50}{2\pi}$$
$$= 7.9577 \text{ cm } (5\text{sf})$$
$$\text{length from } O \text{ to base of cone} = \sqrt{12^2 - 7.9577^2}$$
$$= 8.9819 \text{ cm } (5\text{sf})$$
$$\text{height of cone} = 12 + 8.9819$$
$$= 20.9819$$
$$= 21.0 \text{ cm (3sf)}$$

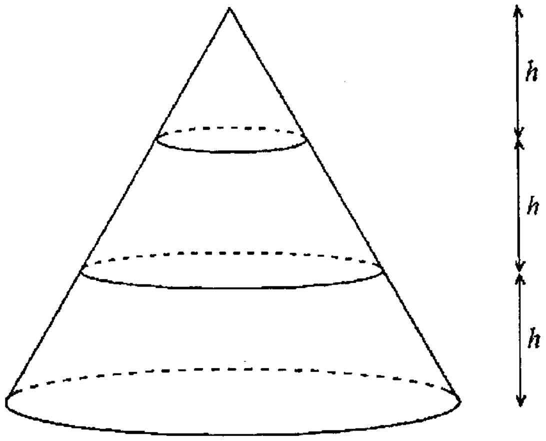
[3]

(b) Find the curved surface area of the cone.

$$\text{slant height of cone} = \sqrt{20.9819^2 + 7.9577^2} \quad or \quad \sqrt{21.0^2 + 7.9577^2}$$
$$= 22.440 \text{ cm} \qquad = 22.457 \text{ cm}$$
$$\text{curved surface area} = \pi(7.9577)(22.440)$$
$$= 561 \text{ cm}^2 \text{ (3sf)}$$

Answer .... cm^2 [2]

(c) After the cone is cut from the steel sphere, the remaining steel is melted down to form part of a solid right circular cone as shown in the following diagram.



The cone comprises 3 layers of equal heights, h cm.
The top and bottom layers are cast from the remaining steel.
The centre section is made from acrylic.

Find the volume of the acrylic used to make the centre layer of the solid cone.

volume of cone in sphere $= \frac{1}{3}\pi(7.9577)^2(20.9819)$ or $\frac{1}{3}\pi(7.9577)^2(21.0)$

$= 1391.4 \text{ cm}^3$ $\quad = 1392.6 \text{ cm}^3$

volume of remaining steel $= \frac{4}{3}\pi(12)^3 - 1391.4$ or $\frac{4}{3}\pi(12)^3 - 1392.6$

$= 5846.8 \text{ cm}^3$ $\quad = 5845.6 \text{ cm}^3$

ratio of volumes $A : A+B : A+B+C : A+C$

$h^3 : (2h)^3 : (3h)^3$

$1 : 8 : 27 : 20$

volume of acrylic $= \frac{7}{20} \times 5846.8$

$= 2050 \text{ cm}^3$

Answer .... cm^3 [4]

6 Plane A travels at an average speed of x km/h for 3 hours 20 minutes and then at an average speed of y km/h for 1 hour 10 minutes.
The plane travels a total distance of 3700 km.

(a) Write down an equation in x and y to represent this information and show that it simplifies to $20x+7y=22200$.

Answer

$$x \times 3\frac{1}{3} + y \times 1\frac{1}{6} = 3700$$

$$\frac{10x}{3} + \frac{7y}{6} = 3700$$

$$20x + 7y = 22200 \text{ (shown)}$$

[1]

Plane B travels at an average speed of x km/h for 2 hours 30 minutes and then at an average speed of y km/h for 1 hour 50 minutes.
It travels 350 km lesser than Plane A.

(b) Write down an equation in x and y to represent this information.

$$x \times 2\frac{1}{2} + y \times 1\frac{5}{6} = 3700 - 350$$

$$\frac{5x}{2} + \frac{11y}{6} = 3350$$

$$15x + 11y = 20100$$

Answer .... [1]

(c) Solve these two equations to find the value of x and the value of y.

Answer

$$20x + 7y = 22200$$

$$x = \frac{22200 - 7y}{20} \qquad -(1)$$

$$15x + 11y = 20100 \qquad -(2)$$

sub (1) into (2):

$$15\left(\frac{22200 - 7y}{20}\right) + 11y = 20100$$

$$333000 - 105y + 220y = 402000$$

$$115y = 69000$$

$$y = 600$$

sub $y = 600$ into (1)

$$x = \frac{22200 - 7600}{20}$$

$$= 900$$

Answer $x =$ ....and $y =$ .... [3]

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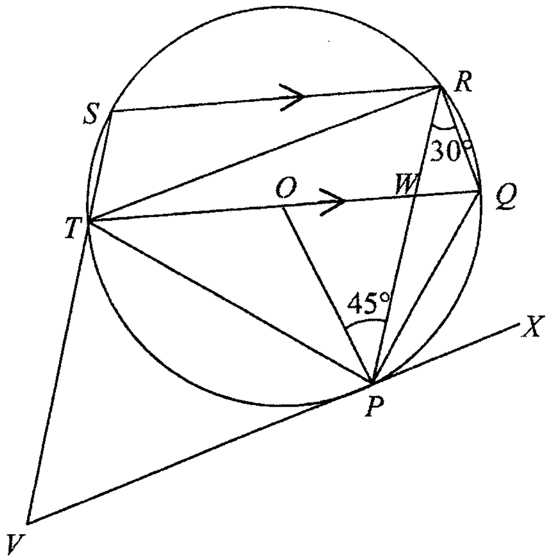
7 In the diagram, P, Q, R, S and T are points on the circle with centre O.
VX is a tangent to the circle at P.
Line SR is parallel to the diameter of the circle TQ.
W lies on TQ and PR.



It is given that $\angle OPR = 45°$ and $\angle PRQ = 30°$.

(a) **(i)** Find angle TOP.
Give a reason for each step of your working.

$\angle POQ = 60°$ ($\angle$ at centre $= 2\angle$s at circumference)
$\angle TOP = 180° - 60°$ (adjacent $\angle$s on a straight line)
$= 120°$

Answer Angle TOP = ... [2]

(ii) Find angle TPV.
Give a reason for each step of your working.

$$\angle OPT = \frac{180° - 120°}{2} \text{(base } \angle\text{s of isosceles } \Delta\text{)}$$
$$= 30°$$
$$\angle OPV = 90° \text{(tangent } \perp \text{ radius)}$$
$$\angle TPV = 90° - 30°$$
$$= 60°$$

Answer Angle TPV = ... [2]

(iii) Find angle TSR.
Give a reason for each step of your working.

$$\angle TSR = 180° - 45° - 30° (\angle\text{s in opposite segments})$$
$$= 105°$$

Answer Angle TSR = ... [1]

(b) Show that $SRWT$ is a parallelogram.

Answer

$$\angle TWR = 60° + 45° (\text{exterior } \angle \text{ of } \Delta)$$
$$= 105°$$
$$= \angle TSR$$
$$\angle STW = 180° - 105° (\text{interior } \angle s,\ SR // TW)$$
$$= 75°$$
$$\angle SRW = 360° - 75° - 105° - 105° (\angle \text{ sum of quadrilateral})$$
$$= 75°$$

Since opposite angles of a quadrilateral are equal, $SRWT$ is a parallelogram.

[3]

8 The variables x and y are connected by the equation $y=\frac{1}{3}x\left(11-x^2\right)$.

Some corresponding values of x and y, correct to 1 decimal places, are given in the following table.

x	–4	–3	–2	–1	0	1	2	3
y	6.7	–2	–4.7	–3.3	0	3.3	p	2

(a) Calculate the value of p.

$p=4.7$

Answer $p=$ .... [1]

(b) On the grid opposite, draw the graph of $y=\frac{1}{3}x\left(11-x^2\right)$ for $-4\le x\le 3$.

[3]

(c) Use your graph to find the solution of $\frac{1}{3}x\left(11-x^2\right)=3$ in the range $-4\le x\le 3$.

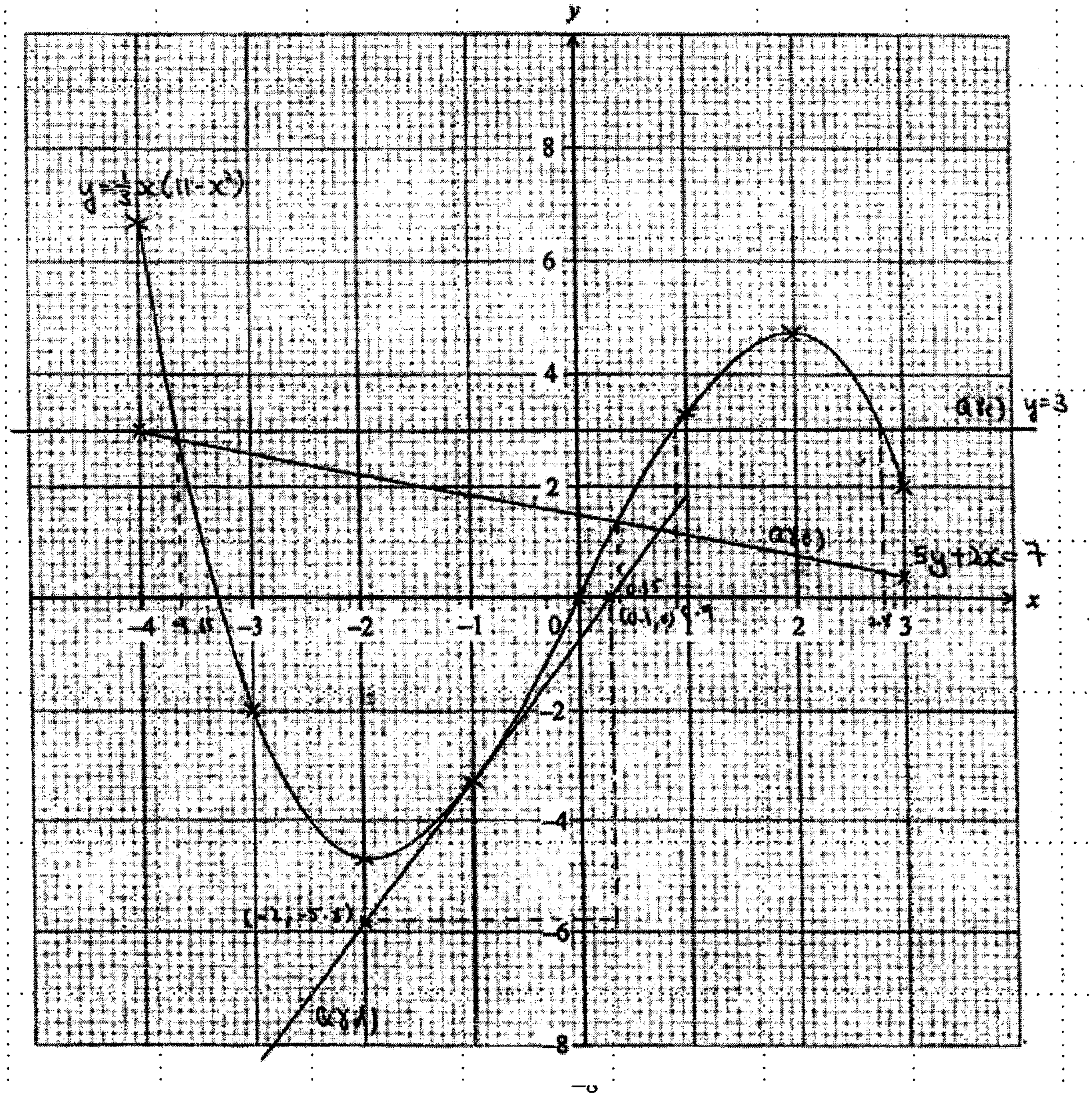
$x=-3.67, 0.88, 2.79$

Answer $x=$ ....or............or........... [2]

(d) By drawing a tangent, find the gradient of the curve at the point $(-1,\ -3.3)$.

Gradient $= 2.33$

Answer .... [2]


y
x
$y=\frac{1}{3}x(11-x^2)$
y=3
5y+2x=7
(-2, -5.8)
(0.3, 0)
-4
-3
-2
-1
0
2
3
8
6
4
2
-2
-4
-6
-8

(e) On the same axes, draw the graph of $5y+2x=7$ for $-4 \le x \le 3$.

[1]

(f) **(i)** Show that the points of intersection of the line and the curve give the solutions of the equation $5x^3-61x+21=0$.

Answer

$5y+2x=7$

$y=\dfrac{-2x+7}{5} \qquad -(1)$

$y=\dfrac{1}{3}x\left(11-x^2\right) \qquad -(2)$

sub (1) into (2):

$\dfrac{-2x+7}{5}=\dfrac{1}{3}x\left(11-x^2\right)$

$-6x+21=55x-5x^3$

$5x^3-61x+21=0$ (shown)

[2]

(ii) Use your graphs to solve the equation $5x^3-61x+21=0$.

$x=-3.6537$ or 0.34771

Answer $x=$ ………….or ………… [1]

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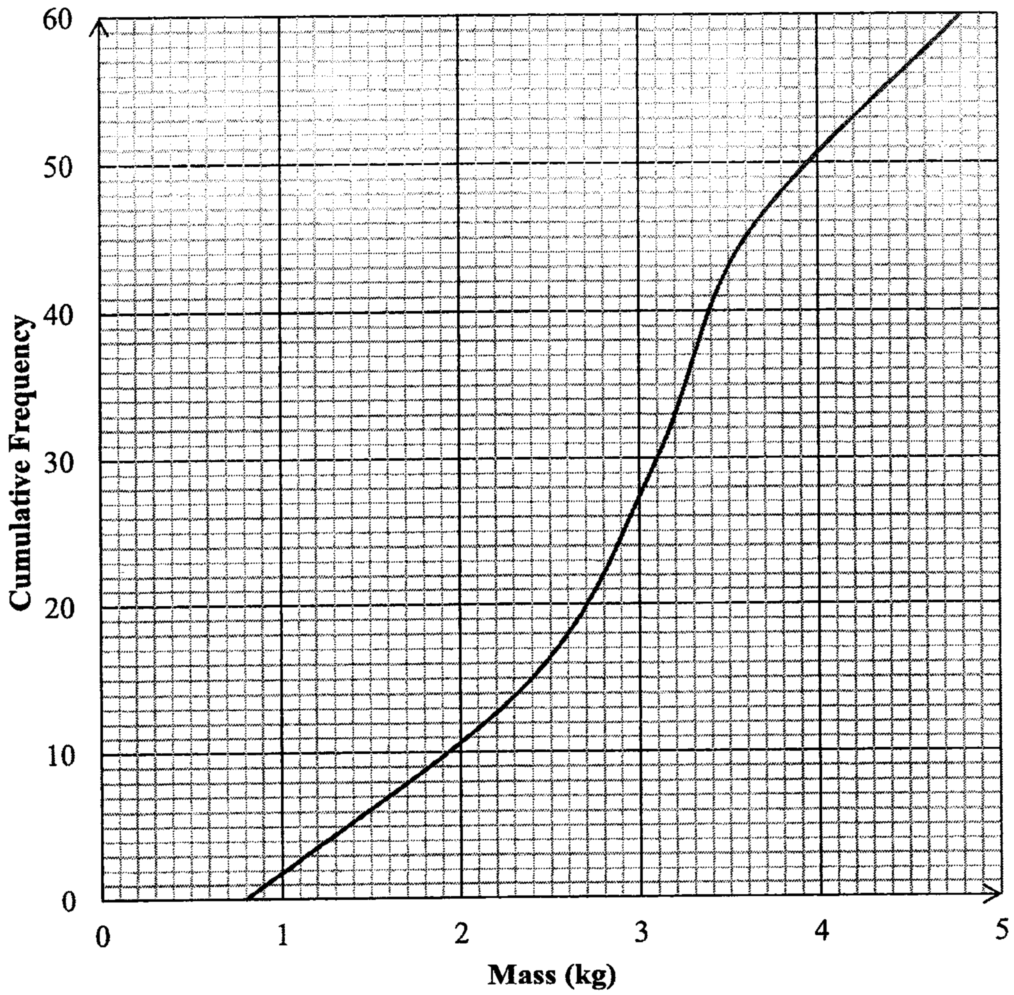
9 60 potato plants produce 5 to 10 potatoes each.
The mass of potatoes produced by each plant were measured.
The cumulative frequency curve below shows the distribution of the masses of the potatoes produced by each plant.



(i) Use the curve to estimate

(a) the median mass,

median mass = 3.1 kg

Answer .... kg [1]

(b) the interquartile range.

interquartile range = 3.6 − 2.4 = 1.2 kg

Answer .... kg [1]

(ii) It was stated that 20% of the potato plants were considered premium plants as they produced greater mass of potatoes.

Find the least mass of potatoes produced for the plant to be 'premium'.

$$\frac{100-20}{100}\times 60=48$$

least mass $=3.8$ kg

Answer .... kg [2]

(iii) The potatoes produced by another group of 60 plants have the same median but smaller interquartile range.

Describe how the cumulative frequency curve will differ from the given curve.

Answer .... The curve will be steeper. ....

....

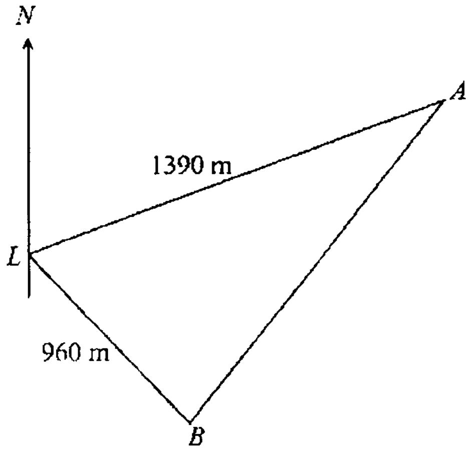
....

....

....

.... [1]

10



ABL is a park on horizontal ground.
A is 1390 m from L on a bearing of $076°$.
B is 960 m from L on a bearing of $138°$.

(a) **(i)** Find AB.

$\angle ALB = 138° - 76°$

$= 62°$

$AB = \sqrt{960^2 + 1390^2 - 2(960)(1390)\cos 62°}$

$= 1265.2$ m (5sf)

$= 1270$ m (3sf)

Answer .... m [2]

(ii) Find angle LAB.

$\sin \angle LAB = \dfrac{\sin 62°}{1265.2} \times 960$

$\angle LAB = 42.064°$ (3dp)

$= 42.1°$ (1dp)

Alternatively,

$\angle LAB = \cos^{-1}\left[\dfrac{1390^2 + 1265.2^2 - 960^2}{2(1390)(1265.2)}\right]$

Answer .... [2]

(iii) Find the area of triangle LAB.

$$\text{area} = \frac{1}{2}(1390)(960)\sin 62^\circ$$
$$= 589100 \text{ m}^2 \text{ (5sf)}$$
$$= 589000 \text{ m}^2 \text{ (3sf)}$$

Alternatively,
Area of $\Delta LAB = \frac{1}{2}(1265.2)(1360)\sin 42.064^\circ$

Answer .... m^2 [2]

(iv) Find the bearing of B from A.

$$\angle LAN_1 = 180^\circ - 76^\circ \text{ (interior } \angle\text{s, } NL \,//\, N_2A)$$
$$= 104^\circ$$
$$\text{bearing of } B \text{ from } A = 360^\circ - 104^\circ - 42.064^\circ \text{ (}\angle\text{s at a point)}$$
$$= 213.936^\circ \text{ (3dp)}$$
$$= 213.9^\circ \text{ (1dp)}$$

Alternatively,
$\angle LAC = 76^\circ$ (alternate $\angle$s, $NL \,//\, N_2A$)
Bearing of B from $A = 180^\circ + (76^\circ - 42.064^\circ)$
$= 213.939^\circ$ (3 dp)
$= 213.9$ (1 dp)

Answer .... ° [2]

(b) T is the top of a tower at L.
The greatest angle of depression from T to the path AB is 5.06°.

Calculate the height of the tower.

$$\text{shortest distance from } L \text{ to } AB = 589100 \div \frac{1}{2} \div 1265.2$$
$$= 931.24 \text{ m (5sf)}$$
$$\tan 5.06^\circ = \frac{LT}{931.24}$$
$$LT = 931.24 \times \tan 5.06^\circ$$
$$LT = 82.5 \text{ m (3sf)}$$

Alternatively,
Shortest distance from L to $AB = 1390 \times \sin 42.063^\circ$

Shortest distance from L to $AB = 960 \times \sin 75.936^\circ$

Answer .... m [3]

(c) Jonah goes on a jog along the edge of the park at a speed of 8.5 km/h.
He starts from L towards A then to B before going back to L.

Calculate the time he takes to jog.
Give your answer in minutes and seconds, corrected to the nearest 10 seconds.

$$\begin{aligned}\text{total distance} &= 960 + 1390 + 1265.2 \\ &= 361.2 \text{ m} \\ \text{time taken} &= 3.615 \div 8.5 \\ &= 0.42532 \text{ h } (5\text{sf}) \\ &= 25.519 \text{ min} \\ &= 25 \text{ min } 30 \text{ s (nearest 10 seconds)}\end{aligned}$$

Answer .... seconds [3]

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11 Mr Ho would like to replace all 60 units of air conditioners in his office building.
He is considering between two brands of air conditioners.

Information about the two brands of air conditioners is shown in the table below.

Brand	Samsing	Potasonic
Price of each air conditioner unit (after GST)	S$1388	S$740
Power consumption per air conditioner unit	3 kW	3.5 kW
Servicing Frequency	Twice a year	Four times a year
Total cost for servicing 60 units	$2100 per servicing	
Warranty*	Two years	Three years
*Warranties cover servicing and maintenance of air conditioner units with free replacement of parts for the stated duration.		

Electricity Tariff from 1 July 2022 is shown in the table below.

SPgroup Empowering the Future of Energy	**Tariff (without GST)**	**Tariff with GST**
kWh* charge (¢ per kWh)	30.17	32.28
* kWh (kilowatt-hour) is a unit of energy equal to one kilowatt (kW) of power sustained for one hour		

The usage of air conditioner units in Mr Ho's office building is shown in the table below.

Days	Usage Time/unit	Number of units used
Monday	8 hours	50
Tuesday to Friday	6 hours each day	60
Saturday and Sunday	No usage	

(a) In view of public holidays, he estimates that the company operates for 51 weeks per year.

Find the usage of all air conditioner units, in hours, for a year.

$51\times(50\times8+60\times6\times4)=93840$ h

Answer .... h [2]

(b) Based on the usage of his office building, which air conditioner model will have a lower cost after 4 years of use?
Justify your decision with calculations.

Answer

	Samsing	Potasonic
Total cost of air conditioners	1388×60 $=\$83280$	740×60 $=\$44400$
Total cost of electricity over 4 years	$3000\times93840\div1000$ $\times4\times0.3228$ $=\$363498.624$	$3500\times93840\div1000$ $\times4\times0.3228$ $=\$424081.728$
Cost of servicing over 4 years	$2\times2\times2100$ $=\$8400$	4×2100 $=\$8400$
Total cost over 4 years	\$455178.624	\$476881.728

Samsing air conditioner model will have a lower total cost after 4 years.

[7]

(c) State one assumption you have made in the above calculations.

Answer ....

1) Price of electricity does not change over 4 years
2) No spare parts needed after warranty ends
3) Aircon usage remains the same for 4 years
4) GST remains the same for 4 years

.... [1]

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