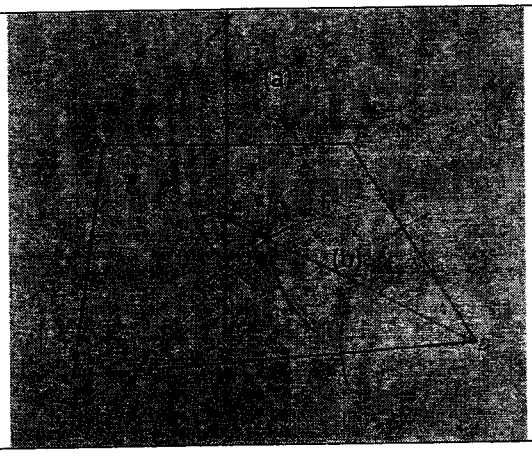
QTSS Prelims 2022 Sec 4E/5N Mathematics Paper 1 Marking Scheme Queens Town

Qn	Solution	Marks allocated	Guidance
1	$10mx-4m^2+6m-15x$ $=2m(5x-2m)-3(5x-2m)$ $=(5x-2m)(2m-3)$	M1 A1	OE
2	Smallest: -3, 35%, 3.142, π, Largest: $\sqrt{36}$	B2	B1 for four correct when one is covered up Accept equivalents.
3	$BC=\sqrt{21.5^2-7.05^2}$ $=20.3$ cm	M1 A1	
4	Interest earned for 5 years $=\$3500\times 2.3\%\times 5$ $=\$402.50$ Total value of her investment $=\$3500+\$402.50$ $=\$3902.50$	M1 A1	
5	$P=\frac{k}{\sqrt[3]{q}}$ $P_{new}=\frac{k}{\sqrt[3]{q_{new}}}$ $P_{new}=\frac{k}{\sqrt[3]{0.125q}}$ $P_{new}=\frac{k}{0.5\sqrt[3]{q}}=2\left(\frac{k}{\sqrt[3]{q}}\right)=2P$ % increase $=\frac{2P-P}{P}\times 100\%$ $=100\%$	M1 A1	
6(a)	11	B1	
6(b)	9	B1	

7	$\sqrt{\frac{2.39\times 8.46}{96.2}}$ $\approx\sqrt{\frac{2\times 8}{100}}$ $=0.4$	 M1 A1	
8	Let the largest base be the face. Height of cuboid $=\frac{1296}{162}$ $=8$ cm Length × breadth = 162 cm^2, where length>8 cm and breadth>8 cm. Hence the dimensions of the cuboid are 18cm by 9 cm by 8cm.	 M1 A1	
9	Time taken for the flight $=\frac{9535.25}{887}$ $=10.75$ hours =10 hours and 45 minutes Time in Istanbul when plane reached Singapore = 01 45 + 10 hours and 45 minutes =12 30 Time difference = 17 30 – 12 30 = 5 hours Singapore is ahead by 5 hours.	 M1 M1 A1	
10a	$\frac{5}{x}+14=7$ $\frac{5}{x}=-7$ $x=-\frac{5}{7}$	 B1	

10a	$\frac{3x-2}{5}-\frac{x+1}{4}$ $=\frac{4(3x-2)}{20}-\frac{5(x+1)}{20}$ $=\frac{12x-8-5x-5}{20}$ $=\frac{7x-13}{20}$	M1 A1	
11	Area of largest square $=(8x)^2$ $=64x^2$ Area of unshaded region $=(5x)^2-(3x)^2$ $=16x^2$ Probability it lies inside the shaded region $=\frac{64x^2-16x^2}{64x^2}$ $=\frac{3}{4}$	 M1 M1 A1	Or $48x^2$[B1] $\frac{48x^2}{64x^2}$[M1]
12(a)	Mean time = 39.8 minutes	B1	Use of calculator
12(bi)	Percentage $=\frac{50}{360}\times 100\%$ $=13.9\%$	 B1	
12(bii)	The **total number of adults who took part in the quiz is unknown** and hence it is not possible to calculate the number of adults who were over 60 years that took part in the quiz.	R1	
13	Exterior angle of polygon $=\frac{2}{9}\times 180°$ $=40°$ Number of sides of polygon $=\frac{360°}{40°}$ $=9$ sides	M1 M1 A1	Equating 9 units to 180° or equivalent

14(a)	$(3x+4k)^2$ $=9x^2+24kx+16k^2$	B2	M1 if they are able to show correct expansion or multiplication frame.
14(b)	$24k=-48$ $k=-2$	B1	
15	$\frac{\text{Area of triangle } CDE}{\text{Area of triangle } BDF}=\left(\frac{1}{2}\right)^2=\frac{1}{4}$ $\frac{\text{Area of triangle } ABF}{\text{Area of triangle } BDF}=\frac{5}{4}$ Area of triangle CED : Area of triangle ABF 1 : 5	M1 M1 A1	
16(a)	$10 \notin P \cap Q$	B1	
16(bi)	$n[(P \cup Q)']=6$	B1	
16(bii)	$n[(P \cup Q) \cap (P \cap Q)']=5$	B1	

17a,b		B1 B1	
17c	3.8cm	B1	Allow e.c.f
18(a)	$\left(81x^{12}\right)^{\frac{3}{4}} = 27x^9$	B2	B1 for $3x^3$ seen or for $27x^k$, $k=0$ or ax^9, $a \neq 0$.
18(b)	$\frac{16^p}{2^q} = 8^r$ $\frac{2^{4p}}{2^q} = 2^{3r}$ $4p - q = 3r$ $q = 4p - 3r$	B2	B1 for 2^{4p} or 2^{3r}
19(a)	$x^2 - 11x + 13 = \left(x - \frac{11}{2}\right)^2 - \frac{69}{4}$	B1	
19(b)	$\left(x - \frac{11}{2}\right)^2 - \frac{69}{4} = 0$ $\left(x - \frac{11}{2}\right)^2 = \frac{69}{4}$ $x - \frac{11}{2} = \pm 4.1533$ $x = 9.65$ or 1.35	M1 A1,A1	Allow M1 ecf

20	$\tan 38° = \frac{44}{BD}$ $BD = \frac{44}{\tan 38} = 56.3174...\text{m}$ $\tan 15° = \frac{AX}{56.3174}$ $AX = 56.3174 \tan 15° = 15.0902...\text{m}$ $AB = 15.0902 + 44$ $= 59.1\text{m}$	M1 M1 M1 A1	 Allow M1 ecf
21(a)	$n = 1$ $m = 1$	B1 B1	
21(b)	$\sqrt{z} = 3^6 \times 5^{12}$ $z = 3^{12} \times 5^{24}$	M1 A1	
22(a)	$\frac{BC}{\sin 120°} = \frac{6.5}{\sin 27°}$ $BC = \frac{6.5 \sin 120°}{\sin 27°}$ $= 12.3993...\text{cm}$ $12.3993^2 = 13.1^2 + 15.2^2 - 2(13.1)(15.2)\cos \angle BDC$ $\angle BDC = \cos^{-1}\left(\frac{12.3993^2 - 13.1^2 - 15.2^2}{-2(13.1)(15.2)}\right)$ $= 51.3°$	M1 M1 M1 A1	 Allow M1 ecf
22(b)	$\angle BAD + \angle BDC = 120 + 51.3 = 171.3° \neq 180°$ Since opposite angles BAD and BDC are not supplementary, they are not angles in opposite segments. Hence it is not possible to draw a circle through the four points.	B1	Allow B1 ecf
23(a)	$S = \begin{pmatrix} 14 & x & 150 \\ 15 & 76 & 143 \end{pmatrix}$	B1	
23(b)	$T = \begin{pmatrix} 14 & x & 150 \\ 15 & 76 & 143 \end{pmatrix} \begin{pmatrix} 3200 \\ 1500 \\ 750 \end{pmatrix}$ $= \begin{pmatrix} 14(3200) + 1500x + 150(750) \\ 15(3200) + 76(1500) + 143(750) \end{pmatrix}$ $= \begin{pmatrix} 157300 + 1500x \\ 269250 \end{pmatrix}$	 B1 B1	 B1 for each element in the matrix

23(c)	$269250 = 157300 + 1500x + 3950$ $x = 72$	B1	
23(d)	The elements in **N** could represent the average of the ticket sales from both the Outward and Return flight.	B1	
24(a)	Correct Shape (U shape) Correct Intercepts ($x-\text{int} = -0.5, 3, y-\text{int} = -3$) Correct minimum point $(1.25, -6.125)$	M1 M1 A1	
24(b)	The graph in (a) will be 7 units lower than the graph $y = 2x^2 - 5x + 4$.	A1	
25(a)	$\text{decceleration} = \frac{9.5}{10}$ $= 0.95\text{m/s}^2$	M1 A1	
25(b)	$\frac{1}{2}(12)(19) = \frac{1}{2}(9 - T + 19)(9.5)$ $228 = 266 - 9.5T$ $T = 4$	M1 M1 A1	
25(c)	Let time be t $\frac{s}{19-t} = 0.95$ $s = 0.95(19-t)$ $\frac{12}{19} = \frac{s}{t}$ $\frac{12}{19} = \frac{0.95(19-t)}{t}$ $12t = 342.95 - 18.05t$ $t = 11.4$ sec	M1 M1 A1	

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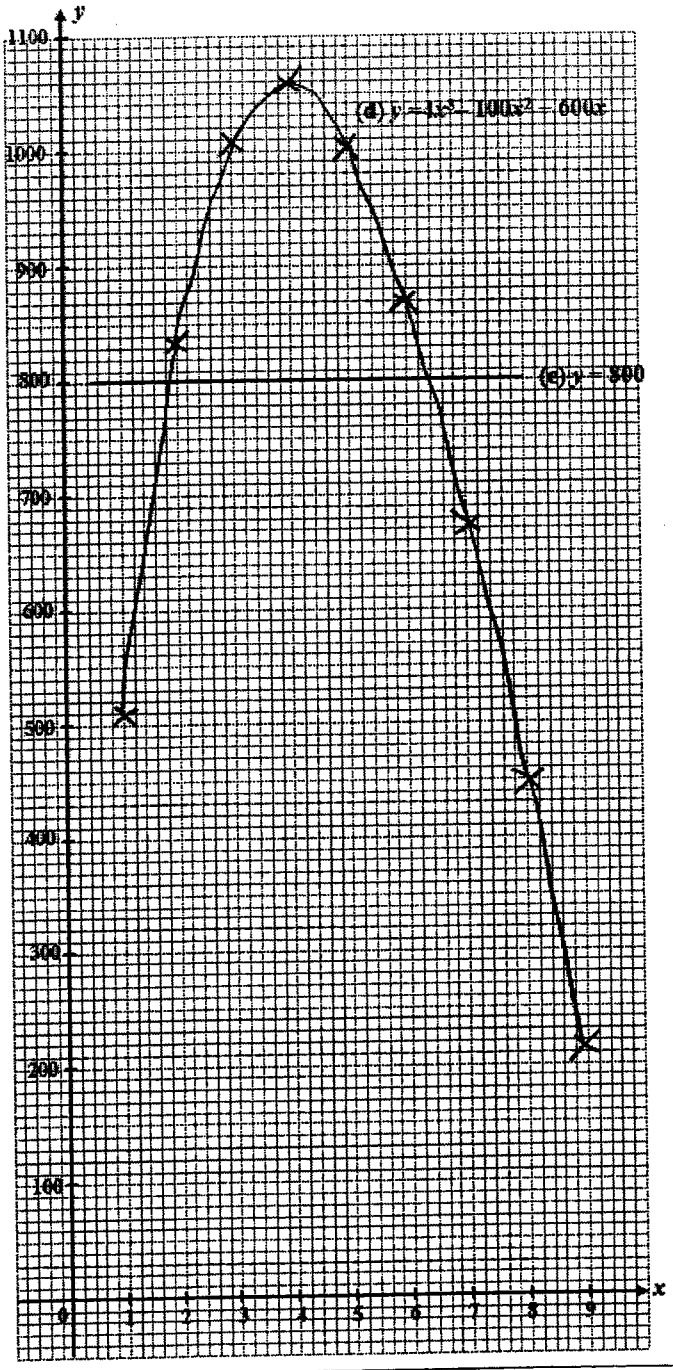
1a	$x < 20$ The largest prime number x is 19.	B1 B1
1b(i)	$q = \frac{5}{6}$ or 0.833	B1
1b(ii)	$p^2 - 4r = \frac{3}{2}q$ $p^2 = \frac{3}{2}q + 4r$ $p = \pm\sqrt{\frac{3}{2}q + 4r}$ or $p = \pm\sqrt{\frac{3q+8r}{2}}$ or $p = \pm\sqrt{\frac{q+\frac{8}{3}r}{\frac{2}{3}}}$	M1 M1 A1
1c	$\frac{x}{(3-2x)^2} + \frac{5(3-2x)}{(3-2x)^2} = 1$ $\frac{15-9x}{(3-2x)^2} = 1$ $4x^2 - 3x - 6 = 0$ $x = 1.656 \approx 1.66$ (2 d.p.) or $x = -0.906 \approx -0.91$ (2 d.p.)	M1 M1 A1 A1
2a	1.91×10^7 (to 3 s.f.)	B1
2b	Percentage increase $= \frac{27.69b - 26.94b}{26.94b} \times 100\%$ $= 2.78\ \%$ (3 s.f.)	M1 A1
2c	Number travelling out in 2019 = (1 + 8.08%) × 9.89 million = 10.689112 million	M1 or OE A1 (exact)
2d	Number travelling out in 2018 $= \frac{8.33 \times 10^5}{1 - 91.9\%}$ $= 1.03 \times 10^7$ (3 s.f.)	M1 or OE A1 (Accept 10.3×10^6)
2e(i)	Deposit $= \frac{20}{100} \times 870 = 174$ Total monthly payments $= 1056 - 174 = 882$ One monthly payment $= \frac{882}{12} = \$73.50$	M1 M1 or their deposit A1
2e(ii)	Price of camera in UK $= \frac{\$510}{0.62} = \$822.58$ Camera in UK is cheaper by = \$870 – \$822.58 = \$47.42	M1 A1

3a(i)	$AB = 30 - 2x$	B1
3a(ii)	$BC = 20 - 2x$	B1
3b	Volume of the box, $y = x(30-2x)(20-2x)$ $y = 4x^3 - 100x^2 + 600x$ (shown)	M1 A1
3c	448	B1
3d		B1 – at least 5 points plotted correctly B1 – all points plotted correctly B1 – smooth curve drawn for $1 \le x \le 9$
3e	6.3 (read accurately based on *their* curve)	B1

3f	The volume of the box y cannot be equal to 1100 cm^3 for all real value of x because the maximum y is below $y = 1100$.	B1
4a	Height $= 3x+5x+2x$ $25 = 10x$ $x = 2.5$ (shown)	M1 M1 A1
4b	Volume up to level M $= \frac{1}{2}\times\frac{4}{3}\pi(5)^3+\pi(5^2)(7)$ $= 261.799 + 549.78$ $= 811.58$ Volume upside down $= \frac{1}{3}\pi(5)^2(7.5)+\pi(5)^2(h)$ $811.58 = 196.35 + 25\pi h$ $615.23 = 25\pi h$ $h = 7.83$ $H = 7.5 + h$ $= 7.5 + 7.83$ $= 15.33 \approx 15.3$ cm (3 s.f.)	M1- correct volume 261.799 or $\frac{250}{3}\pi$ M1- 549.78 or 175π M1- 196.35 or $\frac{125}{2}\pi$ M1 – 615.23 or $\frac{1175}{6}\pi$ M1 – 7.83 A1
5a	Rearrange the equations without solving simultaneous equations Compare gradient of the two lines: $-\frac{20}{a}=4$ $a = -5$	M1 M1 A1
5b	$M(-2\frac{1}{2}, 0)$ Accept $(-\frac{5}{2}, 0)$ or $(-\frac{10}{4}, 0)$ $N(0, 10)$	B1 B1
5c	$MN = \sqrt{10^2+2.5^2} = 10.308$ $\frac{1}{2}\times 10\times 2.5 = \frac{1}{2}\times MN\times d$ $25 = 10.308 \times d$ $d = 2.4253 \approx 2.43$ units (3 s.f.)	M1 M1 A1
5d	NP // MO (NP and MO are both horizontal lines or both gradient = 0) MN // OP (line l is parallel to $4x - y = 0$ or both gradient = 4) $MNPO$ is a parallelogram.	B1 B1

5e	Area of triangle OMN $= \frac{1}{2} \times 10 \times 2.5 = 12.5$ Area of parallelogram $= 2 \times 12.5 = 25$ units^2	M1 or OE A1 (Accept 25.0 if using MN = 10.308 in calculation)
6a	$\angle SRP = 180° - 122° = 58°$ (angles in opposite segments) $\angle RXS = 180° - 28° - 58° = 94°$ (angle sum of triangle)	B1 – reason stated B1 - 94°
6b	$\angle OQP = 58°$ (angles in the same segment) $\angle OPQ = 58°$ (isosceles ΔOQP) $\angle OPV = 90°$ (radius perpendicular to tangent) $\angle QPV = 90° - 58° = 32°$	B1 – reason stated B1 – reason stated B1 – reason stated B1
6c	$\theta = 2 \times 122° = 244°$ Area of major sector $OSRQP = \frac{244°}{360°} \times \pi(5)^2$ or $\frac{1}{2}(5)^2(4.259 \text{ rad})$ $= 53.2$ cm^2 (3 s.f.)	M1 M1 A1
7a	45	B1
7b	T_n for first term $= n^2$ T_n for second term $= 4n$ T_n for the sequence $= n^2 + 4n$	B1 B1
7c	$T_{n+1} - T_n = (n+1)^2 + 4(n+1) - (n^2 + 4n)$ $= n^2 + 2n + 1 + 4n + 4 - n^2 - 4n$ $= 2n + 5$ (shown)	M1 M1 A1
7d	Since n is a positive integer, $2n$ is even number Even number + odd number = odd number.	B1

8a(i)	$\overrightarrow{RS} = \overrightarrow{OS} - \overrightarrow{OR}$ $= \begin{pmatrix} 7 \\ 2 \end{pmatrix} - \begin{pmatrix} -5 \\ -1 \end{pmatrix}$ $= \begin{pmatrix} 12 \\ 3 \end{pmatrix}$ $\lvert RS \rvert = \sqrt{12^2 + 3^2} = 12.4$ units (3 s.f.)	B1 B1
8a(ii)	$RT = xRS$ $\begin{pmatrix} k+5 \\ 6 \end{pmatrix} = x \begin{pmatrix} 12 \\ 3 \end{pmatrix}$ or equating gradient of RT and RS $k + 5 = 2\ (12)$ $k = 19$ $OT = \begin{pmatrix} 19 \\ 5 \end{pmatrix}$	M1 or OE A1
8b(i)	$PQ = \mathbf{q} - \mathbf{p}$ $PR = \frac{1}{5} PQ$ $= \frac{1}{5}\mathbf{q} - \frac{1}{5}\mathbf{p}$	M1 or OE A1
8b(ii)	$OR = OP + PR$ $= \mathbf{p} + \frac{1}{5}\mathbf{q} - \frac{1}{5}\mathbf{p}$ $= \frac{1}{5}\mathbf{q} + \frac{4}{5}\mathbf{p}$ $OM = \frac{1}{2} OR$ $= \frac{1}{2}(\frac{1}{5}\mathbf{q} + \frac{4}{5}\mathbf{p})$ $= \frac{1}{10}(\mathbf{q} + 4\mathbf{p})$	M1 or OE A1
8b(iii)	$RQ = \frac{4}{5} PQ = \frac{4}{5}(\mathbf{q} - \mathbf{p})$ $\overrightarrow{MN} = \overrightarrow{ON} - \overrightarrow{OM} = \frac{1}{2}\mathbf{q} - \frac{1}{10}(\mathbf{q} + 4\mathbf{p}) = \frac{2}{5}(\mathbf{q} - \mathbf{p})$ $\overrightarrow{RQ}$ is scalar multiple of $\overrightarrow{MN}$ so they are parallel and $RQNM$ has one pair of parallel side so $RQNM$ is a **trapezium** (need to be supported by correct reason)	B1 B1 B1

9a(i)(a)	104 g	B1
9a(i)(b)	Q3 – Q1 = 113 – 93 = 20 g	M1 (either 113 or 93) A1
9a(ii)	Percentage delivered = $\frac{172-16}{200}\times 100\%$ = 78%	M1 A1
9a(iii)a	84 g	B1
9a(iii)b	20 g	B1
9b	Number of part-time women workers = $\frac{1}{5}\times 15 = 3$ Total number of male workers = 15 – 7 = 8 Let the number of part-time male workers be x $\frac{8-x}{15}\times\frac{8-x-1}{14}=\frac{1}{5}$ $\frac{(8-x)(7-x)}{210}=\frac{1\times 42}{5\times 42}$ $(8-x)(7-x)=42$ $x^2-15x+14=0$ $x=14$ (rejected) or $x=1$	B3

	Part-time workers	Full-time workers
Men	1	7
Women	3	4

10a(i)	Distance = Speed × Time = 12 km/h × $\frac{150}{60}$ h = 30 km Distance of <u>one</u> cycling session = 10 km	M1 A1
10a(ii)	For 30 mins of leisure cycling = 220 calories For 150 mins, calories used = $\frac{150}{30}\times 220 = 1100$ calories	B1 B1
10b	<u>For one week</u> Duration of moderate-intensity aerobic activity (leisure cycling)	B1 for multiply

	$= 2 \times \frac{12\text{ km}}{12\text{ km/h}}$ $= 2\text{ h}$ $= 120\text{ mins}$ Duration of vigorous-intensity aerobic activity (running) $= 4 \times \frac{5\text{ km}}{11\text{ km/h}}$ $= 1.8181\text{ h}$ $= 109.086\text{ mins}$ Total equivalent combination of moderate- and vigorous- intensity aerobic activity = 120 mins + 2×109.086 mins = 338.172 mins > 300 mins Conclusion: Fomi is correct as he meets the target.	quantity 2 and 4 correctly. B1 $\frac{12\text{ km}}{12\text{ km/h}}$ B1 $\frac{5\text{ km}}{11\text{ km/h}}$ B1 convert to minutes B1 338.172 B1 correct conclusion