Preliminary Examination 2
Secondary Four

ADDITIONAL MATHEMATICS

4047/01

Paper 1

17 August 2016

2 hours

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}ab\sin C$$

Answer all the questions.

1 The equation of the curve is $y = px^q - 8$, where p and q are constants.
Given that the curve passes through the points $(2, -4)$ and $(5, 17)$, find the value of p and of q. [4]

2 The second derivative of y is given by $\frac{d^2y}{dx^2} = 2x + 4$.
Given that $y = 12$ when $x = 3$, and $y = -\frac{1}{3}$ when $x = 2$, find y in terms of x. [4]

3 The equation of a curve is $y = ax^2 - 4x + 2a - 3$, where a is a constant.
Find the range of values of a for which the curve lies completely above the line $y = -1$. [5]

4 The equation of a curve is $y = \frac{3\cos x}{\sin x}$, where $0 < x < \pi$.

(i) Show that the gradient function can be expressed in the form $\frac{k}{\sin^2 x}$, where k is a constant. [2]

(ii) Find the x-coordinates of the points at which the tangents to the curve are perpendicular to the line $2x - 8y = -1$, leaving your answers in exact form. [3]

5 The number of people, N, in a housing estate who contracted influenza during a flu epidemic after t days is modelled by the equation $N = \frac{1000}{1 + 199e^{-0.8t}}$.

(i) Find the initial number of people who contracted influenza during the flu epidemic. [1]

(ii) Given that there are 937 people who contracted influenza after x days, find x correct to the nearest whole number. [3]

(iii) Find the number of people who eventually contracted influenza after a long time. [1]

6 (i) Sketch the curve $y = |4x - x^2|$, indicating the coordinates of the maximum point and of the points where the curve meets the x-axis. [3]

(ii) State the value or range of values of m if the equation $|4x - x^2| = m$ has

(a) 2 solutions, [1]

(b) 3 solutions, [1]

(c) 4 solutions. [1]

7 The function P is defined by $P(x) = 2x^3 + (4 - 2a)x^2 - ax + 6a$, where a is a constant.

(i) Show that $x + 2$ is a factor of $P(x)$. [2]

(ii) Find the other quadratic factor of $P(x)$ in terms of a. [2]

(iii) Find the range of values of a for which the equation $P(x) = 0$ has only 1 real root. [3]

8 The table below shows the experimental values of two variables x and y.
An error was made in recording one of the values of y.

x	2	3	4	5	6
y	5.8	15	30	43.5	74

It is known that x and y are related by an equation $y = ax(x + b) + 2$, where a and b are unknown constants.

(i) Express $y = ax(x + b) + 2$ in a form suitable for drawing a straight line graph. [1]

(ii) Draw a straight line graph for the given data. [3]

(iii) Use your graph to estimate

(a) the value of a and of b, [2]

(b) a value of y to replace the incorrect value. [2]

9 The roots of the quadratic equation $2x^2-4x-1=0$ are α and β.

(i) Find the value of $\alpha^2+\beta^2$. [2]

(ii) Show that the value of $\alpha^3+\beta^3$ is 11. [2]

(iii) Find a quadratic equation whose roots are $\left(\alpha^3+\dfrac{1}{\beta^3}\right)$ and $\left(\beta^3+\dfrac{1}{\alpha^3}\right)$. [4]

10 (i) Express $\dfrac{14x^2-15x+2}{x(2x-1)^2}$ in partial fractions. [5]

(ii) Hence find $\displaystyle\int\frac{14x^2-15x+2}{x(2x-1)^2}\,dx$. [4]

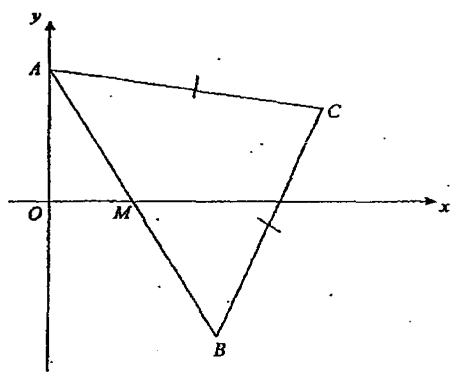
11 A particle P travels in a straight line from a fixed point O with acceleration a m/s^2 given by $a=8t-k$, where t is the time in seconds after passing O, and k is a constant. When P passes O, its velocity is 5 m/s. At $t=2$, its velocity is -21 m/s.

(i) Show that the value of k is 21. [2]

(ii) Find the range of values of t during which P is travelling towards O. [3]

(iii) Given that P comes to instantaneous rest at points A and B, find the distance AB. [4]

12 The diagram, not drawn to scale, shows a triangle ABC, where $AC = BC$ and A lies on the y-axis. M is the mid-point of AB, $OM = 2$ units and $\tan\angle OMC = -\frac{2}{3}$.



(i) Show that the equation of CM is $3y - 2x + 4 = 0$. [2]

(ii) Find the coordinates of B. [4]

(iii) Given that the area of triangle ABC is $\frac{52}{3}$ square units, find the coordinates of C. [4]

End of Paper

2016 Preliminary Examination 2
Additional Mathematics 4047 Paper 1
Mark Scheme

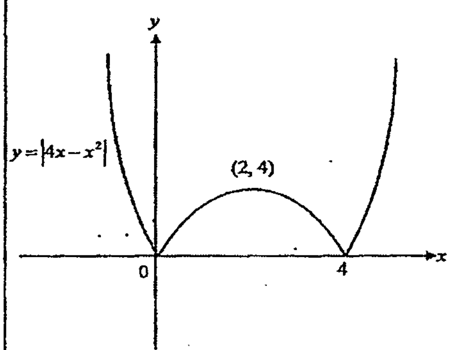
Qn	Working
1	$y = px^q - 8$ $-4 = p(2^q) - 8$ $p(2^q) = 4$ ——(1) $17 = p(5^q) - 8$ $p(5^q) = 25$ ——(2) $(1) \div (2): \dfrac{p(2^q)}{p(5^q)} = \dfrac{4}{25}$ $\dfrac{2^q}{5^q} = \dfrac{4}{25}$ $\left(\dfrac{2}{5}\right)^q = \left(\dfrac{2}{5}\right)^2$ $q = 2$ $p = 1$

Qn	Working
2	$\dfrac{d^2y}{dx^2} = 2x + 4$ $\dfrac{dy}{dx} = x^2 + 4x + c$, where c is a constant. $y = \dfrac{x^3}{3} + 2x^2 + cx + d$, where c and d are constants. When $x = 3, y = 12$ $12 = \dfrac{3^3}{3} + 2(3)^2 + 3c + d$ $3c + d = -15$ ——(1) When $x = 2, y = -\dfrac{1}{3}$ $-\dfrac{1}{3} = \dfrac{2^3}{3} + 2(2)^2 + 2c + d$ $2c + d = -11$ ——(2) Solving, $c = -4$, $d = -3$ Equation of curve: $y = \dfrac{x^3}{3} + 2x^2 - 4x - 3$

Qn	Working
3	$y=ax^2-4x+2a-3$ $ax^2-4x+2a-3>-1$ $ax^2-4x+2a-2>0$ $b^2-4ac<0$ $16-4(a)(2a-2)<0$ $16-8a^2+8a<0$ $a^2-a-2>0$ $(a-2)(a+1)>0$ $a<-1$ or $a>2$ Since curve lies completely above x-axis, $a>0$ $\Longrightarrow a>2$

Qn	Working
4(i)	$y=\frac{3\cos x}{\sin x}$ $\frac{dy}{dx}=\frac{-3\sin^2 x-3\cos^2 x}{\sin^2 x}$ $=\frac{-3(\sin^2 x+\cos^2 x)}{\sin^2 x}$ $=\frac{-3}{\sin^2 x}$ (shown)
(ii)	$2x-8y=-1$ $y=\frac{1}{4}x+\frac{1}{8}$ $\frac{-3}{\sin^2 x}=-4$ $\sin^2 x=\frac{3}{4}$ $\sin x=\pm\frac{\sqrt{3}}{2}$ $\alpha=\frac{\pi}{3}$ $x=\frac{\pi}{3}$ or $\frac{2\pi}{3}$

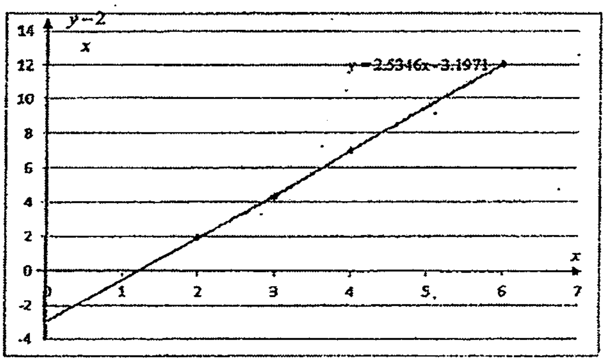
Qn	Working
5(i)	$N=\dfrac{1000}{1+199e^{-0.8t}}$. When $t=0$, $N=\dfrac{1000}{1+199}$. $=5$ The initial number of people who first contacted influenza is 5.
(ii)	$937=\dfrac{1000}{1+199e^{-0.8t}}$. $1+199e^{-0.8t}=\dfrac{1000}{937}$ $e^{-0.8t}=\dfrac{\frac{1000}{937}-1}{199}$ $-0.8t=\ln\left(\dfrac{\frac{1000}{937}-1}{199}\right)$ $t=10$ (to nearest whole number)
(iii)	As $t \Rightarrow \alpha$, $N \Rightarrow \dfrac{1000}{1}=1000$

Qn	Working
6(i)	
(ii)(a)	$m=0$ or $m>4$
(b)	$m=4$
(c)	$0<m<4$

Qn	Working
7(i)	$P(x)=2x^3+(4-2a)x^2-ax+6a$ $P(-2)=2(-2)^3+(4-2a)(-2)^2-a(-2)+6a$ $=-16+16-8a+8a=0$ $(x+2)$ is a factor.
(ii)	Comparing coefficients of x^3 and constant term, $2x^3+(4-2a)x^2-ax+6a=(x+2)(2x^2+px+3a)$ Comparing coefficients of x, $-a=3a+2p$ $p=-2a$ The other quadratic factor of $P(x)$ is $2x^2-2ax+3a$
(iii)	If $P(x)=0$ has only 1 real root, the only real root is -2, hence there are no real roots in $2x^2-2ax+3a=0$. $b^2-4ac<0$ $4a^2-4(2)(3a)<0$ $a^2-6a<0$ $a(a-6)<0$ $0<a<6$

Qn	Working
8i	$\dfrac{y-2}{x} = ax + ab$ where $Y = \dfrac{y-2}{x}$, $X = x$, $m = a$ and Y-intercept $= ab$
ii	(see table and graph below)

$X = x$	2	3	4	5	6
$Y = \dfrac{y-2}{x}$	1.9	4.33	7	8.3	12



iiia

$a = \dfrac{12-1.9}{6-2} = 2.53$

$ab = -3.2 \Rightarrow b = \dfrac{-3.2}{2.53} = -1.26$

iiib

Correct value of $\dfrac{y-2}{x} = 9.5$

Correct value of $y = 9.5 \times 5 + 2 = 49.5$

Qn	Working
9(i)	$2x^2-4x-1=0$ $\alpha+\beta=2$ $\alpha\beta=-\frac{1}{2}$ $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=4-2\left(-\frac{1}{2}\right)$ $=5$
(ii)	$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=2\left(5+\frac{1}{2}\right)=11$ (shown)
(iii)	Sum of roots $=\alpha^3+\beta^3+\frac{\alpha^3+\beta^3}{(\alpha\beta)^3}$ $=11+\frac{11}{\left(-\frac{1}{2}\right)^3}$ $=-77$ Product of roots $=\alpha^3\beta^3+1+1+\frac{1}{(\alpha\beta)^3}$ $=\left(-\frac{1}{2}\right)^3+2+\frac{1}{\left(-\frac{1}{2}\right)^3}$ $=-6\frac{1}{8}$ Equation: $x^2+77x-6\frac{1}{8}=0$ $8x^2+616x-49=0$

Qn	Working
10(i)	$\frac{14x^2-15x+2}{x(2x-1)^2}=\frac{A}{x}+\frac{B}{2x-1}+\frac{C}{(2x-1)^2}$ $14x^2-15x+2=A(2x-1)^2+Bx(2x-1)+Cx$ Sub $x=\frac{1}{2}$ $14\left(\frac{1}{2}\right)^2-15\left(\frac{1}{2}\right)+2=\frac{1}{2}C$ $C=-4$ Sub $x=0$ $A=2$ Sub $x=1$ $14-15+2=A+B+C$ $B=3$ $\frac{14x^2-15x+2}{x(2x-1)^2}=\frac{2}{x}+\frac{3}{2x-1}-\frac{4}{(2x-1)^2}$
(ii)	$\int\frac{14x^2-15x+2}{x(2x-1)^2}\,dx=\int\frac{2}{x}+\frac{3}{2x-1}-\frac{4}{(2x-1)^2}\,dx$ $=2\ln x+\frac{3}{2}\ln(2x-1)+\frac{2}{2x-1}+C$

Qn	Working
11(i)	$a=8t-k$ $v=\int a\ dt$ $=4t^2-kt+C_1$, where C_1 is a constant. When $t=0,\ v=5 \rightarrow C=5$ $v=4t^2-kt+5$ When $t=2,\ v=-21$ $-21=16-2k+5$ $k=21$ (shown)
(ii)	Travelling towards $O \Rightarrow v<0$ $4t^2-21t+5<0$ $(4t-1)(t-5)<0$ $\frac{1}{4}<t<5$
(iii)	$v=4t^2-21t+5$ $S=\int 4t^2-21t+5\,dt$ $=\frac{4t^3}{3}-\frac{21t^2}{2}+5t+C_2$, where C_2 is a constant. When $t=0,\ s=0 \rightarrow C_2=0$ $S=\frac{4t^3}{3}-\frac{21t^2}{2}+5t$ At instantaneous rest, $v=0$ $t=\frac{1}{4}$ or 5 $S\left(\frac{1}{4}\right)=\frac{4\left(\frac{1}{4}\right)^3}{3}-\frac{21\left(\frac{1}{4}\right)^2}{2}+5\left(\frac{1}{4}\right)$ $=\frac{59}{96}$ $S(5)=\frac{4(5)^3}{3}-\frac{21(5)^2}{2}+25$

	$= -\frac{425}{6}$ Distance $AB = \frac{425}{6} + \frac{59}{96}$ $= \frac{6859}{96}$ / $71\frac{43}{96}$ / 71.4 m

Qn	Working
12(i)	Gradient of CM is $\frac{2}{3}$. $M(2, 0)$ Equation of CM: $y = \frac{2}{3}x + c$ $0 = \frac{2}{3}(2) + c$ $c = -\frac{4}{3}$ $y = \frac{2}{3}x - \frac{4}{3}$ $3y - 2x + 4 = 0$
(ii)	Since $AC = BC$ and M is the mid-point of AB, AM is perpendicular to MC. Gradient of AM is $-\frac{3}{2}$ $y - 0 = -\frac{3}{2}(x - 2)$ Equation of AB: $y = -\frac{3}{2}x + 3$ Coordinates of A: $(0, 3)$ Let the coordinates of $B = (a, b)$ $\left(\frac{a+0}{2}, \frac{b+3}{2}\right) = (2, 0)$ $a = 4, b = -3$ $B(4, -3)$
(iii)	Let coordinates of C be (p, q)

Area of $ABC = \frac{1}{2}\begin{vmatrix} 0 & 4 & p & 0 \\ 3 & -3 & q & 3 \end{vmatrix}$

$= \frac{1}{2}|(0+4q+3p)-(12-3p+0)|$

$= \frac{1}{2}|4q+6p-12|$

$\frac{52}{3} = 2q+3p-6$

$2q+3p = \frac{70}{3}$ ———(1)

Since C lies on CM, $q = \frac{2}{3}p - \frac{4}{3}$ ———(2)

Solving, $2\left(\frac{2}{3}p - \frac{4}{3}\right) + 3p = \frac{70}{3}$

$4\frac{1}{3}p = 26$

$p = 6$

$q = \frac{2}{3}(6) - \frac{4}{3} = \frac{8}{3}$

$C\left(6, \frac{8}{3}\right)$

Preliminary Examination 2
Secondary Four

ADDITIONAL MATHEMATICS

Paper 2

4047/02
18 August 2016

2 hours 30 minutes

Additional Materials: Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

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You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

HAND IN QUESTIONS 1 TO 3 SEPARATELY FROM QUESTIONS 4 TO 11.

This document consists of 8 printed pages and 2 cover pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Answer all the questions.

1 It is given that $f(x) = x^3 - 3x^2 + 4x$.

(i) Show that $f(x)$ is an increasing function for all values of x. [3]

(ii) Hence, show that $f(x)$ is positive for all positive values of x. [2]

2 A rectangle has a fixed perimeter of 40 cm. The length of one side, x cm, increases at a constant rate of 0.5 cm/s. Find the rate at which the area is increasing at the instant when $x = 3$. [5]

3 (a) Find the term independent of x in the binomial expansion of $\left(x^2 - \frac{1}{2x^3}\right)^{10}$. [3]

(b) Given that the first 4 terms in the binomial expansion of $\left(2x + \frac{1}{4}\right)^9$, in descending powers of x, are $512x^9 + 576x^8 + ax^7 + bx^6 + \ldots$, where a and b are constants, find

(i) the value of a and of b, [3]

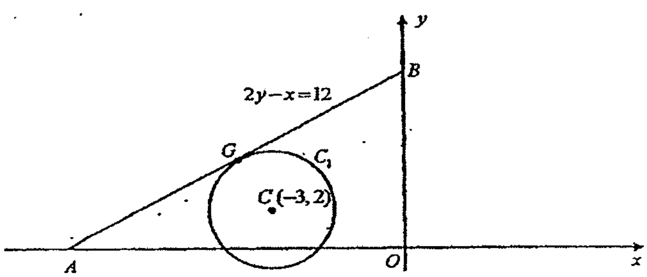
(ii) the coefficient of x^6 in $\left(2x + \frac{1}{4}\right)^9\left(\frac{4}{x} - 1\right)\left(\frac{4}{x} + 1\right)$. [2]

Begin Question 4 on a fresh piece of paper.

4 (a) Given that $\log_3 a = r$, $\log_{27} b = s$ and $\frac{a}{b} = 3^t$, express t in terms of r and s. [3]

(b) Solve $\log_3 x + 3 = 10\log_x 3$. [5]

5 In the diagram below, a circle C_1, with centre at $C(-3, 2)$, touches the line $2y - x = 12$ at the point G.
The line $2y - x = 12$ intersects the x-axis at A and the y-axis at B.



Find

(i) the coordinates of A and of B, [2]

(ii) the equation of the line CG, [2]

(iii) the equation of the circle C_1, [3]

(iv) the equation of the circle C_2 which is a reflection of the circle C_1 in the line AB. [2]

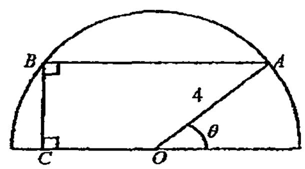
The acute angle between AG and AC is $\theta°$.

(v) Show that $\theta = \tan^{-1}\frac{1}{4}$. [2]

6 (i) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left[e^{2x}(2-3x)\right]$. [3]

(ii) Hence, find $\int_0^{\ln 2} 5xe^{2x}\,\mathrm{d}x$. [5]

7 The diagram below shows a trapezium $ABCO$ inscribed in a semi-circle with centre O and radius 4 units. OA makes an angle of θ radians with the diameter. AB is parallel to the diameter and BC is perpendicular to both lines AB and OC.



(i) Show that the perimeter, y, of trapezium $ABCO$ is given by

$$y = 4(1+\sin\theta+3\cos\theta).$$ [3]

(ii) Find the value of θ for which y has a stationary value and determine whether this value of y is a maximum or a minimum. [4]

(iii) Express the perimeter of the trapezium in the form $y = 4 + R\cos(\theta-\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [2]

(iv) Hence solve the equation $4(1+\sin\theta+3\cos\theta) = 12$, for $0 < \theta < \frac{\pi}{2}$. [2]

8 (i) Prove that $\frac{1-\tan^2\theta}{1+\tan^2\theta}=\cos 2\theta$. [3]

(ii) Use the result in (i) to show that

$$1+x^2=\sqrt{2}x^2-\sqrt{2} \text{ where } x=\tan 67.5°.$$ [2]

(iii) Hence find the values of the constants c and d such that

$$\tan 67.5° = c+d\sqrt{2}.$$ [3]

(iv) Hence show that $\tan 7.5° = \frac{1+\sqrt{2}-\sqrt{3}}{1+\sqrt{3}+\sqrt{6}}$. [3]

9 The temperature, x°C, inside a house t hours after 4 am is given by $x=21-3\cos\left(\frac{\pi t}{12}\right)$ for $0 \le t \le 24$, and the temperature, y°C, outside the house at the same time is given by $y=22-5\cos\left(\frac{\pi t}{12}\right)$ for $0 \le t \le 24$.

(i) Find the temperature inside the house at 8 am. [2]

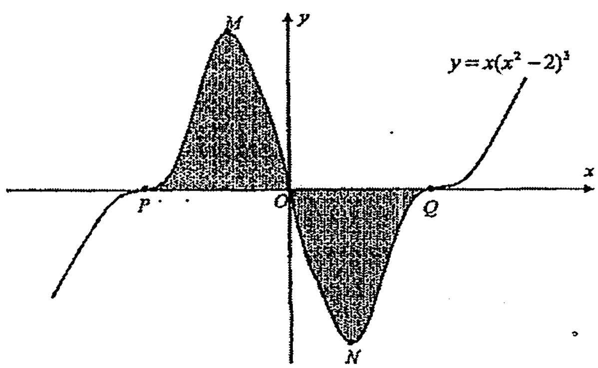
The difference between the temperatures inside and outside of the house is given by $D=x-y$.

(ii) Write down and simplify an expression for D in terms of t for $0 \le t \le 24$. [1]

(iii) Sketch the graph of D against t for $0 \le t \le 24$. [3]

(iv) Determine the time(s) of the day when the temperature inside of the house is equal to the temperature outside the house. Hence find the range of values of t when the temperature inside of the house is less than the temperature outside of the house. [4]

10 The diagram shows the curve $y = x(x^2 - 2)^3$. P and Q are the points of intersection of the curve with the x-axis. M and N are the maximum and minimum points of the curve respectively.



(i) Find the coordinates of P and of Q. [2]

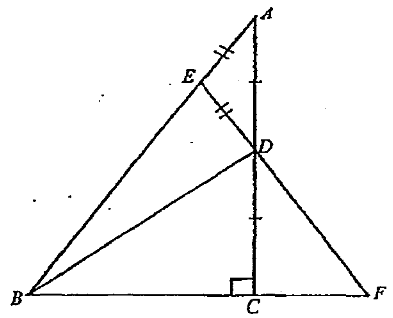
(ii) Find the x-coordinates of M and of N. [4]

(iii) Show that P and Q are stationary points of inflexion of the curve. [2]

(iv) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left[\left(x^2 - 2\right)^4\right]$. [2]

(v) Hence find the total area of the shaded regions. [3]

11 (a) The diagram shows a triangle ABC which has a right angle at C.
The point D is the mid-point of the side AC.
The point E lies on AB such that $AE = DE$.
The line segment ED is produced to meet the line BC produced at F.

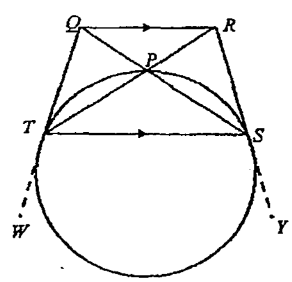


(i) Prove that ΔACB is similar to ΔDCF. [2]

(ii) Explain why ΔEFB is isosceles. [1]

(iii) Show that $EB = 3AE$. [2]

(b) $QRST$ is a trapezium in which QR is parallel to TS and its diagonals meet at P. The circle through T, P and S touches QW, RY at T and S respectively.



Prove that

(i) $\angle RQS = \angle QTR$, [2]

(ii) $QRST$ is a cyclic quadrilateral. [3]

End of Paper

2016 Preliminary Examination 2
Additional Mathematics 4047/2
Mark Scheme

Qn	Working	Marks	Total	Remarks
1i	$f'(x) = 3x^2 - 6x + 4$ $= 3(x-1)^2 + 1 > 0$ $\therefore$ f(x) is an increasing function for all values of x as $f'(x) > 0$.		[3]	
1ii	When $x = 0$, $f(x) = 0$ As x increases, y increases as $f'(x) > 0$ $\therefore$ f(x) is positive for all positive values of x as $f(x) \geq 0$.		[2]	
		Total	5	
2	Length of the other side $= \frac{40-2x}{2} = (20-x)$ cm Area of rectangle, $A = x(20-x)$ $\frac{dA}{dx} = 20 - 2x$ $\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$ $= (20-2x) \times 0.5$ When $x = 3$, $\frac{dA}{dt} = 14 \times 0.5 = 7$ Rate at which area is increasing = 7 cm²/s		[5]	
		Total	5	

Qn	Working	Marks	Total	Remarks
3a	General Term $=\binom{10}{r}(x^2)^{10-r}\left(-\frac{1}{2}x^{-3}\right)^r$ For independent term, $x^{20-2r-3r}=x^0$ $5r=20$ $r=4$ Term independent of $x=\binom{10}{4}\left(-\frac{1}{2}\right)^4=\frac{105}{8}$ or $13\frac{1}{8}$ or 13.125		[3]	
3bi	Term in $x^7=\binom{9}{2}(2x)^7\left(\frac{1}{4}\right)^2=288x^7$ $\therefore a=288$ Term in $x^6=\binom{9}{3}(2x)^6\left(\frac{1}{4}\right)^3=84x^6$ $\therefore b=84$		[3]	
3bii	$\left(2x+\frac{1}{4}\right)^9\left(\frac{4}{x}-1\right)\left(\frac{4}{x}+1\right)$ $=(...+576x^8+...+84x^6+...)\left(\frac{16}{x^2}-1\right)$ $=...9216x^6-84x^6...=...9132x^6+...$ Coefficient of $x^6=9132$		[2]	
		Total	8	

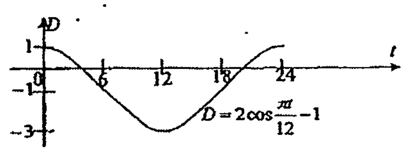
Qn	Working	Marks	Total	Remarks
4(a)	$\log_3 a = r$			
	$a = 3^r$			Substitution
	$\log_{27} b = s$			
	$b = 3^{3s}$			
	$\frac{a}{b} = 3^t$			
	$\frac{3^r}{3^{3s}} = 3^t$			
	$t = r - 3s$		[3]	
(b)	$\log_3 x + 3 = 10 \log_x 3$			
	$\log_3 x + 3 = 10 \frac{\log_3 3}{\log_3 x}$			Change of base
	$\log_3 x + 3 = \frac{10}{\log_3 x}$			
	Let $\log_3 x = a$			
	$a + 3 = \frac{10}{a}$			
	$a^2 + 3a - 10 = 0$			Form quadratic equation
	$(a+5)(a-2) = 0$			
	$a = -5$ or $a = 2$			
	$\log_3 x = -5$ or $\log_3 x = 2$			
	$x = \frac{1}{243}$ or $x = 9$		[5]	
		Total	[8]	

Qn	Working	Marks	Total	Remarks
5i	When $x=0$, $2y=12 \Rightarrow y=6$ $B=(0,6)$ When $y=0$, $x=-12$ $A=(-12,0)$		[2]	
ii	Gradient of $AB=0.5$ Gradient of $CG=-2$ Eqn of CG: $y-2=-2(x+3)$ $y=-2x-4$		[2]	
iii	As G is at the intersection of AB and CG, $-2x-4=\frac{12+x}{2}$ $-4x-8=12+x$ $x=-4$ $y=-2(-4)-4=4$ $G=(-4,4)$ $CG=\sqrt{(2-4)^2+(-3+4)^2}=\sqrt{5}$ Equation of C_1: $(x+3)^2+(y-2)^2=5$		[3]	
iv	Let centre of $C_2=(a,b)$ $\frac{a-3}{2}=-4$ or $\frac{b+2}{2}=4$ $a=-5$ or $b=6$ Equation of C_2: $(x+5)^2+(y-6)^2=5$		[2]	Accept other method
v	$\tan\theta=\frac{GC}{AG}=\frac{\sqrt{5}}{\sqrt{(-12+4)^2+(-4)^2}}$ $=\frac{1}{4}$ $\theta=\tan^{-1}\left(\frac{1}{4}\right)$		[2]	
		Total	11	

Qn	Working	Marks	Total	Remarks
6i	$\frac{d}{dx}\left[\left(e^{2x}(2-3x)\right)\right]$ $=e^{2x}(-3)+(2-3x)(2e^{2x})$ $=e^{2x}-6xe^{2x}$		[3]	
ii	$\int_0^{\ln 2}\left(e^{2x}-6xe^{2x}\right)dx=\left[\left(e^{2x}(2-3x)\right)\right]_0^{\ln 2}$ $=e^{2\ln 2}(2-3\ln 2)-e^0(2)$ $=6-12\ln 2$ or -2.3178 $\therefore 6\int_0^{\ln 2}xe^{2x}dx=12\ln 2-6+\int_0^{\ln 2}e^{2x}dx$ $=12\ln 2-6+\frac{1}{2}\left(e^{2x}\right)_0^{\ln 2}$ $=12\ln 2-\frac{9}{2}$ $\therefore 5\int_0^{\ln 2}xe^{2x}dx=\frac{5}{6}\left(12\ln 2-\frac{9}{2}\right)=10\ln 2-\frac{15}{4}$ or 3.18		[5]	
		Total	8	

Qn	Working	Marks	Total	Remarks
7i	$\sin\theta = \frac{BC}{4} \Rightarrow BC = 4\sin\theta$ $\cos\theta = \frac{OC}{4} \Rightarrow OC = 4\cos\theta$ $y = OA + AB + BC + OC$ $= 4 + 2(4\cos\theta) + 4\sin\theta + 4\cos\theta$ $= 4(1 + \sin\theta + 3\cos\theta)$		[3]	
ii	$\frac{dy}{d\theta} = 4\cos\theta - 12\sin\theta$ For stationary point, $\frac{dy}{d\theta} = 0$ $4\cos\theta - 12\sin\theta = 0$ $\tan\theta = \frac{1}{3} \Rightarrow \theta = 0.32175$ or 0.322 $\frac{d^2y}{d\theta^2} = -4\sin\theta - 12\cos\theta$ When $\theta = 0.32175$, $\frac{d^2y}{d\theta^2} = -4\sin 0.32175 - 12\cos 0.32175 = -12.6 < 0$ Hence y is a maximum.		[4]	
iii	$y = 4 + 12\cos\theta + 4\sin\theta = 4 + R\cos(\theta - \alpha)$ $R = \sqrt{12^2 + 4^2} = \sqrt{160}$ $\tan\alpha = \frac{1}{3}$ $\alpha = 0.32175$ $\therefore y = 4 + \sqrt{160}\cos(\theta - 0.322) = 4 + 12.6\cos(\theta - 0.322)$		[2]	
iv	$4 + \sqrt{160}\cos(\theta - 0.32175) = 12$ $\cos(\theta - 0.32175) = \frac{8}{\sqrt{160}}$ $\alpha = 0.88608$ $(\theta - 0.32175) = 0.88608$ $\theta = 1.21$		[2]	
		Total	11	

Qn	Working	Marks	Total	Remarks
8i	$\text{RHS} = \dfrac{1-\dfrac{\sin^2\theta}{\cos^2\theta}}{1+\dfrac{\sin^2\theta}{\cos^2\theta}}$ $= \dfrac{\dfrac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$ $= \dfrac{\cos 2\theta}{1} = \cos 2\theta$		[3]	
ii	If $x = \tan 67.5°$, $\cos 2(67.5)° = \dfrac{1-\tan^2 67.5°}{1+\tan^2 67.5°}$ $\dfrac{-1}{\sqrt{2}} = \dfrac{1-x^2}{1+x^2}$ or $\dfrac{-\sqrt{2}}{2} = \dfrac{1-x^2}{1+x^2}$ $\sqrt{2}(1-x^2) = -1(1+x^2)$ or $-\sqrt{2}-\sqrt{2}x^2 = 2-2x^2$ $1+x^2 = \sqrt{2}x^2 - \sqrt{2}$ or $2x^2-2 = \sqrt{2}x^2+\sqrt{2}$ $1+x^2 = \dfrac{2x^2-2}{\sqrt{2}} = \sqrt{2}x^2 - \sqrt{2}$		[2]	For stating $\cos 135° = -\cos 45° = \dfrac{-1}{\sqrt{2}}$
iii	$\sqrt{2}x^2 - x^2 = \sqrt{2}+1$ $x^2(\sqrt{2}-1) = \sqrt{2}+1$ $x^2 = \dfrac{\sqrt{2}+1}{\sqrt{2}-1} \times \dfrac{\sqrt{2}+1}{\sqrt{2}+1} = (\sqrt{2}+1)^2$ $\therefore x = \sqrt{2}+1$ $\tan 67.5° = \sqrt{2}+1$ $c = 1, d = 1$		[3]	
iv	$\tan 67.5° = \tan(60°+7.5°)$ $\tan(60°+7.5°) = \sqrt{2}+1$ $\dfrac{\tan 60°+\tan 7.5°}{1-\tan 60° \tan 7.5°} = \sqrt{2}+1$ $\dfrac{\sqrt{3}+\tan 7.5°}{1-\sqrt{3}\tan 7.5°} = \sqrt{2}+1$ $\sqrt{3}+\tan 7.5° = (\sqrt{2}+1)(1-\sqrt{3}\tan 7.5°)$ $\sqrt{3}+\tan 7.5° = \sqrt{2}-\sqrt{6}\tan 7.5° + 1 - \sqrt{3}\tan 7.5°$ $\tan 7.5° = \dfrac{1+\sqrt{2}-\sqrt{3}}{1+\sqrt{3}+\sqrt{6}}$.		[3]	
		Total	11	

Qn	Working	Marks	Total	Remarks
9i	At 8 am, $t=4$, $A=21-3\cos\left(\frac{4\pi}{12}\right)=19.5$ Temp inside the house at 8 am $=19.5\,°C$		[2]	
9ii	$D=x-y$ $=(21-3\cos\frac{\pi t}{12})-(22-5\cos\frac{\pi t}{12})$ $=2\cos\frac{\pi t}{12}-1$		[1]	
(iii)		Shape Points	[3]	
(iv)	When temperature inside or outside is the same, $D=0$ $2\cos\frac{\pi t}{12}-1=0$ $\cos\frac{\pi t}{12}=\frac{1}{2}$ $\alpha=\frac{\pi}{3}$ $\frac{\pi t}{12}=\frac{\pi}{3}$ or $\frac{\pi t}{12}=2\pi-\frac{\pi}{3}$ $t=4$ or $t=20$ The times are 8 am and 12 midnight Hence the interval is $4<t<20$		[4]	
		Total	10	

Qu	Working	Marks	Total	Remarks

10i

When $y=0$, $x(x^2-2)^3=0$

$x=0$ or $x=\pm\sqrt{2}$

$\therefore P=(-\sqrt{2},0)$ and $Q=(\sqrt{2},0)$ **[2]**

ii

$y=x(x^2-2)^3$

$\frac{dy}{dx}=x\left(3(x^2-2)^2(2x)\right)+(x^2-2)^3$

$=(x^2-2)^2(6x^2+x^2-2)$

$=(x^2-2)^2(7x^2-2)$

For stat point, $\frac{dy}{dx}=0$

$(x^2-2)^2(7x^2-2)=0$

$x=\pm\sqrt{2}$ or $x=\pm\sqrt{\frac{2}{7}}$

x- coordinate of $N=\sqrt{\frac{2}{7}}$ or 0.535

x- coordinate of $M=-\sqrt{\frac{2}{7}}$ or -0.535 **[4]**

iii

From part ii, P and Q are stationary points.

	$-\sqrt{2}^-$	$-\sqrt{2}$	$-\sqrt{2}^+$
$\frac{dy}{dx}$	+	0	+
Slope	/	—	/

	$\sqrt{2}^-$	$\sqrt{2}$	$\sqrt{2}^+$
$\frac{dy}{dx}$	+	0	+
Slope	/	—	/

[2]

P and Q are points of inflexion.

iv

$\frac{d}{dx}\left[(x^2-2)^4\right]=4(x^2-2)^3(2x)$

$=8x(x^2-2)^3$ **[2]**

Qn	Working	Marks	Total	Remarks
v	Total area of shaded region $=2\int_{-\sqrt{2}}^{0} x(x^2-2)^3\,dx$ or $=2\left\|\int_{0}^{\sqrt{2}} x(x^2-2)^3\,dx\right\|$ $=2\times\frac{1}{8}\left[(x^2-2)^4\right]_{-\sqrt{2}}^{0}$ $=\frac{1}{4}\left[(0-2)^4-(2-2)^4\right]$ $=4$ sq units		[3]	
		Total	13	

Qn	Working	Marks	Total	Remarks
11ai	$\angle CDF = \angle ADE$ (vertically opposite angles) and $\angle EDA = \angle EAD$ (base angles of isos. Δ) (1) $\angle BAC = \angle CDF$ (2) $\angle DCF = \angle ACB = 90°$ (given) ΔACB is similar to ΔDCF (AA Similarity)		[2]	
ii	$\angle DFC = \angle ABC$ (Corr angles of similar triangles) $\therefore \Delta EFB$ is isosceles.		[1]	
iii	As $AC = 2AD$, $\therefore AB = 2DF$ (ratio of corr sides of similar Δs) $FD = EF - ED = EB - AE$ $AE + BE = 2(EB - AE)$ $3AE = EB$		[2]	
11bi	$\angle RQS = \angle QST$ (alt angles, QR//TS) $\angle QST = \angle QTR$ (tan chord theorem) $\therefore \angle RQS = \angle QTR$		[2]	
ii	Let $\angle WTS = x°$ $\therefore \angle TQR = x°$ (corr angles, QR//TS) $\angle TPS = x°$ (tan chord theorem) $\therefore \angle TSY = x°$ (tan chord theorem) $\therefore \angle TSR = 180° - x°$ (adj. angles on a st line) Since $\angle TSR + \angle TQR = 180°$ $QRST$ is a cyclic quadrilateral. (Angles in opp segments)		[3]	
		Total	10	
	Second Alternative Produce WTQ and YSR to meet at M. $\therefore \Delta MTS$ is isos. (tgts from ext pt are equal) $\therefore \angle QTS$ and $\angle RST$ are equal. $\therefore \angle TQR = 180° - \angle QTS$ (corr angles, $QR//TS$) Since $\angle TSR + \angle TQR = 180°$ $QRST$ is a cyclic quadrilateral. (Angles in opp segments)			