Name:	Register No.:	Class:

PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS

Paper 1

4047/01

17 August 2016

2 hours

Additional Materials: Answer Paper
Mark Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces provided at the top of this page and on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighter, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work and mark sheet securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

1 Sketch the graph of $y = |2x^2 - x - 1|$, indicating the intercepts and the turning point. [3]

2 Find the range of values of c for which the graph $y = x^2 - 3x + cx + 5$ lies entirely above the line $y = x + 1$. [4]

3 Solve the equation $\sin 2x = \sin x$ for $0° \leq x \leq 360°$. [4]

4 The cubic polynomial $f(x)$ has roots $x = \frac{1}{2}$, -3 and h. Given that the coefficient of x^3 is 6 and $f(x)$ has a remainder of -18 when divided by $x + 1$, find the value of h. Hence, find the remainder when $f(x)$ is divided by $2x - 3$. [4]

5 The sides AB and BC of ΔABC are of length $\left(2 + \sqrt{3}\right)$ cm and $\left(4 + \frac{2}{\sqrt{3}}\right)$ cm respectively. Given that $\angle ABC = 60°$, find the area of ΔABC in the form $a + b\sqrt{3}$ where a and b are rational numbers. [4]

6 Solve the following simultaneous equations.

$$\frac{1}{x} + \frac{3}{y} = 1$$

$$\left(\frac{1}{e^2}\right)^{1+2x} \times e^y = e$$ [5]

7 Find, without using a calculator, the exact value of $\frac{\tan 49° - \tan 34°}{1 + \tan 49° \tan 34°}$. [5]

8 (a) It is known that x and y are related by the equation $ax^2 + ky^3 - 120 = 0$, where a and k are non-zero constants. Explain how the value of a and k may be obtained from a suitable straight line graph. [3]

(b) A straight line graph is obtained by plotting $\frac{1}{y}$ against x. Given that the graph passes through the point $(\sqrt{3}, 1)$ and makes an angle of $60°$ with the line $y = 1$, express y in terms of x. [4]

9 Given that $\tan^2 A - 2\tan^2 B = 1$,

(i) show that $\cos^2 B = 2\cos^2 A$, [3]

(ii) find the exact value of $\tan B$ given that A and B are acute angles such that $A + B = \frac{\pi}{2}$. [5]

10 (i) Write down and simplify, in descending powers of x, the first three terms in the expansion of $(x^3 + \frac{2}{x^4})^n$, where $n > 0$. [3]

(ii) Hence find the value of n given that the coefficient of the third term is 7 times that of the second term. [2]

(iii) Using the value of n found in (ii), without expanding $(x^3 + \frac{2}{x^4})^n$, show that there is no constant term in the expansion. [3]

11 An object moves in a straight line, so that, t seconds after passing a fixed point O, its velocity, v m/s, is given by $v = 6t^2 - 22t + 9$. Find

(i) an expression for the displacement from O at any time t, [3]

(ii) the acceleration of the object when it comes to momentary rest the second time, [4]

(iii) the total distance travelled in the first two seconds after passing through O. [2]

12 (i) Express $\frac{9x^2 - 15x + 27}{(2x-5)(x^2+9)}$ in partial fractions.

(Hint: use substitution method) [4]

(ii) Differentiate $\ln(x^2 + 9)$ with respect to x. [1]

(iii) Hence find $\int_3^4 \frac{9x^2 - 15x + 27}{(2x-5)(x^2+9)} dx$. Give your answer in the form $a \ln b$ where a and b are rational numbers to be determined. [4]

13

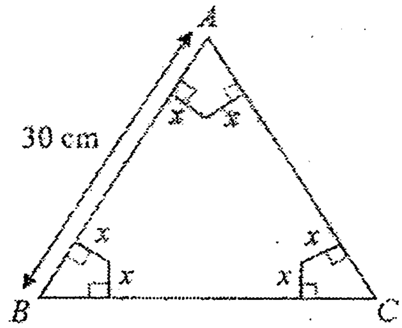


Figure 1

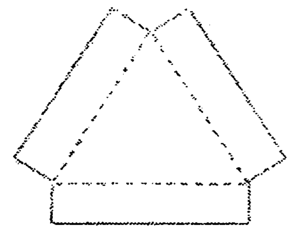


Figure 2

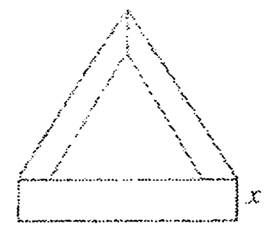


Figure 3

Figure 1 shows a piece of card in the form of an equilateral triangle ABC of side 30 cm. A kite shape is cut from each corner of ΔABC to give the shape as shown in Figure 2. The remaining card shown in Figure 2 is folded along the dotted lines, to form the open triangular box of height x cm, shown in Figure 3.

(i) Show that the volume, V cm^3, of the triangular box is given by

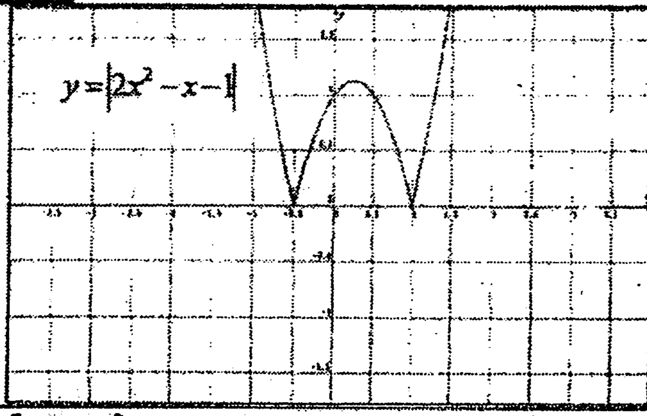
$$V = \frac{\sqrt{3}}{4}x(30 - 2\sqrt{3}x)^2.$$ [4]

(ii) Given that x can vary, find the value of x when V has a stationary value. [4]

(iii) By considering the sign of $\frac{dV}{dx}$, determine whether the volume of the triangular box is a maximum or minimum. [2]

END OF PAPER

Answer Key

1	
2	$0<c<8$
3	$0°, 60°, 180°, 300°, 360°$
4	$94\frac{1}{2}$
5	$4+\frac{5\sqrt{3}}{2}$ cm^2
6	$x=\frac{3}{2}, y=9$ or $x=-\frac{1}{2}, y=1$
7	$2-\sqrt{3}$
8	(a) $y^3=-\frac{a}{k}x^2+\frac{120}{k}$; Plot y^3 against x^2 where gradient $=-\frac{a}{k}$ and y^3-intercept $=\frac{120}{k}$
	(b) $y=\frac{1}{\sqrt{3x-2}}$
9	(i) As shown
	(ii) $\sqrt{\frac{1}{2}}$
10	(i) $x^{3n}+2nx^{3n-7}+2n(n-1)x^{3n-14}+\ldots$
	(ii) 8
	(iii) For constant term, $r=\frac{24}{7}$ is not a positive integer
11	(i) $s=2t^3-11t^2+9t$
	(ii) 16.4 m/s^2
	(iii) 14.0 m
12	(i) $\frac{3}{2x-5}+\frac{3x}{x^2+9}$
	(ii) $\frac{2x}{x^2+9}$
	(iii) $\frac{3}{2}\ln\frac{25}{6}$
13	(i) As shown
	(ii) $\frac{5\sqrt{3}}{3}$
	(iii) Maximum

Name:	Register No.:	Class:

SECONDARY FOUR
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS

Paper 2

4047/02
23 August 2016
2 hours 30 minutes

Additional Materials: Answer Paper
Mark Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces provided at the top of this page and on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighter, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work and mark sheet securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

1 (a) Find the quotient and remainder when $4x^3 - 12x^2 + 7$ is divided by $2x^2 - 3x - 2$. [3]

(b) Let $f(x) = 4x^3 - 12x^2 + 7 + (ax + b)$, where a and b are constants. It is given that $f(x)$ is divisible by $2x^2 - 3x - 2$.

(i) State the value of a and of b. [2]

(ii) Deduce the roots of the equation $f(x) = 0$. [2]

2 α and β are the roots of the equation $x^2 - 4x + 3 = 0$, where α and β are positive integers and $\alpha > \beta$.

(i) Express $\alpha - \beta$ in terms of $\alpha + \beta$ and $\alpha\beta$. [2]

(ii) Without finding the value of α and of β, form a quadratic equation whose roots are $\alpha^2\beta$ and $-\alpha\beta^2$. [5]

3 As part of an experiment, a group of students started a rumour in their school and recorded down the number of students who have heard of the rumour after every hour. There are 500 students in the school. After collecting their data, they propose that the spread of the rumour can be modelled by the equation $N = \frac{500}{1 + 99e^{-2t}}$ where N is the number of students who have heard of the rumour and t is the number of hours after the group of students started the rumour.

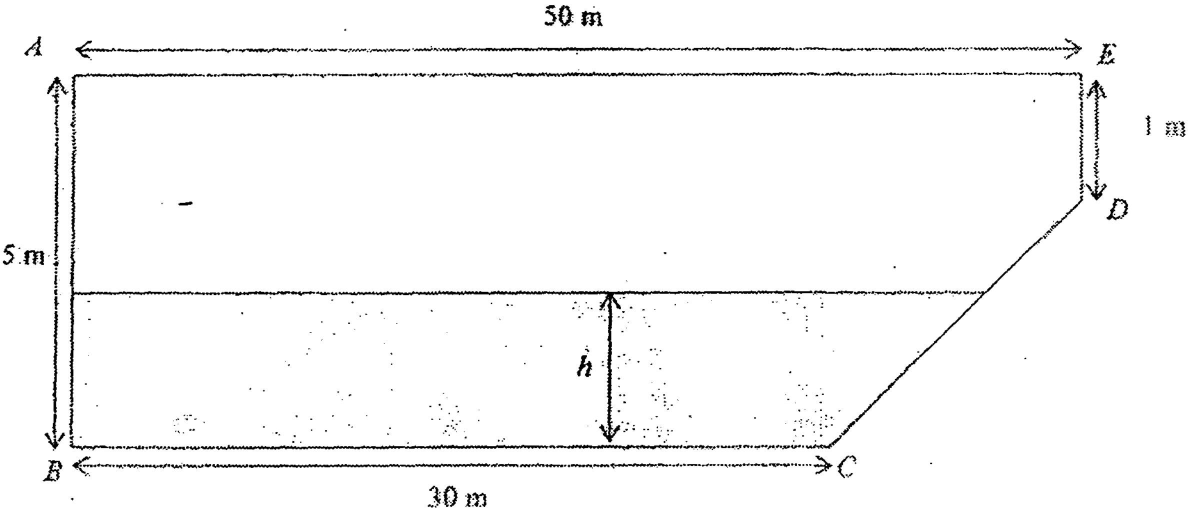
(a) Find the number of students in the group who started the rumour. [1]

(b) How long will it take for the rumour to spread to 300 students? [3]

(c) Find the rate at which the rumour is spreading after 3 hours. [2]

(d) Explain whether the entire school population will hear about the rumour based on the equation modelled by the students. [2]

4



$ABCDE$ is the cross sectional area of a swimming pool with a width of 15 m. AB, BC, DE and AE are 5 m, 30 m, 1 m and 50 m respectively.

(i) Show that the volume of water V, when the swimming pool is filled with water to a depth of h m, is given by $V = \dfrac{900h + 75h^2}{2}$. [3]

(ii) Find the rate of change of the depth of water in the swimming pool when $h = 3.5$ m, given that the swimming pool is filled with water at a rate of $0.3\ \text{m}^3/\text{min}$. [3]

5 (a) Solve the equation $2(4^x) + 3(9^x) = 5(6^x)$. [5]

(b) Solve the equation $\log_9(4x^2 + 3x + 5) - \log_3(x+1) = \dfrac{1}{2}$. [5]

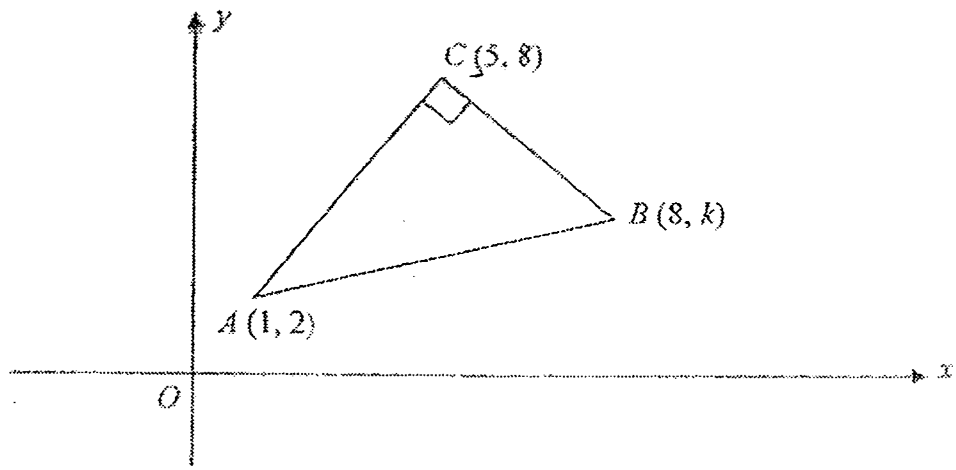
6 Jane researched online for the average monthly temperature at Paradise Island and found that the coldest month on the Island is in January with a temperature of −7°C and the hottest month is in July with a temperature of 45°C. She noticed that the average monthly temperature, T, can be modelled by the equation $T = A\cos Bx + C$, where A, B and C are constants and x is the number of months after January.

(i) Based on the above scenario, show that $T = -26\cos\dfrac{\pi}{6}x + 19$. [3]

(ii) Sketch the graph of $T = -26\cos\dfrac{\pi}{6}x + 19$ for $0 \le x \le 12$. [3]

(iii) Jane would like to visit Paradise Island only when the average monthly temperature is above 25°C. By showing your workings clearly, suggest the months in which Jane should visit the island. [4]

7 The figure shows a right-angled triangle ABC, where the coordinates of A, B and C are $(1, 2)$, $(8, k)$ and $(5, 8)$ respectively and $\angle ACB = 90°$.



(i) Find the value of k. [2]

D is the point of intersection of the perpendicular bisector of AC with the y-axis.

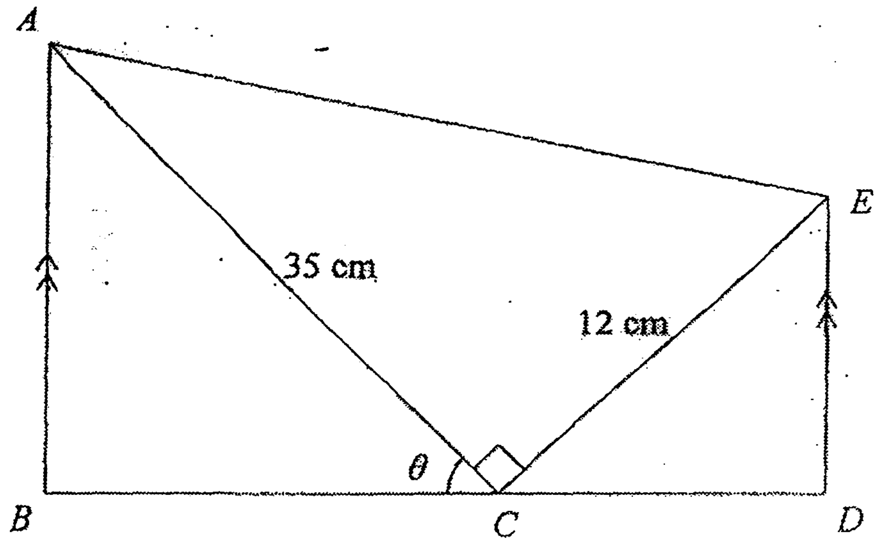
(ii) Find the coordinates of D. [4]

(iii) Determine whether the quadrilateral $ABCD$ is a trapezium. Justify your answer. [2]

$E(-5, -7)$ is a point on CA produced.

(iv) Find the ratio of the area of ΔABC to the area of ΔABE. [2]

8 $ABCDE$ is a trapezium with AB parallel to DE. It is given that $AC = 35$ cm, $CE = 12$ cm, $\angle ACE = 90°$ and $\angle ACB = \theta°$, where θ is an acute angle measured in degrees.



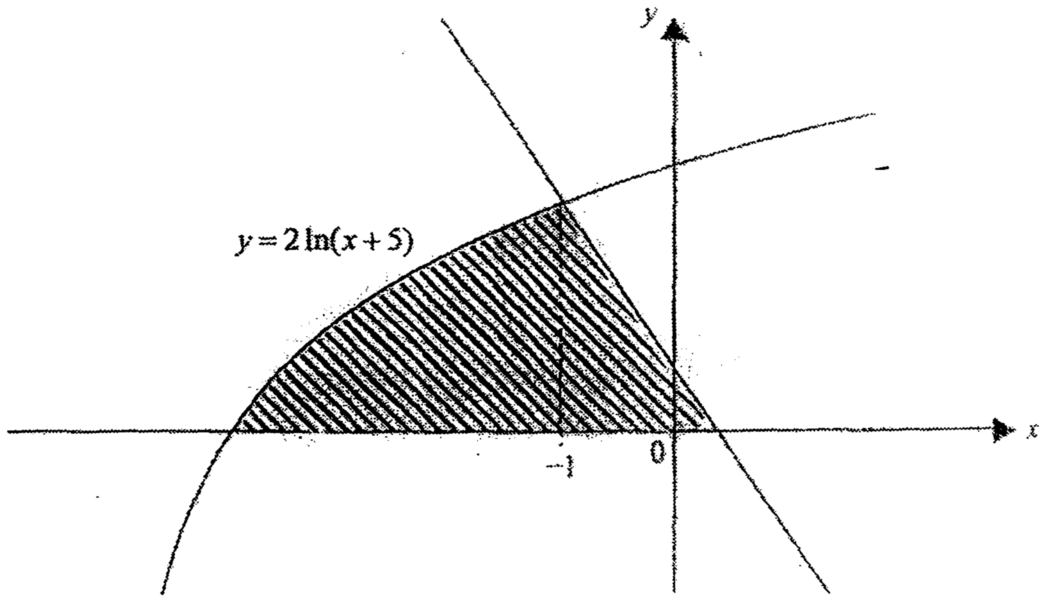
(i) Show that the perimeter, P, of $ABCDE$ is given by $P = 37 + 47\cos\theta + 47\sin\theta$. [3]

(ii) Express P in the form $37 + R\sin(\theta + \alpha)$, where $R > 0$ and α is an acute angle. [3]

(iii) Determine the maximum value of P and the corresponding value of θ. [3]

(iv) Justify with working, if it is possible for the perimeter of $ABCDE = 70$ cm. [3]

9

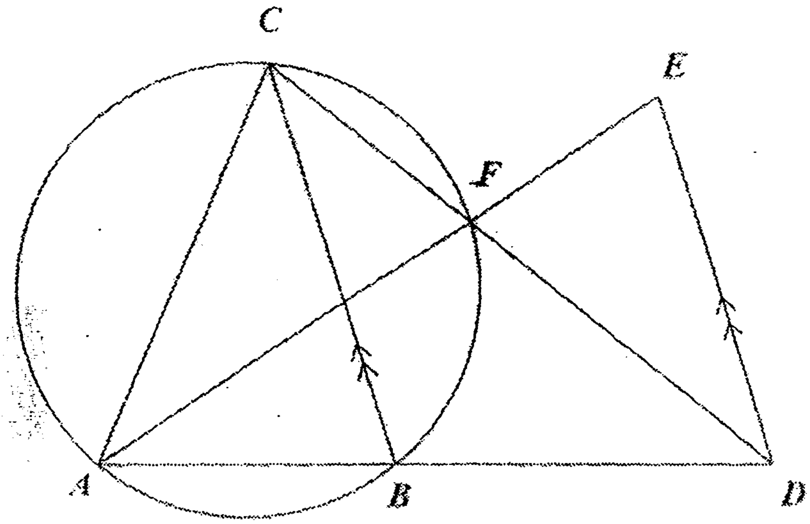


The diagram above shows the curve with equation $y = 2\ln(x+5)$ and the normal to the curve at $x = -1$.

(i) Show that the equation of the normal to the curve $y = 2\ln(x+5)$ at $x = -1$ is $y = -2x + 4\ln 2 - 2$. [3]

(ii) Find the area of the shaded region bounded by the curve, the normal to the curve at $x = -1$ and the x axis, leaving your answer to 2 decimal places. [6]

10



A, B and C are three points on the circumference of a circle. The line AE bisects $\angle BAC$ and intersects the circle ABC at F. D is the point of intersection of AB produced and CF produced. E is a point on AF produced such that DE is parallel to BC.

(i) Prove that DE is a tangent to the circle passing through A, F and D. [3]

(ii) Prove that ΔEDF is similar to ΔEAD. [2]

(iii) Prove that ΔDEF is similar to ΔACF. [2]

(iv) Using your result from (b) and (c), prove that $DE^2 = EF^2 + DF \times CF$. [3]

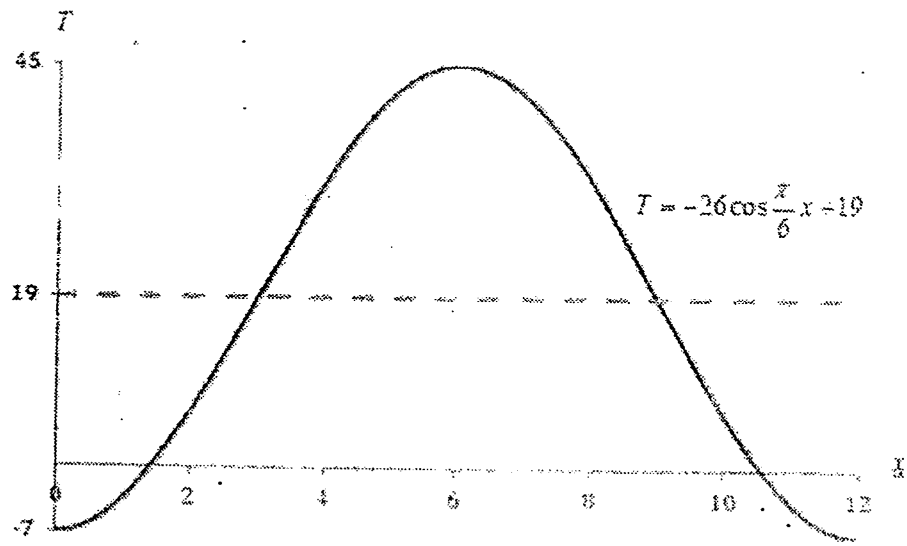
11 (i) Find the equations of the two lines that are tangents to the circle $(x-2)^2 + y^2 = 5$ and pass through the point $(-3, 0)$. [6]

(ii) Find the coordinates of the intersection of the circle with the tangent lines. [3]

(iii) State the number of intersections between the line $y = \frac{1}{4}(x+3)$ and the circle $(x-2)^2 + y^2 = 5$. Justify your answer without finding the intersections. [2]

END OF PAPER

Answer Key

Question	Answer
1 (a)	Quotient: $2x-3$ Remainder: $-5x+1$
(bi)	$a=5, b=-1$
(bii)	$x=1\frac{1}{2},\ 2,\ -\frac{1}{2}$
2 (i)	$\therefore \alpha-\beta=\sqrt{(\alpha+\beta)^2-4\alpha\beta}$
(ii)	$x^2-6x-27=0$
3 (a)	5 students
(b)	2.50 hours
(c)	158
4 (ii)	0.000421m/min
5 (a)	$x=-1$ or 0
(b)	$x=1$ or 2
6 (b)	
(c)	May, June, July, August or September
7 (i)	$k=6$
(ii)	$D(0,7)$
(iii)	$ABCD$ is not a trapezium as it does not have one pair of parallel sides.
(iv)	2 : 3
8 (ii)	$P=37+47\sqrt{2}\sin(\theta+45°)$
(iii)	Maximum Value = 103 cm; Corresponding value of $\theta=45°$
(iv)	Not possible
9 (ii)	7.01
11 (i)	$y=\frac{1}{2}x+\frac{3}{2}$ or $y=-\frac{1}{2}x-\frac{3}{2}$
(ii)	(1, 2) or (1, −2)
(iii)	2 intersections