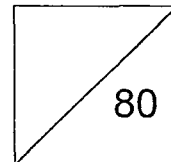


Name of Student

Class

Index Number



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**PRELIMINARY EXAMINATION 2016  
SECONDARY 4 EXPRESS  
ADDITIONAL MATHEMATICS PAPER 1**

4047/01

Date: 22 Aug 2016

Duration: 2 hours

Time: 0800 – 1000

*Additional Materials:* 8 sheets of writing paper

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**READ THESE INSTRUCTIONS FIRST**

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction tape/fluid

Answer **all** the questions.

Write your answers on the separate Answer Paper provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

## Mathematical Formulae

### 1. ALGEBRA

#### Quadratic Equation

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

#### Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

### 2. TRIGONOMETRY

#### Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

#### Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer all the questions.

1 A curve has the equation  $y = 4x^2 - px + p - 3$ , where  $p$  is a constant. Find the range of values of  $p$  for which the curve lies completely above the  $x$ -axis. [4]

2 Solve the equation  $\ln(4^x - 4) - x \ln 2 = \ln 3$ . [4]

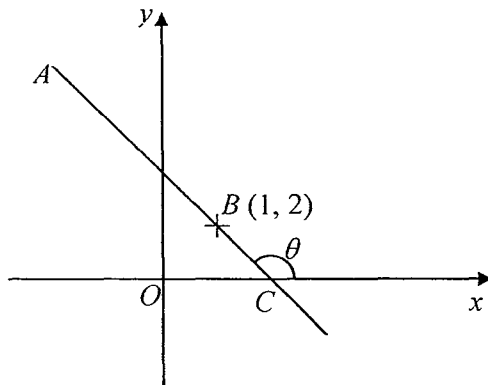
3 A curve has the equation  $y = \frac{1-x}{3x+4}$  for  $x > 0$ .

(i) Obtain an expression for  $\frac{dy}{dx}$ . [2]

(ii) Show that  $y$  is a decreasing function. [1]

(iii) Given that  $y$  decreases at the rate of 0.75 units per second, calculate the rate of change of  $x$  at the instant when  $x = 3$ . [2]

4



The diagram shows a straight line  $ABC$  such that  $AB : BC = 3 : 1$ . The point  $B$  is  $(1, 2)$  and the point  $C$  lies on the  $x$ -axis.  $\theta$  is the angle between the positive  $x$ -axis and the line  $AC$ . Given that  $\tan \theta = -2$ , find

(i) the equation of the line  $AC$ , [1]

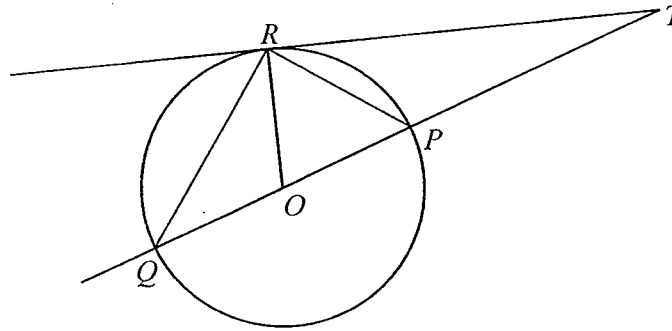
(ii) the coordinates of  $C$  and of  $A$ . [3]

The point  $D$  is such that  $ABOD$  is a parallelogram.

(iii) Find the coordinates of  $D$ . [2]

- 5 In an experiment, a scientist started with 5 000 000 cells and observed that 40% of the cells are dying every minute. The number of cells remaining,  $N$ , after  $t$  minutes, is given by  $N = Ae^{kt}$ , where  $A$  and  $k$  are constants.
- (i) Find the value of  $A$  and of  $k$ . [4]
- (ii) Find the value of  $t$  when the number of cells decreases to 2000. [2]
- 6 (i) Sketch the curve  $y = |x^2 - 4|$  for  $-2 \leq x \leq 3$ . [3]
- (ii) Find the  $x$ -coordinates of the points of intersection of the curve  $y = |x^2 - 4|$  and the line  $y = 6$ . [3]

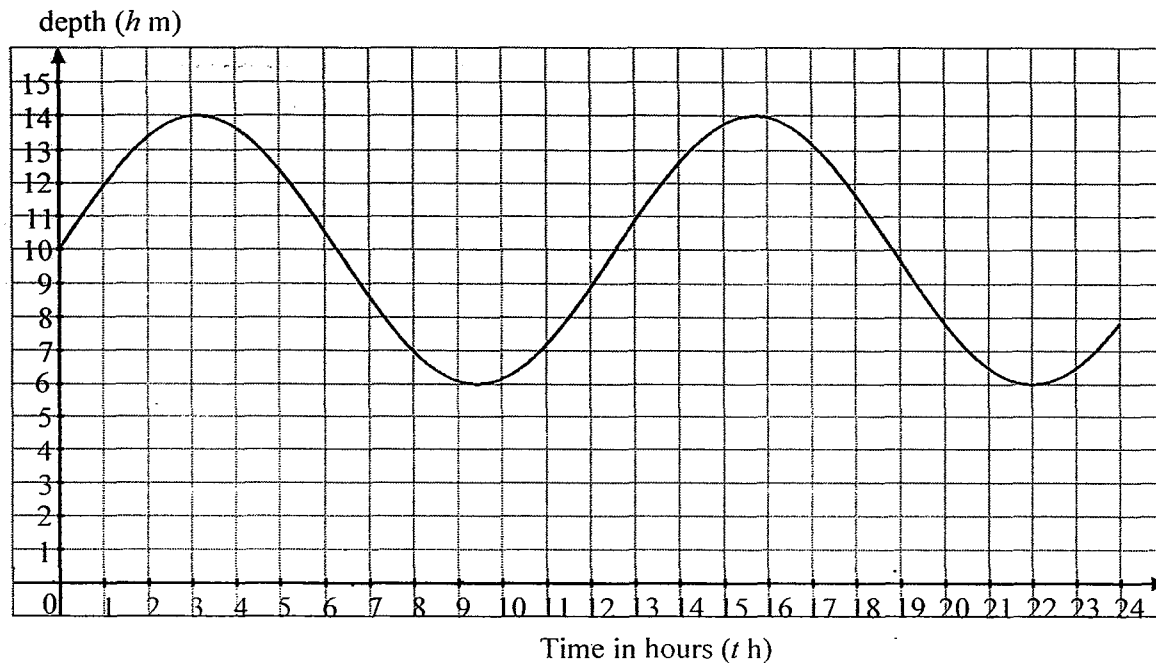
7



The diagram shows a circle, centre  $O$ . The point  $R$  lies on the circle and  $TR$  is a tangent to the circle. The line  $TQ$  passes through  $O$  and intersects the circle at  $P$  and  $Q$ .

- (i) Prove that triangles  $TRP$  and  $TQR$  are similar. [2]
- (ii) Prove that  $TP \times TQ = OT^2 - OR^2$ . [4]

8



The diagram shows the graph of the depth of water,  $h$  metres, in a harbour on a particular day, which is modelled by the equation,  $h = a \sin \frac{1}{2}t + b$ , where  $a$  and  $b$  are constants and  $t$  is the time in hours after midnight.

(i) State the period of  $h$ . [1]

(ii) Use the graph to find the value of  $a$  and of  $b$ . [2]

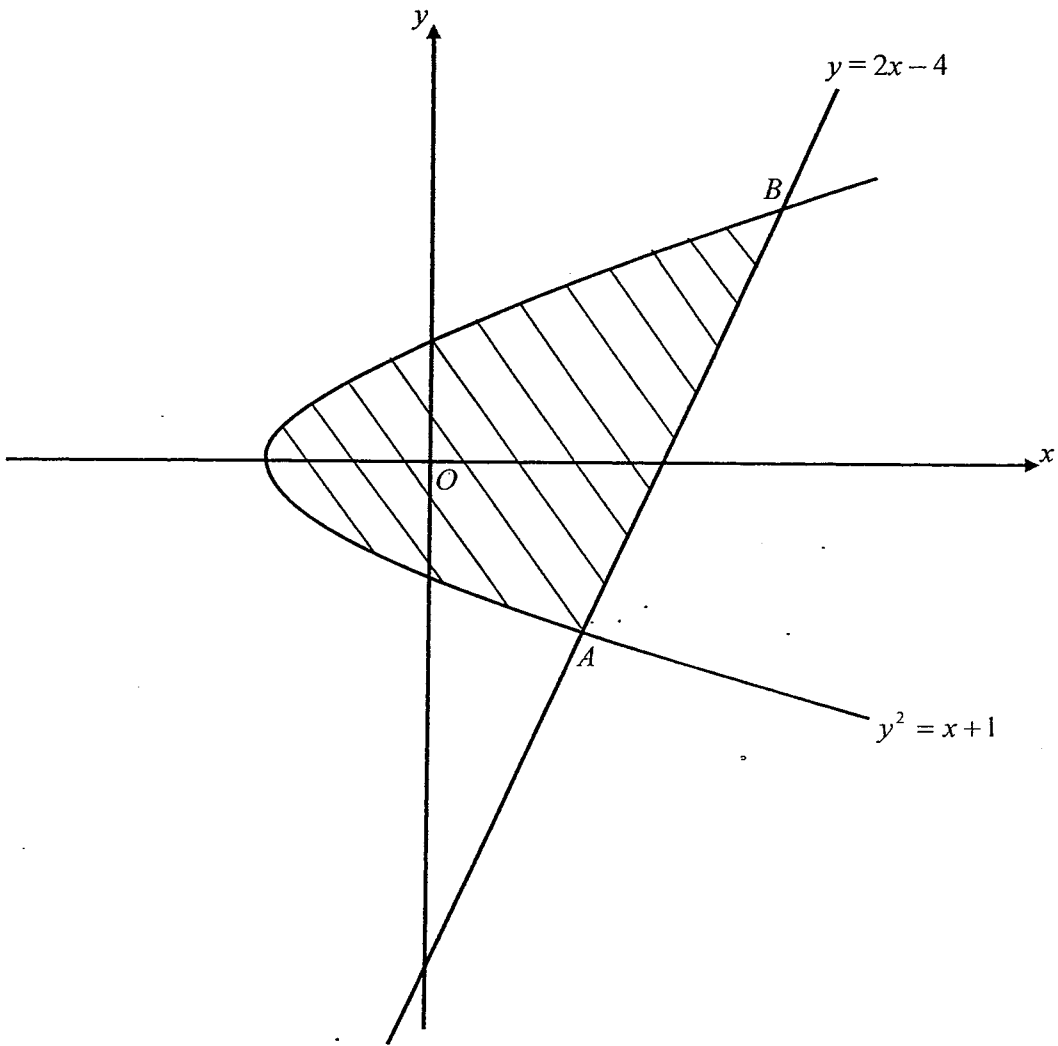
The harbour gates are closed when the depth of the water is less than seven metres. An alarm rings when the gates are opened or closed.

(iii) Using the values of  $a$  and  $b$  found in (ii), calculate the values of  $t$  when the alarm rings on this particular day. [4]

(iv) Hence find the total length of time when the harbour gates are closed. [1]

9 (i) Show that  $\frac{\sin \theta}{1 + \frac{1}{\sec \theta}} + \cot \theta = \operatorname{cosec} \theta$ . [4]

(ii) Hence find, in degrees, the smallest value of  $\theta$  such that  $\frac{\sin 2\theta}{1 + \frac{1}{\sec 2\theta}} + \cot 2\theta = 6 \cos 2\theta$ . [4]



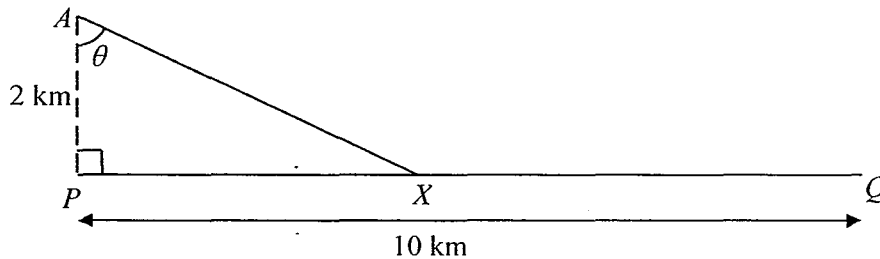
The diagram shows part of the curve  $y^2 = x + 1$ . The line  $y = 2x - 4$  intersects the curve at points  $A$  and  $B$ . Find

- (i) the coordinates of  $A$  and of  $B$ , [4]
- (ii) the area of the shaded region. [4]

11 A particle moves in a straight line, so that,  $t$  seconds after leaving a fixed point  $O$ , its velocity,  $v \text{ m s}^{-1}$  is given by  $v = 2 + 5t - 3t^2$ . The particle comes to instantaneous rest at the point  $Q$ . Find

- (i) the acceleration of the particle at  $Q$ , [4]
- (ii) the distance  $OQ$ , [3]
- (iii) the total distance travelled by the particle in the time interval  $t = 0$  to  $t = 3$ . [2]

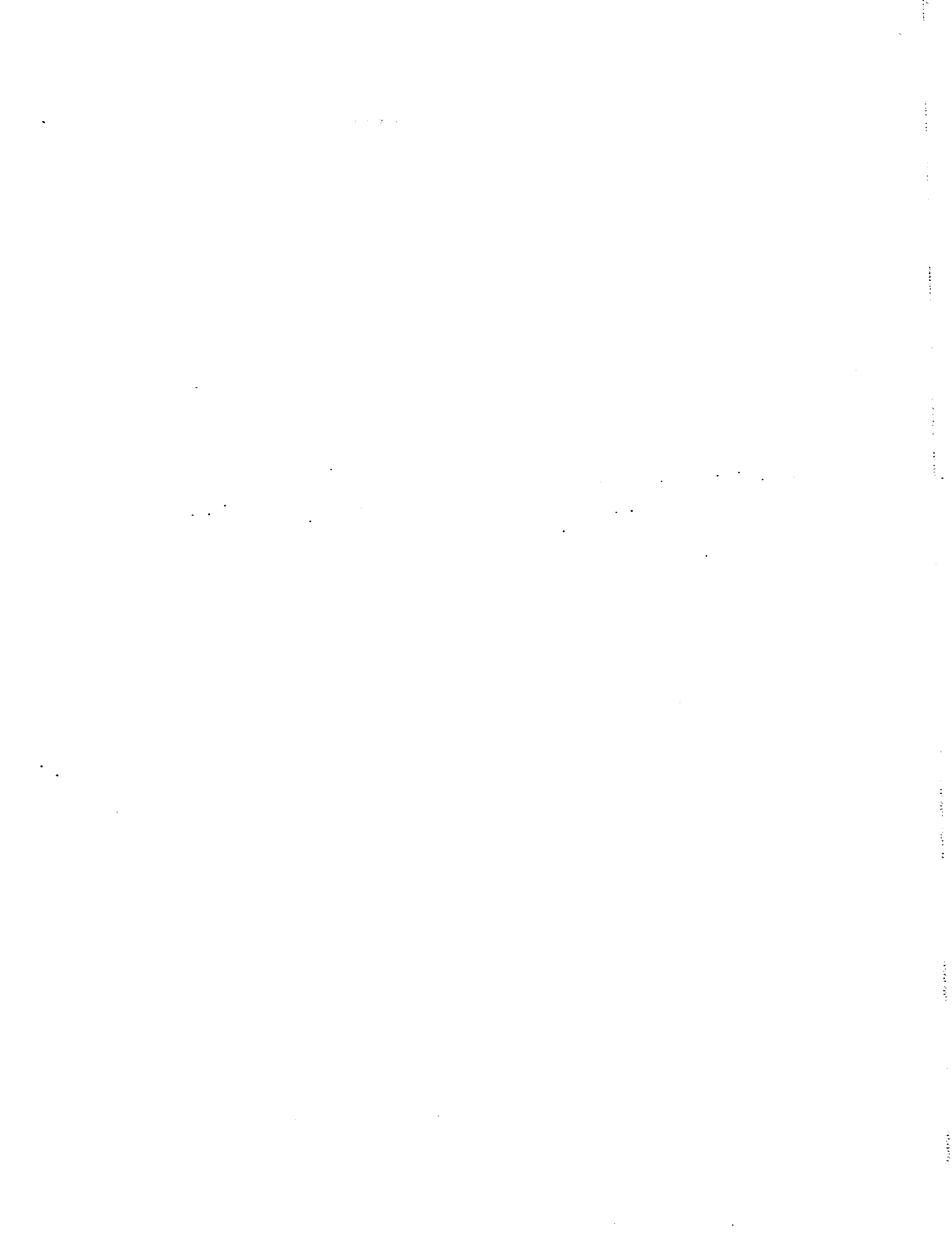
12



The diagram shows a straight road  $PQ$ , of length 10 km. A man is at point  $A$ , where  $AP$  is perpendicular to  $PQ$  and  $AP$  is 2 km. He travels in a straight line to meet the road at point  $X$ , where angle  $PAX = \theta$  radians. The man travels at 3 km/h along  $AX$  and 5 km/h along  $XQ$ . He takes  $T$  hours to travel from  $A$  to  $Q$ .

- (i) Show that  $T = \frac{2 \sec \theta}{3} + 2 - \frac{2 \tan \theta}{5}$ . [4]
- (ii) Given that  $\theta$  can vary, show that  $T$  has a stationary value when  $PX = 1.5$  km. [6]

End of paper

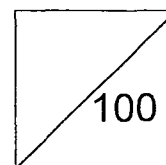




Name of Student

Class

Index Number



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**PRELIMINARY EXAMINATION 2016  
SECONDARY 4 EXPRESS  
ADDITIONAL MATHEMATICS PAPER 2**

4047/02

**Date:** 17 Aug 2016

**Duration:** 2 h 30 min

**Time:** 1100 – 1330

*Additional Materials:* **8 sheets of writing paper**

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**READ THESE INSTRUCTIONS FIRST**

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction tape/fluid.

Answer **all** the questions.

Write your answers on the separate Answer Paper provided.

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You are reminded of the need for clear presentation in your answers.

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The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **100**.

## Mathematical Formulae

### 1. ALGEBRA

#### Quadratic Equation

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

#### Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$ .

### 2. TRIGONOMETRY

#### Identities

$$\sin^2 A + \cos^2 A = 1$$

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$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

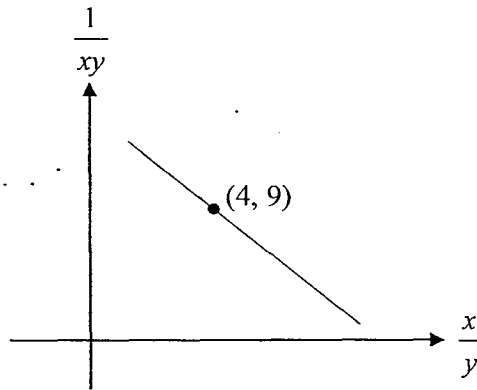
$$\Delta = \frac{1}{2} ab \sin C$$

Answer all the questions.

1 Given that, for all values of  $x$ ,  $x^5 - 2x^3 + 2x^2 + 4x - 3 = Ax + B + (x^2 - 1)Q(x)$ , where  $Q(x)$  is a polynomial,

- (i) state the degree of the polynomial,  $Q(x)$ , [1]
- (ii) find the remainder of  $x^5 - 2x^3 + 2x^2 + 4x - 3$ , when divided by  $x^2 - 1$ , in terms of  $x$ . [5]

2



The diagram shows part of a straight line graph drawn to represent the equation  $y = \frac{ax^2 + b}{cx}$ , where  $a$ ,  $b$  and  $c$  are integers. Given that the line passes through

$(4, 9)$  and has gradient  $-\frac{1}{4}$ , find

- (i) the value of  $\frac{y}{x}$  where the straight line cuts the horizontal axis, [3]
- (ii) the value of  $a$ , of  $b$  and of  $c$ . [3]

3

In the expansion  $\left(2x^2 + \frac{3}{x}\right)^n$ , in descending powers of  $x$ , the ratio of the coefficients of the third and first term is  $81 : 1$ .

- (i) Find the value of  $n$ . [3]
- (ii) Write down the first three terms of the expansion. [2]
- (iii) Find the term that is independent of  $x$ . [2]

4 (i) Express  $\frac{11-7x}{3x^2+11x-4}$  in partial fractions. [3]

(ii) Hence evaluate  $\int_1^2 \frac{11-7x}{9x^2+33x-12} dx$ . [4]

5 (i) Solve  $2x^3 + x^2 - 5x + 2 = 0$ . [4]

(ii) Hence solve  $16\tan^3\theta + 4\tan^2\theta - 10\tan\theta + 2 = 0$ , where  $0^\circ \leq \theta \leq 90^\circ$ . [4]

6 A curve is such that  $\frac{dy}{dx} = \frac{e^{5x} + 1}{e^{3x}}$  and  $(0, \frac{1}{2})$  is a point on the curve.

(i) Explain why the curve has no stationary points. [2]

(ii) Find the value of  $y$  when  $x = 2$ . [6]

7 The equation of a curve is  $y = \frac{(x-3)^2}{2x+5}$ .

(i) Find an expression for  $\frac{dy}{dx}$  and obtain the coordinates of the stationary points. [5]

(ii) Find an expression for  $\frac{d^2y}{dx^2}$  and hence determine the nature of these stationary points. [4]



Diagram 1

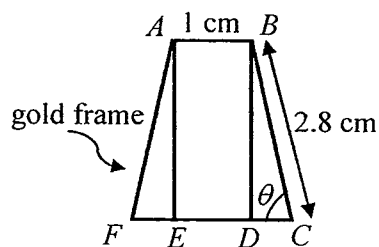


Diagram 2

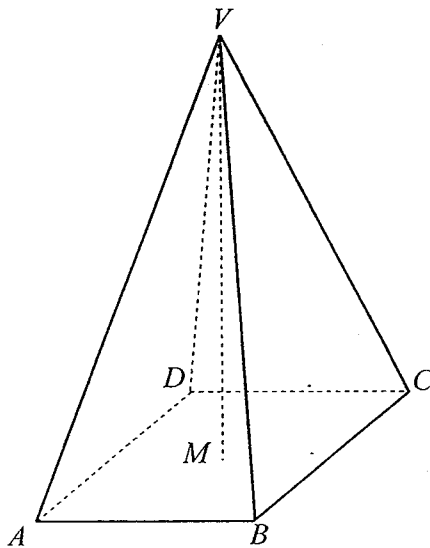
Diagram 1 shows the front view of a pendant which can be modelled as a regular trapezium. Diagram 2 shows the back view of the modelled pendant with the gold frame that is used to hold the pendant. Trapezium  $ABCF$ , line  $AE$  and  $BD$  form the structure of the gold frame.

$AB = DE = 1$  cm,  $AF = BC = 2.8$  cm and  $\angle AFE = \angle BCD = \theta$ .

- (i) Show that the total length of the structure that form the gold frame,  $P$ , is  $(5.6\sin\theta + 5.6\cos\theta + 7.6)$  cm. [2]
- (ii) Express  $P$  in the form  $R\sin(\theta + \alpha) + 7.6$ , where  $R > 0$  and  $\alpha$  is an acute angle. [4]
- (iii) Given that the perimeter of the gold frame is 15 cm, find the values of  $\theta$ . [3]

9 Do not use a calculator in this question.

- (i) Express  $\frac{7\sqrt{2}}{3\sqrt{2}-2}$  in the form  $a+b\sqrt{2}$ , where  $a$  and  $b$  are integers. [2]



The diagram shows a right pyramid with a square base of side  $\frac{7\sqrt{2}}{3\sqrt{2}-2}$  cm.

Given that the height,  $VM$ , of the pyramid is  $\frac{1}{2}BD^2$ , find

- (ii) an expression for  $BD^2$  in the form  $c+d\sqrt{2}$ , where  $c$  and  $d$  are integers, [3]
- (iii) the volume of the pyramid in the form  $p+q\sqrt{2}$ , where  $p$  and  $q$  are rational numbers. [4]
- 10 (a) A circle, whose equation is  $x^2 + y^2 - 10x + 8y + 5 = 0$ , has centre  $C$ .
- (i) Find the centre of the circle,  $C$ . [1]
- (ii) Explain why point  $P(4, -11)$  lies outside of the circle. [3]
- (iii) A line drawn through  $P$  is tangent to the circle at point  $T$ . Find the length of  $PT$ . [2]
- (b) The equation of a curve is  $y = x^2 - 7x + 10$ . Point  $A$  is a point on the curve and it lies on the  $y$ -axis. Find the equation of the normal at point  $A$ . [4]

11 (a) Given that  $y = \tan x$ , show that  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx}y = 0$ . [4]

(b) (i) Find  $\int_0^\pi 8\cos^2\left(\frac{x}{2}\right)dx$ . [3]

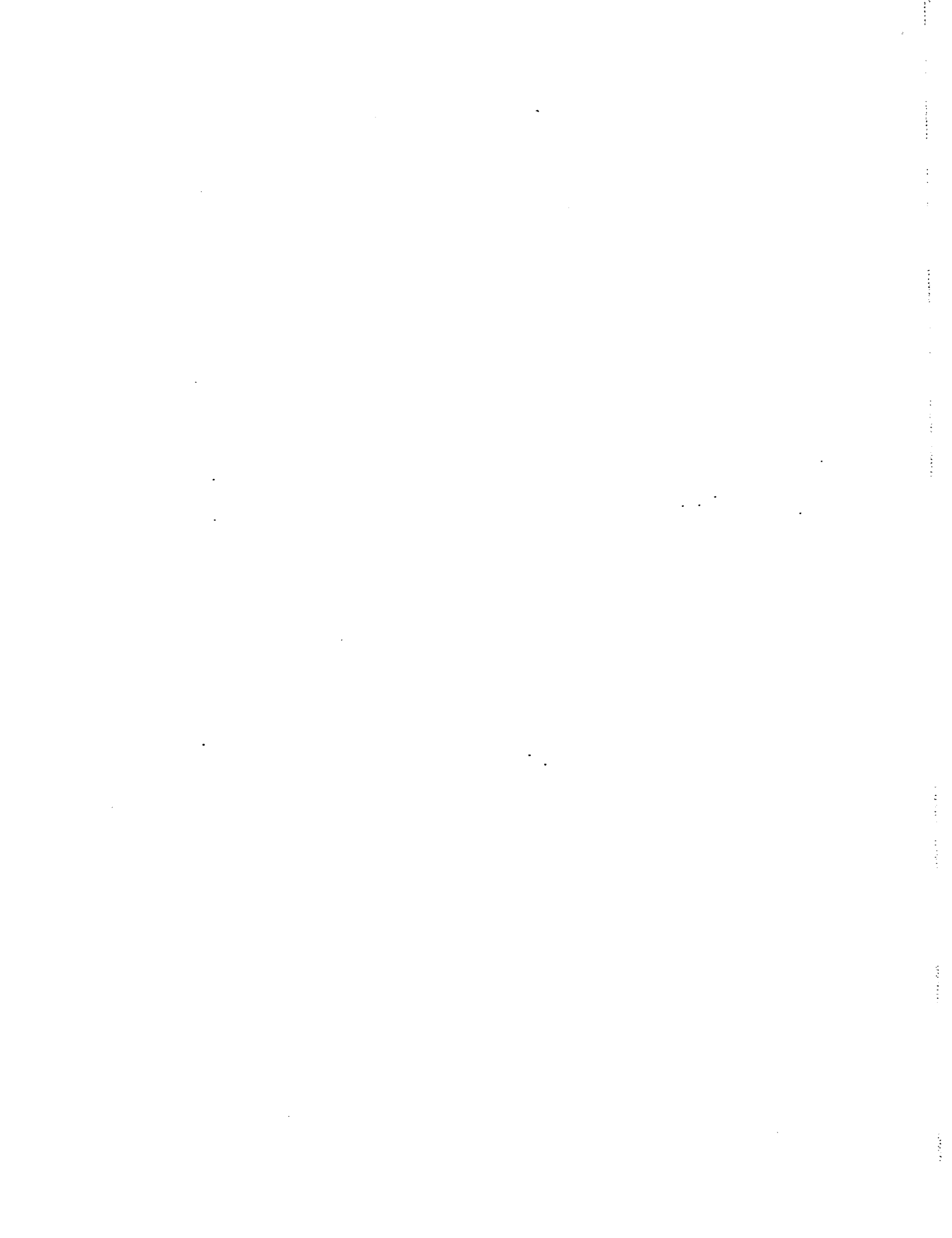
(ii) Hence find  $\int_0^\pi \left[3 - \sin^2\left(\frac{x}{2}\right)\right]dx$ . [3]

12 The roots of the quadratic equation  $3x^2 - 7x + 4 = 0$  are  $2\alpha + \beta$  and  $\alpha + 2\beta$ .

(i) Find the value of  $\alpha + \beta$ . [3]

(ii) Show that the value of  $\alpha\beta = \frac{10}{81}$ . [3]

(iii) Find a quadratic equation whose roots are  $\frac{1}{2}\alpha + \beta$  and  $\alpha + \frac{1}{2}\beta$ . [5]





### Answers

- 1 (i) 3  
(ii)  $3x - 1$
- 2 (i)  $\frac{y}{x} = \frac{1}{40}$   
(ii)  $a = 1, b = 4$  and  $c = 40$
- 3 (i)  $n = -8$  (rejected) or  $n = 9$   
(ii)  
 $512x^{18} + 6912x^{15} + 41472x^{12} + \dots$   
(iii) 489888
- 4 (i)  
$$\frac{11-7x}{3x^2+11x-4} = \frac{2}{3x-1} - \frac{3}{x+4}$$
  
(ii) 0.0213
- 5 (i)  $x = 1, x = \frac{1}{2}, x = -2$   
(ii)  $\theta \approx 14.0^\circ, 26.6^\circ$
- 6 (i) For all real values of  $x$ ,  
 $e^{2x} > 0$  and  $e^{-3x} > 0$ ,  
 $\therefore \frac{dy}{dx} > 0, \frac{dy}{dx}$  can never be zero.  
 $\therefore$  the curve has no stationary point.  
(ii)  $y \approx 27.6$
- 7 (i)  $(3,0)$  and  $(-8,-11)$   
(ii)  $(3,0)$  is a min. pt.  
 $(-8,0)$  is a max. pt.
- 8 (ii)  $P = 7.92\sin(\theta + 45^\circ) + 7.6$   
(iii)  $\theta \approx 24.1^\circ, 65.9^\circ$
- 9 (i)  $3 + \sqrt{2}$   
(ii)  $BD^2 = 22 + 12\sqrt{2}$   
(iii)  $\frac{193}{3} + 44\sqrt{2}$  cm<sup>3</sup>

- 10(a) (i)  $(5,-4)$   
(ii) radius = 6  
Length of  $PC = \sqrt{50}$   
 $\approx 7.07$   
Since length of  $PC$  is longer than radius of circle, thus, the point  $P$  is outside of the circle.  
(iii) 3.74 units  
(b)  $y = \frac{1}{7}x + 10$
- 11(a)  $y = \tan x$   
 $\frac{dy}{dx} = \sec^2 x$   
 $\frac{d^2y}{dx^2} = 2\sec x \cdot \sec x \cdot \tan x$   
 $\frac{d^2y}{dx^2} = 2\frac{dy}{dx}y$   
 $\frac{d^2y}{dx^2} - 2\frac{dy}{dx}y = 0$  (shown)
- 11(b) (i)  $4\pi$   
(ii)  $\frac{5\pi}{2}$
- 12 (i)  $\alpha + \beta = \frac{7}{9}$   
(iii)  $x^2 - \frac{7}{6}x + \frac{1}{3} = 0$

Additional Mathematics  
Preliminary Examination 2016  
Marking Scheme

1  $y = 4x^2 - px + p - 3$

$$b^2 - 4ac < 0$$

$$(-p)^2 - 4(4)(p-3) < 0$$

M1

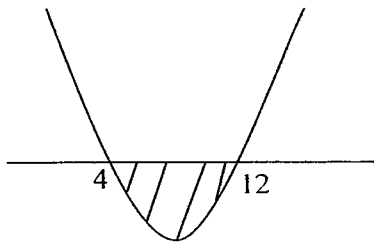
$$p^2 - 16p + 48 < 0 \quad \text{correct quadratic form}$$

M1

Finding the solution of quadratic:  $p = 4$  or  $12$

DM1

$$(p-12)(p-4) < 0$$



$$4 < p < 12$$

A1

2  $\ln(4^x - 4) - x \ln 2 = \ln 3$

$$\ln(4^x - 4) - \ln 2^x = \ln 3$$

$$\ln \frac{4^x - 4}{2^x} = \ln 3 \quad \text{applying quotient law}$$

M1

$$\frac{4^x - 4}{2^x} = 3$$

$$2^{2x} - 3(2^x) - 4 = 0 \quad \text{correct quadratic equation}$$

M1

**Or substituting  $y = 2^x$  to get  $y^2 - 3y - 4 = 0$**

$$(y-4)(y+1) = 0$$

$$y = 4 \text{ or } y = -1$$

$$2^x = 4 \text{ or } 2^x = -1 (\text{rej})$$

M1

$$x = 2$$

A1

3 (i)  $y = \frac{1-x}{3x+4}$

$$\frac{dy}{dx} = \frac{(-1)(3x+4) - (1-x)(3)}{(3x+4)^2}$$

M1

$$= \frac{-7}{(3x+4)^2}$$

A1

(ii) Since  $(3x+4)^2 > 0$  and  $\frac{-7}{(3x+4)^2} < 0$ ,

$y$  is a decreasing function for all real values of  $x$

}  
}

B1

(iii)  $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$$-0.75 = \frac{-7}{(3x+4)^2} \times \frac{dx}{dt}$$

M1

When  $x = 3$ ,  $\frac{dx}{dt} = \frac{-3}{4} \times \frac{169}{-7} = 18\frac{3}{28}$  units / sec

A1

(or 18.1 units / sec)

4 (i)  $y - 2 = -2(x - 1)$

$$y = -2x + 4$$

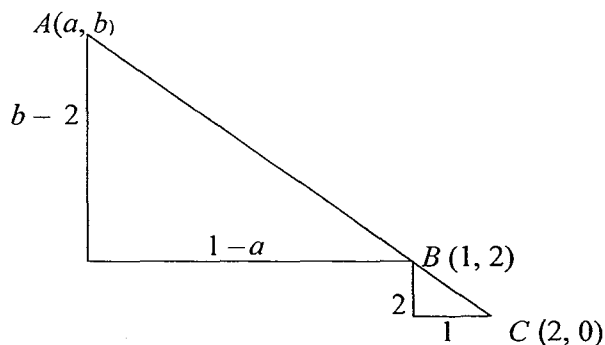
B1

(ii) when  $y = 0$ ,  $x = 2$

$\therefore$  Coordinates of  $C = (2, 0)$

B1

Let the coordinates of  $A$  be  $(a, b)$ .



Apply similar triangle ratios

$$\frac{1-a}{1} = \frac{3}{1} \quad \text{and} \quad \frac{b-2}{2} = \frac{3}{1}$$

M1

$$a = -2 \quad \text{and} \quad b = 8$$

A1

$\therefore$  Coordinates of  $A = (-2, 8)$

[Or apply distance formula

Subst  $x = a$  into  $y = -2x + 4$

$$y = -2a + 4$$

Distance of  $AB = 3$  Distance of  $BC$

$$\sqrt{(a-1)^2 + (-2a+4-2)^2} = 3\sqrt{(1-2)^2 + (2-0)^2} \quad \text{M1}$$

$$a^2 - 2a + 1 + 4a^2 - 8a + 4 = 9(5)$$

$$5a^2 - 10a - 40 = 0$$

$$5(a-4)(a+2) = 0$$

$$a = 4(\text{rej}) \quad \text{or} \quad a = -2$$

$$b = -8$$

$\therefore$  Coordinates of  $A = (-2, 8)$

A1]

(iii) Let the point  $D$  be  $(h, k)$   
mid-point of  $BD =$  mid-point of  $AO$

$$\left( \frac{h+1}{2}, \frac{k+2}{2} \right) = \left( \frac{-2+0}{2}, \frac{8+0}{2} \right)$$

M1

$$\frac{h+1}{2} = -1, \quad \frac{k+2}{2} = 4$$

$$h = -3, \quad k = 6$$

A1

$D(-3, 6)$

5 (i)  $N = Ae^{kt}$

When  $t = 0$ ,  $N = 5\,000\,000$

$$5\,000\,000 = Ae^{k(0)}$$

$$A = 5\,000\,000$$

B1

When  $t = 1$ ,  $N = \frac{60}{100} \times 5\,000\,000$

$$= 3\,000\,000$$

$$3\,000\,000 = 5\,000\,000e^{k(1)}$$

M1

$$e^k = \frac{3}{5}$$

$$k = \ln \frac{3}{5}$$

M1

$$= -0.5108 \approx -0.511$$

A1

(ii)  $2000 = 5000000e^{-0.5108t}$

$$e^{-0.5108t} = \frac{2}{5000}$$

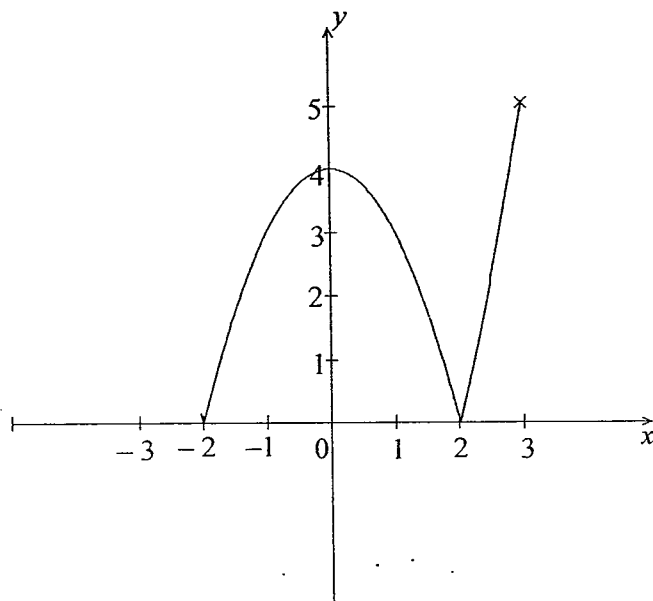
$$-0.5108t = \ln \frac{2}{5000}$$

M1

$t = 15.3$  min

A1

6 (i)



Correct shape

B1

$x$  - intercepts and turning point shown correctly

B1

end point (3, 5) shown clearly

B1

(ii)  $|x^2 - 4| = 6$

$x^2 - 4 = 6$  or  $x^2 - 4 = -6$

M1

$x^2 = 10$  or  $x^2 = -2$  (rej)

$x = 3.16$  or  $-3.16$

A2

- 7 (i)  $\angle RTP = \angle QTR$  (common angle)  
 $\angle TRP = \angle TQR$  ( $\angle$ s in the alternate segment or tangent chord thm)  
 $\therefore \triangle TRP$  and  $\triangle TQR$  are similar. (AA similarity)

}  
 }  
 }

B1

B1

- (ii) Since  $\triangle TRP$  and  $\triangle TQR$  are similar,

$$\frac{TR}{TQ} = \frac{TP}{TR}$$

$$\Rightarrow TR^2 = TP \times TQ \quad \text{----- (1)}$$

M1

$\angle ORT = 90^\circ$  (tangent  $\perp$  radius)

M1

$\Rightarrow \triangle ORT$  is a right angled triangle.

By Pythagoras theorem,

$$OT^2 = OR^2 + TR^2$$

$$TR^2 = OT^2 - OR^2 \text{ -----(2)} \quad \text{M1}$$

subst (1) into (2)

$$OT^2 - OR^2 = TP \times TQ \text{ (shown)} \quad \text{A1}$$

8 (i) period =  $\frac{2\pi}{\frac{1}{2}} = 4\pi$  B1

(ii) When  $t = 0$ ,  $10 = a \sin 0 + b$

$$\Rightarrow b = 10 \quad \text{B1}$$

$$\text{max value} = 14 \text{ when } \sin \frac{1}{2}t = 1$$

$$\Rightarrow a + 10 = 14$$

$$a = 4 \quad \text{B1}$$

(iii)  $4 \sin \frac{1}{2}t + 10 = 7$  M1

$$\sin \frac{1}{2}t = -\frac{3}{4}$$

$$\alpha = 0.8480 \text{ (accept } 0.84806)$$

$$\frac{1}{2}t = \pi + 0.8480, 2\pi - 0.8480, \pi + 0.8480 + 2\pi, 2\pi - 0.8480 + 2\pi \quad \text{M2}$$

(M1 for each cycle)

$$= 3.989, 5.435, 10.27, 11.71$$

$$t = 7.978, 10.87, 20.54, 23.42$$

$$\approx 7.98 \text{ h, } 10.9 \text{ h, } 20.5 \text{ h, } 23.4 \text{ h} \quad \text{A1}$$

(iv) Length of time the gates are closed =  $(10.87 - 7.978) + (23.42 - 20.54)$

$$= 5.772 \text{ h} \approx 5.77 \text{ h} \quad \text{B1}$$

$$9 \quad (i) \quad \frac{\sin \theta}{1 + \frac{1}{\sec \theta}} + \cot \theta = \operatorname{cosec} \theta$$

$$= \frac{\sin \theta}{1 + \cos \theta} + \frac{\cos \theta}{\sin \theta} \quad \text{M1}$$

$$= \frac{\sin^2 \theta + \cos \theta + \cos^2 \theta}{\sin \theta(1 + \cos \theta)} \quad \text{M1}$$

$$= \frac{1 + \cos \theta}{\sin \theta(1 + \cos \theta)} \quad (\text{Applying the identity } \sin^2 \theta + \cos^2 \theta = 1) \quad \text{M1}$$

$$= \frac{1}{\sin \theta} = \operatorname{cosec} \theta \quad \text{A1}$$

$$(ii) \quad \frac{\sin 2\theta}{1 + \frac{1}{\sec 2\theta}} + \cot 2\theta = 6 \cos 2\theta$$

$$\operatorname{cosec} 2\theta = 6 \cos 2\theta \quad \text{M1}$$

$$\frac{1}{\sin 2\theta} = 6 \cos 2\theta$$

$$6 \sin 2\theta \cos 2\theta = 1$$

$$3(2 \sin 2\theta \cos 2\theta) = 1$$

$$3 \sin 4\theta = 1 \quad (\text{applying double angle formula}) \quad \text{M1}$$

$$\sin 4\theta = \frac{1}{3}$$

$$\alpha = 19.47^\circ$$

$$4\theta = 19.47^\circ \quad \text{M1}$$

$$\theta = 4.87^\circ \approx 4.9^\circ \quad \text{A1}$$



10 (i)  $y^2 = x + 1$  ----- (1)  
 $y = 2x - 4$  ----- (2)

Subst (2) into (1)

$(2x - 4)^2 = x + 1$  M1

$4x^2 - 16x + 16 - x - 1 = 0$

$4x^2 - 17x + 15 = 0$  M1

$(4x - 5)(x - 3) = 0$

$x = 1\frac{1}{4}$  or 3 A1

$y = -1\frac{1}{2}$  or 2

$A(1\frac{1}{4}, -1\frac{1}{2}), B(3, 2)$  A1

(ii) From (2),  $x = \frac{y + 4}{2}$   
 $= \frac{y}{2} + 2$

Area =  $\int_{-\frac{3}{2}}^2 \left[ \left( \frac{y}{2} + 2 \right) - (y^2 - 1) \right] dy$  M2  
(M1 M1)

$= \int_{-\frac{3}{2}}^2 \left[ \left( \frac{y}{2} - y^2 + 3 \right) \right] dy$

$= \left[ \frac{y^2}{4} - \frac{y^3}{3} + 3y \right]_{-\frac{3}{2}}^2$  M1

$= \left( 1 - \frac{8}{3} + 6 \right) - \left( \frac{9}{16} + \frac{9}{8} - \frac{9}{2} \right)$

$= 7\frac{7}{48}$  units<sup>2</sup> (Accept 7.15 units<sup>2</sup>) A1

**Alternative Method**

$$[\text{Area} = \underbrace{\int_{-1}^3 (x+1)^{\frac{1}{2}} dx - \frac{1}{2} \times 1 \times 2}_{\text{M1}} + \underbrace{\left| \int_{-1}^{\frac{5}{4}} -(x+1)^{\frac{1}{2}} dx \right| + \frac{1}{2} \times \frac{3}{4} \times \frac{3}{2}}_{\text{M1}}]$$

M1

M1

$$= \left[ \frac{2}{3} (x+1)^{\frac{3}{2}} \right]_{-1}^3 - 1 + \left[ \frac{2}{3} (x+1)^{\frac{3}{2}} \right]_{-1}^{\frac{5}{4}} + \frac{9}{16} \quad \text{M1}$$

$$= \frac{16}{3} - 1 + \frac{9}{4} + \frac{9}{16}$$

$$= 7 \frac{7}{48} \text{ units}^2 \quad \text{A1 ]}$$

**Accept other logical methods**

11 (i)  $v = 2 + 5t - 3t^2$

At instantaneously at rest  $\Rightarrow v = 0$

$$2 + 5t - 3t^2 = 0 \quad \text{M1}$$

$$3t^2 - 5t - 2 = 0$$

$$(3t+1)(t-2) = 0$$

$$t = -\frac{1}{3} \text{ (rej) or } t = 2 \quad \text{A1}$$

$$\text{acceleration} = \frac{dv}{dt}$$

$$= 5 - 6t \quad \text{M1}$$

$$\text{At } t = 2, \text{ acceleration} = 5 - 6(2) = -7 \text{ m/s}^2 \quad \text{A1}$$

(ii)  $s = \int (2 + 5t - 3t^2) dt$

$$= 2t + \frac{5t^2}{2} - \frac{3t^3}{3} + c \quad \text{M1}$$

when  $t = 0$  and  $s = 0$ ,  $c = 0$

$$s = 2t + \frac{5t^2}{2} - t^3 \quad \text{M1}$$

$$\text{At } t = 2, s = \frac{5(2)^2}{2} - (2)^3 + 2(2) = 6 \text{ m} \quad \text{A1}$$

[OR  $\int_0^2 (2 + 5t - 3t^2) dt$

$$= \left[ 2t + \frac{5t^2}{2} - \frac{3t^3}{3} \right]_0^2 \quad (\text{M1 for integration, M1 for the limits})$$

$$= 6 \text{ m} \quad \text{A1}]$$

(iii) At  $t = 3, s = \frac{5(3)^2}{2} - (3)^3 + 2(3)$

$$= 1\frac{1}{2} \text{ m} \quad \text{M1}$$

Total distance travelled =  $6 + 6 - 1\frac{1}{2}$

$$= 10\frac{1}{2} \text{ m} \quad \text{A1}$$

[OR  $\int_2^3 (2 + 5t - 3t^2) dt$

$$\left[ \left[ 2t + \frac{5t^2}{2} - \frac{3t^3}{3} \right]_2^3 \right] \text{ M1}$$

$$= 4\frac{1}{2} \text{ m} \quad \text{M1}$$

Total distance travelled =  $6 + 4\frac{1}{2} = 10\frac{1}{2} \text{ m} \quad \text{A1}]$

12 (i)  $\cos \theta = \frac{2}{AX}$

$$AX = \frac{2}{\cos \theta}$$

$$= 2 \sec \theta \text{ km} \quad \text{M1}$$

Time taken for  $AX = \frac{2 \sec \theta}{3} \text{ h}$

$$\tan \theta = \frac{PX}{2}$$

$$PX = 2 \tan \theta \text{ km} \quad \text{M1}$$

$$XQ = 10 - 2 \tan \theta \quad \text{M1}$$

$$\text{Time taken for } XQ = \frac{10 - 2 \tan \theta}{5} \text{ h}$$

$$T = \frac{2 \sec \theta}{3} + \frac{10 - 2 \tan \theta}{5}$$

$$= \frac{2 \sec \theta}{3} + 2 - \frac{2 \tan \theta}{5} \quad (\text{shown})$$

A1

$$(ii) \quad T = \frac{2 \sec \theta}{3} + 2 - \frac{2 \tan \theta}{5}$$

$$= \frac{2}{3 \cos \theta} + 2 - \frac{2 \tan \theta}{5}$$

$$\frac{dT}{d\theta} = \frac{0(\cos \theta) - 2(-3 \sin \theta)}{9 \cos^2 \theta} - \frac{2}{5} \sec^2 \theta$$

$$= \frac{2 \sin \theta}{3 \cos^2 \theta} - \frac{2}{5} \sec^2 \theta$$

M2

M1 M1

For stationary value of  $T$ ,  $\frac{dT}{d\theta} = 0$

$$\frac{2 \sin \theta}{3 \cos^2 \theta} - \frac{2}{5} \sec^2 \theta = 0$$

M1

$$\frac{2 \sin \theta}{3 \cos^2 \theta} - \frac{2}{5 \cos^2 \theta} = 0$$

$$\frac{10 \sin \theta - 6}{5 \cos^2 \theta} = 0$$

$$\Rightarrow 10 \sin \theta - 6 = 0$$

$$\sin \theta = \frac{3}{5}$$

M1

$$\theta = 0.6435$$

M1

$$PX = 2 \tan 0.6435$$

$$= 1.5 \text{ m} \quad (\text{shown})$$

A1

[OR  $PX = 1.5$

$$2 \tan \theta = 1.5$$

M1

$$\tan \theta = 0.75$$

$$\theta = 0.6435$$

M1

$$\text{When } \theta = 0.6435, \quad \frac{dT}{d\theta} = \frac{2 \sin 0.6435}{3 \cos^2 0.6435} - \frac{2}{5 \cos^2 0.6435} \text{ M1}$$

$$= 0 \text{ (shown) A1}$$

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Qn	Workings	Marks allocation
1	<p>(i) degree of polynomial, <math>Q(x) = 3</math></p> <p>(ii) <math>x^5 - 2x^3 + 2x^2 + 4x - 3 = Ax + B + (x^2 - 1)Q(x)</math>                      subst. <math>x = 1</math>,  <math>1 - 2 + 2 + 4 - 3 = A + B + 0</math>  <math>A + B = 2</math> ----- (1)</p> <p>subst. <math>x = -1</math>,  <math>-1 + 2 + 2 - 4 - 3 = -A + B</math>  <math>-A + B = -4</math> ----- (2)</p> <p>(1) + (2), <math>2B = -2</math>  <math>B = -1</math></p> <p>subst. <math>B = -1</math> into (1), <math>A - 1 = 2</math>  <math>A = 3</math></p> <p>The remainder is <math>3x - 1</math>.</p> <p>Alternate Method: long division</p> $x^5 - 2x^3 + 2x^2 + 4x - 3 = 3x - 1 + (x^2 - 1)(x^3 - x + 2)$	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1 each for correct A and B value</p> <p>A1</p> <p>2 m for remainder                      3 m for quotient                      (1 m for each term)</p>
2	<p>(i) <math>\frac{1}{xy} = -\frac{1}{4}\left(\frac{x}{y}\right) + C</math>                      subst. (4, 9), <math>9 = -\frac{1}{4}(4) + C</math>  <math>C = 10</math></p> <p>Graph cuts at horizontal axis <math>\rightarrow \frac{1}{xy} = 0</math></p> $0 = -\frac{1}{4}\left(\frac{x}{y}\right) + 10$ $\frac{y}{x} = \frac{1}{40}$ <p>(ii)</p> $\frac{1}{xy} = -\frac{1}{4}\left(\frac{x}{y}\right) + 10$ $1 = -\frac{1}{4}(x^2) + 10xy$	<p>M1</p> <p>M1</p> <p>A1</p>

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Qn	Workings	Marks allocation
	$10xy = 1 + \frac{1}{4}x^2$ $40xy = 4 + x^2$ $y = \frac{4+x^2}{40x}, \text{ thus } a=1, b=4 \text{ and } c=40$	B3, 1 m each, working must be seen
3	<p>(i) First term = <math>2^n</math></p> <p>Coeff. of third term = <math>\binom{n}{2}(2x^2)^{n-2}\left(\frac{3}{x}\right)^2</math></p> $= \frac{n(n-1)}{2}(2^{n-2}3^2)(x^2)^{n-2}\left(\frac{1}{x}\right)^2$ <p>Thus, <math>\frac{\frac{n(n-1)}{2}(2^{n-2}3^2)}{2^n} = 81</math></p> $n(n-1) = \frac{81}{2^{-3}3^2}$ $n^2 - n - 72 = 0$ $(n+8)(n-9) = 0$ $n+8 = 0 \text{ or } n-9 = 0$ $n = -8 \text{ (rejected) } n = 9$ <p>(ii) <math>\left(2x^2 + \frac{3}{x}\right)^9</math></p> $= 512x^{18} + \binom{9}{1}(2x^2)^8\left(\frac{3}{x}\right) + \binom{9}{2}(2x^2)^7\left(\frac{3}{x}\right)^2 + \dots$ $= 512x^{18} + 6912x^{15} + 41472x^{12} + \dots$ <p>(iii) <math>T_{r+1} = \binom{9}{r}(2x^2)^{9-r}\left(\frac{3}{x}\right)^r</math></p> $\rightarrow 2(9-r) - r = 0$ $r = 6$ <p>Term independent of <math>x = \binom{9}{6}(2)^{9-6}(3)^6</math></p> $= 489888$	<p>M1</p> <p>M1, o.e., formulating eqn</p> <p>A1, must reject negative value</p> <p>B2, minus 1 m for 1 error</p> <p>M1, o.e. (e.g. expansion)</p> <p>A1</p>
4	<p>(i) <math>\frac{11-7x}{3x^2+11x-4} = \frac{11-7x}{(3x-1)(x+4)}</math></p>	

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Qn	Workings	Marks allocation
	$\frac{11-7x}{(3x-1)(x+4)} = \frac{A}{3x-1} + \frac{B}{x+4}$ $11-7x = A(x+4) + B(3x-1)$ <p>subst <math>x = -4</math>, <math>11+28 = B(-13)</math>  <math>B = -3</math></p> <p><math>x = 0</math>, <math>11 = 4A + 3</math>  <math>4A = 8</math>  <math>A = 2</math></p> <p>Therefore, <math>\frac{11-7x}{3x^2+11x-4} = \frac{2}{3x-1} - \frac{3}{x+4}</math></p> <p>(ii)</p> $\int_1^2 \frac{11-7x}{9x^2+33x-12} dx$ $= \int_1^2 \frac{11-7x}{3(3x^2+11x-4)} dx$ $= \int_1^2 \frac{11-7x}{3(3x-1)(x+4)} dx$ $= \frac{1}{3} \int_1^2 \frac{2}{3x-1} - \frac{3}{x+4} dx$ $= \frac{1}{3} \left[ \frac{2}{3} \ln(3x-1) - 3 \ln(x+4) \right]_1^2$ $= \frac{1}{3} \left[ \frac{2}{3} \ln(5) - 3 \ln(6) \right] - \frac{1}{3} \left[ \frac{2}{3} \ln(2) - 3 \ln(5) \right]$ $= \frac{1}{3} \left[ \frac{2}{3} \ln\left(\frac{5}{2}\right) + 3 \ln\left(\frac{5}{6}\right) \right]$ $= \frac{2}{9} \ln\left(\frac{5}{2}\right) + \ln\left(\frac{5}{6}\right)$ $\approx 0.0213$	<p>M1</p> <p>A1</p> <p>A1</p> <p>minus 1m if not written in partial fractions form</p> <p>M1, o.e.</p> <p>M1, integrating ln</p> <p>[M1, subst]</p> <p>A1</p>
5	<p>(i) let <math>f(x) = 2x^3 + x^2 - 5x + 2</math></p> <p><math>f(1) = 0</math>          therefore, <math>x-1</math> is a factor of <math>f(x)</math></p> $2x^3 + x^2 - 5x + 2 = (x-1)(2x^2 + ax - 2)$ <p>comparing coefficient of <math>x</math>, <math>-5 = -a - 2</math>  <math>a = 3</math></p> <p>therefore, <math>f(x) = (x-1)(2x^2 + 3x - 2)</math></p>	<p>M1</p> <p>M1, o.e.</p>

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Qn	Workings	Marks allocation
	$= (x-1)(2x-1)(x+2)$ $2x^3 + x^2 - 5x + 2 = 0$ $(x-1)(2x-1)(x+2) = 0$ $x-1=0 \text{ or } 2x-1=0 \text{ or } x+2=0$ $x=1 \qquad x=\frac{1}{2} \qquad x=-2$ <p>(ii) <math>16 \tan^3 \theta + 4 \tan^2 \theta - 10 \tan \theta + 2 = 0</math>  <math>2(2 \tan \theta)^3 + (2 \tan \theta)^3 - 5(2 \tan \theta) + 2 = 0</math></p> <p>By comparing, <math>x = 2 \tan \theta</math>,  <math>(2 \tan \theta - 1)(4 \tan \theta - 1)(2 \tan \theta + 2) = 0</math></p> $2 \tan \theta - 1 = 0 \text{ or } 4 \tan \theta - 1 = 0 \text{ or } 2 \tan \theta + 2 = 0$ $\tan \theta = \frac{1}{2} \quad \text{or} \quad \tan \theta = \frac{1}{4} \quad \text{or} \quad \tan \theta = -1 \text{ (rejected)}$ $\theta \approx 26.6^\circ \qquad \theta \approx 14.0^\circ$	<p>A2, minus 1m for 1 error</p> <p>M1, or identify <math>x = 2 \tan \theta</math></p> <p>M1 (factorised)</p> <p>A2, minus 1 m if <math>\tan \theta = -1</math> not rejected</p>
6	<p>(i) <math>\frac{dy}{dx} = \frac{e^{5x} + 1}{e^{3x}}</math>  <math>\frac{dy}{dx} = e^{2x} + e^{-3x}</math></p> <p>when <math>\frac{dy}{dx} = 0</math>,</p> $e^{2x} + e^{-3x} = 0$ $e^{2x} = -e^{-3x}$ $e^{2x} \div e^{-3x} = -1$ $e^{5x} = -1$ <p><math>x</math> is undefined, thus the curve does not have stationary points.</p> <p><u>OR</u></p> $e^{5x} = -1 \text{ (rejected)}$ <p>Since <math>e^{5x} &gt; 0</math> for all values of <math>x</math>, hence the curve does not have stationary points</p> <p><u>OR</u></p> <p>For all real values of <math>x</math>, <math>e^{2x} &gt; 0</math> and <math>e^{-3x} &gt; 0</math>,</p> $\therefore \frac{dy}{dx} > 0, \frac{dy}{dx} \text{ can never be zero.}$ $\therefore \text{the curve has no stationary point.}$	<p>M1, o.e.</p> <p>A1, conclusion</p> <p>M1 A1</p>



Qn	Workings	Marks allocation
	<p>(ii) <math>\frac{dy}{dx} = e^{2x} + e^{-3x}</math></p> $y = \frac{e^{2x}}{2} - \frac{e^{-3x}}{3} + c$ <p>subst. <math>(0, \frac{1}{2})</math>,</p> $\frac{1}{2} = \frac{e^{2(0)}}{2} - \frac{e^{-3(0)}}{3} + c$ $\frac{1}{2} = \frac{1}{2} - \frac{1}{3} + c$ $c = \frac{1}{3}$ <p>Eqn of curve is <math>y = \frac{e^{2x}}{2} - \frac{e^{-3x}}{3} + \frac{1}{3}</math></p> <p>when <math>x = 2</math>, <math>y = \frac{e^{2(2)}}{2} - \frac{e^{-3(2)}}{3} + \frac{1}{3}</math></p> $y = \frac{e^4}{2} - \frac{1}{3e^6} + \frac{1}{3}$ $y \approx 27.6$	<p>M2, integrate exponential</p> <p>M1</p> <p>M1</p> <p>M1, subst into eqn of curve]</p> <p>A1</p>
7	<p>(i) <math>y = \frac{(x-3)^2}{2x+5}</math></p> $\frac{dy}{dx} = \frac{(2x+5)(2)(x-3) - (x-3)^2(2)}{(2x+5)^2}$ $= \frac{(2x+5)(2x-6) - (x^2 - 6x + 9)(2)}{(2x+5)^2}$ $= \frac{4x^2 - 12x + 10x - 30 - 2x^2 + 12x - 18}{(2x+5)^2}$ $= \frac{2x^2 + 10x - 48}{(2x+5)^2}$ <p>For stationary points, <math>\frac{dy}{dx} = 0</math></p> $\frac{(2x+5)(2)(x-3) - (x-3)^2(2)}{(2x+5)^2} = 0$ $(2x+5)(2)(x-3) - (x-3)^2(2) = 0$ $(x-3)(4x+10-2x+6) = 0$ $(x-3)(2x+16) = 0$ $x-3=0 \quad \text{or} \quad 2x+16=0$ $x=3 \quad \text{or} \quad x=-8$	<p>M2</p> <p>M1, o.e.</p> <p>A1, for x coordinates</p>

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Qn	Workings	Marks allocation
	<p>subst. <math>x = 3</math>, into <math>y = \frac{(x-3)^2}{2x+5}</math>, <math>y = 0</math></p> <p>subst. <math>x = -8</math>, into <math>y = \frac{(x-3)^2}{2x+5}</math>, <math>y = \frac{(-8-3)^2}{2(-8)+5}</math>, <math>y = -11</math></p> <p>The stationary points are <math>(3,0)</math> and <math>(-8,-11)</math>.</p> <p>(ii) <math>\frac{dy}{dx} = \frac{2x^2 + 10x - 48}{(2x+5)^2}</math></p> <p><math>\frac{d^2y}{dx^2} = \frac{(2x+5)^2(4x+10) - (2x^2 + 10x - 48)(2)(2x+5)(2)}{(2x+5)^4}</math></p> <p>when <math>x = 3</math>, <math>\frac{d^2y}{dx^2} = \frac{2662 - 0}{14641} = \frac{2}{11} &gt; 0</math>, <math>(3,0)</math> is a min. pt.</p> <p>when <math>x = -8</math>, <math>\frac{d^2y}{dx^2} = \frac{-2662 - 0}{14641} = -\frac{2}{11} &lt; 0</math>, <math>(-8,0)</math> is a max. pt.</p>	<p>A1, for <math>y</math> coordinates                      [minus 1m if not written in coordinates form]</p> <p>M2</p> <p>A1</p> <p>A1</p>
8	<p>(i) Perimeter of pendent  <math>= 1 + 1 + 2 \times 2.8 + 2 \times 2.8 \sin \theta + 2 \times 2.8 \cos \theta</math>  <math>= (5.6 \sin \theta + 5.6 \cos \theta + 7.6) \text{ cm}</math> (Shown)</p> <p>(ii) <math>R = \sqrt{5.6^2 + 5.6^2}</math>  <math>= \sqrt{62.72}</math>  <math>\approx 7.92</math></p> <p><math>\tan \alpha = \frac{5.6}{5.6}</math>  <math>\alpha = 45^\circ</math></p> <p><math>P = 7.92 \sin(\theta + 45^\circ) + 7.6</math></p> <p>(iii) <math>15 = 7.92 \sin(\theta + 45^\circ) + 7.6</math>  <math>7.4 = 7.92 \sin(\theta + 45^\circ)</math>  <math>\sin(\theta + 45^\circ) = \frac{185}{198}</math>                      Basic angle <math>= 69.1223^\circ</math>  <math>\theta + 45^\circ = 69.1223^\circ, 180^\circ - 69.1223^\circ</math>  <math>\theta = 24.1223^\circ, 65.8777^\circ</math>  <math>\theta \approx 24.1^\circ, 65.9^\circ</math></p>	<p>M1 A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>(minus 1 m if student did not express in this form)</p> <p>M1</p> <p>M1 (basic angle)</p> <p>A1</p>

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Qn	Workings	Marks allocation
9	<p>(i) <math display="block">\frac{7\sqrt{2}}{3\sqrt{2}-2} \times \frac{3\sqrt{2}+2}{3\sqrt{2}+2}</math> <math display="block">= \frac{7\sqrt{2}(3\sqrt{2}+2)}{18-4}</math> <math display="block">= \frac{42+14\sqrt{2}}{14}</math> <math display="block">= 3 + \sqrt{2}</math></p> <p>(ii) by Pythagoras Theorem,  <math>BD^2 = AB^2 + AD^2</math>                      Using part (i) answer,  <math>BD^2 = (3 + \sqrt{2})^2 + (3 + \sqrt{2})^2</math>  <math>BD^2 = 2(3 + \sqrt{2})^2</math>  <math>BD^2 = 2(9 + 6\sqrt{2} + 2)</math>  <math>BD^2 = 22 + 12\sqrt{2}</math></p> <p>(iii) Volume of pyramid  <math>= \frac{1}{3} \times \text{base area} \times \text{height}</math>  <math>= \frac{1}{3} \times (3 + \sqrt{2})^2 \times \frac{1}{2}(22 + 12\sqrt{2})</math>  <math>= \frac{1}{3} \times (11 + 6\sqrt{2}) \times (11 + 6\sqrt{2})</math>  <math>= \frac{1}{3}(121 + 132\sqrt{2} + 72)</math>  <math>= \frac{1}{3}(193 + 132\sqrt{2})</math>  <math>= \frac{193}{3} + 44\sqrt{2} \text{ cm}^3</math></p>	<p>M1, rationalise</p> <p>A1</p> <p>M1, formulating</p> <p>A2, A1 for 22 and A1 for <math>12\sqrt{2}</math></p> <p>M1, subst. correct values</p> <p>M1, correct expansion</p> <p>A2, A1 for <math>\frac{193}{3}</math>, A1 for <math>44\sqrt{2}</math></p>
10	<p>(a) (i) centre <math>C = \left( \frac{-10}{-2}, \frac{8}{-2} \right)</math>  <math>= (5, -4)</math></p> <p>(ii) radius <math>= \sqrt{5^2 + 4^2} - 5</math>  <math>= 6</math></p> <p>Length of <math>PC = \sqrt{(5-4)^2 + (-4+11)^2}</math>  <math>= \sqrt{50}</math>  <math>\approx 7.07</math></p>	<p>B1</p> <p>M1 (o.e.)</p> <p>M1</p>

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Qn	Workings	Marks allocation
	<p>Since <b>length of PC is longer than radius</b> of circle, thus, the point <i>P</i> is outside of the circle.</p> <p>(iii) by Pythagoras' Theorem,  <math display="block">PT = \sqrt{50 - 6^2}</math> <math display="block">= \sqrt{14}</math> <math display="block">\approx 3.74 \text{ units}</math></p> <p>(b) point A = (0, 10)</p> $\frac{dy}{dx} = 2x - 7$ <p>when <math>x = 0</math>, <math>\frac{dy}{dx} = -7</math></p> <p>gradient of normal = <math>\frac{1}{7}</math></p> <p>equation of normal is <math>y = \frac{1}{7}x + 10</math></p>	<p>A1 (find length PC and conclude)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p>
11	<p>(a) <math>y = \tan x</math></p> $\frac{dy}{dx} = \sec^2 x$ $\frac{d^2 y}{dx^2} = 2 \sec x \cdot \sec x \cdot \tan x$ $\frac{d^2 y}{dx^2} = 2 \frac{dy}{dx} y$ $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} y = 0 \text{ (shown)}$ <p><u>Alternate solution</u></p> $\frac{dy}{dx} = \sec^2 x$ $= \frac{1}{\cos^2 x}$ $\frac{d^2 y}{dx^2} = \frac{0 - 2 \cos x (-\sin x)}{\cos^4 x}$ $= 2 \sec x \cdot \sec x \cdot \tan x$ <p>LHS = <math>2 \sec x \cdot \sec x \cdot \tan x - 2 \sec^2 x \tan x</math>  <math>= 0</math>  <math>= \text{RHS}</math></p>	<p>M1</p> <p>M2, 1m for <math>2 \sec x</math>, 1m for <math>\sec x \cdot \tan x</math> (o.e.)</p> <p>A1</p>

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Qn	Workings	Marks allocation
	<p>(b) (i) <math>\int_0^{\pi} 8 \cos^2\left(\frac{x}{2}\right) dx = 4 \int_0^{\pi} 2 \cos^2\left(\frac{x}{2}\right) dx</math></p> $= 4 \int_0^{\pi} (\cos x + 1) dx$ $= 4[\sin x + x]_0^{\pi}$ $= 4[0 + \pi - (0 - 0)]$ $= 4\pi$ <p>(ii) <math>\int_0^{\pi} \left[3 - \sin^2\left(\frac{x}{2}\right)\right] dx = \int_0^{\pi} \left[2 + 1 - \sin^2\left(\frac{x}{2}\right)\right] dx</math></p> $= \int_0^{\pi} \left[2 + \cos^2\left(\frac{x}{2}\right)\right] dx$ $= \int_0^{\pi} 2 dx + \int_0^{\pi} \cos^2\left(\frac{x}{2}\right) dx$ $= [2x]_0^{\pi} + \frac{4\pi}{8}$ $= \frac{5\pi}{2}$	<p>M1, using  <math>\cos x = 2 \cos^2\left(\frac{x}{2}\right) - 1</math></p> <p>M1, integrate</p> <p>A1</p> <p>M1 (apply identity)</p> <p>M1</p> <p>A1</p>
12	<p>(i) sum of roots, <math>2\alpha + \beta + \alpha + 2\beta = 3\alpha + 3\beta</math></p> $= 3(\alpha + \beta)$ $= -\frac{7}{3}$ $= \frac{7}{3}$ <p>product of roots, <math>(2\alpha + \beta)(\alpha + 2\beta) = 2\alpha^2 + 4\alpha\beta + \alpha\beta + 2\beta^2</math></p> $= 2\alpha^2 + 5\alpha\beta + 2\beta^2$ $= \frac{4}{3}$ $\alpha + \beta = \frac{1}{3} \left(\frac{7}{3}\right)$ $= \frac{7}{9}$ <p>(ii) from product of roots, <math>2\alpha^2 + 4\alpha\beta + \alpha\beta + 2\beta^2 = \frac{4}{3}</math></p> $2\alpha^2 + 4\alpha\beta + 2\beta^2 + \alpha\beta = \frac{4}{3}$ $2(\alpha^2 + 2\alpha\beta + \beta^2) + \alpha\beta = \frac{4}{3}$	<p>M1 (<math>3(\alpha + \beta)</math>)</p> <p>M1</p> <p>A1</p> <p>M1, o.e.</p>

Qn	Workings	Marks allocation
	$2(\alpha + \beta)^2 + \alpha\beta = \frac{4}{3}$ $2\left(\frac{7}{9}\right)^2 + \alpha\beta = \frac{4}{3}$ $\alpha\beta = \frac{4}{3} - 2\left(\frac{7}{9}\right)^2$ $\alpha\beta = \frac{10}{81} \text{ (shown)}$	M1    A1
(iii)	<p>sum of roots, <math>\frac{1}{2}\alpha + \beta + \alpha + \frac{1}{2}\beta = \frac{3}{2}(\alpha + \beta)</math></p> $= \frac{3\left(\frac{7}{9}\right)}{2}$ $= \frac{7}{6}$	M1
	<p>Product of roots,</p> $\left(\frac{1}{2}\alpha + \beta\right)\left(\alpha + \frac{1}{2}\beta\right) = \frac{1}{2}\alpha^2 + \frac{1}{4}\alpha\beta + \alpha\beta + \frac{1}{2}\beta^2$ $= \frac{1}{2}\alpha^2 + \frac{5}{4}\alpha\beta + \frac{1}{2}\beta^2$ $= \frac{1}{2}(\alpha^2 + \beta^2) + \frac{5}{4}\alpha\beta$ $= \frac{1}{2}\left((\alpha + \beta)^2 - 2\alpha\beta\right) + \frac{5}{4}\alpha\beta$ $= \frac{1}{2}\left(\left(\frac{7}{9}\right)^2 - 2\left(\frac{10}{81}\right)\right) + \frac{5}{4}\left(\frac{10}{81}\right)$ $= \frac{1}{3}$	M1    M1  M1
	<p>The quadratic equation is <math>x^2 - \frac{7}{6}x + \frac{1}{3} = 0</math></p>	A1, accept $6x^2 - 7x + 2 = 0$