SINGAPORE CHINESE GIRLS' SCHOOL
PRELIMINARY EXAMINATION 2018
SECONDARY FOUR
O-LEVEL PROGRAMME

ADDITIONAL MATHEMATICS **4047/01**
Paper 1

Wednesday **1 August 2018** **2 hours**

Additional Materials: Answer Paper
Graph Paper
Cover Page

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved electronic scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1. A rectangle has a length of $(6\sqrt{3}+3)$ cm and an area of 66 cm^2. Find the perimeter of the rectangle in the form $(a+b\sqrt{3})$ cm, where a and b are integers. [3]

2. On the same axes sketch the curves $y^2 = 225x$ and $y = 15x^3$. [3]

3. **(i)** Find the exact value of 15^x, given that $25^{x+2} = 36 \times 9^{1-x}$. [3]

(ii) Hence, find the value of x, giving your answer to 2 decimal places. [2]

4. **(a)** Given that $\log_3 y - \log_3 x = 1 + \log_3(x+y)$, express y in terms of x. [3]

(b) Solve the equation $\log_3(8-x) + \log_3 x = 2\log_9 15$. [4]

5. The equation of a curve is $y = \dfrac{x-4}{\sqrt{2x+5}}$.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ can be expressed in the form $\dfrac{ax+b}{(2x+5)^{\frac{3}{2}}}$ where a and b are constants. [3]

(ii) Given that y is increasing at a rate of 0.4 units per second, find the rate of change of x when $x = 2$. [2]

6. The roots of the quadratic equation $4x^2 + x - m = 0$, where m is a constant, are α and β.

The roots of the quadratic equation $8x^2 + nx + 1 = 0$, where n is a constant, are $\dfrac{1}{\alpha^3}$ and $\dfrac{1}{\beta^3}$.

(i) Show that $m = -8$ and hence find the value of n. [5]

(ii) Find a quadratic equation whose roots are $\alpha + 2$ and $\beta + 2$. [4]

7. **(i)** Show that $\dfrac{2-2\sec^2 x}{(1+\cos x)(1-\cos x)} = -2\sec^2 x$. [3]

(ii) Hence find, for $-\pi \le x \le \pi$, the values of x in radians for which

$$\frac{2-2\sec^2 x}{(1+\cos x)(1-\cos x)} = 4\tan x.$$ [4]

8. The temperature, T °C, of a container of liquid decreases with time, t minutes. Measured values of T and t are given in the table below.

t (min)	10	20	30	40
T (°C)	58.5	41.6	34.7	31.9

It is known that T and t are related by the equation $T = 30 + pe^{-qt}$, where p and q are constants.

(i) On a graph paper, plot $\ln(T-30)$ against t for the given data and draw a straight line graph. [3]

(ii) Use you graph to estimate the value of p and of q. [4]

(iii) Explain why the temperature of the liquid can never drop to 30°C. [1]

9. Given that $y = 2xe^{1-x}$, find

(i) $\dfrac{dy}{dx}$, [2]

(ii) p for which $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + pe^{1-x} = 0$, [4]

(iii) the range of values of x for which y is an increasing function. [3]

10. An open rectangular cake tin is made of thin sheets of steel which costs \$2 per 1000 cm^2. The tin has a square base of length x cm, a height of h cm and a volume of 4000 cm^3.

(i) Show that the cost of steel, C, in dollars, for making the cake tin is given by

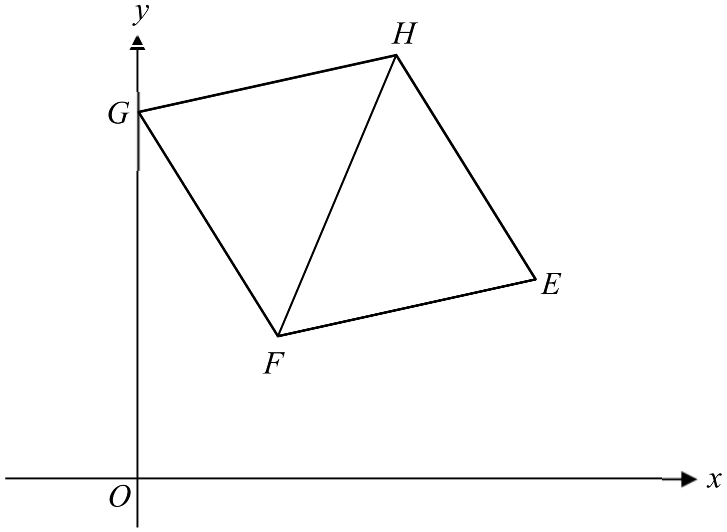
$$C = \frac{x^2}{500} + \frac{32}{x}.$$ [2]

Given that x can vary,

(ii) find the value of x for which C has a stationary value, [3]

(iii) explain why this value of x gives the minimum value of C. [3]

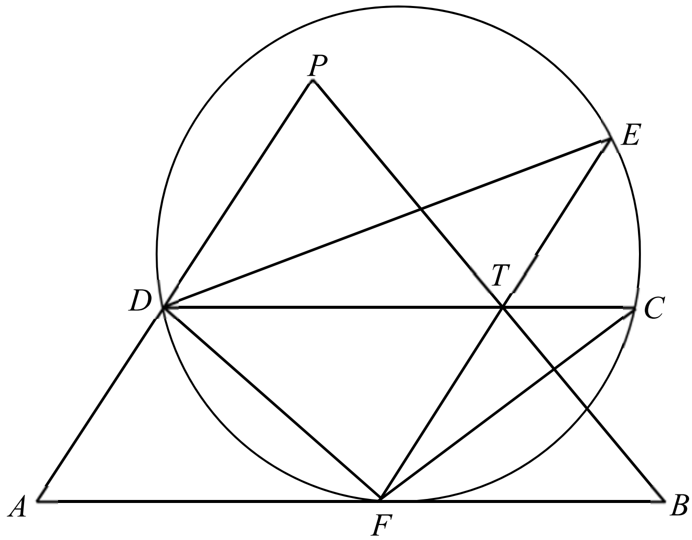
11. The diagram shows a kite *EFGH* with *EF* = *EH* and *GF* = *GH*. The point *G* lies on the *y*-axis and the coordinates of *F* and *H* are (2, 1) and (6, 9) respectively.



The equation of *EF* is $y = \frac{x}{8} + \frac{3}{4}$.

Find
(i) the equation of *EG*, [4]
(ii) the coordinates of *E* and *G*, [3]
(iii) the area of the kite *EFGH*. [2]

12.



The diagram shows a circle passing through points *D*, *E*, *C* and *F*, where *FC* = *FD*.
The point *D* lies on *AP* such that *AD* = *DP*.
DC and *EF* cut *PB* at *T* such that *PT* = *TB*.

(i) Show that *AB* is a tangent to the circle at point *F*. [3]

(ii) By showing that triangle *DFT* and triangle *EFD* are similar, show that
$$DF^2 - FT^2 = FT \times ET .$$ [4]

End of Paper 1

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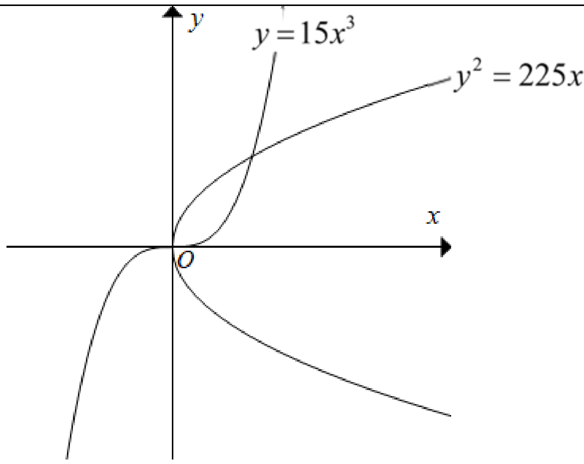
Paper 1

1.	Breadth $= \dfrac{66}{6\sqrt{3}+3}$ or $\dfrac{22}{2\sqrt{3}+1}$ $= \dfrac{66}{6\sqrt{3}+3} \times \dfrac{6\sqrt{3}-3}{6\sqrt{3}-3}$ $\quad \dfrac{22}{2\sqrt{3}+1} \times \dfrac{2\sqrt{3}-1}{2\sqrt{3}-1}$ $= \dfrac{66(6\sqrt{3}-3)}{99}$ $\quad \dfrac{22(2\sqrt{3}-1)}{11}$ $= 4\sqrt{3}-2$ cm Perimeter $= 2(6\sqrt{3}+3+4\sqrt{3}-2)$ $= 20\sqrt{3}+2$ cm
2.	
3. (i)	$25^{x+2} = 36 \times 9^{1-x}$ $(5^{2x})(5^4) = \dfrac{2^2 \times 9^2}{3^{2x}}$ $(5^{2x})(3^{2x}) = \dfrac{2^2 \times 9^2}{25^2}$ $(15^x)^2 = \dfrac{2^2 \times 9^2}{25^2}$ $15^x > 0,\ 15^x = \dfrac{18}{25}$
(ii)	$15^x = \dfrac{18}{25}$ $x \lg 15 = \lg\left(\dfrac{18}{25}\right)$ $x = \dfrac{\lg\left(\dfrac{18}{25}\right)}{\lg 15}$ $= -0.12$
4. (a)	$\log_3 y - \log_3 x = 1 + \log_3(x+y)$

		$\log_3 \frac{y}{x} = \log_3 3 + \log_3 (x+y)$ $\frac{y}{x} = 3(x+y)$ $y = 3x^2 + 3xy$ $y - 3xy = 3x^2$ $y(1-3x) = 3x^2$ $y = \frac{3x^2}{1-3x}$
	(b)	$\log_3(8-x) + \log_3 x = 2\log_9 15$ $\log_3[x(8-x)] = \frac{2\log_3 15}{\log_3 9}$ $\log_3[x(8-x)] = \frac{2\log_3 15}{2\log_3 3}$ $8x - x^2 = 15$ $x^2 - 8x + 15 = 0$ $(x-3)(x-5) = 0$ $x = 3, \quad 5$
5.	(i) (ii)	$\frac{dy}{dx} = \frac{(2x+5)^{\frac{1}{2}}(1) - \frac{1}{2}(x-4)(2x+5)^{-\frac{1}{2}}(2)}{2x+5}$ $= \frac{(2x+5)^{-\frac{1}{2}}(2x+5-x+4)}{2x+5}$ $= \frac{x+9}{(2x+5)^{\frac{3}{2}}}$ When $x = 2$, $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $0.4 = \frac{2+9}{(4+5)^{\frac{3}{2}}} \times \frac{dx}{dt}$ $\frac{dx}{dt} = 0.4 \times \frac{27}{11}$ $= \frac{54}{55}$ or 0.982 unit per second

6.	**(i)**	$\alpha+\beta=-\frac{1}{4}$ $\alpha\beta=-\frac{m}{4}$ $\frac{1}{(\alpha\beta)^3}=\frac{1}{8}$ $\alpha\beta=2$ $\therefore -\frac{m}{4}=2$ $m=-8$ $\frac{1}{\alpha^3}+\frac{1}{\beta^3}=-\frac{n}{8}$ $\frac{\alpha^3+\beta^3}{\alpha^3\beta^3}=-\frac{n}{8}$ $\alpha^3+\beta^3=-n$ $-n=(\alpha+\beta)\left[(\alpha+\beta)^2-3\alpha\beta\right]$ $n=-\left(-\frac{1}{4}\right)\left[\left(-\frac{1}{4}\right)^2-6\right]$ $=-\frac{95}{64}$
	(ii)	Sum of roots $=\alpha+\beta+4$ $=\frac{15}{4}$ Product of roots $=(\alpha+2)(\beta+2)$ $=\alpha\beta+2(\alpha+\beta)+4$ $=2+2\left(-\frac{1}{4}\right)+4$ $=\frac{11}{2}$ New equation: $x^2-\frac{15}{4}x+\frac{11}{2}=0$ or $4x^2-15x+22=0$

7. (i)

$$\frac{2-2\sec^2 x}{(1+\cos x)(1-\cos x)} = \frac{-2(\sec^2 x-1)}{\sin^2 x}$$
$$= \frac{-2\tan^2 x}{\sin^2 x}$$
$$= \frac{-2\left(\frac{\sin^2 x}{\cos^2 x}\right)}{\sin^2 x}$$
$$= \frac{-2}{\cos^2 x}$$
$$= -2\sec^2 x$$

(ii)

$$\frac{2-2\sec^2 x}{(1+\cos x)(1-\cos x)} = 4\tan x$$
$$-2\sec^2 x = 4\tan x$$
$$-\frac{1}{\cos^2 x} = \frac{2\sin x}{\cos x}$$
$$-1 = 2\sin x\cos x$$
$$\sin 2x = -1$$
$$2x = -\frac{\pi}{2}, \frac{3\pi}{2}$$
$$x = -\frac{\pi}{4}, \frac{3\pi}{4}$$

8. (i)

t (min)	10	20	30	40
$\ln(T-30)$	3.35	2.45	1.55	0.64

(ii)

$$T = 30 + pe^{-qt}$$
$$\ln(T-30) = \ln p - qt$$
$$\ln p = 4.25$$
$$p = e^{4.25} = 70.1$$

$$-q = \text{gradient}$$
$$= \frac{0.65-4.25}{40}$$
$$= -0.09$$

(ii)

Since $e^{-qt} > 0$, $T = 30 + 70e^{-0.09t} > 30$

Hence, $T > 30$ for all values of t.

9.	(i)	$\dfrac{dy}{dx} = 2e^{1-x} - 2xe^{1-x}$
	(ii)	$\dfrac{d^2y}{dx^2} = -2e^{1-x} - 2e^{1-x} + 2xe^{1-x}$ $= -4e^{1-x} + 2xe^{1-x}$ $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + pe^{1-x} = 0$ $-pe^{1-x} = \dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx}$ $= -4e^{1-x} + 2xe^{1-x} + 2(2e^{1-x} - 2xe^{1-x})$ $= -4e^{1-x} + 2xe^{1-x} + 4e^{1-x} - 4xe^{1-x}$ $= -2xe^{1-x}$ $p = 2x$
	(iii)	When $\dfrac{dy}{dx} > 0$, $2e^{1-x} - 2xe^{1-x} > 0$ $2e^{1-x}(1-x) > 0$ Since $e^{1-x} > 0$ for all x, $1 - x > 0$ $x < 1$
10.	(i)	$x^2h = 4000 \Rightarrow h = \dfrac{4000}{x^2}$ $C = \dfrac{2}{1000} \times (x^2 + 4hx)$ $= \dfrac{2}{1000}\left(x^2 + 4x \times \dfrac{4000}{x^2}\right)$ $= \dfrac{x^2}{500} + \dfrac{32}{x}$
	(ii)	$\dfrac{dC}{dx} = \dfrac{x}{250} - \dfrac{32}{x^2}$ When $\dfrac{dC}{dx} = 0$, $\dfrac{x}{250} - \dfrac{32}{x^2} = 0$ $x^3 = 8000$ $x = 20$
	(iii)	$\dfrac{d^2C}{dx^2} = \dfrac{1}{250} + \dfrac{64}{x^3}$ When $x = 20$, $\dfrac{d^2C}{dx^2} = \dfrac{3}{250} > 0$ Since, $\dfrac{d^2C}{dx^2} > 0$ when $x = 20$, C has a minimum value.

11.	**(i)**	Gradient of $FH = \dfrac{9-1}{6-2} = 2$ Gradient of $EG = -\dfrac{1}{2}$
	(ii)	Midpoint of $FH = \left(\dfrac{2+6}{2}, \dfrac{1+9}{2}\right)$ $= (4, 5)$ Equation of EG: $y - 5 = -\dfrac{1}{2}(x-4)$ $y = -\dfrac{x}{2} + 7$
	(iii)	$-\dfrac{x}{2}x + 7 = \dfrac{x}{8} + \dfrac{3}{4}$ $\dfrac{5x}{8} = \dfrac{25}{4} \Rightarrow x = 10$ $y = 2$ Coordinate of $E = (10, 2)$ $y = -\dfrac{x}{2} + 7$ When $x = 0$, $y = 7$ Coordinate of $G = (0, 7)$
	(iv)	Area of $EFGH$ $= \dfrac{1}{2}\begin{vmatrix} 0 & 2 & 10 & 6 & 0 \\ 7 & 1 & 2 & 9 & 7 \end{vmatrix}$ $= \dfrac{1}{2}\left[(4+90+42)-(14+10+12)\right]$ $= 50 \text{ unit}^2$ **<u>Alternative Method</u>** Area of $EFGH = \dfrac{1}{2} \times HF \times GE$ $= \dfrac{1}{2} \times \sqrt{4^2+8^2} \times \sqrt{10^2+5^2}$ $= 50 \text{ units}^2$

12.	**(i)**	*DT* is parallel to *AB*. (Midpoint Theorem) $\angle AFD = \angle TDF$ (alt angles) $= \angle FED$ Since $\angle AFD$ and $\angle FED$ satisfies the alternate segment theorem, *AB* is a tangent at *F*.
	(ii)	$\angle DFE$ is common. $\angle TDF = \angle DCF$ (base angles of an isos triangle) $\angle DCF = \angle DEF$ (angles in the same segment) $\therefore DFT$ and *EFD* are similar triangles (AA) $\frac{DF}{EF} = \frac{FT}{FD}$ $DF^2 = FT \times EF$ $= FT \times (ET + TF)$ $= FT^2 + FT \times ET$ $DF^2 - FT^2 = FT \times ET$

SINGAPORE CHINESE GIRLS' SCHOOL
PRELIMINARY EXAMINATION 2018
SECONDARY FOUR
O-LEVEL PROGRAMME

ADDITIONAL MATHEMATICS **4047/02**
Paper 2

Friday **3 August 2018** **2 hours 30 minutes**

Additional Materials: Answer Paper
Cover Page

READ THESE INSTRUCTIONS FIRST

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Write in dark blue or black pen on both sides of the paper.
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Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved electronic scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

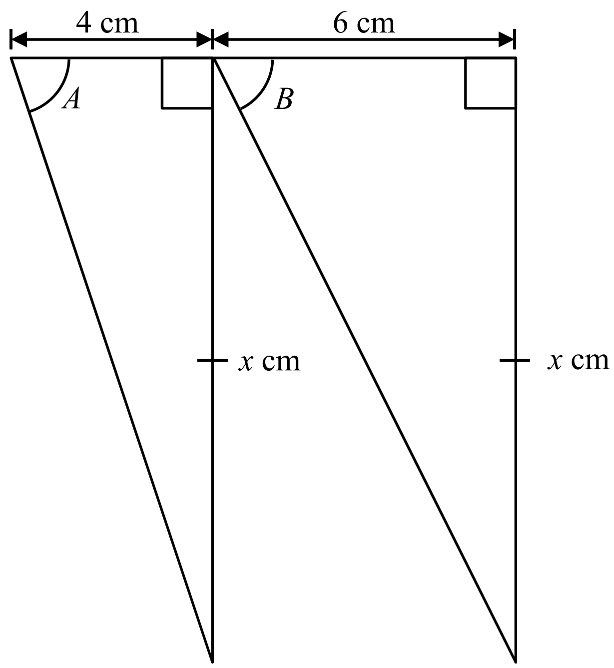
1. **(a)** In the expansion of $(3x-1)(1-kx)^7$ where k is a non-zero constant, there is no term in x^2. Find the value of k. [4]

(b) In the binomial expansion of $\left(\dfrac{2}{x^3}-x^2\right)^{12}$, in ascending powers of x, find the term in which the power of x first becomes positive. [4]

2. **(a)** Explain why the curve $y=px^2+2x-p$ will always cut the line $y=-1$ at two distinct points for all real values of p. [4]

(b) Find the values of a such that the curve $y=ax^2+x+a$ lies below the x-axis. [4]

3. **(a)** The diagram shows two right-angled triangles with the same height x cm.
One triangle has a base of 4 cm and the other triangle has a base of 6 cm.
Angles A and B are such that $A + B = 135°$.



Find the value of x. [4]

(b) The current y (in amperes), in an alternating current (A.C.) circuit, is given by

$y=170\sin(kt)$, where t is the time in seconds.

The period of this function is $\dfrac{1}{60}$ second.

(i) Find the amplitude of y. [1]

(ii) Find the exact value of k in radians per second. [1]

(iii) For how long in a period is $y > 85$? [3]

4. The function $g(x) = 2x^4 + x^3 + 4x^2 + hx - k$ has a quadratic factor $2x^2 + 3x + 1$.

(i) Find the value of h and of k. [5]

(ii) Determine, showing all necessary working, the number of real roots of the equation $g(x) = 0$. [4]

5. The function f is defined by $f(x) = 4 + 2x - 3x^2$.

(i) Find the value of a, of b and of c for which $f(x) = a + b(x+c)^2$. [4]

(ii) State the maximum value of f(x) and the corresponding value of x. [2]

(iii) Sketch the curve of $y = |f(x)|$ for $-1 \le x \le 2$, indicating on your graph the coordinates of the maximum point. [3]

(iv) State the value(s) of k for which $|f(x)| = k$ has

(a) 1 solution, [1]

(b) 3 solutions. [1]

6. **(i)** Find $\dfrac{d}{dx}\left[(\ln x)^2\right]$. [2]

(ii) Using the result from part **(i)**, find $\displaystyle\int \frac{3x^3 - 5\ln x}{x}\,dx$ and hence show that

$$\int_1^e \frac{3x^3 - 5\ln x}{x}\,dx = e^3 - \frac{7}{2}.$$ [4]

7. **(i)** Show that $\dfrac{d}{dx}(\sec x) = \sec x \tan x$. [2]

(ii) Given that $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$, find the value of n for which $y = e^{\tan x}$ is a solution of the equation

$$\frac{d^2y}{dx^2} = (1 + \tan x)^n \frac{dy}{dx}.$$ [7]

8. A circle passes through the points $A(2, 6)$ and $B(5, 5)$, with its centre lying on the line $3y = -x + 5$.

(i) Find the perpendicular bisector of AB. [3]

(ii) Find the equation of the circle. [4]

CD is a diameter of the circle and the point P has coordinates $(-2, -1)$.

(iii) Determine whether the point P lies inside the circle. [2]

(iv) Is angle CPD a right angle? Explain. [1]

9. **(i)** Given that $\frac{x^2-4x+1}{x^2-6x+9} = A + \frac{B}{x-3} + \frac{C}{(x-3)^2}$, where A, B and C are constants, find the value of A, of B and of C. [4]

(ii) Hence, find the coordinates of the turning point on the curve, $y = \frac{x^2-4x+1}{x^2-6x+9}$ and determine the nature of this turning point. [6]

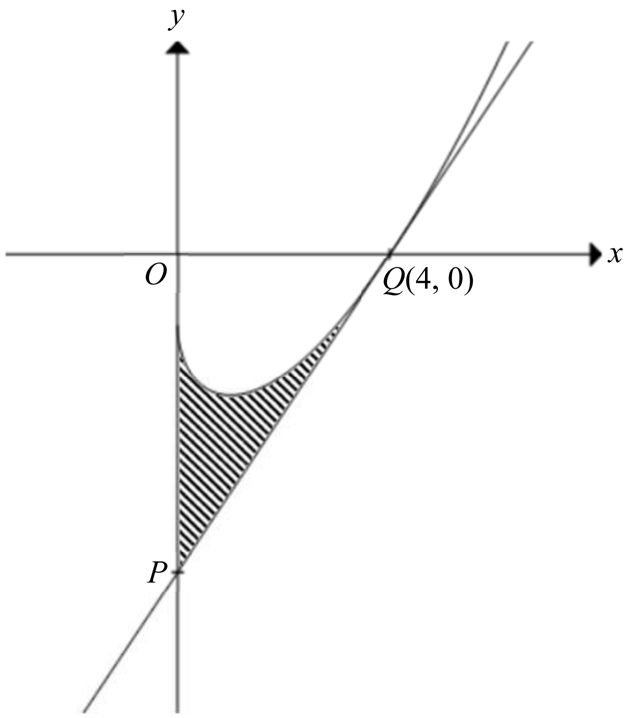
10. A particle starts from rest at O and moves in a straight line with an acceleration of a ms^{-2}, where $a = 2t - 1$ and t is the time in seconds since leaving O.

(i) Find the value of t for which the particle is instantaneously at rest. [4]

(ii) Show that the particle returns to O after $1\frac{1}{2}$ seconds. [4]

(iii) Find the distance travelled in the first 4 seconds. [2]

11. The diagram below shows part of a curve $y = \mathrm{f}(x)$. The curve is such that $\mathrm{f}'(x) = x^{\frac{1}{2}} - x^{-\frac{1}{2}}$ and it passes through the point $Q(4, 0)$. The tangent at Q meets the y-axis at the point P.



(i) Find $\mathrm{f}(x)$. [3]

(ii) Show that the y-coordinate of P is -6. [3]

(iii) Find the area of the shaded region. [4]

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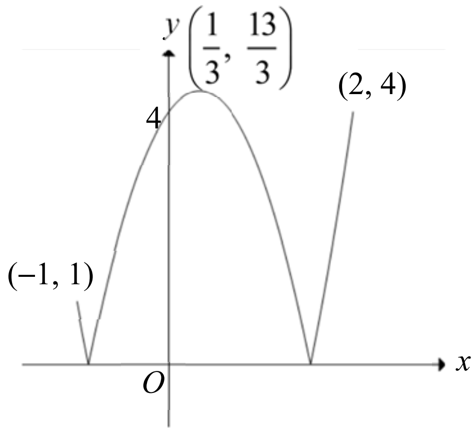
Solution

1.	**(a)**	$(3x-1)(1-kx)^7$ $=(3x-1)(1-7kx+21k^2x^2+...)$ $-21k-21k^2=0$ $-21k(1+k)=0$ $k\neq 0,\ k=-1$
	(b)	$T_{r+1}=\binom{12}{r}\left(\frac{2}{x^3}\right)^{12-r}(-x^2)^r$ $=\binom{12}{r}(2^{12-r})(-1)^r x^{5r-36}$ $5r-36>0$ $r>7.2$ $r=8$ $T_9=\binom{12}{8}(2^4)(-1)^8x^{40-36}$ $=7920x^4$
2.	**(a)**	$px^2+2x-p=-1$ $px^2+2x+1-p=0$ $D=4-4(p)(1-p)$ $=4p^2-4p+4$ $=4(p^2-p+1)$ or $4p^2-4p+1+3$ $=4\left[\left(p-\frac{1}{2}\right)^2+\frac{3}{4}\right]$ $(2p-1)^2+3$ $=4\left(p-\frac{1}{2}\right)^2+3>0$ $(2p-1)^2+3>0$ Since $\left(p-\frac{1}{2}\right)^2\geq 0$ or $(2p-1)^2\geq 0$, the discriminant > 0, the curve will always cut the the line at two distinct points for all real values of p.
	(b)	$D=1-4a^2<0$ $D=1-4a^2<0$ and $a<0$ $(1+2a)(1-2a)<0$ or $4a^2-1>0$ $(2a-1)(2a+1)>0$ $a<-\frac{1}{2}$ or $a>\frac{1}{2}$ $\therefore a<-\frac{1}{2}$

3.	**(a)**	$\tan A = \frac{x}{4}$, $\tan B = \frac{x}{6}$ $\tan(A+B) = \tan 135°$ $\frac{\tan A + \tan B}{1 - \tan A \tan B} = -1$ $\frac{x}{4} + \frac{x}{6} = -1 + \left(\frac{x}{4}\right)\left(\frac{x}{6}\right)$ $6x + 4x = -24 + x^2$ $x^2 - 10x - 24 = 0$ $(x-12)(x+2) = 0$ $x = 12, \ -2$ (NA)
	(b)	$y = 170\sin(kt)$
	(i)	Amplitude = 170 or 170 A
	(ii)	$k = 2\pi \div \frac{1}{60}$ $= 120\pi$
	(iii)	When $y = 85$, $170\sin(120\pi t) = 85$ $\sin(120\pi t) = \frac{85}{170} = \frac{1}{2}$ $120\pi t = \frac{\pi}{6}, \ \frac{5\pi}{6}$ $t = \frac{1}{720}, \ \frac{5}{720}$ Duration $= \frac{5}{720} - \frac{1}{720}$ $= \frac{1}{180}$ seconds

4.	**(i)**	$g(x)=2x^4+x^3+4x^2+hx-k$ $2x^2+3x+1=(2x+1)(x+1)$ $g\left(-\frac{1}{2}\right)=0$ $2\left(-\frac{1}{2}\right)^4+\left(-\frac{1}{2}\right)^3+4\left(-\frac{1}{2}\right)^2+h\left(-\frac{1}{2}\right)-k=0$ $1-\frac{h}{2}-k=0$ $k+\frac{h}{2}=1 \quad \ldots\ldots(1)$ $g(-1)=0$ $2-1+4-h-k=0$ $h+k=5 \quad \ldots\ldots(2)$ $\frac{h}{2}=4$ $h=8$ $k=-3$ **Alternative method** $2x^4+x^3+4x^2+hx-k=(2x^2+3x+1)(x^2+bx-k)$ Comparing coefficient of x^3, $1=2b+3$ $b=-1$ Comparing coefficient of x^2, $4=-2k+3b+1$ $k=-3$ Comparing coefficient of x, $h=b-3k$ $h=8$
	(ii)	Let $g(x)=(2x^2+3x+1)(x^2+bx+3)$ Comparing coefficient of x, $8=9+b$ $b=-1$ $g(x)=(2x^2+3x+1)(x^2-x+3)$ $g(x)=0$ $(2x+1)(x+1)(x^2-x+3)=0$ $x=-\frac{1}{2}$, $x=-1$ or $x^2-x+3=0$ $b^2-4ac = 1-12<0$ $=-11<0$ No real roots. Hence, $g(x)=0$ has only 2 real roots

5.	**(i)**	$\mathrm{f}(x) = 4 + 2x - 3x^2$ $= -3\left(x^2 - \frac{2x}{3}\right) + 4$ $= -3\left[\left(x - \frac{1}{3}\right)^2 - \frac{1}{9}\right] + 4$ $= -3\left(x - \frac{1}{3}\right)^2 + \frac{13}{3}$ $a = \frac{13}{3}$, $b = -3$, $c = -\frac{1}{3}$
	(ii)	Max value $= \frac{13}{3}$ or $4\frac{1}{3}$ at $x = \frac{1}{3}$
	(iii)	
	(iv)(a)	$k = \frac{13}{3}$
	(b)	$1 < k \leq 4$

6.	(i)	$\frac{d}{dx}(\ln x)^2 = 2\ln x\left(\frac{1}{x}\right)$ $= \frac{2\ln x}{x}$
	(ii)	$\int \frac{3x^3 - 5\ln x}{x}\,dx = \int 3x^2\,dx - \int \frac{5\ln x}{x}\,dx$ $= x^3 - \frac{5}{2}(\ln x)^2 + C$ $\int_1^e \frac{3x^3 - 5\ln x}{x}\,dx = \left[x^3 - \frac{5}{2}(\ln x)^2\right]_1^e$ $= e^3 - \frac{5}{2}(\ln e)^2 - 1$ $= e^3 - \frac{7}{2}$
7.	**(i)**	$\frac{d}{dx}(\sec x) = \frac{d}{dx}[(\cos x)^{-1}]$ $= (-1)(\cos x)^{-2}(-\sin x)$ $= \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$ $= \sec x \tan x$
	(ii)	$\frac{dy}{dx} = \frac{d}{dx}(e^{\tan x})$ $= \sec^2 x e^{\tan x}$ $\frac{d^2y}{dx^2} = \frac{d}{dx}(\sec^2 x e^{\tan x})$ $= e^{\tan x}(2\sec x)(\sec x \tan x) + \sec^2 x(\sec^2 x e^{\tan x})$ $= \sec^2 x e^{\tan x}(2\tan x + \sec^2 x)$ $= (1 + 2\tan x + \tan^2 x)\frac{dy}{dx}$ $= (1 + \tan x)^2 \frac{dy}{dx}$ $\therefore n = 2$

8.	**(i)**	Midpoint of $AB = \left(\frac{2+5}{2}, \frac{6+5}{2}\right)$ $= \left(\frac{7}{2}, \frac{11}{2}\right)$ Gradient of $AB = \frac{5-6}{5-2}$ $= -\frac{1}{3}$ Gradient of perpendicular bisector = 3 Equation of perpendicular bisector, $y - \frac{11}{2} = 3\left(x - \frac{7}{2}\right)$ $y = 3x - 5$
	(ii)	From (i) $y = 3x - 5$ ……(1) The centre also lies on $3y = -x + 5$ ……(2) Substitute (1) into (2), $3(3x-5) = -x + 5$ $x = 2$ $y = 1$ Centre of circle, (2, 1) Radius of circle $= \sqrt{(2-5)^2 + (1-5)^2}$ $= \sqrt{25}$ $= 5$ units Equation of circle, $(x-2)^2 + (y-1)^2 = 25$ Or $x^2 + y^2 - 4x - 2y - 20 = 0$
	(iii)	Distance between the Centre and P $= \sqrt{(2+2)^2 + (1+1)^2}$ $= 2\sqrt{5}$ units < 5 units $\therefore P$ lies inside the circle. If angle $CPD = 90°$, P should lie on the circle. (Right angle in a semicircle) Hence, angle CPD cannot be 90°

9.	**(i)**	$\dfrac{x^2-4x+1}{x^2-6x+9}$ Using Long Division, $\dfrac{x^2-4x+1}{x^2-6x+9}=1+\dfrac{2x-8}{(x-3)^2}$ Let $\dfrac{2x-8}{(x-3)^2}=\dfrac{B}{x-3}+\dfrac{C}{(x-3)^2}$. $\dfrac{2x-8}{(x-3)^2}=\dfrac{B(x-3)+C}{(x-3)^2}$ $2x-8=B(x-3)+C$ Comparing coefficient of x, $B=2$ Let $x=3$, $6-8=C$ $C=-2$ $A=1$
	(ii)	$\dfrac{dy}{dx}=\dfrac{d}{dx}\left[1+2(x-3)^{-1}-2(x-3)^{-2}\right]$ $=-2(x-3)^{-2}-2(-2)(x-3)^{-3}$ $=-2(x-3)^{-3}(x-3-2)$ $=-\dfrac{2(x-5)}{(x-3)^3}$ or $\dfrac{10-2x}{(x-3)^3}$ or $-\dfrac{2}{(x-3)^2}+\dfrac{4}{(x-3)^3}$ When $\dfrac{dy}{dx}=0$, $-\dfrac{2(x-5)}{(x-3)^3}=0$ $x=5$ When $x=5$, $y=\dfrac{3}{2}$ Turning point, $\left(5,\ \dfrac{3}{2}\right)$.
		<u>**Alternative Method**</u> When $\dfrac{dy}{dx}=0$, $-\dfrac{2}{(x-3)^2}+\dfrac{4}{(x-3)^3}=0$ $\dfrac{4}{(x-3)^3}=\dfrac{2}{(x-3)^2}$ $2(x-3)^2=(x-3)^3$ Since $x\neq 3$, $2=x-3$ $x=5$ When $x=5$, $y=\dfrac{3}{2}$ Turning point, $\left(5,\ \dfrac{3}{2}\right)$.

	$\frac{d^2y}{dx^2} = \frac{-2(x-3)^3 + 2(3)(x-5)(x-3)^2}{(x-3)^6}$ $= \frac{-2(x-3)+6(x-5)}{(x-3)^4}$ $= \frac{4x-24}{(x-3)^4}$ or $\frac{4}{(x-3)^3} - \frac{12}{(x-3)^4}$ When $x = 5$, $\frac{d^2y}{dx^2} = -\frac{1}{4} < 0$ $\left(5, \frac{3}{2}\right)$ is maximum point. **Alternative method** <table><tr><td>x</td><td>5^-</td><td>5</td><td>5^+</td></tr><tr><td>$\frac{dy}{dx}$</td><td>+</td><td>0</td><td>–</td></tr><tr><td>Slope</td><td>/</td><td>–</td><td>\</td></tr></table>$\left(5, \frac{3}{2}\right)$ is maximum point.

10. (i)	$v = \int (2t-1)\mathrm{d}t$ $= t^2 - t + C$ When $t = 0$, $v = 0$, $C = 0$ $\therefore v = t^2 - t$ When $v = 0$, $t^2 - t = 0$ $t(t^2 - 1) = 0$ $t = 0$ (NA), 1	**M1** **A1** **M1** **A1**	 **A0 (If no C)**
(ii)	$s = \int (t^2 - t)\mathrm{d}t$ $= \frac{t^3}{3} - \frac{t^2}{2} + D$ When $t = 0$, $s = 0$, $D = 0$ $\therefore s = \frac{t^3}{3} - \frac{t^2}{2}$ When $s = 0$, $\frac{t^3}{3} - \frac{t^2}{2} = 0$ $2t^3 - 3t^2 = 0$ $t^2(2t-3) = 0$ $t = 0, \frac{3}{2}$ Hence, the particle returns to O after $1\frac{1}{2}$ seconds. **<u>Alternative method</u>** When $t = \frac{3}{2}$, $s = \frac{1.5^3}{3} - \frac{1.5^2}{2}$ $= 0$ Hence, the particle returns to O after $1\frac{1}{2}$ seconds.	**M1** **A1** **M1** **A1** **M1** **A1**	 **A0 (If no C)** **Answer with conclusion** **Answer with conclusion**
(iii)	When $t = 1$, $s = \frac{1^3}{3} - \frac{1^2}{2} = -\frac{1}{6}$ When $t = 4$, $s = \frac{4^3}{3} - \frac{4^2}{2} = 13\frac{1}{3}$ Distance travelled $= 13\frac{1}{3} + 2\left(\frac{1}{6}\right)$ $= 13\frac{2}{3}$ m	 **M1** **A1**	 **(10 marks)**

11.	**(i)**	$f'(x) = x^{\frac{1}{2}} - x^{-\frac{1}{2}}$		
		$f(x) = \int (x^{\frac{1}{2}} - x^{-\frac{1}{2}})dx$	**M1**	
		$= \frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + C$	**A1**	
		At (4, 0), $\frac{2}{3}(4)^{\frac{3}{2}} - 2(4)^{\frac{1}{2}} + C = 0$		
		$C = -\frac{4}{3}$		
		$f(x) = \frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} - \frac{4}{3}$	**A1**	
	(ii)	At Q, $f'(x) = \frac{dy}{dx} = 4^{\frac{1}{2}} - 4^{-\frac{1}{2}}$		
		$= \frac{3}{2}$	**B1**	
		Equation of PQ, $y = \frac{3}{2}(x-4)$	**M1**	
		$y = \frac{3}{2}x - 6$		
		$\therefore$ at P, $y = -6$	**A1**	
	(iii)	Area of shaded region		
		$= \frac{1}{2} \times 4 \times 6 + \int_0^4 \left(\frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} - \frac{4}{3}\right) dx$ or		
		$= \frac{1}{2} \times 4 \times 6 - \left\| \int_0^4 \left(\frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} - \frac{4}{3}\right) dx \right\|$	**M1**	**Difference**
		$= 12 + \left[\frac{\frac{2}{3}x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{2x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{4}{3}x\right]_0^4$	**M1**	**Integral + limits**
		$= 12 + \left[\frac{4}{15}x^{\frac{5}{2}} - \frac{4}{3}x^{\frac{3}{2}} - \frac{4}{3}x\right]_0^4$		
		$= 12 + \left[\frac{4}{15}(4)^{\frac{5}{2}} - \frac{4}{3}(4)^{\frac{3}{2}} - \frac{4}{3}(4)\right]$		
		$= 12 - \frac{112}{15}$	**M1**	
		$= \frac{68}{15}$ unit^2 or $4\frac{8}{15}$ unit^2 or 4.53 unit^2	**A1**	
				(10 marks)

End of Paper 2