CANDIDATE NAME		INDEX NUMBER	

Anglo-Chinese School (Barker Road)

PRELIINARY EXAMINATION 2019
SECONDARY FOUR EXPRESS / FIVE NORMAL ACADEMIC

ADDITIONAL MATHEMATICS 4047
PAPER 1

2 HOURS

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

This question paper consists of 12 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

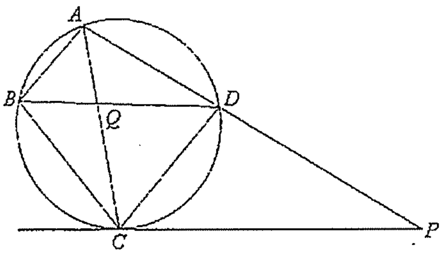
$$\Delta = \frac{1}{2} ab \sin C$$

Answer all the questions

1 The quadratic equation $x^2 - 9x + 8 = 0$ has roots $\alpha\beta^2$ and $\alpha^2\beta$. Find the quadratic equation whose roots are α and β with integer coefficients. [4]

2 **Without using a calculator**, find the value of the integers a and b for which the solution of the equation $x\sqrt{24} = x\sqrt{2} + \sqrt{6}$ is $\dfrac{a+\sqrt{b}}{11}$. [5]

3 In a diagram, A, B, C and D are points on the circle. The tangent at C meets AD produced at P. The chords AC and BD intersect at Q. The line BQD bisects angle ABC.



Prove that

(i) $\angle DCP = \angle ACD$, [3]

(ii) ΔPCD is similar to ΔPAC, [2]

(iii) $PC^2 = PA \times PD$. [1]

4 The function f is defined, for $x \geq 0$, by $f(x) = 5\sin\left(\frac{x}{2}\right) - 1$.

(i) State the minimum value of f(x). [1]

(ii) State the amplitude and period of f(x). [2]

(iii) Find the smallest value of x such that f(x) = 0. [2]

(iv) Sketch the graph of $y = 5\sin\left(\frac{x}{2}\right) - 1$ for $0 \leq x \leq 4\pi$. [2]

5 (i) Given that the coefficient of x^2 in the expansion of $\left(2x-\frac{k}{x}\right)^8$ in ascending powers of x is -1792, find the value of the constant k. [4]

(ii) Hence, find the coefficient of x^4 in the expansion of $(2+x^2)\left(2x-\frac{k}{x}\right)^8$. [3]

6 (i) Prove that $\frac{1-\sin x}{\cos x}+\frac{\cos x}{1-\sin x}=2\sec x$. [3]

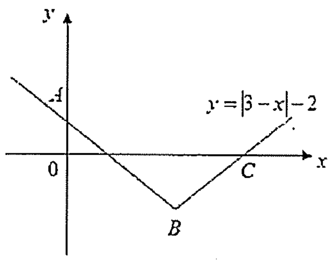
(ii) Hence, solve the equation $\frac{1-\sin 2x}{\cos 2x}+\frac{\cos 2x}{1-\sin 2x}=\tan 80^\circ$ for $0^\circ \le x \le 360^\circ$. [4]

7 The equation of a curve is $y=4x^2+4kx+12-k$, where k is a constant.

(i) Find the range of values of k for which the curve lies completely above the x-axis. [4]

(ii) In the case where $k=1$, find the equation of the tangent to the curve which is parallel to the x-axis. [3]

8



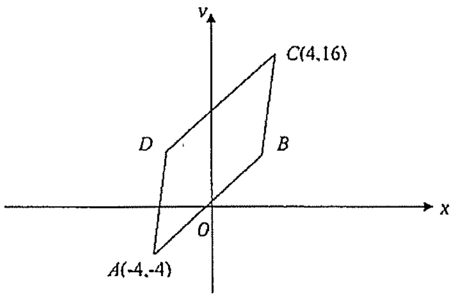
The diagram shows part of the graph of $y=|3-x|-2$.

(i) Find the coordinates of A, of B and of C. [3]

(ii) Find the range of k for which the equation $|3-x|-2=kx$ has 2 solutions. [2]

(ii) Solve the equation $|3-x|-2=3x$. [3]

9 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a parallelogram $ABCD$ in which A is (-4,-4) and C is (4, 16).

The line BD is parallel to the x-axis and the line AD has the equation $y = 11x + 28$.

(i) Find the equation of the line BC. [2]

(ii) Find the coordinates of B and of D. [5]

(ii) Hence or otherwise, find the area of parallelogram $ABCD$. [2]

10 Without using a calculator, solve, for x and y, the simultaneous equations

$$64^x \times 4^y = 1,$$
$$\log_{\frac{1}{2}}(x-3y-6)+2=\log_2 x.$$

[10]

11 In the recent years, the release of greenhouse gases has accelerated the melting of glaciers and thus, resulted in a rise of global temperatures. With minimal actions taken to prevent global warming, the average temperature, T°C, projected to rise after x years from 2016, is given by $T = 31(1.5)^{kx}$, where k is a constant.

(i) Given that the projected average temperature in Singapore in 2019 is 35 °C, find the value of k correct to 1 decimal place. [2]

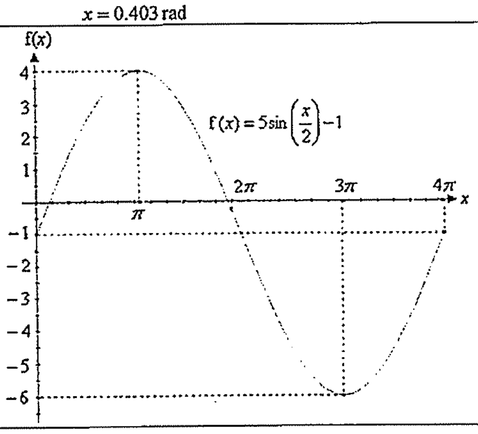
(ii) Find the average temperature in Singapore in 2016. [1]

(iii) In which year will the average temperature in Singapore increase by at least 15% of its temperature in 2016? [5]

(iii) Sketch the graph of T against x. [2]

End Of Paper

		ADDITIONAL MATH PAPER 1 MARKING SCHEME	
1	$x^2-9x+8=0$ $\alpha^2\beta+\beta^2\alpha=9$ $\alpha^3\beta^3=8$ $\alpha\beta=2$ $\alpha\beta(\alpha+\beta)=9$ $2(\alpha+\beta)=9$ $\alpha+\beta=\frac{9}{2}$ So equation with α and β as roots is $x^2-\frac{9}{2}x+2=0$ $2x^2-9x+4=0$ (Integer coefficients)		
2	$x\sqrt{24}=x\sqrt{2}+\sqrt{6}$, $x\sqrt{24}-x\sqrt{2}=\sqrt{6}$ $x(\sqrt{24}-\sqrt{2})=\sqrt{6}$ $x=\frac{\sqrt{6}}{\sqrt{2}(\sqrt{12}-1)}$ $x=\frac{\sqrt{6}}{\sqrt{2}(\sqrt{12}-1)}\times\frac{\sqrt{12}+1}{\sqrt{12}+1}$ $x=\frac{\sqrt{36}+\sqrt{3}}{12-1}$ $x=\frac{6+\sqrt{3}}{11}$ So $a=6$ and $b=3$		
3	(i)	$\measuredangle DCP=\measuredangle CBD$ (tangent chord theorem) $\measuredangle ABD=\measuredangle CBD$ (bisected angles of triangle ABC) $\measuredangle ABD=\measuredangle ACD$ (angles in same segment) So $\measuredangle DCP=\measuredangle ACD$ (shown)	
	(ii)	$\measuredangle CPD=\measuredangle APC$ (shared angle) $\measuredangle PCD=\measuredangle PAC$ (tangent chord theorem) So by AA similarity test, $\triangle PCD$ similar to $\triangle PAC$	
	(iii)	Since $\triangle PCD$ similar to $\triangle PAC$ from ii), $\frac{PC}{PA}=\frac{PD}{PC}$ $PC^2=PA\times PD$ (shown)	
4	(i)	Min. $=-6$	

	(ii)	Amp: 5 Period: 4π	
	(iii)	$5\sin\left(\frac{x}{2}\right)-1=0$ $\sin\left(\frac{x}{2}\right)=\frac{1}{5}$ $\frac{x}{2}=0.2014\text{ rad}$ $x=0.403\text{ rad}$	
	(iv)	 	
5	(i)	To find coefficient of x^2	

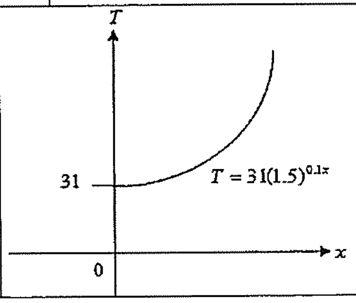
		$\binom{8}{r}(2x)^{8-r}\left(-\frac{k}{x}\right)^r=\binom{8}{r}(2)^{8-r}(-k)^r x^{8-2r}$ 8-2r = 2 r = 3 comparing coefficients, then -1792 = $\binom{8}{3}(2)^5(-k)^3$ = -1792k k = 1	
	(ii)	From (a) sub k=1, $(2+x^2)\left(2x-\frac{1}{x}\right)^8$ To find coefficient of x^4 *of* $\left(2x-\frac{1}{x}\right)^8$, 8-2r = 4 r = 2 $\binom{8}{2}(2)^6(-1)^2=1792$ Coefficient of x^4 of $(2+x^2)\left(2x-\frac{1}{x}\right)^8$: 2(1792) + 1(-1792) = 1792	

6	(i)	$LHS=\frac{1-\sin x}{\cos x}+\frac{\cos x}{1-\sin x}$ $=\frac{1-2\sin x+\sin^2 x+\cos^2 x}{\cos x(1-\sin x)}$ $=\frac{2-2\sin x}{\cos x(1-\sin x)}$ $=\frac{2(1-\sin x)}{\cos x(1-\sin x)}$ $=2\sec x=RHS(proven)$	
	(ii)	$2\sec 2x=\tan 80^{\circ}$ $\cos 2x=\frac{2}{\tan 80^{\circ}}=0.3527$ basic angle $=69.4^{\circ}$ $2x=69.4^{\circ},290.6^{\circ},429.4^{\circ},650.6^{\circ}$ $x=34.7^{\circ},145.3^{\circ},214.7^{\circ},325.3^{\circ}$	
7	(i)	$y=4x^2+4kx+12-k$ $b^2-4ac<0,$ $(4k)^2-4(4)(12-k)<0$ $16k^2-16(12)+16k<0$ $k^2+k-12<0$ $(k+4)(k-3)<0$ $-4<k<3$	
	(ii)	When $k=1,\ y=4x^2+4x+11$ Let the tangent parallel to the x-axis be $y=m$ Then $m=4x^2+4x+11$ $4x^2+4x+11-m=0$ $b^2-4ac=0$ $16-16(11-m)=0$ $16(1-11+m)=0$ $1-11+m=0$ $m=10$ So the equation of the tangent parallel to the x-axis when k=1 is $y=10$	

8	(i)	A(0,1) Sub $y = 0$, $\lvert 3-x \rvert -2 = 0$ $\lvert 3-x \rvert = 2$ $x = 1$(rej) or 5 so C(5,0) x-coordinate of B $= \frac{1+5}{2}$ $= 3$ y-coordinate of B: $y = \lvert 3-3 \rvert -2$ $= -2$ So B(3,-2)	
	(ii)	Gradient of $OB = \frac{-2-0}{3-0}$ $= -\frac{2}{3}$ So $-\frac{2}{3} < k < 1$	
	(iii)	$\lvert 3-x \rvert -2 = 3x$ $\lvert 3-x \rvert = 3x+2$ $3-x = 3x+2$ or $-3+x = 3x+2$ $4x = 1$ or $2x = -5$ $x = 0.25$ or -2.5(Rej)	

9	(i)	Gradient BC = 5 Eqn BC: y = 5x + c Sub (4, 16), c = -4 Eqn BC: y = 5x + -4	
9	(ii)	Since *ABCD* is a parallelogram, the diagonals *AC* and *BD* bisect each other. Midpt *AC* = Midpt *BD* $= (\frac{-4+4}{2}, \frac{-4+16}{2})$ = (0, 6) Since *BD* is parallel to the *x*-axis, *y*- coordinates of both *B* and *D* is 6. For *D*, sub *y* = 6 into *y* = 5*x* + 16, *x* = -2 so D = (-2, 6) sub y = 6 into y = 5x-4 x = 2 B = (2, 6) So *B* = (2, 6), and *D* = (-2, 6)	
	(iii)	The coordinates of *A*, *B*, *C* and *D* are in an anticlockwise direction. $\text{Area of } ABCD = \frac{1}{2}\begin{vmatrix} -4 & 2 & 4 & -2 & -4 \\ -4 & 6 & 16 & 6 & -4 \end{vmatrix}$ $= \frac{1}{2}[(-24+32+24+8)-(-8+24-32-24)]$ $= \frac{1}{2}(80)$ $= 40 \text{ units}^2$ Hence, the area of the parallelogram *ABCD* is 38 units².	

10	$64^x \times 4^y = 1$ $2^{6x} \times 2^{2y} = 2^0$ $6x+2y=0$ $3x+y=0$ $y=-3x---(1)$ $\log_{\frac{1}{2}}(x-3y-6)+2=\log_2 x$ $\frac{\log_2(x-3y-6)}{\log_2 \frac{1}{2}}=\log_2 x-\log_2 4$ $-\log_2(x-3y-6)=\log_2 x-\log_2 4$ $\log_2(x-3y-6)=\log_2 4-\log_2 x$ $\log_2(x-3y-6)=\log_2 \frac{4}{x}$ $x-6-3y=\frac{4}{x}---(2)$ Sub (1) into (2) $x-6+9x=\frac{4}{x}$ $10x-6=\frac{4}{x}$ $10x^2-6x=4$ $5x^2-3x-2=0$ $(5x+2)(x-1)=0$ $x=-\frac{2}{5}$ or 1 $y=\frac{6}{5}$ or -3 So $x=-\frac{2}{5}$, $y=\frac{6}{5}$ or $x=1, y=-3$	

11	(i)	When $x=3$, $T=35$, $35=31(1.5)^{k(3)}$ $k=0.1(1dp)$	
	(ii)	When $t=0$, $T=31(1.5)^0$ $=31°$	
	(ii)	115% of initial temp $=\frac{115}{100}\times 31$ $=35.65°C$ When $T=35.65°C$, $35.65=31(1.5)^{0.1x}$ $\frac{35.65}{31}=(1.5)^{0.1x}$ $\lg\left(\frac{35.65}{31}\right)=\lg(1.5)^{0.1x}$ $\lg(1.15)=0.1x\lg(1.5)$ $0.1x=\frac{\lg 1.15}{\lg 1.5}$ $x=\frac{\frac{\lg 1.15}{\lg 1.5}}{0.1}$ $=3.45(3.s.f)$ So the temperature will increase by at least 15% in 2020.	
	(iii)		

CANDIDATE NAME

INDEX NUMBER

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PRELIMINARY EXAMINATION 2019
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ADDITIONAL MATHEMATICS 4047
PAPER 2

2 HOUR 30 MINUTES

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

Answer all the questions

1 It is given that $x = \sin^{-1}\left(\frac{5}{13}\right)$, where $0° \le x \le 90°$. Without evaluating x, find the exact value of

(i) $\cos(x+45°)$, [2]

(ii) $\tan 2x$. [2]

2 (i) Sketch on the same diagram the graphs of $y^2 = 5x$ and $y = -\frac{1}{x}$. [3]

(ii) Hence, write down the number of solutions of the equation $\sqrt{5x} = -\frac{1}{x}$. [1]

3

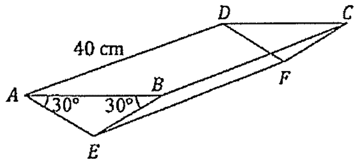


Figure 1

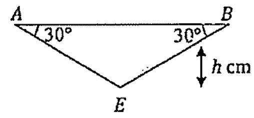


Figure 2

Figure 1 shows an open tank in the shape of a triangular prism. The vertical ends ABE and DCF are congruent triangles. It is given that $\angle ABE = \angle BAE = 30°$ and the length of AD is 40 cm. The tank is fixed in position with the open top $ABCD$ horizontal.
Water is poured into the empty tank at a constant rate of $200\,\text{cm}^3\text{s}^{-1}$.
Figure 2 shows the depth of water, t seconds after pouring starts, is h cm.

(i) Show that, when the depth of water in the tank is h cm, the volume, V cm^3, of water in the tank is given by $V = \left(40\sqrt{3}\right)h^2$. [3]

(ii) Find the rate at which h is increasing when $h = 5$. [4]

4 The polynomial $2x^3 - 3ax^2 + ax + b$ has a factor $(x-1)$ and when divided by $(x+2)$, a remainder of -54 is obtained.

(i) Find the value of a and of b. [4]

(ii) Using the values of a and b in (i), factorise the polynomial completely. [2]

(iii) Hence, or otherwise, solve $\frac{16}{x^3} - \frac{36}{x^2} + \frac{6}{x} + 4 = 0$. [2]

5 The points $(10,-3)$ and $(10,9)$ are on the circumference of a circle. The line $y = 13$ is a tangent to the circle.

(i) Show that the radius of the circle is 10 units. [2]

(ii) Find the coordinates of the centre of the circle, if the x-coordinate of the centre is less than 5. [2]

(iii) Determine if the point (1, -5) lie on the circle, inside the circle or outside the circle. [2]

(iv) Find the equation of the circle. [2]

(v) Find the equations of the tangents to the circle which are parallel to the y-axis. [2]

6 (i) Express $\cos\theta + 2\sin\theta$ as a single trigonometric ratio. [4]

(ii) Find the maximum value of $(\cos\theta + 2\sin\theta)^2$ and the corresponding values of θ where $0^\circ \le \theta \le 360^\circ$. [3]

(iii) Find all the angles between 0° and 360° inclusive that will satisfy the equation

$$\cos\theta + 2\sin\theta = 2.15.$$

[3]

7 A particle travels in a straight line so that t seconds after passing a fixed point O, its velocity, v cms^{-1}, is given by $v = ke^{2t} - 2t$ where k is a constant.
The particle is instantaneously at rest at the first second.

(i) Find the exact value of k. [2]

(ii) Show that the particle reaches minimum velocity when $t = 1 - \ln\sqrt{2}$. [5]

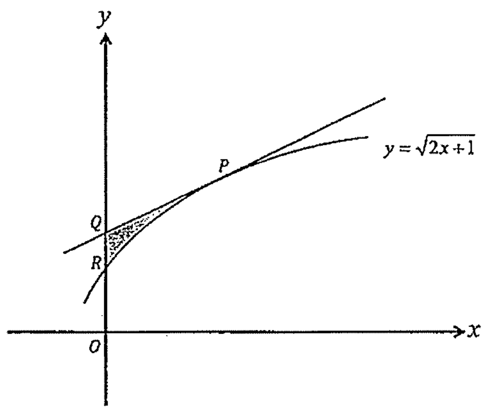
(iii) Find the distance of the particle from O when the particle is at its minimum velocity. [5]

8 (i) Write $\frac{4x^2+8x}{x^2+4x-12}$ as a partial fraction in the form $a-\frac{b}{x+6}+\frac{c}{x-2}$, where a, b and c are integers. [4]

(ii) Differentiate $2x\ln(x^2+4x-12)$ with respect to x. [3]

(iii) Using the results in part (i) and part (ii), determine $\int \ln(x^2+4x-12)\,dx$. [4]

9



The diagram shows part of the curve $y=\sqrt{2x+1}$, cutting the y-axis at R. The tangent at the point P on the curve cuts the y-axis at Q.

(i) Given that the gradient of the normal to the curve at P is -2, find the coordinates of P. [4]

(ii) Calculate the area of the shaded region. [7]

10 A rectangle of area y m^2 has sides of length x m and $(Ax + B)$ m, where A and B are constants and x and y are variables. Values of x and y are given in the table below.

x	50	100	150	200	250
y	3250	9000	17250	24000	41250

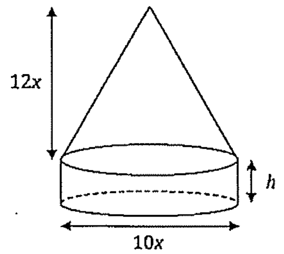
It is believed that an error was made in recording one of the values of y.

(i) Using suitable variables, draw, on the graph grid provided on the next page, a straight line graph and hence estimate the value of each of the constants A and B. [6]

(ii) Identify the incorrect reading and use the straight line graph to estimate a value of y to replace the incorrect recording of y. [3]

(iii) On the same diagram, draw the straight line representing the equation $y = x^2$ and explain the significance of the value of x given by the point of intersection of the two lines. [2]

11



The diagram shows an open mould in the shape of a cylinder and a right circular cone. The diameter of the cylinder is $10x$ cm and its height is h cm. The vertical height of the cone is $12x$ cm.

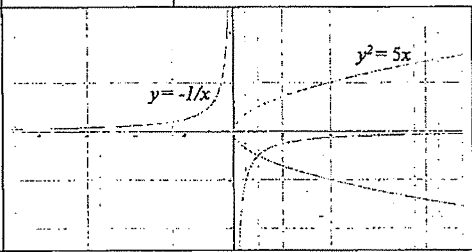
(i) Given that the mould is made from a plastic sheet whose area is 430π cm^2, express h in terms of x. [3]

(ii) Show that the volume, V cm^3, of the mould is given by $V = 1075\pi x - \frac{125}{2}\pi x^3$. [3]

(iii) Hence, calculate the value of x for which the volume has a stationary value. [3]

(iv) Using the answer in (iii), find the corresponding value of h, and determine whether the volume is a maximum or a minimum. [3]

End Of Paper

		ADDITIONAL MATH PAPER 2 MARKING SCHEME	
1	(i)	$\cos(x+45°)=\cos x\cos 45°-\sin x\sin 45°$ $=(\frac{12}{13})(\frac{1}{\sqrt{2}})-(\frac{5}{13})(\frac{1}{\sqrt{2}})$ $=\frac{7}{13\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$ $=\frac{7\sqrt{2}}{26}$	
	(ii).	$\tan(2x)=\frac{2\tan x}{1-\tan^2 x}$ $=\frac{2(\frac{5}{12})}{1-(\frac{5}{12})^2}$ $=1\frac{1}{119}$	
2	(i)		
	(ii)	0 solutions	

3	(i)	Area of the shaded triangle in Figure 2 $= \frac{2\sqrt{3}h(h)}{2}$ $= \sqrt{3}h^2$ Volume of the water, $V = \sqrt{3}h^2(40)$ $= 40\sqrt{3}h^2$ (shown)		
	(ii)	Given that $\frac{dV}{dt} = 200 \text{ cm}^3\text{s}^{-1}$, $\frac{dV}{dh} = 80\sqrt{3}h$ When $h = 5, \frac{dV}{dh} = 400\sqrt{3}$, $\frac{dh}{dt} = 200 \div 400\sqrt{3}$ $\frac{dh}{dt} = \frac{\sqrt{3}}{6} \text{ cm s}^{-1}$		
4	(i)	Let $f(x) = 2x^3 - 3ax^2 + ax + b$. $f(1) = 0$ $2(1)^3 - 3a(1)^2 + a(1) + b = 0$ $2 - 3a + a + b = 0$ $-2a + b = -2$ -(1) $f(-2) = -54$ $2(-2)^3 - 3a(-2)^2 + a(-2) + b = -54$ $-16 - 12a - 2a + b = -54$ $-14a + b = -38$ -(2) Subtracting (2) from (1), $(-2a + b) - (-14a + b) = -2 - (-38)$ $12a = 36$ $a = 3$ Substituting $a = 3$ into (1), $-2(3) + b = -2$ $b = 4$ Hence, the values of a and b are 3 and 4.		

	(ii)	$f(x)=2x^3-3(3)x^2+3x+4$ $=2x^3-9x^2+3x+4$ $=(x-1)(2x^2-7x-4)$ $=(x-1)(2x+1)(x-4)$ $=(2x+1)(x-1)(x-4)$		
	(iii)	Let $y=\frac{2}{x}$. $2y^3-9y^2+3y+4=0$ $(2y+1)(y-1)(y-4)=0$ $2y+1=0$ A $y-1=0$ or $y-4=0$ $2y=-1$, $y=-\frac{1}{2}$, $\frac{2}{x}=-\frac{1}{2}$, $x=-4$ $y=1$, $\frac{2}{x}=1$, $x=2$ $y=4$, $\frac{2}{x}=4$, $x=\frac{1}{2}$		
5	(i)	Mid-point of chord joining $(10,-3)$ and $(10,9)$ $=\left(10,\frac{9-3}{2}\right)$ $=(10,3)$ Centre lies on $y = 3$, i.e. the y-coordinates is 3. Radius of circle $=13-3$ $=10$ units (Shown)		
	(ii)	$(10-x)^2+(9-3)^2=10^2$ $10-x=\pm 8$ $x=2$ or 18 (reject) Centre is (2, 3)		
	(iii)	$\sqrt{(2-1)^2+(3-(-5))^2}=8.06(3.s.f)$ <10 Therefore the point (1,-5) lies within the circle.		
	(iv)	$(x-2)^2+(y-3)^2=100$		
	(v)	$x=-8$ $x=12$		

6	(i)	$R=\sqrt{(1)^2+(2)^2}$ $=\sqrt{5}$ $R\sin\alpha=2$ $R\cos\alpha=1$ $\tan\alpha=\frac{2}{1}$ $\alpha=63.43°$ $\approx 63.4°$ $y=\sqrt{5}\cos(\theta-63.4°)$		
	(ii)	$\max(\cos\theta+2\sin\theta)^2=5$ $\theta=63.4°,243.4°$		
	(iii)	$\sqrt{5}\cos(\theta-63.43°)=2.15$ $\cos(\theta-63.43°)=\frac{2.15}{\sqrt{5}}$ $\alpha=15.94°$ $(\theta-63.43°)=-15.94°,\ 15.94°$ $\theta=47.5°$ or $79.3°$		
7	(i)	$ke^2-2=0$ $k=\frac{2}{e^2}$		
	(ii)	$v=2e^{2t-2}-2t$ $a=4e^{2t-2}-2$ $4e^{2t-2}=2$ $2t-2=\ln\left(\frac{1}{2}\right)$ $t=1-\frac{1}{2}\ln 2$ $t=1-\ln\sqrt{2}$ $\frac{d^2v}{dt^2}=8e^{2t-2}>0$ For all values of t, hence minimum.		

	(iii)	$S=e^{2t-2}-t^2+c$ $S=0, t=0, c=-\frac{1}{e^2}$ $S=e^{2t-2}-t^2-\frac{1}{e^2}$ $t=1-\ln\sqrt{2}, S=-0.0623cm$ $dist=0.0623cm$		
8	(i)	$\frac{4x^2+8x}{x^2+4x-12}=\frac{4(x^2+4x-12)-8x+48}{(x+6)(x-2)}$ $=4-\frac{8x-48}{(x+6)(x-2)}$ By partial fractions, $\frac{8x-48}{(x+6)(x-2)}=\frac{A}{x+6}+\frac{B}{x-2}$ $8x-48=A(x-2)+B(x+6)$ $sub x=2, B=-4$ $sub x=-6, A=12$ So $\frac{4x^2+8x}{x^2+4x-12}=4-\frac{12}{x+6}+\frac{4}{x-2}$		
	(ii)	$\frac{d}{dx}[2x\ln(x^2+4x-12)]=2\ln(x^2+4x-12)+2x\left(\frac{2x+4}{x^2+4x-12}\right)$ $=2\ln(x^2+4x-12)+\frac{4x^2+8x}{x^2+4x-12}$		
(iii) Integrate both sides in (ii),				

		$\int \frac{d}{dx}[2x\ln(x^2+4x-12)]\,dx=\int\left[2\ln(x^2+4x-12)+\frac{4x^2+8x}{x^2+4x-12}\right]dx$ $2x\ln(x^2+4x-12)+c=\int 2\ln(x^2+4x-12)\,dx+\int\frac{4x^2+8x}{x^2+4x-12}\,dx$ $=2\int\ln(x^2+4x-12)\,dx+\int 4-\frac{12}{x+6}+\frac{4}{x-2}\,dx$ $2x\ln(x^2+4x-12)+c-\int 4-\frac{12}{x+6}+\frac{4}{x-2}\,dx=2\int\ln(x^2+4x-12)\,dx$ $2x\ln(x^2+4x-12)-4x+12\ln\|x+6\|-4\ln\|x-2\|+C=2\int\ln(x^2+4x-12)\,dx$ $\int\ln(2x+1)\,dx=x\ln(x^2+4x-12)-2x+6\ln\|x+6\|-2\ln\|x-2\|+\frac{C}{2}$		
9	(i)	$y=\sqrt{2x+1}$ $\frac{dy}{dx}=\frac{1}{2}(2x+1)^{-\frac{1}{2}}(2)$ $\frac{dy}{dx}=\frac{1}{\sqrt{2x+1}}$ $\frac{1}{2}=\frac{1}{\sqrt{2x+1}}$ $\sqrt{2x+1}=2$ $2x+1=4$ $x=\frac{3}{2}$ Sub $x=\frac{3}{2}$ into $y=\sqrt{2x+1}$ $y=2$ Coordinates of $P\left(\frac{3}{2},2\right)$		
	(ii)	Equation of the tangent: $y-2=\frac{1}{2}\left(x-\frac{3}{2}\right)$ $y=\frac{1}{2}x+\frac{5}{4}$ To find coordinate of Q, let $x=0$. $y=\frac{1}{2}(0)+\frac{5}{4}$ Coordinates of $Q\left(0,\frac{5}{4}\right)$		

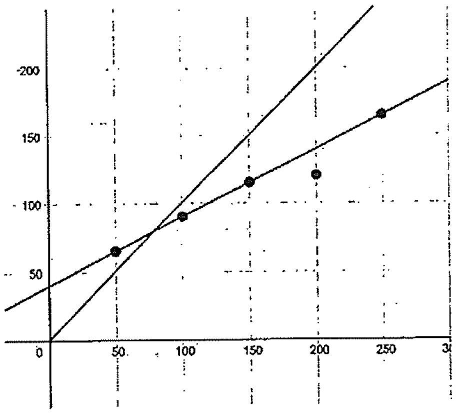
		To find coordinate of R, let $x=0$. $y=\sqrt{2(0)+1}$ $y=1$ Coordinates of $R(0,1)$ Area of trapezium: $=\frac{1}{2}\left(\frac{5}{4}+2\right)\left(\frac{3}{2}\right)$ $=\frac{39}{16}\ sq\ units$ Area under curve from $x=0$ to $x=\frac{3}{2}$ $=\int_0^{\frac{3}{2}}(2x+1)^{\frac{1}{2}}dx$ $=\left[\frac{(2x+1)^{\frac{3}{2}}}{\frac{3}{2}(2)}\right]_0^{\frac{3}{2}}$ $=\left[\frac{(2x+1)^{\frac{3}{2}}}{3}\right]_0^{\frac{3}{2}}$ $=\left\{\left[\frac{8}{3}\right]-\left[\frac{1}{3}\right]\right\}$ $=\frac{7}{3}\ sq\ units$ Area under shaded regions: $=\frac{39}{16}-\frac{7}{3}$ $=\frac{5}{48}\ sq\ units$		

10

$y = x(Ax + B)$

$\frac{y}{x} = Ax + B$

x	50	100	150	200	250
$\frac{y}{x}$	65	90	115	120	165



Gradient = 1/2

A = 1/2

From graph, y-intercept = 40, so B = 40

(ii) Error at x-value = 200

More suitable value of V at x=200,

From graph,

$\frac{y}{x} = 140$

$y = 140(200)$

$= 28000$

(iii) $y = x^2$

$\frac{y}{x} = x$ (draw)

The intersection of this graph with the original graph is the point in which the length x = *Ax+B*, in otherwords, the rectangle is a square.

11	(i)	Let the slant height of the cone be l cm. By Pythagoras' Theorem, $l^2 = (12x)^2 + \left(\frac{10x}{2}\right)^2$ $= 144x^2 + 25x^2$ $= 169x^2$ $l = \pm 13x$ Since $l > 0$, $l = 13x$. Total surface area of mould $= 430\pi$ cm^2 $2\pi(5x)h + \pi(5x)(13x) = 430\pi$ $10xh + 65x^2 = 430$ $10xh = 430 - 65x^2$ $h = \frac{430 - 65x^2}{10x}$ $h = \frac{43}{x} - \frac{13x}{2}$		
	(ii)	$V = \frac{1}{3}\pi(5x)^2(12x) + \pi(5x)^2\left(\frac{43}{x} - \frac{13}{2}x\right)$ $= 100\pi x^3 + 25\pi x^2\left(\frac{43}{x} - \frac{13}{2}x\right)$ $= 100\pi x^3 + 1075\pi x - \frac{325}{2}\pi x^3$ $= 1075\pi x - \frac{125}{2}\pi x^3$ (shown)		
	(iii)	$V = 1075\pi x - \frac{125}{2}\pi x^3$ $\frac{dV}{dx} = 1075\pi - \frac{375}{2}\pi x^2$		

		For stationary values, $\frac{dV}{dx}=0$. $1075\pi-\frac{375}{2}\pi x^2=0$ $\frac{375}{2}x^2=1075$ $x^2=\frac{86}{15}$ $x=\pm\sqrt{\frac{86}{15}}$ $=\pm 2.3944$ $=\pm 2.39$ (correct to 3 sig. fig.) Since $x>0$, $x=2.39$. Hence, the value of x is 2.39.		
	(iv)	When $x=2.3944$, $h=\frac{43}{2.3944}-\frac{13}{2}(2.3944)$ $=2.39$ (correct to 3 sig. fig.) $\frac{dV}{dx}=1075\pi-\frac{375}{2}\pi x^2$ $\frac{d^2V}{dx^2}=-375\pi x<0$ At x=2.3944 By the second derivative test, the volume, V, is a maximum. Hence, the corresponding value of h is 2.39 and the volume is a maximum.		