_______________________ (   )   CLASS: 4 (   )

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2019
ADDITIONAL MATHEMATICS PAPER 1 [4047/01]

S4

03 September 2019 Tuesday **2 hours**

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a 2B pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters and glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiners' Use

Question	Marks	Question	Marks	Question	Marks
1		7		Table of Penalties	
2		8			
3		9			
4		10		Units	
5		11		Presentation	
6				Accuracy	
Parent's Name & Signature: Date:		Total:		80	

This document consists of 17 printed pages and 1 blank page.

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1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

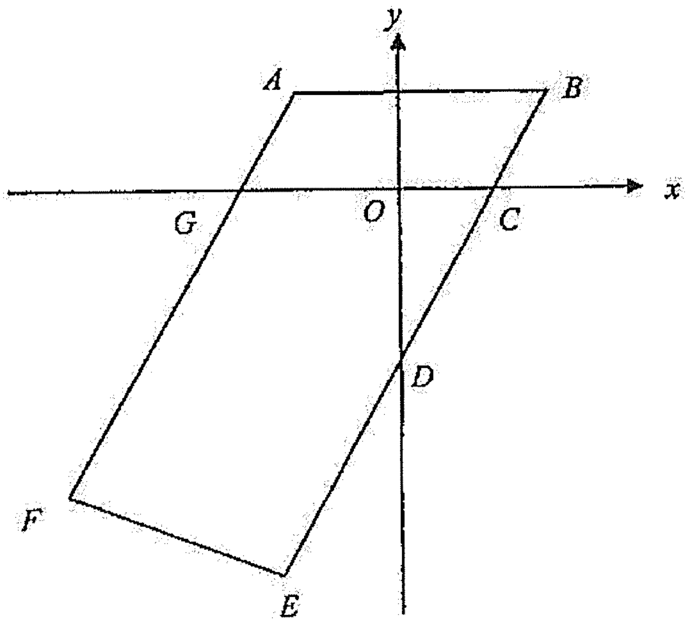
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\text{Area of } \Delta = \frac{1}{2}ab\sin C$$

1 Rashid bought a hot bowl of soup and he left it to cool on the table. The temperature of the soup, $T°$ C, at time, t minutes, is given by Newton's Law of Cooling formula, $T = Ae^{-kt} + T_a$, where A and k are positive constants and T_a is the ambient temperature or the temperature of the surroundings. When the temperature of the soup was first taken, its temperature was $81°C$ and the ambient temperature was $31°C$. After 10 minutes, the temperature of the soup was $51°C$, with no change in the ambient temperature.

(i) Calculate the value of A and of k. [3]

(ii) If Rashid wants the soup to be at most $35°C$ when he drinks it, determine the minimum number of minutes he has to wait, assuming no change to the ambient temperature. [2]

2 **Solutions to this question by accurate drawing will not be accepted.**



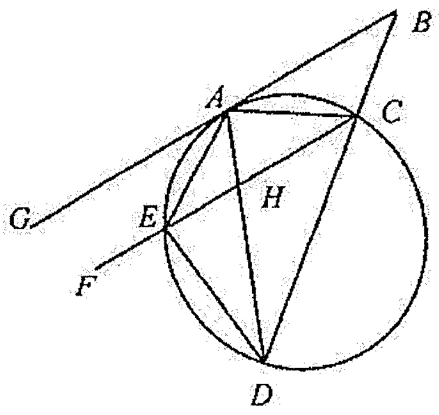
In the diagram, O is the origin. The points, C and G, lie on the x-axis. The line $BCDE$ is parallel to the line AGF and perpendicular to the line EF. The coordinates of A, B and D are $(-2,3)$, $(3,k)$ and $(0,-3)$ respectively. The length of BD is $\sqrt{45}$, and $\frac{BD}{AF}=\frac{2}{3}$.

(i) Find the value of k. [3]

(ii) Show that the coordinates of F are $\left(-\frac{13}{2}, -6\right)$. [2]

(iii) Find the coordinates of E. [3]

3 In the diagram, $ACDE$ is a cyclic quadrilateral. Lines GAB and $FEHC$ are parallel, and line GAB is a tangent to the circle at A. Lines AD and EC meet at H.



Prove that

(i) triangle ABD and triangle CBA are similar, [2]

(ii) triangle ACH and triangle ADC are similar, [2]

(iii) AD bisects angle CDE, [1]

(iv) $AB \times AH = AC \times BC$. [2]

4 (i) Express $12\sin\theta\cos\theta - 8\cos^2\theta + 7$ in the form $A\sin 2\theta + B\cos 2\theta + C$, where A, B and C are constants. [2]

(ii) Solve $12\sin\theta\cos\theta - 8\cos^2\theta + 7 = 0$ for $0^\circ < \theta < 180^\circ$. [5]

(4)

5 (a) Given that $y = \frac{x^2}{e^x}$, find the range of values of x for which y is an increasing function. [4]

(b) The equation of a curve is $y = (x-1)\ln(1-x)$. Find the exact x-coordinate of the point at which the normal is parallel to the y-axis. [4]

6 A particle P leaves a fixed point O and moves in a straight line so that, t seconds after leaving O, its velocity v cm s^{-1} is given by $v = t^2 - 14t + 48$. Calculate

(i) the minimum velocity of P, [2]

(ii) the values of t when P is instantaneously at rest, [2]

(iii) the distance travelled by P in the first 10 seconds. [4]

(iv) Show that the particle will not return to O. [1]

7 A right circular cone of depth 40 cm and radius 10 cm is held with vertex downwards. It contains water which leaks out through a hole at a rate of 8 cm^3s^{-1}. Find the rate at which the water level is decreasing when the radius of the surface of the water is 4 cm. [6]

(6)

8 Determine the number of solutions for the equation

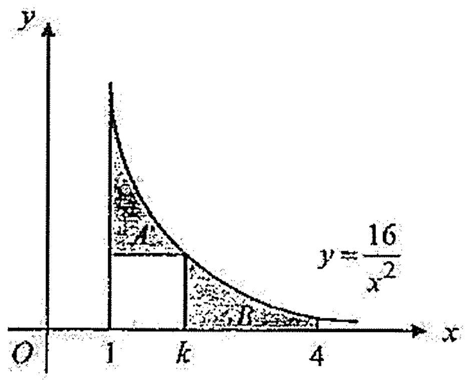
$$2+6\log_8 x=\frac{\log_5(9x-15)}{\log_5 2}.$$

[5]

9 Find the range of values of x such that the curve $y=2x-x^2$ lies above the curve $y=2x^2+17x-42$. [4]

10 (a) Find, in terms of π, the value of $\int_0^{\pi} \sin^2 x \cos^2 x \, dx$. [4]

(b) The diagram shows part of the curve $y = \dfrac{16}{x^2}$. Also shown are lines perpendicular to the x-axis at the points with x-coordinates 1, k and 4.



Given that the areas of the regions marked A and B are equal, find the value of k.

[7]

11 The table shows experimental values of two variables, x and y, which are connected by an equation of the form $y^m x = k$, where k and m are constants.

x	2	4	6	8	10
y	6.25	1.56	0.694	0.391	0.250

(i) Plot $\ln y$ against $\ln x$, and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of k and of m. [4]

(iii) By adding a suitable straight line to the same diagram, find the solution to the pair of simultaneous equations $y^m x = k$ and $y = \sqrt[3]{x}$. [3]

End of Paper

1 The roots of the equation $3x^2 - 6x + 5 = 0$ are α and β. Given that the roots of $x^2 + px + q = 0$ are $\dfrac{\beta^2}{\alpha}$ and $\dfrac{\alpha^2}{\beta}$, find the value of p and of q. [5]

2 (i) Show that $-x^2+x-(1+h^2)$ is always negative for all values of h. [2]

(ii) Find the possible values of k for which the line $y=2x+k$ is tangent to the curve $y^2=1-2x^2$. [3]

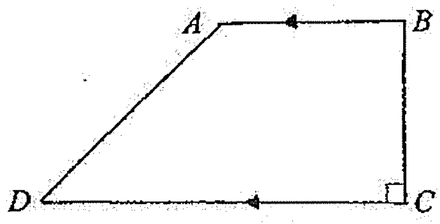
(iii) Find the range of values of x which satisfies $x+2 \leq x^2 < 16$. [4]

3 (i) Solve the equation $x-\sqrt{1-2x}=-7$. [3]

(ii) Solve the simultaneous equations

$$\frac{3^x}{9^{1-y}} = 81 \quad \text{and} \quad 4^x(2^{3y}) = \frac{1}{\sqrt{8}}.$$ [5]

(iii) The trapezium $ABCD$, where AB is parallel to DC, and has an area of $12+6\sqrt{10}$ cm^2.



Given that the length of AB is $\sqrt{2}+\sqrt{5}$ cm and the length of DC is 2 times of AB, find,

(a) the height, BC of the trapezium in the form $a\sqrt{b}$, where a and b are integers. [4]

(iii) **(b)** the exact value of AD^2 in the form $c+d\sqrt{10}$, where c and d are integers. [2]

4 **(i)** The function $f(x)=x^3+ax^2-2x-36$ is divisible by $x-2$. Find the value of a. [1]

(ii) Given that $x^3+5x^2-2x-24=(x+1)^2(x+b)+cx-27$ for all values of x, find the

value of b and of c.
[3]

(iii) The graph of a cubic polynomial expression, $y=f(x)$ has a coefficient of w for x^3.

This graph cuts the x-axis at $(-3, 0)$, $(-1, 0)$, $(4, 0)$ and the y-axis at $(0, 24)$.

Find an expression for $f(x)$.
[3]

5 (i) Sketch the graph of $y=\left|x^2+4x\right|$ showing the x-intercepts and the coordinates of the turning point. [3]

[Turn over

(ii) State the number of solutions to the equation $\left|x^2+4x\right|=x+4$ by drawing a suitable line on the same axes. [2]

6 (i) Find the middle term of $\left(x-\frac{1}{2x^2}\right)^8$.

[2]

(ii) The first 3 terms of $(2a+x)(1-3x)^n$ is $4-59x+bx^2$. Find the value of a, of n and

of b.

[5]

(13)

7. The points $A(5, -7)$ and $B(6, 0)$ lie on a circle, with centre C.

Given that the point C lies on the line $y = 5x - 13$.

(i) Find the equation of the circle.
[7]

A second circle with radius r units and centre P, also passes through the points A and B.

(ii) Find the exact smallest possible value of r. [2]

(iii) State the coordinates of centre P. [1]

14

8 (i) Show that $\cot\theta + \tan\theta = 2\operatorname{cosec} 2\theta$.
[4]

(ii) Hence, solve the equation $\cot\frac{x}{2} + \tan\frac{x}{2} = \operatorname{cosec}^2 x - 3$, giving your answers in radians and within the principal range.
[5]

9 (i) Sketch, on the same axes, the graphs of $y = 2\sin x - 3$ and $y = 4|\cos x|$ for $-\pi \le x \le \pi$.

Hence, deduce the value of m for which the equation $2\sin x - 3 = 4|\cos x| + m$ has 1 solution in this range.

[5]

15

(ii) A student reasoned that since the range of values of y for both equations $y = \sin x$ and $y = \cos x$ are between -1 and 1 inclusive, then the range of values of the expression $8 \sin x + 5 \cos x$ can be obtain as follow

$$-8 \leq 8 \sin x \leq 8$$

$$-5 \leq 5 \cos x \leq 5$$

$$\Rightarrow \quad -13 \leq 8 \sin x + 5 \cos x \leq 13$$

(a) Without performing any calculations, explain why this reasoning is incorrect.

[1]

(b) Find the range of values of $8 \sin x + 5 \cos x$.

[4]

10 (i) Express $\frac{2x-18}{x^3+6x^2+9x}$ in partial fraction.

[5]

(16)

[Turn o

(ii) Hence, or otherwise, find the equation of the curve where the gradient is

$\frac{x-9}{x^3+6x^2+9x}$ and passes through the point (3, ln 2).

[5]

11 Given that $y=3(x-1)^4-4(x-1)^3+5$, find and determine the nature of the stationary points.

[6]

17

12 (i) Sketch the graph of $y = 3\ln(x - 2)$, clearly showing the asymptote and the point where the curve crosses the x-axis.
[3]

(ii) Find $\frac{d}{dx}\left(\cos^3\frac{x}{2}\right)$ and hence evaluate $\int_0^{\pi}\cos^2\frac{x}{2}\sin\frac{x}{2}\ dx$

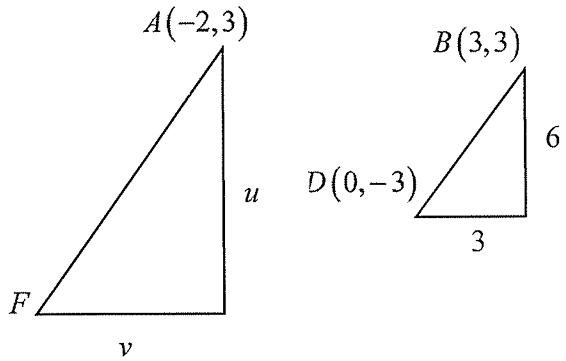
[5]

(18)

AM-2019-AHS-Prelim-P1 – Marking Scheme

1(i)	$T = A e^{-kt} + T_a$ $t = 0$, $T_a = 31$, $T = 81$ $81 = A e^{-k\times 0} + 31$ $A = 50$ $t = 10$, $T_a = 31$, $T = 51$ $51 = 50 e^{-k\times 10} + 31$ $e^{-10k} = \frac{20}{50}$ $k = -\frac{1}{10} \times \ln\left(\frac{2}{5}\right)$ $= 0.091629$ $= 0.0916$
(ii)	$T = 50 e^{-0.091629t} + 31$ $35 = 50 e^{-0.091629t} + 31$ $e^{-0.091629t} = \frac{4}{50}$ $-0.091629t = \ln\left(\frac{2}{25}\right)$ $t = \frac{1}{-0.091629} \times \ln\left(\frac{2}{25}\right)$ $= 27.565$ $= 27.6$ Rashid must wait for at least 27.6 minutes before he can drink the soup.

2(i)	$BD = \sqrt{45}$ $\sqrt{(3-0)^2 + (k-(-3))^2} = \sqrt{45}$ $9 + k^2 + 6k + 9 = 45$ $k^2 + 6k - 27 = 0$ $(k+9)(k-3) = 0$ $k = -9(\text{NA})$, $k = 3$
(ii)	$A(-2,3)$, $B(3,3)$, $D(0,-3)$



$$\frac{AF}{BD}=\frac{3}{2} \qquad \frac{AF}{BD}=\frac{3}{2}$$

$$\frac{u}{6}=\frac{3}{2} \qquad \frac{v}{3}=\frac{3}{2}$$

$$u=9 \qquad v=\frac{9}{2}$$

Coordinates of $F=\left(-2-\frac{9}{2},\ 3-9\right)=\left(-\frac{13}{2},-6\right)$

(iii)

Gradient of $BCDE=\dfrac{3-(-3)}{3-0}=2$

Equation of line $BCDE$ is $y=2x-3$

Gradient of $EF=-\dfrac{1}{2}$

Equation of line EF

$$y-(-6)=-\frac{1}{2}\left(x-\left(-\frac{13}{2}\right)\right)$$

$$y=-\frac{1}{2}x-\frac{37}{4}$$

Solving the pair of simultaneous equations

$$2x-3=-\frac{1}{2}x-\frac{37}{4}$$

$$\frac{5}{2}x=-\frac{37}{4}+3$$

$$x=-\frac{5}{2}$$

$$y=2\left(-\frac{5}{2}\right)-3$$

$$=-8$$

Coordinates of $E=\left(-\frac{5}{2},-8\right)$

3(i)	$\angle CAB = \angle CDA$ (Alternate Segment Theorem) And $\angle BDA = \angle CDA$ (same angle) $\angle ABC = \angle ABD$ (Common angle) Triangle *ABD* is similar to triangle *CBA*. (AA)
(ii)	$\angle CAB = \angle CDA$ (Alternate Segment Theorem) $\angle CAB = \angle ACH$ (Alternate angles, *GAB//FEHC*) Hence $\angle ACH = \angle CDA$ $\angle HAC = \angle DAC$ (Common angle) Triangle *ACH* is similar to triangle *ADC*. (AA)
(iii)	From (ii), $\angle ACH = \angle CDA$ $\angle ACH = \angle ADE$ (Angles in the same segment) Hence $\angle ADE = \angle CDA$ Therefore *AD* bisects angle *CDE*.
(iv)	Triangle *ABD* is similar to triangle *CBA*. $\frac{AB}{BC} = \frac{AD}{AC}$ Triangle *ACH* is similar to triangle *ADC*. $\frac{AC}{AH} = \frac{AD}{AC}$ Hence $\frac{AB}{BC} = \frac{AC}{AH}$ $AB \times AH = AC \times BC$

4(i)	$12\sin\theta\cos\theta - 8\cos^2\theta + 7 = 6(2\sin\theta\cos\theta) - 8\cos^2\theta + 7$ $= 6\sin 2\theta - 8\left(\frac{\cos 2\theta + 1}{2}\right) + 7$ $= 6\sin 2\theta - 4\cos 2\theta + 3$
(ii)	$6\sin 2\theta - 8\cos^2\theta + 7 = 0$ $6\sin 2\theta - 4\cos 2\theta + 3 = 0$ Let $6\sin 2\theta - 4\cos 2\theta = R\sin(2\theta - \alpha)$ $R = \sqrt{6^2 + 4^2} = \sqrt{52}$ $\tan\alpha = \frac{4}{6}$ $\alpha = 33.690°$ $\sqrt{52}\sin(2\theta - 33.690°) + 3 = 0$

	$\sin\left(2\theta-33.690^\circ\right)=-\frac{3}{\sqrt{52}}$ basic angle $=24.583^\circ$ $2\theta-33.690^\circ=-24.583^\circ$ or $2\theta-33.690^\circ=180^\circ+24.583^\circ$ $\theta=4.553^\circ$ $\quad\theta=119.137^\circ$ $\theta=4.6^\circ$ $\quad=119.1^\circ$

5(a)	$y=\frac{x^2}{e^x}$ $\frac{dy}{dx}=\frac{e^x(2x)-x^2e^x}{\left(e^x\right)^2}$ $\frac{dy}{dx}=\frac{2x-x^2}{e^x}$ Since y is an increasing function, $\frac{dy}{dx}>0$ $\frac{2x-x^2}{e^x}>0$ Since $e^x>0,\ 2x-x^2>0$ $x(2-x)>0$ $0<x<2$
(b)	$y=(x-1)\ln(1-x)$ $\frac{dy}{dx}=(x-1)\frac{1}{1-x}(-1)+(1)\ln(1-x)$ $\frac{dy}{dx}=1+\ln(1-x)$ Given that normal is parallel to the y-axis,

	$\frac{dy}{dx}=0$ $1+\ln(1-x)=0$ $\ln(1-x)=-1$ $1-x=e^{-1}$ $x=1-\frac{1}{e}$ $x=\frac{e-1}{e}$

6(i)	$v=t^2-14t+48$ $\frac{dv}{dt}=2t-14$ For minimium velocity, $\frac{dv}{dt}=0$ $2t-14=0$ $t=7$ Minimum $v=7^2-14(7)+48=-1\text{ m / s}$
(ii)	$v=t^2-14t+48$ When P is instantaneous at rest, $v=0$ $t^2-14t+48=0$ $t=6$ or $t=8$

(iii)	$v = t^2 - 14t + 48$ $s = \int t^2 - 14t + 48\, dt$ $s = \frac{t^3}{3} - 7t^2 + 48t + c$ When $t = 0, s = 0, \; c = 0$ $s = \frac{t^3}{3} - 7t^2 + 48t$ When $t = 6, \; s = 108$ When $t = 8, \; s = 106\frac{2}{3}$ When $t = 10, \; s = 113\frac{1}{3}$ Total distance $= 108 + \left(108 - 106\frac{2}{3}\right) + \left(113\frac{1}{3} - 106\frac{2}{3}\right) = 116$ m
(iv)	$s = \frac{t^3}{3} - 7t^2 + 48t$ When P returns to O, $s = 0$ $\frac{t^3}{3} - 7t^2 + 48t = 0$ $\frac{t}{3}\left(t^2 - 21t + 144\right) = 0$ $t = 0$ or $t^2 - 21t + 144 = 0$ Discriminant $= (-21)^2 - 4(1)(144)$ $= -135 < 0$ The particle does not return to O.

7	By similar triangles, $\frac{10}{40} = \frac{r}{h}$ $r = \frac{h}{4}$ $V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi\left(\frac{h}{4}\right)^2 h$ $= \frac{1}{48}\pi h^3$ $\frac{dV}{dh} = \frac{1}{16}\pi h^2$ When $r = 4$, $h = 16$ cm. Rate at which the volume is decreasing, $\frac{dV}{dt} = -8$ Using chain rule, $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ $-8 = \frac{1}{16}\pi(16)^2 \times \frac{dh}{dt}$ $\frac{dh}{dt} = -\frac{1}{2\pi}$ Rate at which the water level is decreasing is $\frac{1}{2\pi}$ cm s^{-1}.

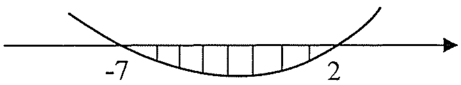
8 Determine the number of solutions for the equation

$$2+6\log_8 x=\frac{\log_5(9x-15)}{\log_5 2}$$

[5]

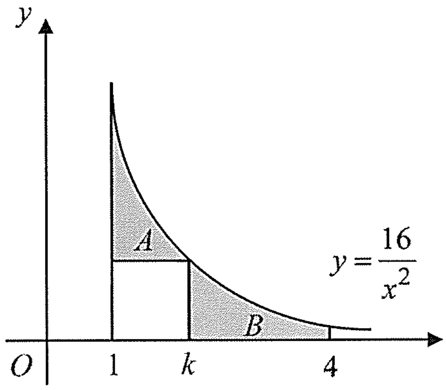
8	$2+6\log_8 x=\frac{\log_5(9x-15)}{\log_5 2}$ $2+\frac{6\log_2 x}{\log_2 8}=\log_2(9x-15)$ $2+\frac{6\log_2 x}{\log_2 2^3}=\log_2(9x-15)$ $2+\frac{6\log_2 x}{3\log_2 2}=\log_2(9x-15)$ $2+2\log_2 x=\log_2(9x-15)$ $2+2\log_2 x=\log_2(9x-15)$ $2+\log_2 x^2=\log_2(9x-15)$ $\log_2(9x-15)-\log_2 x^2=2$ $\log_2\frac{(9x-15)}{x^2}=2$ $\frac{(9x-15)}{x^2}=2^2$ $9x-15=4x^2$ $9x-15=4x^2$ $4x^2-9x+15=0$ Discriminant $=(-9)^2-4(4)(15)$ $=-159<0$ The equation has no real roots, hence there are zero solutions.	**M2 – apply rules of logarithms and change of base to obtain the quadratic equation** **A1 – quadratic equation** **M1** **A1**

9 Find the range of values of x such that the curve $y = 2x - x^2$ lies above the curve $y = 2x^2 + 17x - 42$. [4]

9	$2x - x^2 > 2x^2 + 17x - 42$ $2x - x^2 - 2x^2 - 17x + 42 > 0$	M1	
	$-3x^2 - 15x + 42 > 0$ $3x^2 + 15x - 42 < 0$ $x^2 + 5x - 14 < 0$ $(x-2)(x+7) < 0$	M1	
		M1	
	$-7 < x < 2$	A1	

10 **(a)** Find, in terms of π, the value of $\int_0^{\pi} \sin^2 x \cos^2 x \, dx$. [4]

(b) The diagram shows part of the curve $y = \dfrac{16}{x^2}$. Also shown are lines perpendicular to the x-axis at the points with x-coordinates 1, k and 4.



Given that the areas of the regions marked A and B are equal , find the value of k. [7]

10(a)	$\int_0^{\pi} \sin^2 x \cos^2 x \, dx$ $= \int_0^{\pi} (\sin x \cos x)^2 \, dx$ $= \int_0^{\pi} \left(\frac{1}{2}\sin 2x\right)^2 dx$ $= \frac{1}{4}\int_0^{\pi} \sin^2 2x \, dx$ $= \frac{1}{4}\int_0^{\pi} \frac{1-\cos 4x}{2} \, dx$ $= \frac{1}{8}\left[x - \frac{\sin 4x}{4}\right]_0^{\pi}$ $= \frac{\pi}{8}$	M1 M1 M1 A1

(b)	When $x = k$, $y = \dfrac{16}{k^2}$	
	Area of A	
	$= \int_1^k \dfrac{16}{x^2}\,dx - (k-1)\left(\dfrac{16}{k^2}\right)$	M1
	$= \left[-\dfrac{16}{x}\right]_1^k - \left(\dfrac{16}{k} - \dfrac{16}{k^2}\right)$	
	$= -\dfrac{16}{k} - \left(-\dfrac{16}{1}\right) - \dfrac{16}{k} + \dfrac{16}{k^2}$	
	$= -\dfrac{32}{k} + 16 + \dfrac{16}{k^2}$	A1
	Area of B	
	$= \int_k^4 \dfrac{16}{x^2}\,dx$	M1
	$= \left[-\dfrac{16}{x}\right]_k^4$	
	$= -\dfrac{16}{4} - \left(-\dfrac{16}{k}\right)$	
	$= -4 + \dfrac{16}{k}$	A1
	Area of A = Area of B	
	$-\dfrac{32}{k} + 16 + \dfrac{16}{k^2} = -4 + \dfrac{16}{k}$	
	$\dfrac{16}{k^2} - \dfrac{48}{k} + 20 = 0$	
	$16 - 48k + 20k^2 = 0$	M1
	$5k^2 - 12k + 4 = 0$	
	$(k-2)(5k-2) = 0$	M1
	$k = 2$ or $k = 0.4$ (N.A.)	A1

11 The table shows experimental values of two variables, x and y, which are connected by an equation of the form $y^m x = k$, where k and m are constants.

x	2	4	6	8	10
y	6.25	1.56	0.694	0.391	0.250

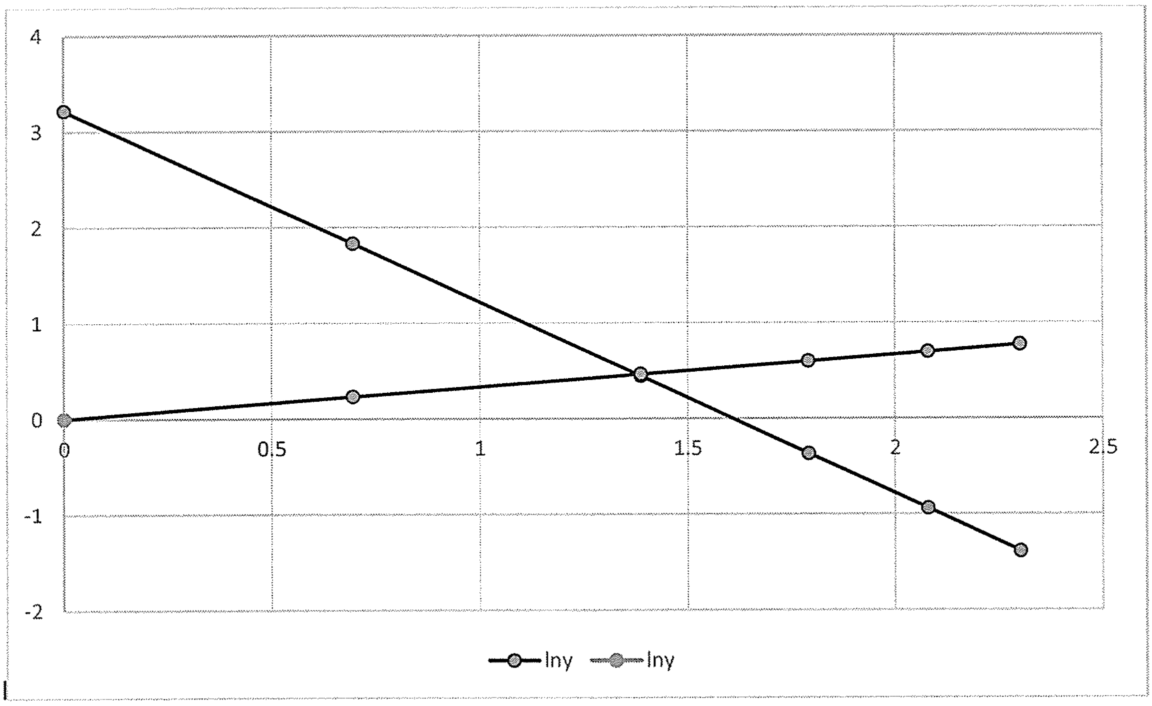
(i) Plot $\ln y$ against $\ln x$, and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of k and of m. [4]

(iii) By adding a suitable straight line to the same diagram, find the solution to the pair of simultaneous equations $y^m x = k$ and $y = \sqrt[3]{x}$. [3]

11(i)

$\ln x$	0.693	1.386	1.792	2.079	2.303
$\ln y$	1.833	0.445	-0.365	-0.939	-1.386



(ii)

$$y^m x = k$$

$$\ln\left(y^m x\right) = \ln k$$

$$\ln y^m + \ln x = \ln k$$

$$m \ln y = -\ln x + \ln k$$

$$\ln y = -\frac{1}{m}\ln x + \frac{1}{m}\ln k$$

Gradient of the line $= -\dfrac{1}{m}$

Y-intercept $= \dfrac{1}{m}\ln k$

Gradient of the line $= -2$ $\left(\text{Accept } -1.7 \text{ to } -2.3\right)$

$$-\frac{1}{m} = -2$$

$$m = 0.5 \quad \left(\text{Accept } 0.435 \text{ to } 0.588\right)$$

Y-intercept =3.217859 $\left(\text{Accept } 3.1 \text{ to } 3.3\right)$

$$\frac{1}{m}\ln k = 3.217589$$

$$\frac{1}{0.5}\ln k = 3.217589$$

$$\ln k = 0.5 \times 3.217589$$

$$k = e^{0.5\times 3.217589}$$

$$= 5 \quad \left(\text{Accept } 3.8 \text{ to } 6.2\right)$$

(iii)

$$y = \sqrt[3]{x}$$

$$y = x^{\frac{1}{3}}$$

$$\ln y = \frac{1}{3}\ln x$$

$\ln x$	0	1	2
$\ln y$	0	0.333	0.667

$$\ln x = 1.2 \quad \left(\text{Accept } 1.0 \text{ to } 1.4\right)$$

At intersection point, $x = e^{1.2}$

$$= 3.32 \quad \left(\text{Accept } 2.7 \text{ to } 4.1\right)$$

AM-2019-AHS-SA2-P2-Marking Scheme

1	$\alpha+\beta=\frac{6}{3}$ $\qquad \alpha\beta=\frac{5}{3}$ $=2$ Sum of roots : $\frac{\beta^2}{\alpha}+\frac{\alpha^2}{\beta}=-p$ $-p=\frac{\alpha^3+\beta^3}{\alpha\beta}$ $-p=\frac{(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)}{\alpha\beta}$ $-p=\frac{(\alpha+\beta)\left[(\alpha+\beta)^2-2\alpha\beta-\alpha\beta\right]}{\alpha\beta}$ $-p=\frac{2\left[(2)^2-3\times\frac{5}{3}\right]}{\frac{5}{3}}$ $p=\frac{6}{5}$ $\left[\text{or } 1\frac{1}{5} \text{ or } 1.2\right]$ Product of roots: $\frac{\beta^2}{\alpha}\times\frac{\alpha^2}{\beta}=q$ $q=\frac{\alpha^2\beta^2}{\alpha\beta}$ $q=\alpha\beta$ $q=\frac{5}{3}$ $\left[\text{or } 1\frac{2}{3}\right]$

2(i)	$D=b^2-4ac$ *of* $-x^2+x-(1+h^2)$ $=1^2-4(-1)(-1-h^2)$ $=1-4-4h^2$ $=-3-4h^2$ <0	**alternative** $-x^2+x-(1+h^2)$ $=-(x^2-x)-(1+h^2)$ $=-\left[(x-\frac{1}{2})^2-\frac{1}{4}\right]-(1+h^2)$ $=-(x-\frac{1}{2})^2+\frac{1}{4}-(1+h^2)$ $=-(x-\frac{1}{2})^2-(\frac{3}{4}+h^2)$ <0

Since D ($b^2 - 4ac$) is always negative for all values of h and coefficient of x^2 is -1, then $-x^2 + x - h^2 - 1$ is always negative.

2(ii)

Subst $y = 2x + k$ into $y^2 = 1 - 2x^2$

$$(2x+k)^2 = 1 - 2x^2$$

$$4x^2 + 4kx + k^2 = 1 - 2x^2$$

$$6x^2 + 4kx + k^2 - 1 = 0$$

For line to be tangent to curve, discriminant =0

$$(4k)^2 - 4(6)(k^2 - 1) = 0$$

$$16k^2 - 24k^2 + 24 = 0$$

$$8k^2 = 24$$

$$k = \pm\sqrt{3}$$

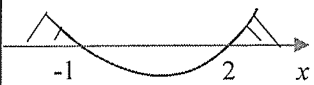
2(iii)

$x + 2 \le x^2 < 16$

$x + 2 \le x^2$

$x^2 - x - 2 \ge 0$

$(x-2)(x+1) \ge 0$

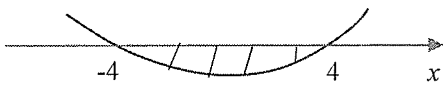


$x \le -1$ *or* $x \ge 2$

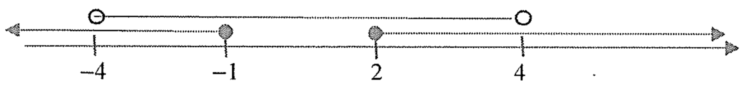
$x^2 < 16$

$x^2 - 16 < 0$

$(x+4)(x-4) < 0$



$-4 < x < 4$



Therefore the range of values of x are

$-4 < x \le -1$ or $2 \le x < 4$

3(i)	$x-\sqrt{1-2x}=-7$ $x+7=\sqrt{1-2x}$ Square both sides $x^2+14x+49=1-2x$ $x^2+16x+48=0$ $(x+4)(x+12)=0$ $x=-4 \quad or\ x=-12(reject)$

3(ii)	$\dfrac{3^x}{9^{1-y}}=81$ and $4^x(2^{3y})=\dfrac{1}{\sqrt{8}}$ $\dfrac{3^x}{9^{1-y}}=81$ ------------------(1) $4^x(2^{3y})=\dfrac{1}{\sqrt{8}}$ ------------------(2) Eq(1) $3^x=9^{1-y}(81)$ $3^x=3^{2-2y}(3^4)$ $\therefore\ x=6-2y$ ------------------(3) Eq(2) $2^x(2^{3y})=2^{-\frac{3}{2}}$ $2^{2x+3y}=2^{-\frac{3}{2}}$ $\therefore\ 2x+3y=-\dfrac{3}{2}$ ------------------(4) Sub Eq(3) into Eq(4) $2(6-2y)+3y=-\dfrac{3}{2}$ $12-4y+3y=-\dfrac{3}{2}$ $y=\dfrac{27}{2}$ $\left[\text{or } 13\dfrac{1}{2} \text{ or } 13.5\right]$ Sub $y=\dfrac{27}{2}$ into Eq(3) $x=6-2(\dfrac{27}{2})$ $=-21$

3(iii)	Area of Trapezium $=12+6\sqrt{10}$
	$\frac{1}{2}(AB+CD)Height=12+6\sqrt{10}$
(a)	$Height=\frac{2\left(12+6\sqrt{10}\right)}{3\left(\sqrt{2}+\sqrt{5}\right)}$
	$=\frac{\left(24+12\sqrt{10}\right)}{3\left(\sqrt{2}+\sqrt{5}\right)}\times\frac{\sqrt{2}-\sqrt{5}}{\sqrt{2}-\sqrt{5}}$
	$=\frac{24\sqrt{2}+12\sqrt{20}-24\sqrt{5}-12\sqrt{50}}{3(2-5)}$
	$=\frac{24\sqrt{2}+12(2\sqrt{5})-24\sqrt{5}-12(5\sqrt{2})}{-9}$
	$=\frac{-36\sqrt{2}}{-9}$
	$=4\sqrt{2}cm$

(b)	$AD^2=\left(\sqrt{2}+\sqrt{5}\right)^2+\left(4\sqrt{2}\right)^2$
	$=2+2\left(\sqrt{2}\right)\left(\sqrt{5}\right)+25+16(2)$
	$=59+2\sqrt{10}$

4(i)	$f(2)=0$
	$(2)^3+a(2)^2-2(2)-36=0$
	$8+4a-4-36=0$
	$4a=32$
	$a=8$

(ii)	$x^3+5x^2-2x-24=(x+1)^2(x+b)+cx-27$ $Let\ x=-1$ $-1+5+2-24=-c-27$ $c=-27+1-5-2+24$ $c=-9$ $Let\ x=0$ $-24=(1)^2b-27$ $b=3$ ~~~~~~~~~~~~~~~~~~~~**Alternatively** ~~~~~~~~~~~~~~~ Using comparison method, $x^3+5x^2-2x-24=(x^2+2x+1)(x+b)+cx-27$ $x^3+5x^2-2x-24=x^3+bx^2+2x^2+2bx+x+b+cx-27$ $x^3+5x^2-2x-24=x^3+(b+2)x^2+(2b+1+c)x+b-27$ By comparing constant $\therefore -24=b-27$ $b=3$ By comparing coefficient of x $-2=2(3)+1+c$ $c=-9$

4(iii)	$f(x)=w(x+3)(x+1)(x-4)$ At (0, 24) $24=w(3)(1)(-4)$ $w=-2$ $\therefore f(x)=-2(x+3)(x+1)(x-4)$

5(i)	(−2, 4) x $y = x + 4$ Deduct 1 mark if axes and graphs are not labelled. G1 for correct shape of $y = \lvert x^2 + 4x \rvert$. G1 for indicating -4 and 0 as x-intercepts along the x-axis. G1 for indicating coordinates of turning point (-2,4) on the graph. G1 for correct sketch of $y=x+4$
5(ii)	B1 for stating **3 solutions. No B1 marks if graph is not drawn.**

6(i)	Middle term of $\left(x - \frac{1}{2x^2}\right)^8$ is $T_5 = \binom{8}{4} x^4 \left(-\frac{1}{2x^2}\right)^4$ $= \frac{35}{8x^4}$

6(ii)	$(2a+x)(1-3x)^n$ $= (2a+x)[1^n + \binom{n}{1}1^{n-1}(-3x) + \binom{n}{2}1^{n-2}(-3x)^2 + ...]$ $= (2a+x)[1 - 3nx + \frac{9n(n-1)x^2}{2} + ...]$ $= 2a - 6anx + 9an(n-1)x^2 + x - 3nx^2 + ...$ $= 2a + (1-6an)x + [9an(n-1) - 3n]x^2$ Comparing expression with $(4 - 59x + bx^2)$, Constant: $2a = 4$

	$a = 2$ Coeff. of x: $1 - 6an = -59$ $1 - 6(2)n = -59$ $n = 5$ Coeff. of x^2: $9an(n-1) - 3n = b$ $9(2)(5)(5-1) - 3(5) = b$ $b = 345$

7(i)	Mid-point of $AB = \left(\frac{5+6}{2}, \frac{-7+0}{2}\right)$ $= \left(\frac{11}{2}, -\frac{7}{2}\right)$ Gradient of $AB = \frac{-7-0}{5-6}$ $= 7$ Equation of the perpendicular bisector of AB $y - \left(-\frac{7}{2}\right) = -\frac{1}{7}\left(x - \frac{11}{2}\right)$ $y = -\frac{1}{7}x - \frac{19}{7}$ $y = -\frac{1}{7}x - \frac{19}{7}$ ....(1) $y = 5x - 13$ .... (2) (1)=(2), $-\frac{1}{7}x - \frac{19}{7} = 5x - 13$ $36x = 72$ $x = 2$ sub $x = 2$ into (2), $y = 5(2) - 13$ $= -3$ Centre (2, – 3) Radius $= \sqrt{(2-6)^2 + (-3-0)^2}$ = 5 units Equation of the circle: $(x-2)^2 + (y+3)^2 = 25$ [OR $x^2 + y^2 - 4x + 6y - 12 = 0$]

(ii)	The exact smallest possible value of r $= \frac{1}{2} \times \sqrt{(5-6)^2 + (-7-0)^2}$ $= \frac{\sqrt{50}}{2}$ $= \frac{5\sqrt{2}}{2}$	
(iii)	Coordinates of the centre P are (5.5, –3.5) [OR $\left(\frac{11}{2}, -\frac{7}{2}\right)$]	B1

8(i)	Method 1 LHS = $\cot\theta + \tan\theta$ $= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$ $= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$ $= \frac{1}{\sin\theta\cos\theta}$ $= \frac{2}{2\sin\theta\cos\theta}$ $= \frac{2}{\sin 2\theta}$ $= 2\,\text{cosec}\,2\theta$ $= RHS$ (shown)		Method 2 LHS = $\cot\theta + \tan\theta$ $= \frac{1}{\tan\theta} + \tan\theta$ $= \frac{1+\tan^2\theta}{\tan\theta}$ $= \frac{\sec^2\theta}{\tan\theta}$ $= \frac{1}{\cos^2\theta} \times \frac{\cos\theta}{\sin\theta}$ $= \frac{1}{\sin\theta\cos\theta}$ $= \frac{2}{2\sin\theta\cos\theta}$ $= \frac{2}{\sin 2\theta}$ $= 2\,\text{cosec}\,2\theta$ $= RHS$ (Shown)

(ii)	$\cot\frac{x}{2} + \tan\frac{x}{2} = \text{cosec}^2 x - 3$ $2\,\text{cosec}\,x = \text{cosec}^2 x - 3$ $\text{cosec}^2 x - 2\,\text{cosec}\,x - 3 = 0$ $(\text{cosec}\,x - 3)(\text{cosec}\,x + 1) = 0$ $\text{cosec}\,x = 3$ or $\text{cosec}\,x = -1$ $\frac{1}{\sin x} = 3$ $\quad$ $\frac{1}{\sin x} = -1$ $\sin x = \frac{1}{3}$ $\quad$ $\sin x = -1$ $x = 0.33983$ $\quad$ $x = -\frac{\pi}{2}$ (or -1.5707) Ans: $x \approx 0.340, -\frac{\pi}{2}$ (or -1.57)	M[1] -- QE M[1] – factorization / formula M[1] – in terms of sine A[1, 1] --- deduct 1 mk if ans in degree.

$y = 4\lvert\cos x\rvert$ $y = 2\sin x - 3$ 9(i)
For $2\sin x - 3 = 4\lvert\cos x\rvert + m$ to have 1 solution in the range $-\pi \le x \le \pi$. $m = -1$

9(iia)	The maximum & minimum values of each curve do not occur at the same value of x. [max & min pts of each curves do not occur simultaneously.] **Accept solution where students sketch the graphs to explain.

9 (ii) Method 1		Method 2	
$\sqrt{8^2+5^2} = \sqrt{89}$, $\tan\alpha = \frac{5}{8}$ $\alpha = 32.005°$ $8\sin x + 5\cos x$ $= \sqrt{89}\sin(x - 32.0°)$ $\therefore$ The range of $8\sin x + 5\cos x$ $-\sqrt{89} \le 8\sin x + 5\cos x \le \sqrt{89}$		Let $y = 8\sin x + 5\cos x$ $\frac{dy}{dx} = 8\cos x - 5\cos x$ let $\frac{dy}{dx} = 0$, $8\cos x - 5\sin x = 0$ $\tan x = \frac{8}{5}$ $x = 57.994°, 237.994°$	

Or $-9.4339 \leq 8\sin x + 5\cos x \leq 9.4339$ $-9.43 \leq 8\sin x + 5\cos x \leq 9.43$		when $x = 57.994°$, $8\sin 57.994° + 5\cos 57.994°$ $= 9.4339$ when $x = 237.994°$, $8\sin 237.994° + 5\cos 237.994°$ $= -9.4339$ $\therefore -9.43 \leq 8\sin x + 5\cos x \leq 9.43$	

10(i)	$\dfrac{2x-18}{x^3+6x^2+9x}=\dfrac{2x-18}{x(x+3)^2}$ Let $\dfrac{2x-18}{x(x+3)^2}=\dfrac{A}{x}+\dfrac{B}{x+3}+\dfrac{C}{(x+3)^2}$ $2x-18\equiv A(x+3)^2+Bx(x+3)+Cx$ let $x=0$, $-18=A(9)$ $A=-2$ let $x=-3$, $2(-3)-18=C(-3)$ $C=8$ let $x=1$, $2(1)-18=(-2)(4)^2+B(1)(4)+(8)(1)$ $B=2$ $\therefore \dfrac{2x-18}{x(x+3)^2}=-\dfrac{2}{x}+\dfrac{2}{x+3}+\dfrac{8}{(x+3)^2}$

(ii)	Given that $\frac{dy}{dx}=\frac{x-9}{x^3+6x^2+9x}$ $y=\frac{1}{2}\int\frac{2x-18}{x(x+3)^2}\,dx$ $=\frac{1}{2}\int(-\frac{2}{x}+\frac{2}{x+3}+\frac{8}{(x+3)^2})\,dx$ $=\frac{1}{2}\left[-2\ln x+2\ln(x+3)+\frac{8(x+3)^{-1}}{(-1)(1)}\right]+c$ $=\frac{1}{2}\left[-2\ln x+2\ln(x+3)-\frac{8}{(x+3)}\right]+c$ $=-\ln x+\ln(x+3)-\frac{4}{(x+3)}+c$ $=\ln\frac{x+3}{x}-\frac{4}{(x+3)}+c$ Sub (3, ln 2) into the above eqn $\ln 2=\ln\frac{3+3}{3}-\frac{4}{3+3}+c$ $c=\frac{2}{3}$ Hence the equation of the curve is $y=\ln\frac{x+3}{x}-\frac{4}{(x+3)}+\frac{2}{3}$ OR $y=\ln(x+3)-\ln x-\frac{4}{(x+3)}+\frac{2}{3}$

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$y = 3(x-1)^4 - 4(x-1)^3 + 5$

$\frac{dy}{dx} = 12(x-1)^3 - 12(x-1)^2$

Let $\frac{dy}{dx} = 0$,

$12(x-1)^3 - 12(x-1)^2 = 0$

$12(x-1)^2[(x-1)-1)] = 0$

$(x-1)^2 = 0$ or $(x-2) = 0$

$x = 1$ $\quad$ $x = 2$

when $x = 1$, $y = 3(1-1)^4 - 4(1-1)^3 + 5$

$= 5$

x	< 1	$= 1$	> 1
Sketch of tangent	\	—	\

$\therefore$ (1, 5) is a point of inflexion

when $x = 2$, $y = 3(2-1)^4 - 4(2-1)^3 + 5$

$= 4$

x	< 2	$= 2$	> 2
Sketch of tangent	\	—	/

$\therefore$ (2, 4) is a minimum point

12(i)

$y = 3\ln(x-2)$

The range of value of x is $x > 2$.

The curve crosses the x-axis at the point (3, 0)

y

$y = 3\ln(x-2)$

0 2 3 x

12(ii)	$\frac{d}{dx}\left(\cos^3\frac{x}{2}\right)$ $=(3\cos^2\frac{x}{2})(-\sin\frac{x}{2})(\frac{1}{2})$ $=-\frac{3}{2}\cos^2\frac{x}{2}\sin\frac{x}{2}$ $\int_0^{\pi}\cos^2\frac{x}{2}\sin\frac{x}{2}\ dx$ $=-\frac{2}{3}\int_0^{\pi}\left(-\frac{3}{2}\right)\cos^2\frac{x}{2}\sin\frac{x}{2}\ dx$ $=-\frac{2}{3}\left[\cos^3\frac{x}{2}\right]_0^{\pi}$ $=-\frac{2}{3}\left[\cos^3\frac{\pi}{2}-\cos^3 0\right]$ $=-\frac{2}{3}\left[(0)^3-(1)^3\right]$ $=\frac{2}{3}$