Name:	Class:	Class Register Number:

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

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PRELIMINARY EXAMINATION 2019
SECONDARY 4

ADDITIONAL MATHEMATICS 4047/01

Paper 1 **Tuesday 3 September 2019**

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

For Candidate's Use	For Examiner's Use
Question Number	Marks Obtained
1	
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Total Marks	

This document consists of **19** printed pages and **1** blank page.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 **(a)** Giving your answer in radians and in terms of π, state the principal value of

(i) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$, [1]

(ii) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$. [1]

(b) State the values between which the principal value of $\cos^{-1} x$ must lie. [1]

2 Without using a calculator, find the values of the integers a and b for which the solution of the equation $x\sqrt{8}=\sqrt{2}-x\sqrt{5}$ is $\frac{a-\sqrt{b}}{3}$. [5]

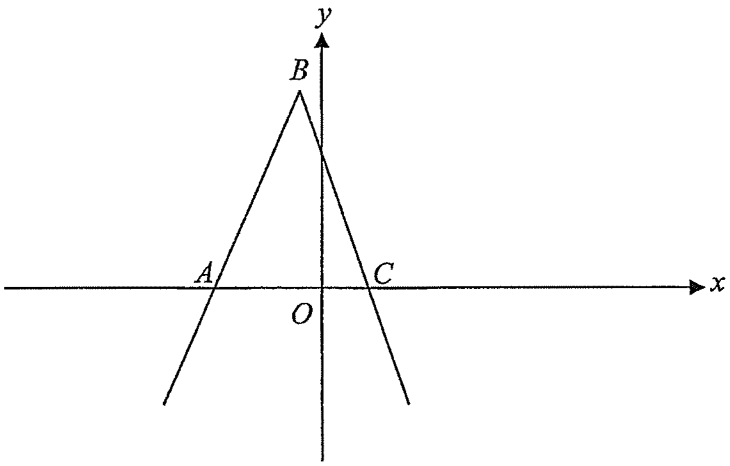
[Turn over

3 (a) It is given that $\log_b\left(x^3y\right)=p$ and $\log_b\left(\dfrac{y}{x^2}\right)=q$.

(i) Express $\log_b x$ in terms of p and q. [3]

(ii) Hence, express $\log_b(xy)$ in terms of p and q. [3]

(b)



The diagram shows part of the graph of $y = 5 - |3x + 1|$.

(i) Find the coordinates of A, B and C. [3]

A line of gradient m, where $m \neq 0$, passes through the origin.

(ii) In the case where $m = -1$, find the coordinates of any point of intersection of the line and the graph of $y = 5 - |3x+1|$. [3]

(iii) In the case where $m > 0$, determine the set of values of m for which the line intersects the graph of $y = 5 - |3x+1|$ in two points. [1]

[Turn over

4 A curve is such that $\frac{dy}{dx} = 20(5x - k)^3$, where k is a non-zero constant.

(i) Given that the curve has a minimum point at $\left(-\frac{3}{5}, 7\right)$, find the value of k. [3]

(ii) Using the value of k found in **(i)**, find the equation of the curve. [4]

[Turn over

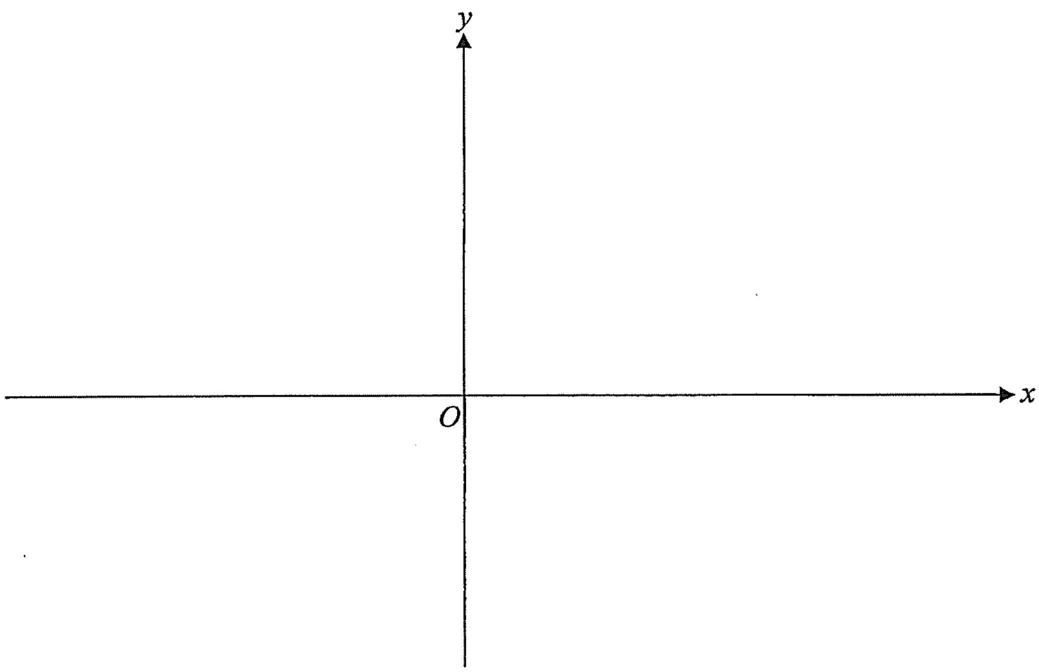
5 Given that $y = p - q\sin 2x$, where p and q are positive integers,

(i) state the period of y. [1]

Given that the maximum and minimum values of y are 5 and -1 respectively, find

(ii) the value of p and of q. [3]

(iii) Using the values of p and q found in part (ii), sketch the graph of $y = p - q\sin 2x$ for $-180° \leq x \leq 180°$ on the axes provided below. [3]



6 (i) Differentiate xe^{3x} with respect to x. [2]

(ii) Using the result from part (i), find $\int xe^{3x}\,dx$ and hence show that $\int_0^1 xe^{3x}\,dx = \frac{2e^3+1}{9}$. [4]

[Turn over

7 Two obtuse angles A and B are such that $\tan(2A+B)=9$ and $\sin B=\frac{1}{\sqrt{10}}$.

(i) Without using a calculator, find the value of $\tan B$. [2]

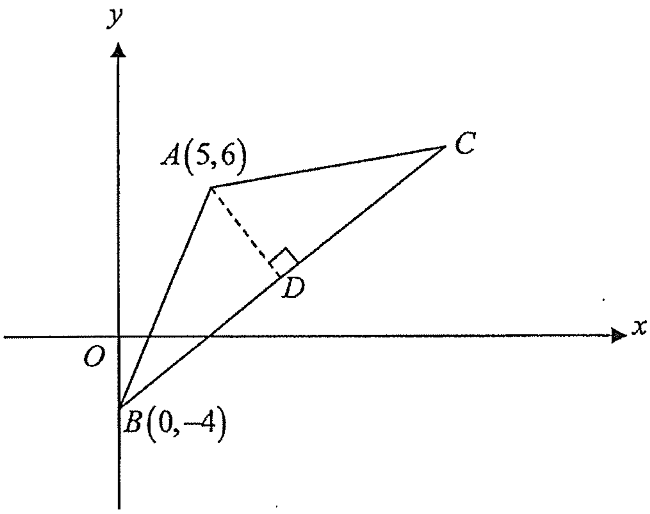
(ii) With workings clearly shown, explain why $135° < A < 180°$. [4]

8 The equation of a curve is $y = \dfrac{2x+1}{x+3}$, where $x \neq -3$.

(i) Explain why the curve has no stationary points. [3]

(ii) Find the equation of the normal to the curve at the point where $x = 2$. [3]

9



The diagram shows a triangle ABC in which the points A and B are $(5,6)$ and $(0,-4)$ respectively. D is a point on BC such that angle ADC is $90°$ and the equation of BC is $y=\frac{3}{4}x-4$.

(i) Find the equation of AD. [3]

(ii) Find coordinates of D. [2]

(iii) Given that $AB = AC$, find the coordinates of C. [2]

(iv) Find the area of triangle ABC. [2]

[Turn over

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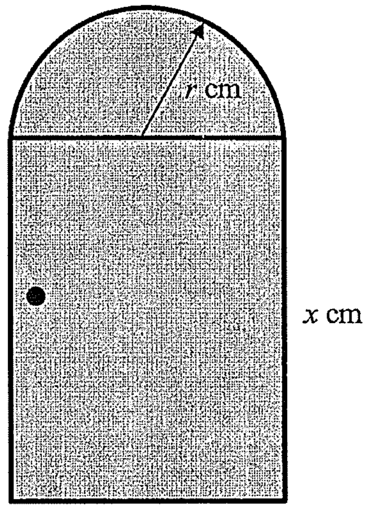


A miniature wooden door of perimeter 60 cm is designed as shown above. It consists of a semicircle of radius r cm and a rectangle of height x cm.

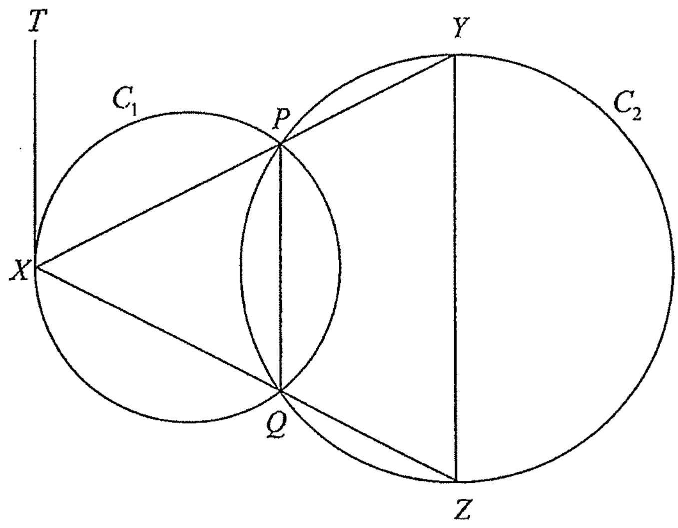
(i) By expressing x in terms of r, show that the area of the door, A cm^2, is given by

$$A = r^2\left(\frac{60}{r} - 2 - \frac{\pi}{2}\right).$$ [3]

(ii) Given that r can vary, show that A has a stationary value when $r = \dfrac{k}{4+\pi}$, where k is a constant to be found and determine whether this value of A is a maximum or minimum. [5]

[Turn over

11



Two circles, C_1 and C_2, intersect at *P* and *Q*, as shown in the diagram.
TX is a tangent to C_1 at *X*. The points *Y* and *Z* lie on the circumference of C_2 such that *XPY* and *XQZ* are straight lines.

(i) Prove that *TX* is parallel to *YZ*. [3]

It is given that □ $TXY = 60°$ and □ $QPZ = 30°$.

(ii) Prove that QY bisects □ XYZ. [3]

(iii) Given further that P and Q are the midpoints of XY and XZ respectively, explain if YZ is the diameter of C_2. [4]

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PRELIMINARY EXAMINATION 2019
SECONDARY 4

ADDITIONAL MATHEMATICS 4047/02

Paper 2 **Tuesday 17 Sept 2019**

Candidates answer on the Question Paper. **2 hours 30 minutes**

Additional Materials: Graph Paper

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

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1 The roots of the quadratic equation $x^2+3x+4=0$ are $\alpha-1$ and $\beta-1$.

(i) Find the value of $\alpha^2+\beta^2$. [4]

(ii) Find a quadratic equation whose roots are $\alpha+2\beta$ and $\beta+2\alpha$. [3]

2 (i) Express $\dfrac{-7x^2+19x-6}{x^2(2x-3)}$ in partial fractions. [5]

(ii) Hence, find $\int \frac{-21x^2+57x-18}{2x^2(2x-3)}\,\mathrm{d}x$. [5]

3 (i) Find the range of values of k for which the turning point of the graph of $y = -2x^2 + kx - 3k$ lies below the x-axis. [3]

(ii) Write down the value(s) of k such that the turning point of the graph is on the x-axis. [1]

(iii) In the case of $k = 5$, find the range of values of x such that $\dfrac{x^2 - 6x - 7}{-2x^2 + kx - 3k} < 0$. [3]

4 The total population of the world, P (in millions), has been increasing each year. According to analysts, the world population can be modelled by an equation of the form

$$P = P_0 e^{kt},$$

where P_0 and k are constants and t is the number of years since 2000. The table below gives values of P and t for some of the years 2005 to 2020.

Year	2005	2010	2015	2020
t years	5	10	15	20
P (in millions)	6492	6861	7241	7657

(i) **Answer only this part question on a piece of graph paper.**
Plot $\ln P$ against t and draw a straight line graph. [3]

(ii) Use the graph to estimate the value of P_0 and of k. [3]

(iii) The world population is projected to reach 11.2 billion in 2100. Determine, with working, whether analysts expect the rate of growth of the population over time from 2020 to decrease, increase or remain constant. [2]

5 **(i)** Show that $\operatorname{cosec}\theta - 4\sin\theta = 4\cot\theta$ can be expressed as $4\cos^2\theta - 4\cos\theta - 3 = 0$. [3]

(ii) Hence solve the equation $\operatorname{cosec} 2x - 4\sin 2x = 4\cot 2x$ for $0° < x < 360°$. [5]

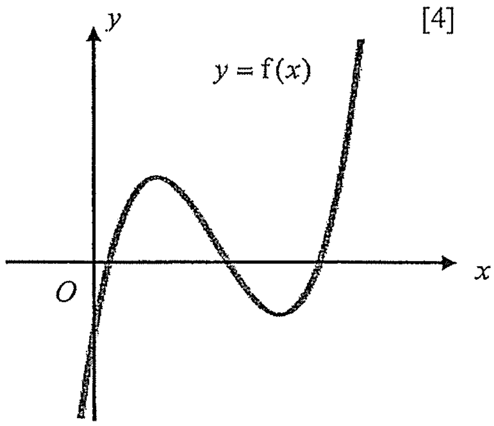
6 **(i)** By considering the general term in the binomial expansion of $\left(3x-\frac{1}{2x}\right)^8$, explain why there are only even powers of x in this expansion. [3]

(ii) Find the constant term and the term in x^2 in $\left(3x-\frac{1}{2x}\right)^8$. [2]

(iii) Find the coefficient of x^2 in the expansion of $\left(2+x+8x^2\right)\left(3x-\dfrac{1}{2x}\right)^8$. [2]

7 **(i)** Factorise the polynomial $f(x) = x^3 - 3x^2 + 7x - 5$. [3]

(ii) A student tried to sketch the graph of $y = f(x)$ as shown below. By providing your working clearly, explain if the student is correct. [4]



It is given that $f(x) = (x+2)g(x) + k$ for all real values of x, where $g(x)$ is a polynomial in x and k is a constant.

(iii) Write down the degree of $g(x)$. [1]

(iv) Find the value of k. [1]

(v) Explain clearly whether $f(x)$ is divisible by $g(x)$. [1]

8 **(a)** The function f is defined, for all values of x, by

$$f(x) = e^{-x}(3x-1).$$

Find the range of values of x for which f is a decreasing function. [5]

(b) A bottle of oil has been spilled onto a flat surface. The oil spill spreads itself out in a circular patch at a steady rate of 2π cm^2/s . Find

(i) the radius of the patch 8 seconds after the oil has been spilled, [2]

(ii) the rate of increase of the radius at this instant. [3]

9 A particle travels in a straight line so that its velocity, v cm/s, is given by $v = t^2 + kt - 3$ where t is the time in seconds after leaving a fixed point O.

(i) Given that the particle has a minimum velocity at $t = 1$, show that $k = -2$. [2]

(ii) Find the time when the particle is instantaneously at rest. [3]

(iii) Find the total distance travelled by the particle in the first 5 seconds. [5]

(iv) Explain clearly why the particle will continue to move away from O after $t = 5$. [2]

10 A circle, C_1, has a diameter AB where A is the point $(-3,-6)$ and B is the point $(9,10)$.

(i) Find the radius and the coordinates of the centre of C_1. [3]

(ii) Hence, write down the equation of C_1. [1]

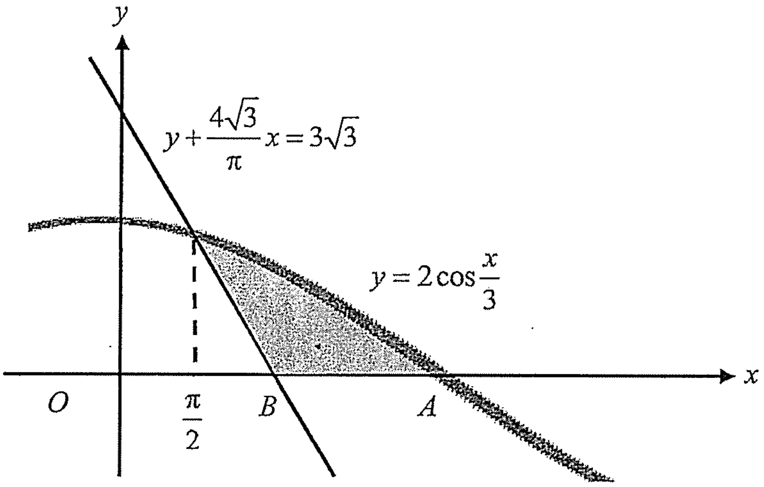
(iii) Find the equation of the tangent to C_1 at B. [3]

The lowest point of C_1 is *D*. The tangents to the circle at *B* and *D* intersect at a point *E*.

(iv) Show that the coordinates of *E* are $(33, -8)$. [3]

(v) Another circle, C_2, with centre *E*, is such that it touches C_1 at one point below and to the right of the centre of C_1. Find the radius of C_2 in the form $a\sqrt{b}+c$, where *a*, *b* and *c* are integers. [2]

11 The diagram shows the line $y+\frac{4\sqrt{3}}{\pi}x=3\sqrt{3}$ and part of the curve $y=2\cos\frac{x}{3}$. The curve intersects the x-axis at the point A. The line intersects the curve at $x=\frac{\pi}{2}$ and the x-axis at the point B.



(i) Show that the x-coordinate of A is $\frac{3\pi}{2}$ and find the x-coordinate of B in terms of π. [3]

(ii) Find the total area of the shaded region bounded by the curve, the x-axis and the line $y+\dfrac{4\sqrt{3}}{\pi}x=3\sqrt{3}$. [6]