Name			Class	

南华中学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2019

Subject	:	Additional Mathematics
Paper	:	4047/01
Level	:	Secondary Four Express
Date	:	29 August 2019
Duration	:	2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correcting fluid / tape.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiner's Use

This paper consists of 26 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

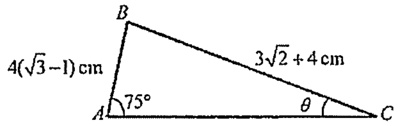
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

1



The diagram shows a triangle ABC in which $AB = 4(\sqrt{3}-1)$ cm, $BC = 3\sqrt{2}+4$ cm, $\angle BAC = 75°$ and $\angle BCA = \theta$. Given that $\sin 75° = \frac{\sqrt{2}}{4}\left(\sqrt{3}+1\right)$, find, **without using a calculator**, the value of $\sin\theta$ in the form of $a + b\sqrt{2}$ where a and b are integers. [4]

2 Evaluate, without using a calculator, $\tan\left[\cos^{-1}\left(-\frac{8}{17}\right)\right]$. [3]

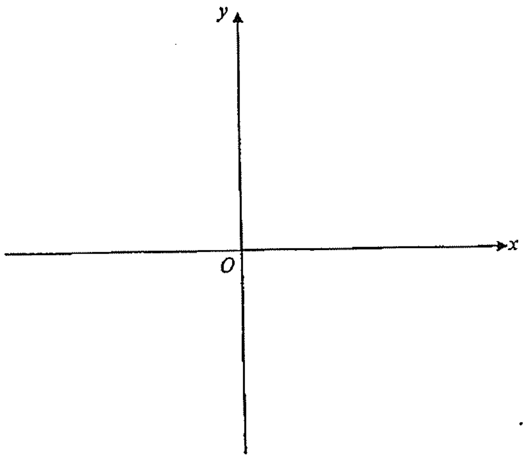
3 A function is defined by the equation $y=\frac{\sin x}{1-\cos x}$ for $0 \le x \le 2\pi$ where $x \ne a$.

(i) State the exact values of a. [1]

(ii) Explain, with reasons, whether the function is increasing or decreasing. [5]

4 A curve has equation $y = e^{2x-1}$.

(i) Sketch the graph of $y = e^{2x-1}$. [2]



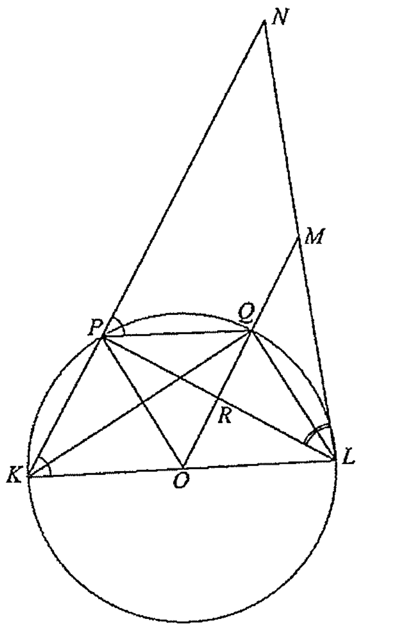
(ii) The curve $y = e^{2x-1}$ and $y = e^{k-x}$ meet at point R where $x = 1$. Find the value of k. [2]

5 (a) Find the range of values of n for which $9x^2+8nx+2n^2>8$ for all real values of x. [3]

(b) A curve has equation $y=(x+3)(x^2-3x+6)$. Explain why $y=(x+3)(x^2-3x+6)$ is always positive for $x>-3$. [3]

6 The coefficient of x^3 in the cubic polynomial $g(x)$ is a, where $a > 0$. The repeated roots of the equation $g(x) = 0$ are 2. Find the value of a if $g(x)$ has a remainder of $-\frac{9}{2}$ and 28 when divided by $(x+1)$ and $(x-4)$ respectively. [4]

7

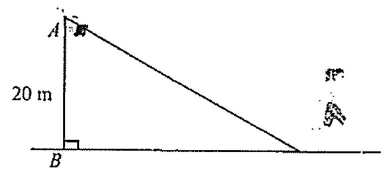


The diagram shows a circle with centre O, diameter KL. NML is a tangent to the circle at L and M is the midpoint of NL. The lines KN and OM cut the circle at P and Q respectively. The lines PL and OQ intersect at R. The line LQ bisects $\angle RLM$ and $\angle NPQ = \angle NKL$.

(i) Prove that $OKPQ$ is a rhombus. [3]

(ii) Prove that $KQ \times RQ = LQ \times LR$. [3]

8



In the diagram, a surveillance camera is mounted at a point A that is 20 m above a point B. A runner runs from point B along a straight course at a speed of 4 m/s The surveillance camera tracks the motion of the runner by panning upwards and downwards at point A. Find the rate of change of the angle that the surveillance camera makes with AB when the runner is 15 m from B. Give your answer in radians per second. [5]

9 The equation $2x^2+x-4=0$ has roots α and β. The equation $16x^2+21x+p=0$ has roots $\frac{1+q\beta^2}{\alpha}$ and $\frac{1+q\alpha^2}{\beta}$. Without finding the values of α and β, find the values of p and q.

[6]

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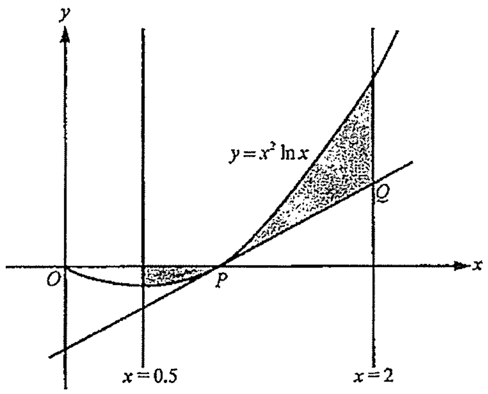
10 (i) Find $\frac{d}{dx}\left(x^3 \ln x\right)$. [2]

(ii) Hence find $\int x^2 \ln x \, dx$. [2]

(iii)



The diagram shows the lines $x=0.5$, $x=2$ and part of the curve $y=x^2 \ln x$. The curve intersects the x-axis at the point P and the tangent to the curve at P meets the line $x=2$ at point Q. Find the total area of the shaded region. [6]

11 The table shows experimental values of two variables x and y.

x	0.1	0.5	1	1.5	2
y	–5.95	1.63	0.83	0.61	0.5

It is known that x and y are related by the equation $\frac{\sqrt{x}}{y}=ax+\frac{b\sqrt{x}}{a}$, where a and b are constants.

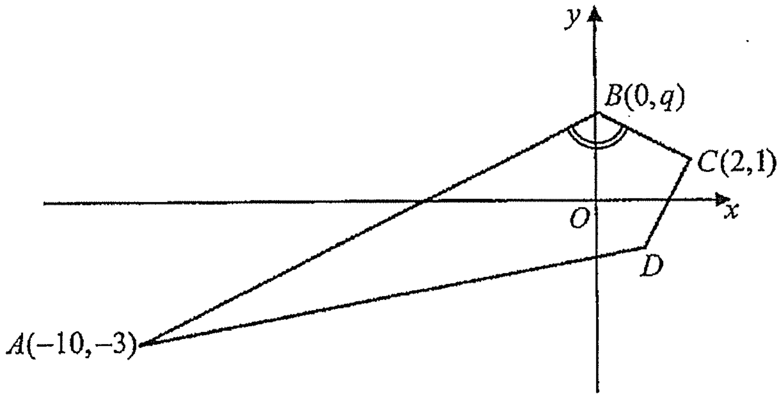
(i) Plot $\frac{1}{y}$ against $\sqrt{x}$ to obtain a straight line graph. [2]

11(i)

(ii) Use your graph to estimate the values of a and b. [4]

(iii) If, instead, a straight line is obtained by plotting $\frac{1}{y\sqrt{x}}$ against $\frac{1}{\sqrt{x}}$, find the gradient of the line. [2]

12



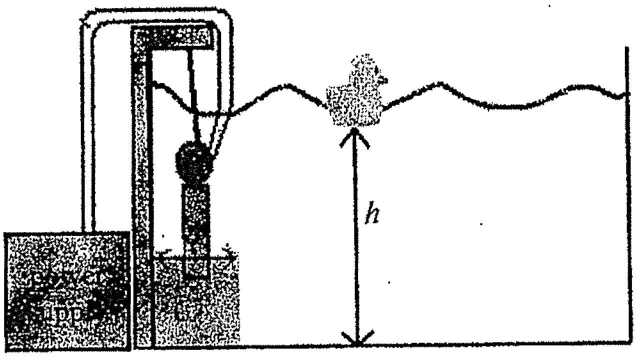
The diagram shows a kite with vertices $A(-10,-3)$, $B(0,q)$, $C(2,1)$ and D. It is given that angle ABO is equal to angle OBC.

(i) Show that $q=2$. [4]

(ii) Find the coordinates of the point D. [4]

(iii) Find the area of the kite $ABCD$. [2]

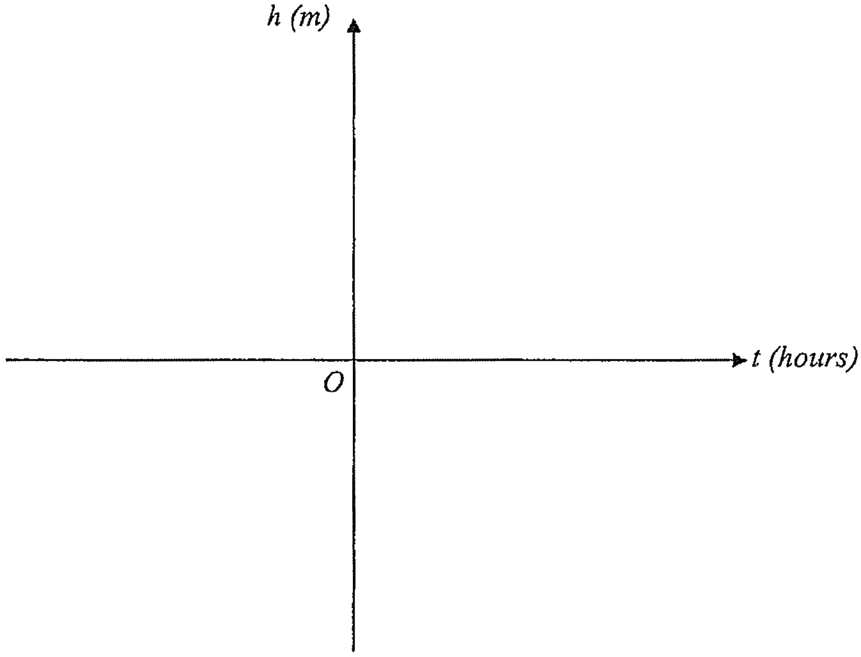
13



To study the effects of water waves, a wave generator and a rubber duck were placed in a water tank as shown in the diagram above. The height, h metres, from the bottom of water tank was modelled by $h = a\sin(kt) + b$, where t is the time in hours after midnight and a, b and k are constants. The motion of the rubber duck was observed for 36 hours. The minimum height of 1.5 m from bottom of water tank was first recorded at 06 00. The maximum height of 2.5 m was first recorded at 18 00.

(i) Find the values of a, b and k. [3]

(ii) Using the values found in (i), sketch the graph of $h = a\sin(kt) + b$ for $0 \le t \le 36$. [2]



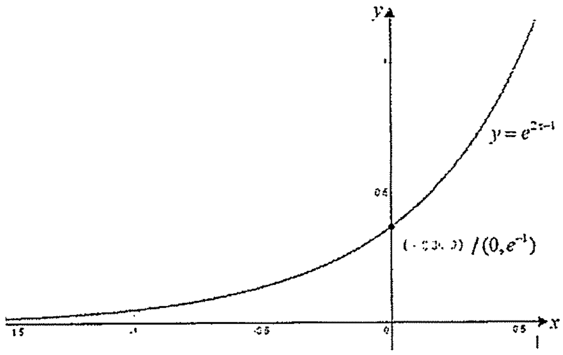
(iii) Find the range of values of t such that the rubber duck is above 2.1 m. [3]

~ End of Paper ~

Answer Key

1	$6-4\sqrt{2}$	2	$-\frac{15}{8}$
3(i)	$a=0,\ 2\pi$	3(ii)	Decreasing function

4(i)

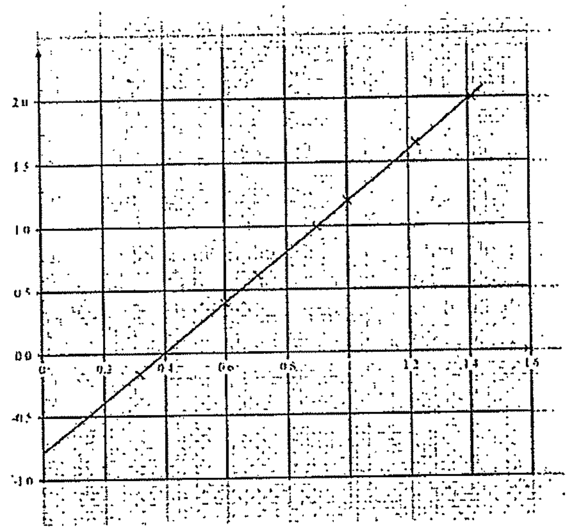


4(ii)	$k=2$	5(a)	$n<-6$ or $n>6$
6	$a=\frac{3}{2}$	8	0.128 rad / s
9	$p=-6,\ q=-1$	10(i)	$x^2+3x^2\ln x$

10(ii) $\int x^2 \ln x\, dx=\frac{1}{3}x^3\ln x-\frac{1}{9}x^3+c_2$, where c_2 is an arbitrary constant

10(iii) 0.639 unit^2 (3 s.f.)

11(i)



11(ii) $a=2$ [1.5 to 2.5], $b=-1.6$ [-2.125 to -1.125]

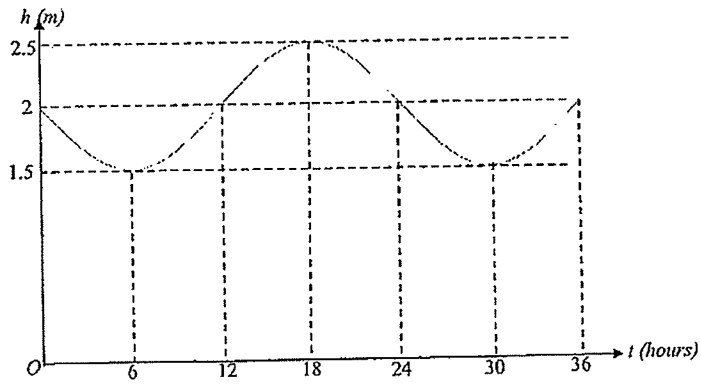
11(iii) -0.8

12(ii) $D(1,-1)$

12(iii) 20 units^2

13(i) $a=-0.5$, $b=2$, $k=\frac{\pi}{12}$

13(ii)



13(iii) $12.8<t<23.2$

Name	( )	Class	

南华中学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2019

Subject	**:**	**Additional Mathematics**
Paper	**:**	**4047/02**
Level	**:**	**Secondary Four Express**
Date	**:**	**2 September 2019**
Duration	**:**	**2 hours 30 minutes**

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You may use a soft pencil for any diagrams or graphs.
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The total number of marks for this paper is 100.

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Mathematical Formulae

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Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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Identities

$$\sin^2 A + \cos^2 A = 1$$

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$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

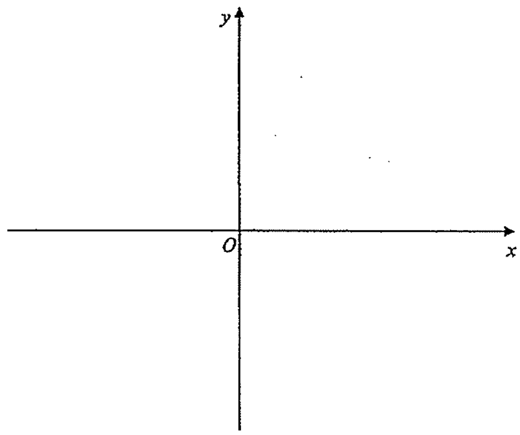
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta ABC = \frac{1}{2}bc \sin A$$

1 (a) Without using a calculator, find the value of 9^x, given that $\frac{1}{3}(5^x)(3^{2x}+81)=45^x$. [4]

(b) Sketch the graph of $y=2\log_4 x-1$ for $x>0$. [2]



(c) Solve the equation $\log_3 \frac{1}{9}\sqrt{x} = 1 + 2\log_x 81$. [5]

2 In a natural habitat, the population of a certain species of snails is given by $P = 0.8\left(Ae^{kt} + 500\right)$, where A and k are constants and t is the time in years starting from 1 January 2010. Over a period of 8 years from 1 January 2010 to 31 December 2017, the population decreased from 50 000 to 19 000.

(i) Calculate the values of A and of k. [3]

(ii) Calculate the year in which the population is 30% more compared to 31 Dec 2017. [3]

(iii) Explain, with justification, the expected population of the snails over a long period of time. [2]

3 (i) Express $\dfrac{4x^4-4x^3+23x^2-24x+5}{x(2x-1)^2}$ in partial fractions. [5]

(ii) Hence find $\int \frac{4x^4-4x^3+23x^2-24x+5}{5x(2x-1)^2}\,dx$. [4]

4 (a) (i) Write down the general term in the binomial expansion of $\left(\frac{6}{x^2}-\frac{x}{2}\right)^{15}$. [1]

(ii) Write down the power of x in this general term. [1]

(iii) Hence, determine the coefficient of x^{-9} in the expansion of $\left(\frac{6}{x^2}-\frac{x}{2}\right)^{15}$. [2]

(iv) Hence determine the coefficient of x^{-9} in $\left(\frac{2}{x^2}-\frac{x}{6}\right)^{15}$. [2]

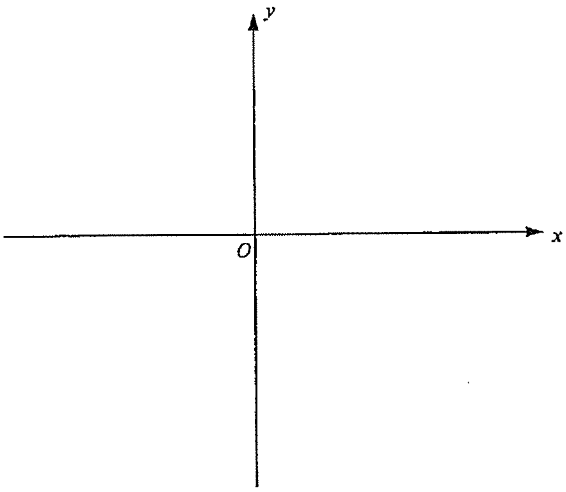
(b) The coefficient of x^2 in the expansion, in ascending powers of x, of $(1+x)^n(5-2x)^3$ [5]
is 3210. Find the value of n, where n is a positive integer.

5 (a) Solve the equation $4-|2x+3|=x$. [3]

(b) A curve has the equation $y=(2x-1)^2-9$.

(i) Explain why the lowest point on the curve has coordinates $\left(\frac{1}{2},-9\right)$. [1]

(ii) Sketch the graph of $y=\left|(2x-1)^2-9\right|$. [3]



(iii) Determine the set of values of m such that $\left|(2x-1)^2-9\right|=mx-2$ has no solution. [2]

6 The coordinates of the points A, B and C are $(0,7)$, $(-1,0)$ and $(6,-1)$ respectively.

(i) Show that AB is perpendicular to BC. [2]

(ii) Explain why A, B and C lie on the circumference of a circle, C_1 with diameter AC. [1]

(iii) Find the centre of C_1. [1]

(iv) The tangent to C_1 at point B is also a tangent to another circle, C_2. Given that the centre of C_2, lies on both the y-axis and the perpendicular bisector of BC, find the equation of C_2.
[8]

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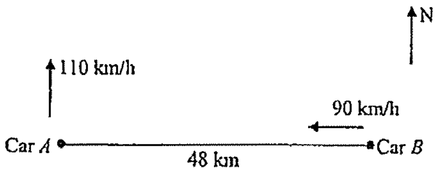
7 (i) Prove that $\sec 3x\left(\sin 3x - 2\sin^3 3x\right) = \tan 3x \cos 6x$. [4]

(ii) Hence find, for $0 \le x \le \frac{\pi}{3}$, the values of x in radians for which

$$-2\sin\frac{3}{2}x\cos\frac{3}{2}x = \sec 3x\left(\sin 3x - 2\sin^3 3x\right).$$ [6]

8



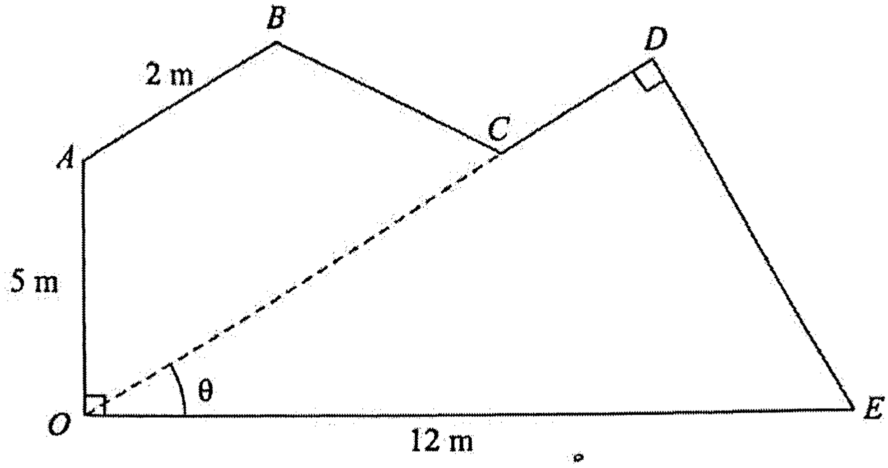
The diagram shows Car B, which is 48 km due east of Car A. Both cars start moving at the same time. Car A travels due north at a constant speed of 110 km/h while Car B travels due west at a constant speed of 90 km/h.

(i) The distance between Car A and Car B at time t hours after the cars started moving is denoted by L km. Express L in the form of $\sqrt{pt^2+(q-rt)^2}$ where p, q and r are constants. [3]

(ii) Given that t can vary, find the stationary value of L. [5]

(iii) Determine whether this value stationary value of L gives the maximum or minimum distance between Car A and Car B. [1]

9



For a theatre production, a panel is constructed by joining an isosceles trapezium and a right- angled triangle together.
It is given that $OA = 5$ m, $OE = 12$ m and OA is perpendicular to the base OE. OCD is perpendicular to DE and makes an angle θ with the base OE. AB and OC are the parallel sides of the trapezium $OABC$ and $AB = 2$ m.
The total length of the edges of the panel $OABCDE$ is represented by S.

(i) Show that $S = 12\cos\theta + 2\sin\theta + 22$. [3]

(ii) Express S in the form of $R\cos(\theta-\alpha)+Q$, where Q is a constant, $R>0$ and $0°<\alpha<90°$. [4]

(iii) A 30 m LED strip is placed along all the edges of the panel *OABCDE*. Triangle *ODE* of the panel is to be painted in black. Calculate the area to paint. [4]

10 A particle X, moves in a straight line with velocity, v m/s, given by $v = 2t^2 + kt + 63$, where k is a constant and t is the time in seconds, measured from the start of the motion. Its initial displacement from a fixed point O is -8 m. The minimum velocity of X occurs at $t = 5.75$.

(i) Find the minimum velocity of X. [2]

(ii) Find the values of t for which the particle is at instantaneous rest. [2]

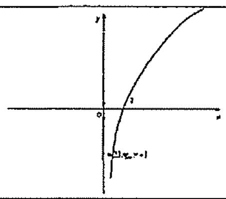
(iii) Find the distance travelled by particle X when $t = 7$. [3]

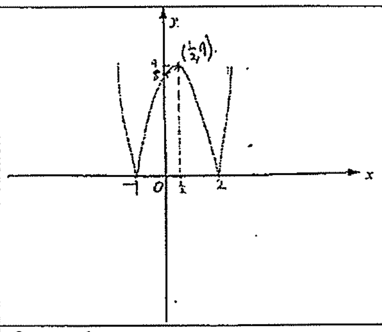
Another particle Y starts its motion at the same time as particle X and moves in a straight line with an initial velocity of 6 m/s from O. Its acceleration, a m/s^2, is given by $a = \frac{3}{5}t$.

(iv) Show that particle Y will not change its direction of motion. [3]

~End of Paper~

Answer Key:

1a	40.5
b	
c	6561 or $\frac{1}{9}$
2(i)	-0.123
(ii)	t = 5.81967 (must have). In year 2015
(iii)	As t becomes very large (over a long period of time), $e^{-0.122604t}$ approaches 0. Then expected population is $P = 0.8\ (0+500)$ $= 400$
3(i)	$x+\frac{5}{x}+\frac{1}{2x-1}-\frac{3}{(2x-1)^2}$
(ii)	$\frac{1}{10}x^2+\ln x+\frac{1}{10}\ln(2x-1)+\frac{3}{10(2x-1)}+c,$
4a(i)	$\binom{15}{r}(6)^{15-r}\left(-\frac{1}{2}\right)^r\left(x^{-30+3r}\right)$
a(ii)	Power of $x = -30+3r$
a(iii)	-84440070
a(iv)	$-\frac{1430}{243}$
b	9
5a	$x=\frac{1}{3}$ or -7
b(i)	For $x \in \mathbb{R}$, $(2x-1)^2 \geq 0$ $(2x-1)^2-9 \geq -9$ $y \geq -9$ At $y=-9, x=\frac{1}{2}$. Hence lowest point is $\left(\frac{1}{2},-9\right)$

b(ii)	
b(iii)	$-2 < m < 1$
6(i)	Product of the gradients of AB and BC $= (7)\times(-\frac{1}{7}) = -1$ $\therefore$ AB is perpendicular to BC.
(ii)	From (i), AB is perpendicular to BC implies that $\angle ABC = 90°$. Due to $\angle$ in a semi-circle, AC is the diameter of the circle, and A, B and C are points on the circumference of the same circle.
(iii)	(3,3)
(iv)	$x^2 + (y+18)^2 = 100.$
7(ii)	$0, \frac{\pi}{3}, \frac{\pi}{9}$
8(i)	$L = \sqrt{12100t^2 + (48-90t)^2}$
(ii)	37.1
(iii)	L is minimum
9(i)	$S = 12\cos\theta + 2\sin\theta + 22$
(ii)	$S = 12.2\cos(\theta - 9.5°) + 22$
(iii)	32.2 m^2
10(i)	-3.125 m/s
(ii)	7 or 4.5
(iii)	117 m
(iv)	For all values of t, since $\frac{3}{10}t^2 > 0$, $v > 0$. Particle will not change its direction of motion since velocity is always positive.

Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS 4047/01
Paper 1

26 August 2019 Monday 2 hours

2019 SECONDARY FOUR EXPRESS
PRELIMINARY EXAMINATIONS

INSTRUCTIONS TO CANDIDATES

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

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For Examiner's Use														
Qn	1	2	3	4	5	6	7	8	9	10	11	12	13	*Marks Deducted*
Marks														

Category	Accuracy	Units	Symbols	Others
Question No.				

Total Marks
80

Setter: Mdm Manju Manoharan
Vetter: Mr Tan Lip Sing

This question paper consists of 16 printed pages and 0 blank pages.

Mathematical Formulae

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Binomial Theorem

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!\,r!}=\frac{n(n-1)\ldots\ldots\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1.$$
$$\sec^2 A=1+\tan^2 A.$$
$$\operatorname{cosec}^2 A=1+\cot^2 A$$
$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}.$$
$$a^2=b^2+c^2-2bc\cos A.$$
$$\Delta=\frac{1}{2}ab\sin C.$$

Answer all questions in the space provided.

1 The roots of the equation $x^2 - 12x + 25 = 0$ are $2\alpha + \beta$ and $2\beta + \alpha$.

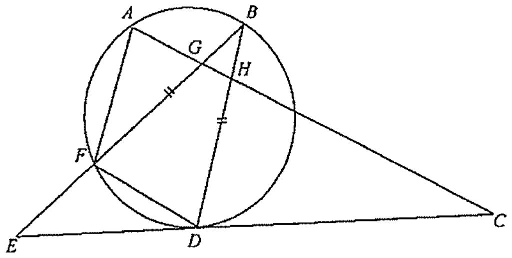
(i) Find the value of $\alpha + \beta$ and $\alpha\beta$. [4]

(ii) Hence, form an equation whose roots are α and β. [1]

2 The line $y = x + 4$ meets the curve $y^2 = 7 - 2x$ at the points P and Q. Find the equation of the perpendicular bisector of PQ. [5]

3 Express $\frac{5x+1}{(3-2x)(x^2+2)}$ in partial fractions. [5]

4 The diagram below shows a circle with points A, B, D and F on its circumference where $BF = BD$. The tangent to the circle at D meets BF extended at E and AH extended at C.



Prove that

(i) triangle EFD is similar to triangle EDB, [2]

(ii) $ED^2 - EF^2 = EF \times DB$. [3]

5 (a) Given that $\frac{4^x}{2^{x+1}} = \frac{3}{5^x}$, evaluate 10^x, [3]

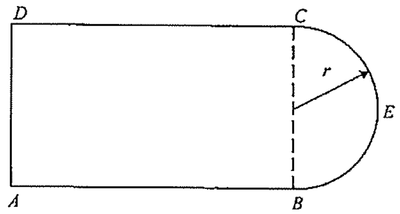
(b) A rectangular block has a square base of side $\left(1+\sqrt{3}\right)$ cm. The volume of the block is $\left(14+8\sqrt{3}\right)\text{cm}^3$. **Without using a calculator**, find the height of the rectangle, in cm, in the form $\left(a+b\sqrt{3}\right)$, where a and b are integers. [3]

6 A curve has the equation f(x), where $f'(x)=\frac{2x}{x^2-1}$ for $x>0$.

(i) Explain whether $f'(x)$ is an increasing or decreasing function. [3]

(ii) Given that $f(2)=8$, obtain an expression for f(x), leaving your answer in exact form. [3]

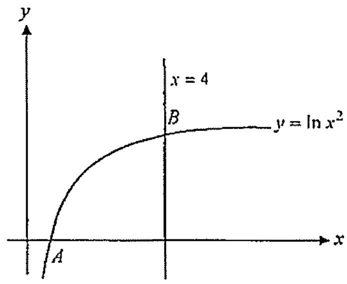
7 A dog park consists of a rectangle $ABCD$ and a semicircles of radius r m. The perimeter of the park is 200 m.



(i) Express the length of AB in terms of r. [2]

(ii) Given that r can vary, find the radius of the semicircle such that the dog park has the largest possible area. [4]

8 The diagram below shows part of the curve $y = \ln x^2$ for $x > 0$.



(i) Find the exact coordinates of A and of B. [2]

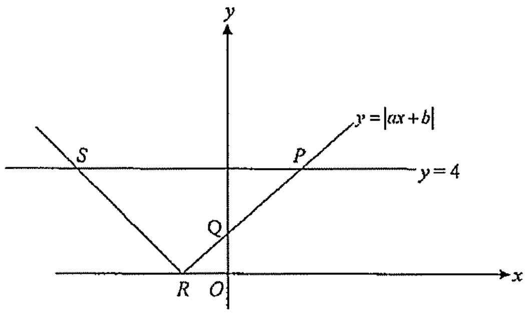
(ii) Determine the area of the shaded region bounded by the curve, the line $x = 4$, and the x-axis. [4]

9 The equation of a circle, C is $x^2+y^2-kx+(k+2)y+c=0$, where k is a constant and $c \geq 0$. The centre of the circle lies on the line $2x+5y+14=0$.

(i) Find the value of the constant k. [4]

(ii) Hence, find the range of values of c. [2]

10



The diagram shows the line $y=4$ and the graph of $y=|ax+b|$, where a and b are positive constants. The graph crosses the line $y=4$ at the points P and S, crosses the y-axis at Q and meets the x-axis at R.

(i) Given that the x-coordinate of P is 1.5 and that $PQ : PR = 3 : 4$, find, with full explanation, the x-coordinate of S. [4]

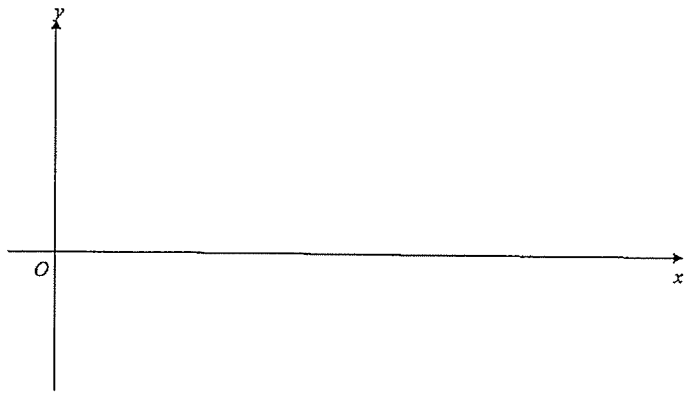
(ii) Find the value of a and of b. [3]

11 (i) Show that $4\cos^2 x - 2\sin^2 x$ can be written as $a\cos 2x + b$, where a and b are integers. [2]

Hence,

(ii) state the period, in radians, and amplitude of $4\cos^2 x - 2\sin^2 x$, [2]

(iii) sketch the graph of $y = 4\cos^2 x - 2\sin^2 x$ for $0 \le x \le 2\pi$ radians. [2]



12 (a) Find all the values of x between 0° and 360° for which $\sqrt{8}\sin(x-30°)=-1$. [3]

(b) Given that $2<x<6$, find the values of x for which $2\tan^2 x-13=\sec x$. [4]

13 An object at A, with an initial displacement of 3 m from a fixed point O, travels in a straight line so that its velocity, v m/s, is given by $v = t^2 - 5t + 6$ where t is the time in seconds after leaving A.

(i) Find the values of t when the object comes to an instantaneous rest. [2]

(ii) Find the acceleration of the object at $t = 5$ s. [2]

(iii) Obtain an expression, in terms of t, for the displacement of the object from O after t seconds. [2]

(iv) Find the average speed of the object in the first 5 seconds. [4]

END OF PAPER

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS 4047/01

Paper 1

26 August 2019 Monday 2 hours

2019 SECONDARY FOUR EXPRESS
PRELIMINARY EXAMINATIONS

MARKING SCHEME

Setter: Mdm Manju Manoharan
Vetter: Mr Tan Lip Sing

This question paper consists of 15 printed pages and 1 blank page.

Answer all questions in the space provided.

1 The roots of the equation $x^2 - 12x + 25 = 0$ are $2\alpha + \beta$ and $2\beta + \alpha$.

(i) Find the value of $\alpha + \beta$ and $\alpha\beta$. [4]

(ii) Hence, form an equation whose roots are α and β. [1]

(i) Sum of roots $= 2\alpha + \beta + 2\beta + \alpha$ $3\alpha + 3\beta = 12$ $\alpha + \beta = 4$	B1
Pdt of roots $= (2\alpha + \beta)(2\beta + \alpha)$ $5\alpha\beta + 2(\alpha^2 + \beta^2) = 25$	M1
$5\alpha\beta + 2\left[(\alpha + \beta)^2 - 2\alpha\beta\right] = 25$	M1 [Seen $(\alpha + \beta)^2 - 2\alpha\beta$]
$\alpha\beta + 2(\alpha + \beta)^2 = 25$ $\alpha\beta = 25 - 2(4)^2$ $= -7$	A1
(ii) Equation: $x^2 - 4x - 7 = 0$	B1✓ [follow through]

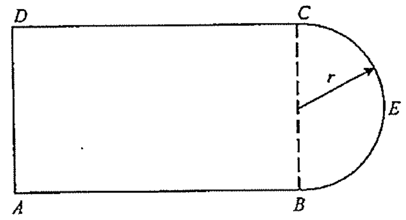
6 A curve has the equation f (x), where $f'(x)=\dfrac{2x}{x^2-1}$ for $x>0$.

(i) Explain whether $f'(x)$ is an increasing or decreasing function. [3]

(ii) Given that $f(2)=8$, obtain an expression for f (x). Leave your answer in exact form. [3]

(i) $f'(x)=\dfrac{2x}{x^2-1}$ $f''(x)=\dfrac{(x^2-1)(2)-(2x)(2x)}{(x^2-1)^2}$ $=\dfrac{-2x^2-2}{(x^2-1)^2}$ $=\dfrac{-2(x^2+1)}{(x^2-1)^2}$ For all $x>0$, $x^2+1>0$ and $(x^2-1)^2>0$ $\therefore \dfrac{-2(x^2+1)}{(x^2-1)^2}<0$ Hence, $f'(x)$ is a decreasing function for all $x>0$.	M1 [use of quotient rule accurately] M1[either numerator or denominator > 0 seen] A1 [with conclusion]
(ii) $f'(x)=\dfrac{2x}{x^2-1}$ $f(x)=\int\dfrac{2x}{x^2-1}dx$ $=\ln(x^2-1)+c$ When f (2) = 8, $8=\ln(4-1)+c$ $c=8-\ln 3$ $\therefore f(x)=\ln(x^2-1)+8-\ln 3$	M1 M1 [Finding the constant, c] A1 [Only accept exact form]

7 A dog park consists of a rectangle $ABCD$ and a semicircles of radius r m. The perimeter of the park is 200 m.

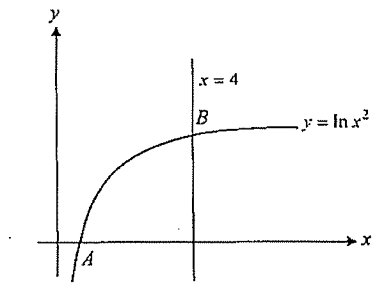


(i) Express the length of AB in terms of x. [2]

(ii) Given that x can vary, find the radius of the semicircle such that the dog park has the largest possible area. [4]

(i) $200 = 2AB + 2r + \frac{1}{2}(2\pi r)$ $AB = \frac{200 - 2r - \pi r}{2}$ $= 100 - r - \frac{1}{2}\pi r$	M1 A1
(ii) Area of park $= 2r(AB) + \frac{1}{2}(\pi r^2)$ $A = 2r\left(100 - r - \frac{1}{2}\pi r\right) + \frac{1}{2}(\pi r^2)$ $A = 200r - 2r^2 - \pi r^2 + \frac{1}{2}\pi r^2$ $A = 200r - 2r^2 - \frac{1}{2}\pi r^2$ $\frac{dA}{dr} = 200 - 4r - \pi r$ When $\frac{dA}{dr} = 0$, $200 - 4r - \pi r = 0$ $r = \frac{200}{\pi + 4}$ $\frac{d^2A}{dr^2} = -4 - \pi < 0$ $\therefore A$ is maximum	 M1 [with subst] M1 A1 M1

8 The diagram below shows part of the curve $y = \ln x^2$ for $x > 0$.



(i) Find the exact coordinates of A and B. [2]

(ii) Determine the area of the shaded region bounded by the curve, the line $x = 4$, and the x-axis. [4]

(i) $y = \ln x^2$ When $x = 4, y = \ln 16 / 4\ln 2$ When $y = 0, x = \pm 1$ A (1, 0) and B (4, ln16)	B1, 1
(ii) $y = \ln x^2$ $x = e^{\frac{y}{2}}$ $\int_0^{\ln 16} e^{\frac{y}{2}} dy$ $= \left[2e^{\frac{y}{2}}\right]_0^{\ln 16}$ $= 2\left[e^{\ln 4} - 1\right]$ $= 6$ Area of shaded region $= 4\ln 16 - 6$ $= 16\ln 2 - 6$ sq units $= 5.09$ sq units	M1 [making x the subject] M1 [with correct limits from (i)] M1 [seen 4ln16 or equivalent from (i)] A1 [accept all proper exp of 16ln2]

9 The equation of a circle, C is $x^2+y^2-kx+(k+2)y+c=0$, where k is a constant and $c \geq 0$. The centre of the circle lies on the line $2x+5y+14=0$.

(i) Find the value of the constant k. [4]

(ii) Hence, find the range of values of c. [2]

(i) $x^2+y^2-kx+(k+2)y+c=0$ $2g=-k$ and $2f=k+2$ $g=-\frac{k}{2}$ $f=\frac{k+2}{2}$ $\therefore$ Centre $\left(\frac{k}{2},-\frac{k+2}{2}\right)$ Since the centre lies on $2x+5y+14=0$, $2\left(\frac{k}{2}\right)+5\left(-\frac{k+2}{2}\right)+14=0$ $5\left(-\frac{k+2}{2}\right)=-k-14$ $-5k-10=-2k-28$ $3k=18$ $k=6$	B1 [accurate to find g and f] M1 M1 [method shown to simplify] A1
(ii) Since $k=6$, $r=\sqrt{\left(\frac{6}{2}\right)^2+\left(-\frac{6+2}{2}\right)^2-c}$ $r=\sqrt{-c+25}$ $\therefore 0 \leq c < 25$	M1 A1

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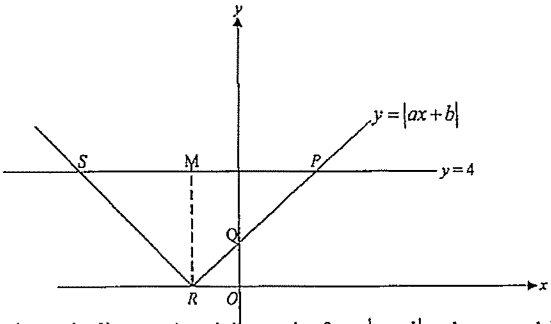


The diagram shows the line $y=4$ and the graph of $y=|ax+b|$, where a and b are positive constants. The graph crosses the line $y=4$ at the points P and S, crosses the y-axis at Q and meets the x-axis at R.

(i) Given that the x-coordinate of P is 1.5 and that $PQ : PR = 3 : 4$, find, with full explanation, the x-coordinate of S. [4]

(ii) Find the value of a and of b. [3]

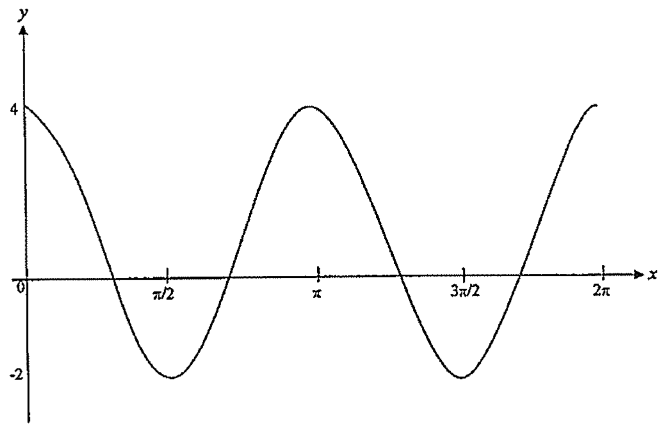
(i) $\frac{PQ}{PR}=\frac{1.5}{PM}$	M1 [use of ratio]
$\frac{3}{4}=\frac{1.5}{PM}$	
$PM=2$ units	
$\therefore x-$coord of $M=-0.5$	A1 [x-coord of M]
$PS=2PM=4$ units	M1[length of PS]
$\therefore x$-coord of $S=-2.5$	A1
(ii) $R(-0.5,0)$	
Gradient of PR $=\frac{0-4}{-0.5-1.5}=2$	M1 [or finding gradient of SR $=-2$]
Eqn of PR:	
$y-4=2(x-1.5)$	M1 [subst into equation]
$y=2x+1$	
$\therefore a=2,\ b=1$	A1

11 (i) Show that $4\cos^2 x - 2\sin^2 x$ can be written as $a\cos 2x + b$, where a and b are integers. [2]

Hence,

(ii) state the period, in radians, and amplitude of $4\cos^2 x - 2\sin^2 x$, [2]

(iii) sketch the graph of $y = 4\cos^2 x - 2\sin^2 x$ for $0 \le x \le 2\pi$ radians. [2]

(i) $4\cos^2 x - 2\sin^2 x$ $= 2(2\cos^2 x) - 2\sin^2 x$ $= 2(\cos 2x + 1) - (1 - \cos 2x)$ $= 2\cos 2x + 2 - 1 + \cos 2x$ $= 3\cos 2x + 1$	M1 [Double angle formula on $\sin^2 x$ or $\cos^2 x$] A1
(ii) Amplitude = 3 Period $= \pi$	B1 B1
(iii)  C1 – Correct shape and amplitude C1 – Correct period	

12 (a) Find all the values of x between 0° and 360° for which $\sqrt{8}\sin(x-30°)=-1$. [3]

(b) Given that $2<x<6$, find the values of x for which $2\tan^2 x-13=\sec x$. [4]

(a) $\sqrt{8}\sin(x-30°)=-1$ $\sin(x-30°)=-\frac{1}{\sqrt{8}}$ For $x-30°$ to exist in this interval, $-30°<x-30°<330°$ basic angle, $\alpha=\sin^{-1}\left(\frac{1}{\sqrt{8}}\right)=20.704°$ $x-30°=-20.704°,180°+20.704°,360-20.704°(rej)$ $x-30°=-20.704°,200.704°$ $x=9.295°,230.704°$ $x=9.3°,230.7°$	M1[basic angle without correct value] M1[for any one correct angle] A1 [correct to 1 d.p]
(b) $2\tan^2 x-13=\sec x$ $2(\sec^2 x-1)-13=\sec x$ $2\sec^2 x-15=\sec x$ $2\sec^2 x-\sec x-15=0$ $(2\sec x+5)(\sec x-3)=0$ $2\sec x+5=0$ or $\sec x=3$ $\sec x=-\frac{5}{2}$ or $\cos x=\frac{1}{3}$ $\cos x=-\frac{2}{5}$ $x=1.2309(\text{rej}),2\pi-1.2309$ $x=\pi-1.159$ (rej), $\pi+1.159$ $x=4.30$, 5.05	M1 [use of identity] M1 M1 [solving for both 1 factors with correct fraction. NO negative] A1

13 An object at A, with an initial displacement of 3 m from a fixed point O, travels in a straight line so that its velocity, v m/s, is given by $v = t^2 - 5t + 6$ where t is the time in seconds after leaving A.

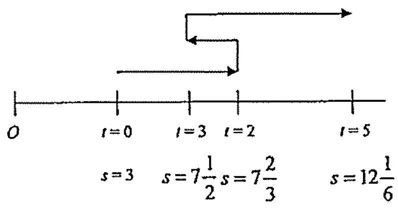
(i) Find the values of t when the object comes to an instantaneous rest. [2]

(ii) Find the acceleration of the object at $t = 5$ s. [2]

(iii) Obtain an expression, in terms of t, for the displacement of the object from O after t seconds. [2]

(iv) Find the average speed of the object in the first 5 seconds. [4]

(i) When at instantaneous rest, $v = 0$ $v = t^2 - 5t + 6$ $t^2 - 5t + 6 = 0$ $(t-3)(t-2) = 0$ $t = 3$ or 2	M1[equate to 0] A1
(ii) $a = \frac{dv}{dt}$ $= 2t - 5$ When $t = 5$, $a = 2(5) - 5$ $= 5$ m/s^2	M1 [Finding a] A1
(iii) $s = \int (t^2 - 5t + 6)\,dt$ $= \frac{t^3}{3} - \frac{5t^2}{2} + 6t + c$ When $t = 0$, $s = 3$, $c = 3$. $\therefore s = \frac{t^3}{3} - \frac{5t^2}{2} + 6t + 3$	M1 A1

(iv) When $t=2$, $s=\frac{2^3}{3}-\frac{5(2)^2}{2}+6(2)+3$ $=7\frac{2}{3}$ When $t=3$, $s=\frac{3^3}{3}-\frac{5(3)^2}{2}+6(3)+3$ $=7\frac{1}{2}$	M1 [finding 2 out 3 relevant s values]
When $t=5$, $s=\frac{5^3}{3}-\frac{5(5)^2}{2}+6(5)+3$ $=12\frac{1}{6}$	M1 [finding 3rd relevant s value]
	
Average speed in first 5 s $=\frac{\left(7\frac{2}{3}-3\right)+\left(7\frac{2}{3}-7\frac{1}{2}\right)+\left(12\frac{1}{6}-7\frac{1}{2}\right)}{5}$	M1 [2 out 3 dist calculated correctly based on above values]
$=\frac{9.5}{5}$ $=1.9$ m/s	A1

[Full 4 marks if equation used does not include $c=3$ from (iii)]

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