TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2019
Secondary 4

CANDIDATE NAME

CLASS | INDEX NUMBER

ADDITIONAL MATHEMATICS **4047/01**

Paper 1

Wednesday 28 August 2019

2 hours

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\text{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 **(a)** Given that $\int_1^2 cx^3 \, dx = \frac{3}{4}$, where c is a constant.

Find the value of $\int_2^3 cx^3 \, dx$. [3]

(b) **(i)** Given that $\left(3^{x+2}\right)\left(5^{x-2}\right) = 15^{2x}$, find the value of 15^x. [3]

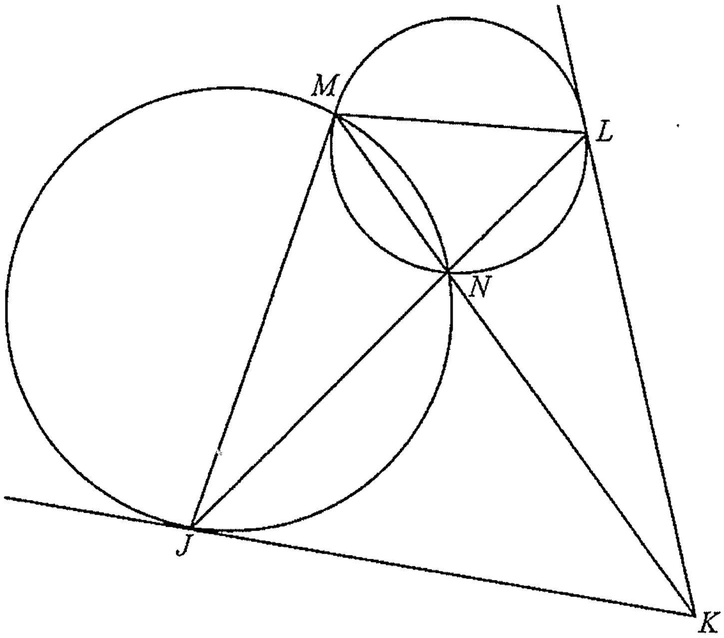
(ii) Hence, solve the equation $\left(3^{x+2}\right)\left(5^{x-2}\right) = 15^{2x}$. [2]

2 P, Q, R and S are the points (0, 9), (–3, –3), (1, –2) and (3, 7) respectively.

(i) Find the equation of the line, l, that is perpendicular to PS and passes through the point R. [3]

(ii) If PQ is extended to meet the line l at T, find the ratio of $PQ : QT$. [4]

3 Two circles intersect at M and N. K is a point on MN produced such that KL and KJ are tangents to the circles at L and J respectively and $KL = KJ$.



Given that LNJ is a straight line, show that

(i) Line KM bisects $\angle LMJ$. [2]

(ii) If MK is perpendicular to LJ, explain why a circle with MK as diameter passes through L and J. [3]

4 Explain why there are always two intersection points between the line $y = a - \frac{5}{2}$ and the curve $y = -2x^2 - ax$ for all real values of x. [5]

5 A moving particle P starts with a velocity of 7 m/s from a point O and moves in a a straight line so that its acceleration after t seconds is given by $a = (20 - 6t)$ m/s^2.

Find

(i) the value of t when the speed is at maximum, [2]

(ii) the total distance travelled by the particle during the fourth second. [4]

6 The roots of the quadratic equation $4x^2 - 8x - 3 = 0$ are α and β.

(i) Find the value of $\alpha^2 + \beta^2$. [3]

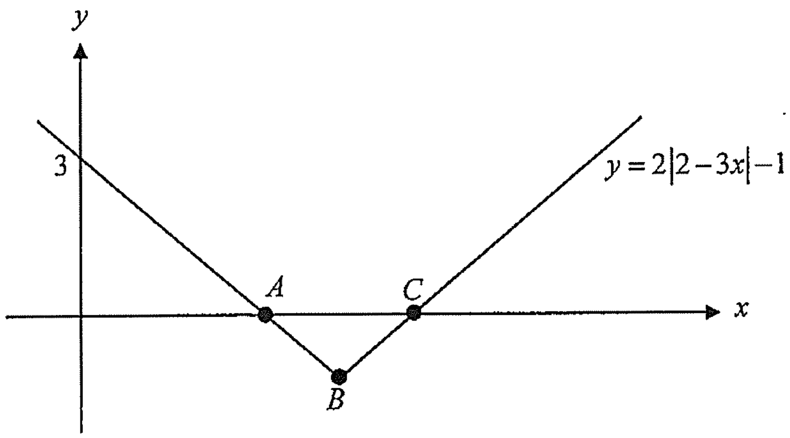
(ii) Find a quadratic equation whose roots are α^3 and β^3. [3]

7 The equation of a curve is $y=(x+3)(ax^2+b)$. The curve passes through the point (0, 9) and when $x=-1$, $\frac{dy}{dx}=\frac{d^2y}{dx^2}=0$. Find

(i) the value of a and of b, [3]

(ii) the stationary point of the curve and determine with working, the nature of the stationary point. [3]

8 The graph $y = 2|2-3x|-1$ is shown below.



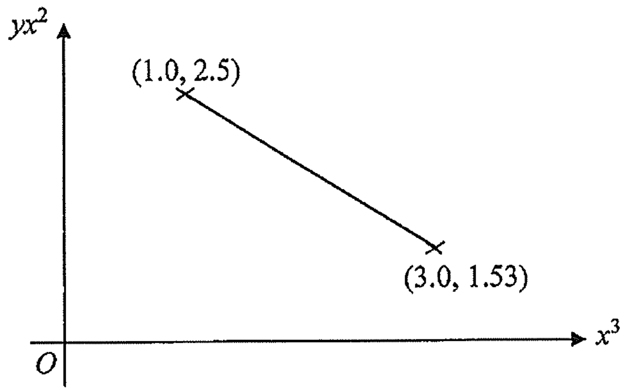
(i) Find the coordinates of A, B and C. [4]

(ii) Determine the number of point(s) of intersection of the line $y = 6x + c$ with $y = 2|2-3x|-1$, given that $c > 3$. Justify your answer. [2]

(iii) Solve the equation $2|2-3x|-1 = 2+6x$. [2]

9 The diagram shows the straight line obtained by plotting yx^2 against x^3.

Variables x and y are related by an equation $y = \frac{p}{x^2} + qx$, where p and q are constants.



(i) Find

(a) the value of p and of q, [4]

(b) the coordinates of the point on the line at which $y = \frac{3}{2x^2}$. [2]

(ii) If the graph of $\frac{y}{x}$ is plotted against $\frac{1}{x^3}$ instead, state the values of the gradient and the $\frac{y}{x}$ - intercept for this graph. [2]

10 **(a)** Solve the equation $2\cot^2 y = \operatorname{cosec} y + 1$ for $0° \le y \le 360°$. [4]

(b) Find the range of values of x between $0°$ and $180°$ which satisfies the inequality

(i) $\sin 2x < 0$, [1]

(ii) $\cos\frac{1}{2}x > 0.5$. [2]

(iii) Hence, state the range of values of x between $0°$ and $180°$ which satisfies both inequalities. [1]

11 An inverted conical vessel initially contains water to a height of 7 cm. A small tap is opened at the vertex and the water leaks such that after t minutes, the rate of decrease of the height, h cm, of the water in the vessel is given by $\frac{3t}{2}$ cm/min.

(i) Find an expression for h in terms of t and hence, calculate the time taken for the vessel to be empty. [3]

The volume of water in the vessel after t minutes is $\frac{\pi}{12}h^3$ cm^3.

(ii) Calculate the rate of change of the volume of water in the vessel when $t = 2$. [4]

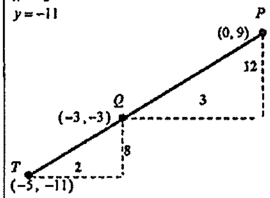
12 The respiration rate R (in litres/min) of a boy can be modelled by the equation $R = 6 - 3\cos\frac{2\pi}{5}t$, where t is the time (in minutes) at which the respiration rate is measured.

(i) Explain why this model suggests that the respiration rate, R (in litres/min) of the boy is $3 \le R \le 9$. [2]

(ii) Respiration rate is considered as healthy if it is more than 7 litres/min. Find the length of time for which the boy's respiration rate is healthy, for every 5 minutes. [4]

End of Paper

Qn	Key Steps
1(a)	$\int_1^2 cx^3 \, dx = \frac{3}{4}$ $\left[\frac{cx^4}{4}\right]_1^2 = \frac{3}{4}$ $\frac{15}{4}c = \frac{3}{4}$ $c = \frac{1}{5}$ $\int_2^3 \frac{1}{5}x^3 \, dx$ $= \left[\frac{1}{20}x^4\right]_2^3$ $= \frac{1}{20}(3)^4 - \frac{1}{20}(2)^4$ $= \frac{13}{4}$
(b)(i)	$(3^{x+2})(5^{x-2}) = 15^{2x}$ $\frac{3^{x+2}}{3^{2x}} \times \frac{5^{x-2}}{5^{2x}} = 1$ $3^{2-x} \times 5^{-2-x} = 1$ $\frac{3^2}{3^x} \times \frac{1}{5^x \times 5^2} = 1$ $\frac{9}{25} \times \frac{1}{3^x \times 5^x} = 1$ $\frac{1}{15^x} = \frac{25}{9}$ $15^x = \frac{9}{25}$
(ii)	$x \lg 15 = \lg \frac{9}{25}$ $x = -0.377$ (3 sf)

2(i)	$m_{PS} = \frac{9-7}{0-3}$ $= -\frac{2}{3}$ $m_{\perp} = \frac{3}{2}$ Equation of line l: $y-(-2) = \frac{3}{2}(x-1)$ $y = \frac{3}{2}x - \frac{7}{2}$
(ii)	$m_{PQ} = \frac{9-(-3)}{0-(-3)}$ $= 4$ Equation of line PQ: $y = 4x+9$ -------- (1) $y = \frac{3}{2}x - \frac{7}{2}$ -------- (2) $(1) = (2)$ $4x + 9 = \frac{3}{2}x - \frac{7}{2}$ $x = -5$ $y = -11$  By counting, $PQ : QT = 3 : 2$

3(i)	$\angle KJN = \angle KLN$ (given) $\angle KJN = \angle JMN$ (tan-chord thm) $\angle KLN = \angle LMN$ (tan-chord thm) $\Rightarrow \angle JMN = \angle LMN$ $\therefore$ Line LON bisects $\angle LMJ$. (shown)
(ii)	Given that $\angle MNL = \angle MNJ = 90°$ and let $\angle LMN = \angle JMN = \angle KLN = \angle KJL = x$, $\therefore \angle NLM = 90° - x$ ($\angle$s sum in ΔNML) $\angle MLK = 90° - x + x$ $= 90°$ $\therefore \angle NJM = 90° - x$ ($\angle$s sum in ΔNMJ) $\angle MJK = 90° - x + x$ $= 90°$ Since $\angle MLK$ and $\angle MJK$ are 90°, they obey $\angle$ in semicircle property. $\Rightarrow MK$ is a diameter of a circle which passes through L and J. (shown)
4	$y = -2x^2 - ax$ -------- (1) $y = a - \frac{5}{2}$ -------- (2) $(1) = (2)$ $-2x^2 - ax = a - \frac{5}{2}$ $-2x^2 - ax - a + \frac{5}{2} = 0$ Discriminant $= (-a)^2 - 4(-2)\left(-a + \frac{5}{2}\right)$ $= a^2 - 8a + 20$ $= a^2 - 8a + \left(\frac{8}{2}\right)^2 + 20 - \left(\frac{8}{2}\right)^2$ $= (a-4)^2 + 4$ $(a-4)^2 \geq 0$ $(a-4)^2 + 4 \geq 4$ $D > 0$ Since $D > 0$, therefore there are always two points of intersection between the line $y = a - \frac{5}{2}$ and the curve $y = -2x^2 - ax$ for all real values of x. (shown)

5(i)	For maximum velocity, $20 - 6t = 0$ $t = 3\frac{1}{3}$ $\frac{d^2v}{dt^2} = -6$ At $t = 3\frac{1}{3}$, velocity is a maximum.
(ii)	$v = \int 20 - 6t \, dt$ $v = 20t - 3t^2 + c$ At $t = 0$, $v = 7$ $\therefore c = 7$ $v = 20t - 3t^2 + 7$ $s = \int 20t - 3t^2 + 7 \, dt$ $s = 10t^2 - t^3 + 7t + c$ At $t = 0$, $s = 0$ $\therefore c = 0$ $s = 10t^2 - t^3 + 7t$ At $t = 3$, $s = 10(3)^2 - (3)^3 + 7(3)$ $s = 84$ m At $t = 4$, $s = 10(4)^2 - (4)^3 + 7(4)$ $s = 124$ m Total distance travelled $= 124 - 84$ $= 40$ m

6(i)

$4x^2-8x-3=0$

$\alpha+\beta=2$

$\alpha\beta=-\frac{3}{4}$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=(2)^2-2\left(-\frac{3}{4}\right)$

$=\frac{11}{2}$

(ii)

$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$

$=(2)\left(\frac{11}{2}-\left(-\frac{3}{4}\right)\right)$

$=\frac{25}{2}$

$\alpha^3\beta^3=(\alpha\beta)^3$

$=\left(-\frac{3}{4}\right)^3$

$=-\frac{27}{64}$

Equation:

$x^2-\frac{25}{2}x-\frac{27}{64}=0$

7(i)

Sub $(0,\ 9)$,

$9=(3)(b)$

$b=3$

$y=(x+3)(ax^2+3)$

$y=ax^3+3ax^2+3x+9$

$\frac{dy}{dx}=3ax^2+6ax+3$

Sub $x=-1$, $\frac{dy}{dx}=0$

$0=3a-6a+3$

$a=1$

(ii)

$\frac{dy}{dx}=3x^2+6x+3$

Sub $x=-1$,

$y=(-1+3)((-1)^2+3)$

$y=8$

$\therefore$ Stationary point $=(-1,\ 8)$

x	-1.1	-1	-0.9
$\frac{dy}{dx}$	>0	$=0$	>0
graph	/	—	/

$\therefore$ Point $(-1,\ 8)$ is a point of inflexion.

8(i)

$y=2|2-3x|-1$

$0=2|2-3x|-1$

$2-3x=\pm\frac{1}{2}$

$2-3x=\frac{1}{2}$ or $2-3x=-\frac{1}{2}$

$x=\frac{1}{2}$ $\qquad$ $x=\frac{5}{6}$

$B=\left(\frac{2}{3},\ -1\right)$

$A=\left(\frac{1}{2},\ 0\right)$

$C=\left(\frac{5}{6},\ 0\right)$

(ii)

The gradient of the right arm of the modulus graph ($y=2|2-3x|-1$) = 6.

Gradient of the line ($y=6x+c$) = 6.

$\therefore$ the line $y=6x+c$ is parallel to the right arm of the graph $y=2|2-3x|-1$ and since $c>3$, there is only 1 point of intersection.

(iii)

$2|2-3x|-1=2+6x$

$|2-3x|=\frac{3}{2}+3x$

$2-3x=\frac{3}{2}+3x$ or $2-3x=-\left(\frac{3}{2}+3x\right)$

$x=\frac{1}{12}$ $\qquad$ (NA)

9(i)(a)

$y=\frac{p}{x^2}+qx$

$yx^2=p+qx^3$

$q=\frac{1.53-2.5}{3.0-1.0}$

$q=-0.485$

Sub $(1.0,\ 2.5)$ and $q=-0.485$,

$2.5=p+(-0.485)(1.0)$

$p=2.985$

(b)

$\therefore yx^2=\frac{3}{2}$

Sub $yx^2=\frac{3}{2}$, $q=-0.485$ and $p=2.985$,

$\frac{3}{2}=2.985+(-0.485)x^3$

$x^3=3.06$ $\quad$ (3 sf)

Coordinates $=\left(3.06,\ \frac{3}{2}\right)$

(ii)

$y=\frac{p}{x^2}+qx$

$\frac{y}{x}=\frac{p}{x^3}+q$

$\therefore$ p = gradient

$q=\frac{y}{x}$-intercept

$\therefore$ Gradient = 2.985

$\frac{y}{x}$-intercept = -0.485

10(a)

$2\cot^2 y=\operatorname{cosec} y+1$

$2(\operatorname{cosec}^2 y-1)=\operatorname{cosec} y+1$

$2\operatorname{cosec}^2 y-\operatorname{cosec} y-3=0$

$(2\operatorname{cosec} y-3)(\operatorname{cosec} y+1)=0$

$\operatorname{cosec} y=\frac{3}{2}$ or $\operatorname{cosec} y=-1$

	$\sin y = \frac{2}{3}$ $\quad$ $\sin y = -1$ Basic angle of $y = 41.810°$ $\quad$ $y = 270°$ $y = 41.8°, 138.2°$ $y = 41.8°, 138.2°, 270°$
(b)(i)	From the graph $y = \sin 2x$, $\therefore \; 90° < x < 180°$
(ii)	Let $\cos\frac{1}{2}x = 0.5$ Basic angle of $\frac{1}{2}x = 60°$ $x = 120°$ $\therefore \; 0° < x < 120°$
(iii)	$90° < x < 120°$
11(i)	$\frac{dh}{dt} = -\frac{3t}{2}$ $h = \int -\frac{3t}{2}\, dt$ $h = -\frac{3t^2}{4} + c$ When $t = 0, h = 7$ $c = 7$

	$\therefore h = -\frac{3t^2}{4} + 7$ Sub $h = 0$, $t = 3.06$ mins
(ii)	$V = \frac{\pi}{12}h^3$ $\frac{dV}{dh} = \frac{\pi}{4}h^2$ $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ $\frac{dV}{dt} = \frac{\pi}{4}h^2 \times \left(-\frac{3t}{2}\right)$ Sub $t = 2$, $h = -\frac{3(2)^2}{4} + 7$ $h = 4$ Sub $t = 2, h = 4$ $\frac{dV}{dt} = \frac{\pi}{4}(4)^2 \times \left(-\frac{3(2)}{2}\right)$ $= -37.7$ cm³/s $\quad$ (3 sf)
12(i)	Maximum $\cos\frac{2\pi}{5}t = 1$ $\left\lvert 3\cos\frac{2\pi}{5}t \right\rvert = 3$ $\therefore$ Minimum $R = 6 - 3$ $= 3$ $\therefore$ Maximum $R = 6 + 3$ $= 9$

	Hence, $3 \le R \le 9$
(ii)	Let $R = 7$, $6 - 3\cos\frac{2\pi}{5}t = 7$ $\cos\frac{2\pi}{5}t = -\frac{1}{3}$ Basic angle of $\frac{2\pi}{5}t = 1.23096$ $\frac{2\pi}{5}t = 1.91063, \; 4.37255$ $t = 1.5204, \; 3.4956$ Duration $= 3.4956 - 1.5204$ $= 1.96$ min $\quad$ (3 sf)

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2019
Secondary 4

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS **4047/02**

Paper 2

Monday 2 September 2019
2 hours 30 minutes

Candidates answer on the Question Paper.
No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

Answer all questions.

1 **(i)** On the same axes sketch the curves $y^2 = 8x$ and $y = x^{-\frac{1}{4}}$. [2]

(ii) Find the value of k for which the x-coordinate of the point of intersection of the two graphs satisfies the equation $x^3 = k$. [2]

(iii) Calculate the coordinates of the point of intersection of the graphs. [2]

2 **(i)** Write down the general term in the binomial expansion of $\left(x^3+\frac{p}{x}\right)^{11}$, where p is a constant. [1]

(ii) Given that $n=1$ and the ratio of the coefficients of $\frac{1}{x^2}$ and $\frac{1}{x^6}$ in the expansion of $x^n\left(x^3+\frac{p}{x}\right)^{11}$ is 5:1, find the value of p. [4]

(iii) Explain why there are no odd powers of x in the expansion of $x^n\left(x^3+\dfrac{p}{x}\right)^{11}$ if n is an odd integer. [3]

3 (i) Express $\dfrac{4x^3+2x^2-5}{2x^3-x^2}$ in partial fractions. [5]

(ii) Hence, find $\int \dfrac{4x^3+2x^2-5}{2x^3-x^2}\,dx$. [3]

4 It is given that $f(x) = ax^3 - 6x^2 + 15x - 1$, where a is a constant.

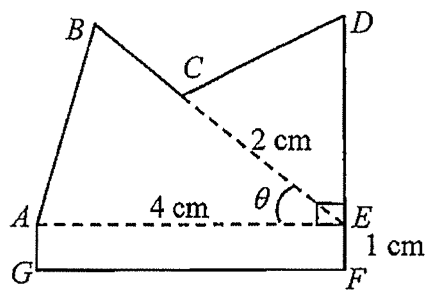
(i) If $a = 1$, show that f(x) is an increasing function for all real values of x. [4]

(ii) Find the range of values of a for which f(x) is an increasing function. [5]

5 (i) Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{e^{3x}}{\sqrt{x-1}}\right)=\frac{e^{3x}(6x-7)}{2\sqrt{(x-1)^3}}$. [4]

(ii) Hence or otherwise, find $\int_2^3 \frac{e^{3x}(6x-7)}{\sqrt{(x-1)^3}}\,dx$. [4]

6



The diagram represents an enclosed space, *ABCDEFG*, which is made up of the rectangle *AEFG*, two isosceles triangles, *ABE* and *CDE*. Angle *AEB* is $\theta°$, angle $AED = 90°$, $AE = BE = 4$ cm, $CE = DE = 2$ cm and $EF = 1$ cm.
The total area of the enclosed space is S cm^2.

(i) Show that $S = 2\cos\theta + 8\sin\theta + 4$. [3]

(ii) Express S in the form $p + R\cos(\theta - \alpha)$, where p is a constant, $R > 0$ and α is acute. [3]

(iii) Find the maximum value of S and the corresponding value of θ. [3]

(iv) Find the value of θ when $S = 11$. [2]

7 (i) Show that $\operatorname{cosec} 2x + \cot 2x = \cot x$. [3]

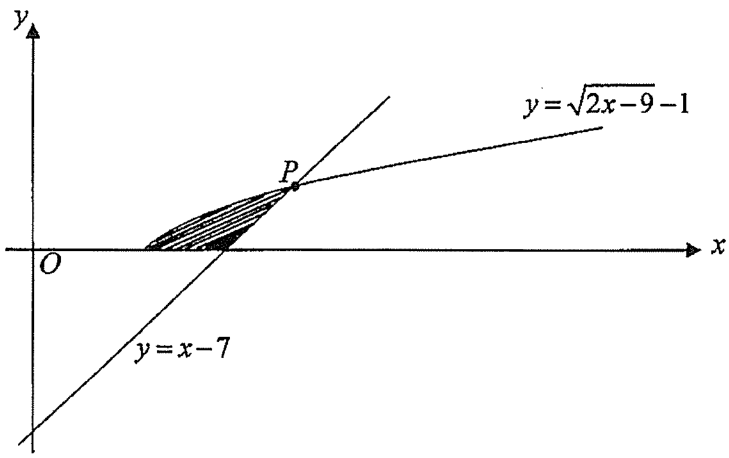
(ii) Hence solve the equation $\frac{1}{2}(\operatorname{cosec} 2x + \cot 2x) = \cos x$ for $0 \le x \le \pi$. [4]

(iii) Given that $3\operatorname{cosec} 2\theta + 3\cot 2\theta = 2\operatorname{cosec} \theta$, without using a calculator,

(a) deduce that $\cos\theta = \frac{2}{3}$, [2]

(b) find the value of $\sin 2\theta$, for $0 < \theta < \frac{\pi}{2}$. [3]

8



The diagram (not drawn to scale) shows part of the curve $y=\sqrt{2x-9}-1$ intersecting the line $y=x-7$ at the point P. Another point Q lies on the curve such that the tangent to the curve at Q is parallel to the line $y=x-7$.

(i) Find the x-coordinate of Q. [4]

(ii) Find the equation of the tangent to the curve at Q. [2]

(iii) Show that the coordinates of the point P is $(9,2)$. [3]

(iv) Find the area of the shaded region. [4]

9 The equation of a circle is $x^2 + y^2 - 4x + 2y - 20 = 0$.

(i) Find the coordinates of the centre, C and the radius of the circle. [3]

Two points A and B lie on the circle such that AC, the radius of the circle is parallel to the x-axis and the line AB passes through the origin.

(ii) Given that the x-coordinate of A is **negative**,

(a) write down the coordinates of A, [2]

(b) find the equation of AB, [2]

(c) find the coordinates of B, [3]

(d) find angle ACB. [2]

10 **(a)** Given that $x = \ln 2$ and $y = \ln 5$,

(i) express $\ln\left(\dfrac{50}{e^3}\right)$ in terms of x and y, [4]

(ii) find the exact value of k such that $\ln k = \dfrac{e^y - e^x}{2}$. [3]

(b) A manufacturer purchases a machine at a cost of $80 000. The value of the machine decreases with time so that its value, $$V$, after t month is given by $V = 80\ 000(10)^{-kt}$, where k is a positive constant. The value of the machine is expected to be $65 000 after $2\frac{1}{2}$ years.

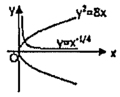
(i) Show that $k \approx 0.003006$. [2]

(ii) Calculate the value, to the nearest $100, of the machine after 20 months. [2]

It is only economical to replace the machine after its value reaches half of its original value.

(iii) Determine, with working, whether it is economical to replace the machine after 9 years. [2]

End of Paper

Qn	Key Steps
1(i)	
(ii)	$8x = x^{-\frac{1}{2}}$ $x^{\frac{3}{2}} = \frac{1}{8}$ $x^3 = \frac{1}{64}$ $k = \frac{1}{64}$
(iii)	$x = \frac{1}{4}, y = \sqrt{8 \times \frac{1}{4}} = \sqrt{2}$ Coordinates $\left(\frac{1}{4}, \sqrt{2}\right)$
2(i)	General term of $\left(x^3 + \frac{p}{x}\right)^{11} = {}^{11}C_r \left(x^3\right)^{11-r} \left(\frac{p}{x}\right)^r$ $= {}^{11}C_r p^r x^{33-4r}$
(ii)	For coefficient of $\frac{1}{x^2}$, $34-4r = -2$ $r = 9$ For coefficient of $\frac{1}{x^6}$, $34-4r = -6$ $r = 10$ $\frac{{}^{11}C_9 p^9}{{}^{11}C_{10} p^{10}} = \frac{5}{1}$ $\frac{5}{p} = \frac{5}{1}$ $p = 1$
(iii)	$x^n\left(x^3 + \frac{p}{x}\right)^{11}$ General term $\Rightarrow {}^{11}C_r p^r x^{33-4r+n}$ $4r$ is always even $\Rightarrow 33-4r$ will be odd. Hence, if n is odd $\Rightarrow 33-4r+n$, which is the sum of two odd integers, will be an even integer. Therefore, there are no odd powers of x.

Qn	Key Steps
3(i)	$2x^3 - x^2 \overline{)4x^3 + 2x^2 + 0x - 5}$ quotient 2 $-\ 4x^3 - 2x^2$ $4x^2 \quad -5$ $\frac{4x^2-5}{x^2(2x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x-1}$ $4x^2 - 5 = Ax(2x-1) + B(2x-1) + Cx^2$ sub x=0 or comparing constant, $B = 5$ sub $x=\frac{1}{2}$, $\frac{1}{4}C = -4 \Rightarrow C = -16$ comparing coefficient of x terms, $2B - A = 0 =$ $\frac{4x^3 + 2x^2 - 5}{2x^3 - x^2} = 2 + \frac{10}{x} + \frac{5}{x^2} - \frac{16}{2x-1}$
(ii)	$\int 2 + \frac{10}{x} + \frac{5}{x^2} + \frac{16}{2x-1} dx$ $= 2x + 10\ln x - \frac{5}{x} - 8\ln(2x-1) + c$

Qn	Key Steps
4(i)	$f(x) = x^3 - 6x^2 + 15x - 1$ $f'(x) = 3x^2 - 12x + 15$ $= 3(x^2 - 4x + 5)$ $= 3[(x-2)^2 + 1]$ $= 3(x-2)^2 + 3$ Since $(x-2)^2 > 0 \Rightarrow f'(x) > 0$ and a>0, $\therefore f(x)$ is an increasing function. Alternatively, Discrimant $= (-12)^2 - 4(3)(15)$ $= -36 < 0$ Since discriminant $< 0 \Rightarrow f'(x) > 0$ and a>0, $\therefore f(x)$ is an increasing function.
(ii)	$f'(x) = 3ax^2 - 12x + 15$ For increasing function, $f'(x) > 0$ $3ax^2 - 12x + 15 > 0$ $\Rightarrow$ Discrimant < 0 $(-12)^2 - 4(3a)(15) < 0$ $144 - 180a < 0$ $a > \frac{4}{5}$

Qn	Key Steps
5(i)	$\frac{d}{dx}\left(\frac{e^{3x}}{\sqrt{x-1}}\right) = \frac{3e^{3x}\sqrt{x-1} - \frac{1}{2}(x-1)^{-\frac{1}{2}}e^{3x}}{x-1}$ $= \frac{e^{3x}}{x-1}\left(3\sqrt{x-1} - \frac{1}{2\sqrt{x-1}}\right)$ $= \frac{e^{3x}}{x-1}\left(\frac{6(x-1)-1}{2\sqrt{x-1}}\right)$ $= \frac{e^{3x}(6x-7)}{2\sqrt{(x-1)^3}}$
5(ii)	$\int_2^3 \frac{e^{3x}(6x-7)}{2\sqrt{(x-1)^3}} dx = \left[\frac{e^{3x}}{\sqrt{x-1}}\right]_2^3$ $\frac{1}{2}\int_2^3 \frac{e^{3x}(6x-7)}{\sqrt{(x-1)^3}} dx = \left[\frac{e^{3x}}{\sqrt{x-1}}\right]_2^3$ $\int_2^3 \frac{e^{3x}(6x-7)}{\sqrt{(x-1)^3}} dx = 2\left[\frac{e^{3x}}{\sqrt{x-1}}\right]_2^3$ $= 2\left[\frac{e^9}{\sqrt{2}} - e^6\right]$ $= 10652.6336$

Qn	Key Steps
6(i)	$S=\frac{1}{2}\times 2^2\sin(90°-\theta)+\frac{1}{2}\times 4^2\sin\theta+4\times 1$ $=\frac{1}{2}\times 4\cos\theta+\frac{1}{2}\times 16\sin\theta+4$ (or draw CX//AB $=2\cos\theta+8\sin\theta+4$ $\Rightarrow \angle XCE=\theta$)
(ii)	Let $2\cos\theta+8\sin\theta+4=R\cos(\theta-\alpha)$ $R=\sqrt{2^2+8^2}$ $=\sqrt{68}\approx 8.25$ (3 sig.fig.) $\tan\alpha=\frac{8}{2}$ $\alpha=75.9637°\approx 76.0°$(1 d.p.) $S=4+8.25\cos(\theta-76.0°)$
(iii)	Max $S=\sqrt{68}+4\approx 12.2\text{cm}^2$ $\cos(\theta-75.9637°)=1$ $\theta-75.9637°=0$ $\theta\approx 76.0°$ (1 d.p)
(iv)	$\sqrt{68}\cos(\theta-75.9637°)+4=11$ $\cos(\theta-75.9637°)=\frac{7}{\sqrt{68}}$ $\theta-75.9637°=-31.9105°, 31.9105°$ (NA) $\theta=44.0532°\approx 44.1°$(1 d.p)
7(i)	LHS $=\text{cosec } 2x+\cot 2x$ $=\frac{1}{\sin 2x}+\frac{\cos 2x}{\sin 2x}$ $=\frac{1+\cos 2x}{\sin 2x}$ $=\frac{1+2\cos^2 x-1}{2\sin x\cos x}$ $=\frac{2\cos^2 x}{2\sin x\cos x}$ $=\frac{\cos x}{\sin x}$ $=\cot x=$ RHS (shown)

Qn	Key Steps
(ii)	$\frac{1}{2}(\text{cosec } 2x+\cot x)=\cos x$ $\cot x=2\cos x$ $\frac{\cos x}{\sin x}=2\cos x$ $\cos x=2\cos x\sin x$ $\cos x(1-2\sin x)=0$ $\cos x=0$ $x=\frac{\pi}{2}$ or $1-2\sin x=0$ $\sin x=\frac{1}{2}$ $x=\frac{\pi}{6},\frac{5\pi}{6}$
(iii)(a)	$3\text{cosec}2\theta+3\cot 2\theta=2\text{cosec}\theta$ $\text{cosec}2\theta+\cot 2\theta=\frac{2}{3}\text{cosec}\theta$ $\cot\theta=\frac{2}{3}\text{cosec}\theta \Rightarrow \frac{\cos\theta}{\sin\theta}=\frac{2}{3\sin\theta}$ $\cos\theta=\frac{2}{3}$
(iii)(b)	$\sin\theta=\frac{\sqrt{5}}{3}$ $\sin 2\theta=2\sin\theta\cos\theta$ $=2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)$ $=\frac{4\sqrt{5}}{9}$
8(i)	$y=\sqrt{2x-9}-1$ $\frac{dy}{dx}=\frac{1}{2}(2x-9)^{-\frac{1}{2}}(2)$ $=\frac{1}{\sqrt{2x-9}}$ tangent at Q parallel to line, $\Rightarrow$ At Q, $\frac{dy}{dx}=1$ $\frac{1}{\sqrt{2x-9}}=1$ $2x-9=1$ $x=5$

Qn	Key Steps
(ii)	$x=5, y=0$ Equation of tangent is $y=x-5$
(iii)	$\sqrt{2x-9}-1=x-7$ $\sqrt{2x-9}=x-6$ $2x-9=(x-6)^2$ $2x-9=x^2-12x+36$ $x^2-14x+45=0$ $(x-9)(x-5)=0$ $x=9$ or $x=5$ (NA) $P=(9,2)$
(iv)	x-intercept of curve $=5$ x-intercept of line $=7$ Area under curve $=\int_5^9\left(\sqrt{2x-9}-1\right)dx$ $=\left[\frac{1}{3}(2x-9)^{\frac{3}{2}}-x\right]_5^9$ $=\left[\frac{1}{3}\left(9^{\frac{3}{2}}-9\right)-9\right]-\left[\frac{1}{3}-5\right]=\frac{14}{3}$ Area of triangle$=\frac{1}{2}(9-7)(2)=2$ Shaded area$=\frac{14}{3}-2=2\frac{2}{3}$ units^2
9(i)	$x^2+y^2-4x+2y-20=0$ $(x-2)^2-4+(y+1)^2-1=20$ $(x-2)^2+(y+1)^2=25$ Centre, C $=(2,-1)$ Radius $=5$
(ii)(a)	A$=(2-5,-1)=(-3,-1)$
(b)	Gradient of AB $=\frac{1}{3}$ Equation of AB is $y=\frac{1}{3}x$

Qn	Key Steps
(c)	$x^2+\left(\frac{1}{3}x\right)^2-4x+2\left(\frac{1}{3}x\right)-20=0$ $\frac{10}{9}x^2-\frac{10}{3}x-20=0$ $x^2-3x-18=0$ $(x-6)(x+3)=0$ $x=6$ or $x=-3$ (rejected) $y=2$ B $=(6,2)$
(d)	CX is extended of AC $\tan\angle BCX=\frac{2-(-1)}{6-2}=\frac{3}{4}$ or $\sin\angle ACB=\frac{3}{5}$; $\cos\angle AC$ $\angle BCX=36.869°$ or 0.64350 $\angle ACB=180-36.869$ or $(\pi-0.64350)$ radians $=143.1°$(1 d.p) or 2.50 radians (3 sig.fig.)
10(a)(i)	$\ln\left(\frac{50}{e^3}\right)$ $=\ln 50-\ln e^3$ $=\ln\left(5^2\times 2\right)-3\ln e$ $=\ln\left(5^2\times 2\right)-3$ $=2\ln 5+\ln 2-3$ $=2y+x-3$
(ii)	$\ln k=\frac{e^y-e^x}{2}$ $\ln k=\frac{e^{\ln 5}-e^{\ln 2}}{2}$ $\ln k=\frac{5-2}{2}$ $\ln k=\frac{3}{2}$ $k=e^{\frac{3}{2}}$

Qn	Key Steps
b(i)	$V = 80000(10)^{-kt}$ $65000 = 80000(10)^{-30k}$ $10^{-30k} = \frac{13}{16}$ $-30k = \lg\frac{13}{16}$ $k = -\frac{1}{30}\lg\frac{13}{16} \approx 0.003006$ (shown)
(ii)	$V = 80000(10)^{-0.003006t}$ $t = 20$ months, $V = 80000(10)^{20(-0.003006)}$ $= 69657.83737$ $\approx \$69700$ (nearest \$100)
(iii)	$80000(10)^{-0.003006t} = 40000$ $10^{-0.003006t} = \frac{1}{2}$ $-0.003006t = \lg\frac{1}{2}$ $t = 100.143 \approx 8.34$ years < 9 years Alternatively, $V = 80000(10)^{108(-0.003006)}$ $= \$37882.79266 < \$40000$ Yes