

Name: _____ ()

Class : Sec _____



TAMPINES SECONDARY SCHOOL
MID-YEAR EXAMINATION 2017
SECONDARY FOUR EXPRESS/4N1
SECONDARY FIVE NORMAL ACADEMIC

MATHEMATICS
PAPER 1
3 May 2017

4048/1
2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

You are expected to use a scientific calculator to evaluate explicit numerical expressions.

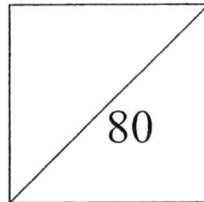
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total marks for this paper is 80.



This paper consists of 20 printed pages

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer all the questions.

1. Find the values of x where $0^\circ \leq x \leq 180^\circ$ for $\sin x = 0.866$, giving your answers correct to the nearest degree.

Answer $x = \dots\dots\dots$ [1]

2. Given that $3^{2017} + 3^{2017} + 3^{2017} = 3^x$, state the value of x .

Answer $x = \dots\dots\dots$ [1]

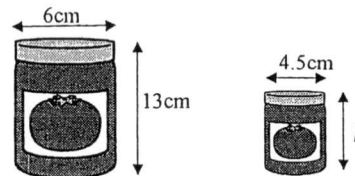
3. (a) Express 50 seconds as a percentage of 5 minutes.

Answer $\dots\dots\dots\%$ [1]

- (b) Express $62\frac{3}{5}\%$ as a decimal.

Answer $\dots\dots\dots$ [1]

4.



The two bottles of pasta sauce in the above diagram are geometrically similar. The larger bottle has a diameter of 6 cm and height 13 cm. The diameter of the smaller bottle is 4.5 cm.

- (a) Find the height, h , of the smaller bottle.

Answer (a) $\dots\dots\dots$ cm [1]

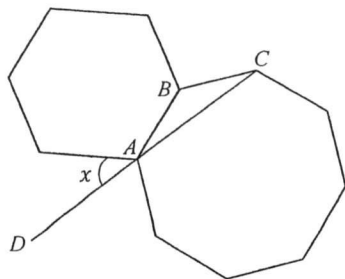
- (b) The larger bottle costs \$6 and the smaller bottle costs \$2.50. Which bottle is more value for money? Justify your answer with calculations.

Answer (b) $\dots\dots\dots$ [2]

5. Solve $x(2x + 7) = 4$.

Answer $x = \dots\dots\dots$ [3]

6.



A regular hexagon and regular octagon share a side AB as shown in the diagram above. DAC is a straight line. Calculate the size of angle x .

Answer $x = \dots\dots\dots^\circ$ [3]

7. The difference between the simple interest and the compound interest earned from a principal amount P , invested for a period of 3 years at 4% per annum is \$48.64. Find the principal amount P .

Answer \$..... [3]

8. Express the following as a single fraction in its simplest form

$$\frac{2}{(x+2)^2} + \frac{4}{2x+4}$$

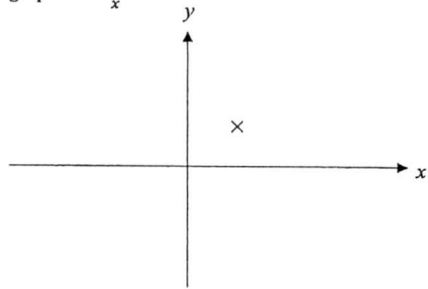
Answer [3]

9. Expand and simplify

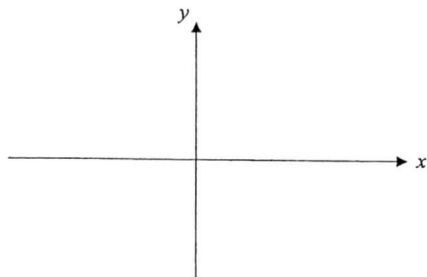
$$(x + 1)^2 - x(x - 2) + 1.$$

Answer [3]

10. (a) In the axes provided below, the point (1,1) has been marked.
Sketch the graph of $y = \frac{1}{x}$. [1]



(b) Sketch the graph of $y = (x + 3)(x - 5)$ in the axes provided, indicating the x -intercept(s) and y -intercept(s) if any. [2]



11.



Diagram 1

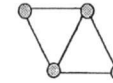


Diagram 2

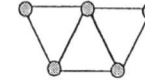


Diagram 3

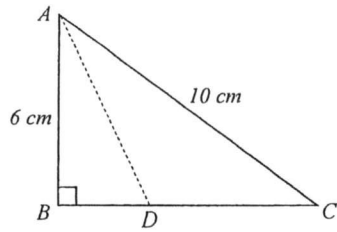
The figure above shows the number of sticks used to form each diagram.

(a) Draw diagram 4 in the space below. [1]

(b) Find an expression in terms of n , the number of sticks used to form Diagram n , reduced to its simplest form.

Answer (b)..... [2]

12.



In the diagram, $\angle ABC = 90^\circ$, $AB = 6$ cm and $AC = 10$ cm.

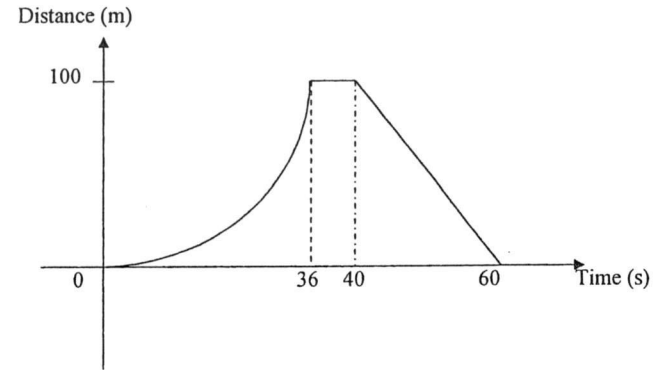
(a) Write down the exact value of $\sin \angle ACB$.

Answer $\sin \angle ACB = \dots\dots\dots$ [1]

(b) D is a point on BC such that $\angle BAD = \angle DAC$. Find the length of BD .

Answer $\dots\dots\dots$ cm [2]

13. The diagram shows the distance-time graph of a cyclist.



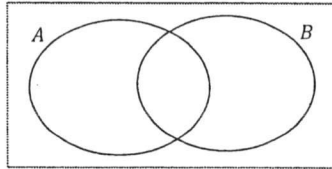
(a) Describe the motion of the cyclist for the first 40 seconds.

Answer (a) $\dots\dots\dots$
 $\dots\dots\dots$ [2]

(b) Calculate the average speed of the cyclist for the entire 60 seconds.

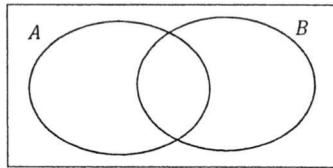
Answer (b) $\dots\dots\dots$ m/s [1]

14. (a)



(i) On the Venn diagram above, shade the region that represents $A' \cap B$. [1]

(ii) On the Venn diagram below, draw the set K where $K \subset B$ and $A \cap K \neq \emptyset$. [1]



(c) Let $\varepsilon = \{x: x \text{ is an integer and } 2 < x \leq 10\}$

$A = \{x: x \text{ is an odd number}\}$

$B = \{x: x \text{ is a prime number}\}$

List the elements of

(i) $A' \cup B$

Answer (b)(i)..... [1]

(ii) $(A \cup B)'$

Answer (b)(ii)..... [1]

15. The arrival frequency of trains at a train interchange is shown in the table below:

Train	Frequency
North Line	Every 3 minutes
South Line	Every 4 minutes
East Line	Every 6 minutes
West Line	Every 7 minutes

If all four trains arrived together at 8am,

(a) find the next time that all four trains will arrive at the same time at the interchange.

Answer (a)..... [2]

(b) Calculate the total number of times all four trains arrive at the interchange from 8am to 8pm inclusive on a particular day.

Answer (b)..... [2]

16. Given that $(x + y)^2 = 1$ and $4xy = 15$,
(a) find an expression for $x^2 + y^2$.

Answer (a)..... [1]

- (b) Hence, find the value of $(2x + 3y)^2 - 5y^2$.

Answer (b)..... [3]

17. (a) Express 1323 as a product of its prime factors.

Answer (a) [1]

- (b) Given that $1323k$ is both a perfect square and a perfect cube, write down the smallest integer value of k .

Answer (b)..... [1]

- (c) The highest common factor of 1323 and another integer x is 9 and the lowest common multiple of both numbers is 2646. Find the value of x .

Answer (c)..... [2]

18. The variables x , y and z are related.
 x is directly proportional to the cube of y and y is inversely proportional to the square root of z .

Given that when $x = 216$, $y = 4$ and $z = 4$,

- (a) find an expression for
 (i) z in terms of y ,

Answer (a)(i)..... [2]

- (ii) y in terms of x .

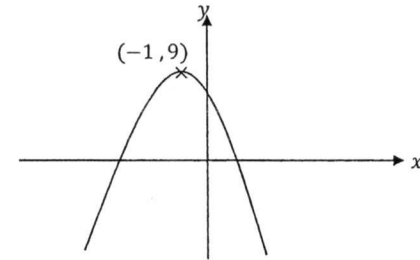
Answer (a)(ii)..... [2]

(b) Hence or otherwise, express z in terms of x and describe the relationship between x and z .

Answer (b)..... [2]

Relationship : [1]

19.



The diagram shows the graph of $y = -x^2 + bx + c$, where b and c are constants. Given that the turning point of the graph is $(-1, 9)$,

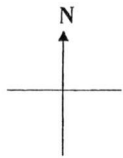
(a) state the equation of line of symmetry of the graph

Answer [1]

(b) find the constants b and c .

Answer $b =$ $c =$ [4]

20. Building A is at a bearing 030° from Building C.
 Building B is at a bearing 120° from Building C.
 Given that $AC = 40\text{m}$ and $BC = 100\text{m}$,
 (a) Use a scale of 1cm to 10m , make an accurate scale drawing to create triangle ABC .
 [2]



×
C

- (b) From your drawing, find the actual distance, in metres, between buildings A and B.

Answer (b).....m [1]

- (c) By drawing a perpendicular bisector and an angle bisector, identify and mark the point P that is equidistant from B and C , and from AC and BC . [2]

21. The table below shows the results of 3 students for their Elementary Mathematics Preliminary Examination.
 The total possible mark for Paper 1 and 2 is 80 and 100 respectively.
 Papers 1 and 2 have equal weightage.

Student	Paper 1	Paper 2
Ali	50	65
Adam	60	58
Alex	30	70

- (a) Represent the data by a 3×2 matrix, A .

Answer (a)..... [1]

- (b) Another matrix Y is represented by $\begin{pmatrix} 0.625 \\ 0.5 \end{pmatrix}$.

- (i) Find AY .

Answer (b)(i)..... [2]

- (ii) Explain how the two numbers in Y came about.

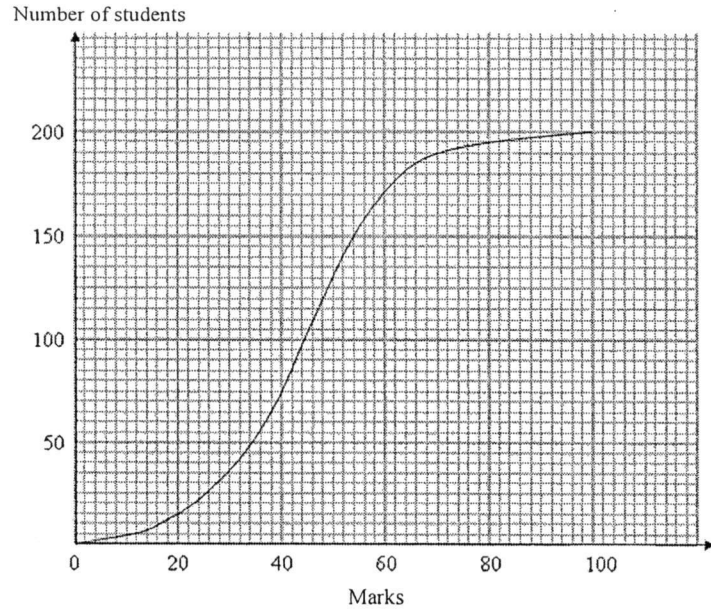
Answer (b)(ii).....
 [2]

- (iii) Describe what the entries in AY represent.

Answer (b)(iii).....
 [1]

22. Part (c) of this question is on the next page.

The cumulative frequency curve below shows the marks obtained by 200 students in a Mathematics examination.

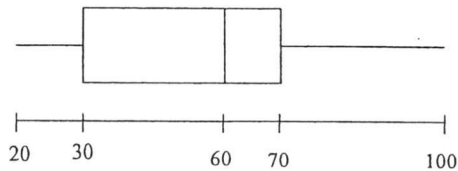


- (a) Use the graph to find
 (i) the median mark
 (ii) the interquartile range.

Answer (a)(i)..... [1]

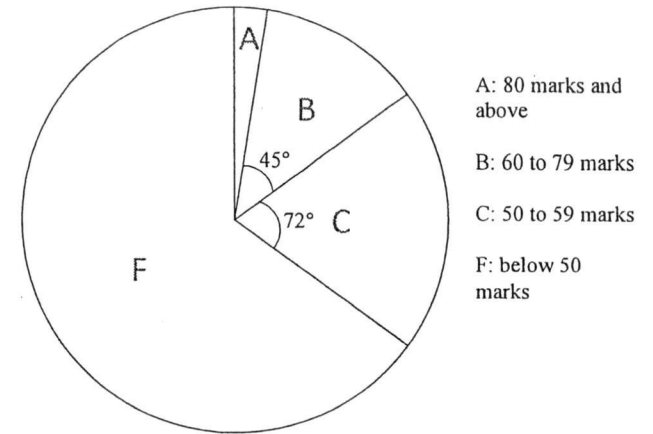
(ii)..... [1]

- (b) The following box and whisker diagram illustrates the marks obtained when the same group of students sat for a Science examination.
 Which subject did the students do better in? Explain why.



Answer (b)
 [2]

- (c) The mathematics examination data can also be represented by a pie chart.
 Calculate the missing angles that represent the areas for A and F.



Answer (c) A:°

F:° [3]

THE END

Answer all the questions.

0.866

For
Examiner's
Use

1. Find the values of x where $0^\circ \leq x \leq 180^\circ$ for $\sin x = 0.866$, giving your answers correct to the nearest degree.

Answer $x = 60^\circ$ or 120° [1]

(B1)
for both

2. Given that $3^{2017} + 3^{2017} + 3^{2017} = 3^x$, state the value of x .

$$3(3^{2017}) = 3^x$$

$$3^{2018} = 3^x$$

Answer $x = 2018$ [1]

(B1)

3. (a) Express 50 seconds as a percentage of 5 minutes.

$$\frac{50}{5 \times 60} \times 100 =$$

$$\frac{50}{3}$$

Answer $16\frac{2}{3}$ or 16.7 % [1]

(B1)

- (a) Express $62\frac{3}{5}$ % as a decimal.

$$\frac{62\frac{3}{5}}{100} =$$

Answer 0.626 [1]

(B1)

Q4 a) $\frac{6}{4.5} = \frac{13}{h}$

$h = 9\frac{3}{4}$ or 9.75 (B1)

→ take $\frac{\text{vol}}{\text{cost}}$ or $\frac{\text{cost}}{\text{vol}}$.

b) $\left(\frac{6}{4.5}\right)^3 = \frac{64}{27}$

(M1) find $\frac{V_1}{V_2}$ & use for comparison.

larger bottle

64 units → \$6

1 unit → \$0.09375

10.6667 ← \$1 unit

smaller bottle

27 units → \$2.50

1 unit → \$0.09259

10.8 unit ← \$1

ANS: smaller bottle (A1)

5 Solve $x(2x + 7) = 4$.

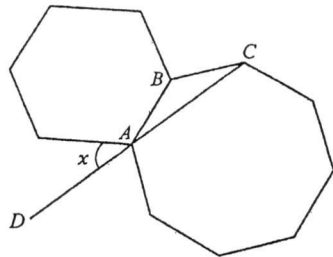
$2x^2 + 7x - 4 = 0$ (M1) expand & move RHS = 0
 $(2x-1)(x+4) = 0$ (M1) factorisation or any relevant mtd.

$x = \frac{1}{2}$ or $x = -4$

(A1) for both.

4

* 2.6



A regular hexagon and regular octagon share a side AB as shown in the diagram above. DAC is a straight line. Calculate the size of angle x .

either 1 (M1)

$$\left\{ \begin{array}{l} \text{int } \angle \text{ of hexagon} = \frac{(6-2) \times 180^\circ}{6} \\ \text{int } \angle \text{ of octagon} = \frac{(8-2) \times 180^\circ}{8} \end{array} \right.$$

$$= 120^\circ \quad x = 180^\circ - 120^\circ - 22.5^\circ$$

$$= 135^\circ \quad = 37.5^\circ \quad \text{(A1)}$$

(M1) $\angle BAC = \frac{180^\circ - 135^\circ}{2} = 22.5^\circ$

Answer $x = 37.5^\circ$ [3]

Answer

* 7.

The difference between the simple interest and the compound interest earned from a principal amount P , invested for a period of 3 years at 4% per annum is \$48.64. Find the principal amount P .

simple interest: $I = \frac{P(4)(3)}{100}$

cpd interest: $I = P\left(1 + \frac{4}{100}\right)^3 - P$

either 1 (M1) and interest for either simple / cpd.

(M1) $P\left(1 + \frac{4}{100}\right)^3 - P - \frac{12P}{100} = 48.64$

recognise that cpd interest is > simple interest \therefore cpd I - simple I = 48.64

$$\frac{76}{15625} P = 48.64$$

$$P = 10000 \quad \text{(A1)}$$

Answer \$10,000 [3]

8. Express the following as a single fraction in its simplest form

$$\frac{2}{(x+2)^2} + \frac{4}{2x+4}$$

$$\frac{2}{(x+2)^2} + \frac{4}{2(x+2)}$$

$$= \frac{4}{2(x+2)^2} + \frac{4(x+2)}{2(x+2)^2}$$

(M1) make denominator e same & multiply numerators by relevant factors

$$= \frac{4 + 4x + 8}{2(x+2)^2}$$

(M1) $= \frac{4x + 12}{2(x+2)^2} = \frac{2x + 6}{(x+2)^2} \quad \text{(A1)}$

combine into 1 fraction

Page 5

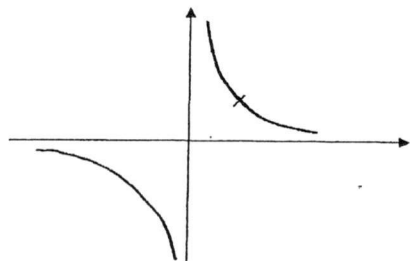
Answer [3]

9. Expand and simplify $(x+1)^2 - x(x-2) + 1$.

$$\begin{aligned}
 &= \overset{\text{(M1)}}{x^2 + 2x + 1} - \overset{\text{(M1)}}{x^2 + 2x} + 1 \\
 &= 4x + 2 \quad \text{(A1)} \\
 &\text{accept } 2(2x+1)
 \end{aligned}$$

Answer [3]

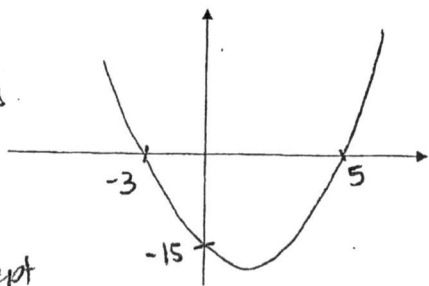
10. (a) In the axes provided below, the point (1,1) has been marked. Sketch the graph of $y = \frac{1}{x}$. [1]



(B1) must pass through (1,1).

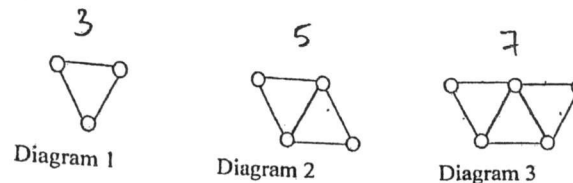
(b) Sketch the graph of $y = (x+3)(x-5)$ in the axes provided, indicating the ~~coordinates of the~~ x-intercept(s) and y-intercept(s) if any. [2]

(B1) shape
 (B1) all intercepts labelled
 * min pt cannot be at y-intercept (if not, -1)



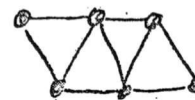
when $y=0$,
 $x = -3$ or 5
 when $x=0$,
 $y = -15$

11.



The figure above shows the number of sticks used to form each diagram.

(a) Draw diagram 4 in the space below. [1]

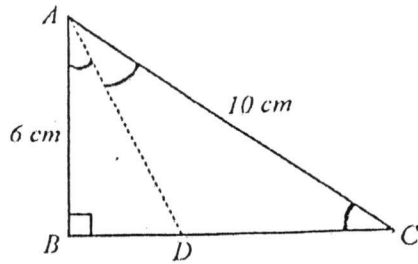


(B1)

(b) Find an expression in terms of n , the number of sticks used to form Diagram n , reduced to its simplest form.

$$\begin{aligned}
 &3 + 2(n-1) \quad \text{(M1)} \\
 &= 3 + 2n - 2 \\
 &= 2n + 1 \quad \text{(A1)} \\
 &\text{or } \text{(B2)}
 \end{aligned}$$

12.



In the diagram, $\angle ABC = 90^\circ$, $AB = 6$ cm and $AC = 10$ cm.
 (a) Write down the exact value of $\sin \angle ACB$.

$$\sin \angle ACB = \frac{3}{5} \text{ or } 0.6$$

Answer [

* (b) D is a point on BC such that $\angle BAD = \angle DAC$. Find the length of BD .

$$\cos \angle CAB = \frac{6}{10}$$

$$\angle CAB = 53.130^\circ \text{ (M1) find } \angle CAB$$

$$\tan \angle BAD = \frac{BD}{6}$$

$$BD = 6 \tan \left(\frac{53.130^\circ}{2} \right)$$

$$= 3$$

accept 3.00

(A1) answer is exact (no rounding off)

Answer cr

13.

a) 0-36sec : move with increasing speed (B1)

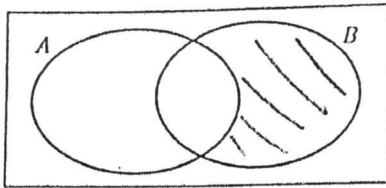
36-40sec : stationary / at rest (B1)

(b)

$$\text{average speed} = \frac{200}{60}$$

$$= 3\frac{1}{3} \text{ m/s. (B1)}$$

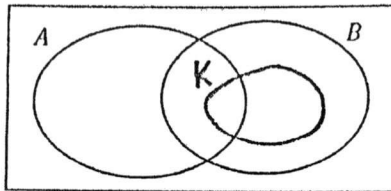
14. (a)



(B1)

(i) On the Venn diagram above, shade the region that represents $A' \cap B$. [1]

(ii) On the Venn diagram below, draw the set K where $K \subset B$ and $A \cap K \neq \emptyset$. [1]



(B1)

- (b) Let $\varepsilon = \{x: x \text{ is an integer and } 2 < x \leq 10\}$
 $A = \{x: x \text{ is an odd number}\}$
 $B = \{x: x \text{ is a prime number}\}$.

List the elements of

(i) $A' \cup B$

J (B1)

Answer (b)(i)..... 3, 4, 5, 6, 7, 8, 10 [1]

(ii) $(A \cup B)'$

(B1)

Answer (b)(ii)..... 4, 6, 8, 10 [1]

15. The arrival frequency of trains at a train interchange is shown in the table below:

Train	Frequency
North Line	Every 3 minutes
South Line	Every 4 minutes
East Line	Every 6 minutes
West Line	Every 7 minutes

If all four trains arrived together at 8am,

(a) find the next time that all four trains will arrive at the same time at the interchange.

$$\begin{array}{r} 3 \overline{) 3, 4, 6, 7} \\ 2 \overline{) 1, 4, 2, 7} \\ 2 \overline{) 1, 2, 1, 7} \\ 7 \overline{) 1, 1, 1, 7} \\ \hline 1, 1, 1, 1 \end{array}$$

LCM = 84 min. (M1)
 = 1h 24 min

Answer (a)..... 9.24 am (A1) [2]

(b) Calculate the total number of times all four trains arrive at the interchange from 8am to 8pm inclusive on a particular day.

12 hours = 720 min.

(M1)
 $\frac{720}{84} = 8\frac{4}{7}$

no. of times = 8 + 1
 = 9

Answer (b)..... 9 (A1) [2]

no: 1.

16. Given that $(x + y)^2 = 1$ and $4xy = 15$,
 (a) find an expression for $x^2 + y^2$.

$$x^2 + 2xy + y^2 = 1$$

$$x^2 + y^2 = 1 - 2xy$$

(B1)

Answer (a)..... [1]

(b) Hence, find the value of $(2x + 3y)^2 - 5y^2$.

$$= 4x^2 + 12xy + 9y^2 - 5y^2 \quad \text{(M1) expand correctly}$$

$$= 4x^2 + 12xy + 4y^2$$

$$= 4(x^2 + y^2) + 12xy \quad \text{(M1) factorise}$$

$$= 4(1 - 2xy) + 12xy$$

$$= 4 - 8xy + 12xy$$

$$= 4 + 4xy$$

$$= 4 + 15$$

$$= 19 \quad \text{(M1)}$$

Answer (b)..... [3]

For
 Examiner's
 Use

17. (a) Express 1323 as a product of its prime factors.

$$\begin{array}{r} 3 \overline{) 1323} \\ 3 \overline{) 441} \\ 3 \overline{) 147} \\ 7 \overline{) 49} \\ 7 \end{array}$$

(B1)

Answer (a)..... $3^3 \times 7^2$ [1]

*(b) Given that $1323k$ is both a perfect square and a perfect cube, write down the smallest integer value of k .

(B1)

Answer (b)..... $3^3 \times 7^4$ or 64827 [1]

*(c) The highest common factor of 1323 and another integer x is 9 and the lowest common multiple of both numbers is ~~1323~~. Find the value of x .

2646

$$\begin{array}{l} x = 2 \times 3^2 \\ 1323 = 3^3 \times 7^2 \\ \hline \text{HCF} = 3^2 \\ \text{LCM} = 2 \times 3^3 \times 7^2 \end{array}$$

(M1) comparing index notation.

$$\begin{array}{l} x = 2 \times 9 \\ = 18 \quad \text{(A1)} \end{array}$$

For
 Examiner's
 Use

18. The variables x , y and z are related. x is directly proportional to the cube of y and y is inversely proportional to the square root of z .

Given that when $x = 216$, $y = 4$ and $z = 4$,

(a) find an expression for
(i) z in terms of y ,

~~11~~
~~11~~
~~11~~
~~11~~
~~11~~

$$y = \frac{k}{\sqrt{z}}$$

$$4 = \frac{k}{\sqrt{4}}$$

$$k = 8 \text{ (M1)}$$

$$x = ky^3$$

$$216 = k(4)^3$$

$$k = \frac{27}{8} \text{ (M1)}$$

Answer (a)(i)..... [1] (2m)

(ii) y in terms of x .

$$y = k\sqrt[3]{x}$$

$$4 = k\sqrt[3]{216}$$

$$k = \frac{4}{6} \text{ (M1)}$$

$$= \frac{2}{3} \text{ (M1)}$$

or

$$x = ky^3$$

$$216 = k(4)^3$$

$$k = \frac{27}{8} \text{ (M1)}$$

Answer (a)(ii)..... [1] (2m)

(b) Hence or otherwise, express z in terms of x and describe the relationship between x and z .

~~11~~
~~11~~
~~11~~
~~11~~
~~11~~

$$z = \left(\frac{27}{8}\sqrt[3]{x}\right)^2 \text{ (M1) ecf.}$$

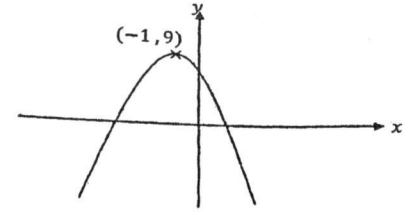
$$= \frac{64}{9}\sqrt[3]{x^2}$$

$$= \frac{144}{3\sqrt{x^2}} \text{ (M1)}$$

Answer (b)..... [2]

Relationship: z is inversely proportional to the cube root of x^2 (ecf) [1]

19.



The diagram shows the graph of $y = -x^2 + bx + c$, where a and b are constants. Given that the turning point of the graph is $(-1, 9)$,
(a) state the equation of line of symmetry of the graph

Answer $x = -1$ [1] (B1)

* (b) find the constants b and c .

$$y = -(x-h)^2 + k$$

$$= -(x+1)^2 + 9 \text{ (M1) sub. max pt into } y = -(x-h)^2 + k$$

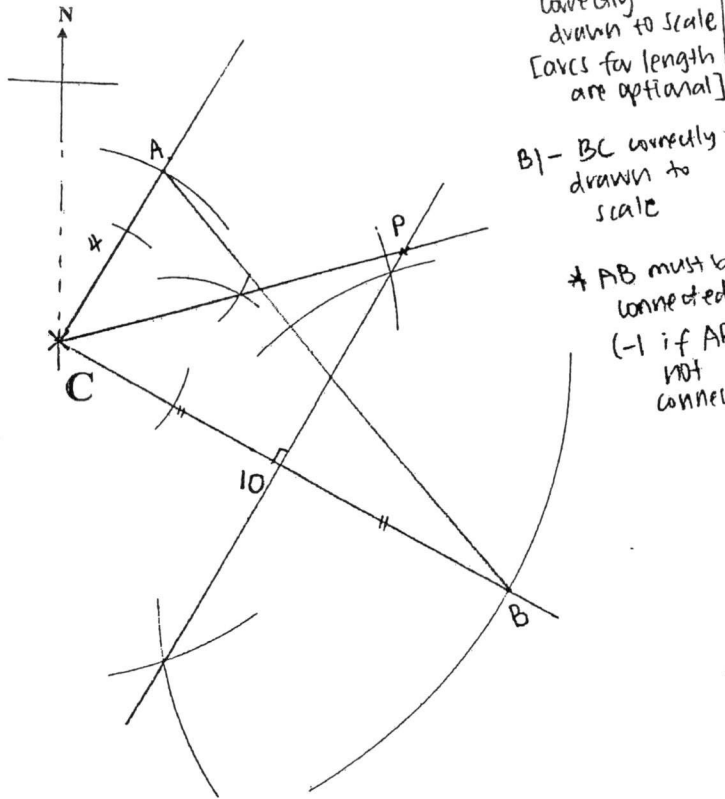
$$= -(x^2 + 2x + 1) + 9 \text{ (M1) expand}$$

$$= -x^2 - 2x - 1 + 9$$

$$= -x^2 - 2x + 8$$

Answer $b = -2$ $c = 8$ [4]

(a) Use a scale of 1cm to 10m, make an accurate scale drawing to create triangle ABC.



B1 - AC correctly drawn to scale
[marks for length are optional]

B1 - BC correctly drawn to scale

* AB must be connected (-1 if AB not connected)

(b) From your drawing, find the actual distance, in metres, between buildings A and B.

B. $AB = 10.8 \text{ cm} \rightarrow 108 \text{ m}$
 $\pm 0.1 \quad \pm 1$

(B1)

Answer (b) ... 108 ... m [1]

(c) By ~~drawing~~ ^{constructing} a perpendicular bisector and an angle bisector, identify and mark the point P that is equidistant from B and C, and from AC and BC. [2]

↑
-1 if not marked

21. The table below shows the results of 3 students for their Elementary Mathematics Preliminary Examination. The total possible mark for Paper 1 and 2 is 80 and 100 respectively. Papers 1 and 2 have equal weightage.

Student	Paper 1	Paper 2
Ali	50	65
Adam	60	58
Alex	30	70

(a) Represent the data by a 3 x 2 matrix, A.

Answer (a) $A = \begin{pmatrix} 50 & 65 \\ 60 & 58 \\ 30 & 70 \end{pmatrix}$ [1]

(B1)

(b) Another matrix Y is represented by $\begin{pmatrix} 0.625 \\ 0.5 \end{pmatrix}$.

(i) Find AY.

$$\begin{pmatrix} 50 & 65 \\ 60 & 58 \\ 30 & 70 \end{pmatrix} \begin{pmatrix} 0.625 \\ 0.5 \end{pmatrix}$$

$$= \begin{pmatrix} 50(0.625) + 65(0.5) \\ 60(0.625) + 58(0.5) \\ 30(0.625) + 70(0.5) \end{pmatrix} \text{ (M)}$$

$$= \begin{pmatrix} 63.75 \\ 66.5 \\ 53.75 \end{pmatrix} \text{ (A)}$$

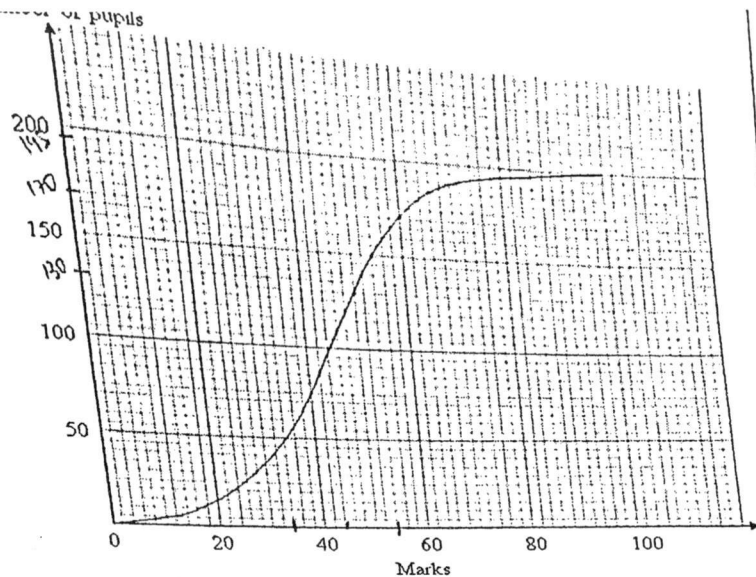
Answer (b)(i) [2]

* (ii) ~~What do the 2 numbers in Y represent?~~
 Explain how the entries in Y came about.

Answer (b)(ii) $0.625 = \frac{1}{80} \times 100 \times 50\%$ (B1)
 $0.5 = \frac{1}{100} \times 100 \times 50\%$ (B1) [2]

* (iii) Describe what the entries in PY represent.

Answer (b)(iii) Total marks for the whole exam for each student respectively. (B1) [1]

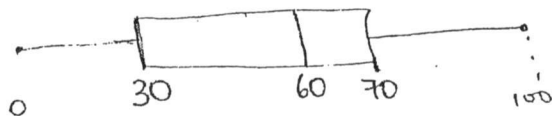


- (a) Use the graph to find
 (i) the median mark
 (ii) the interquartile range.

54 - 34

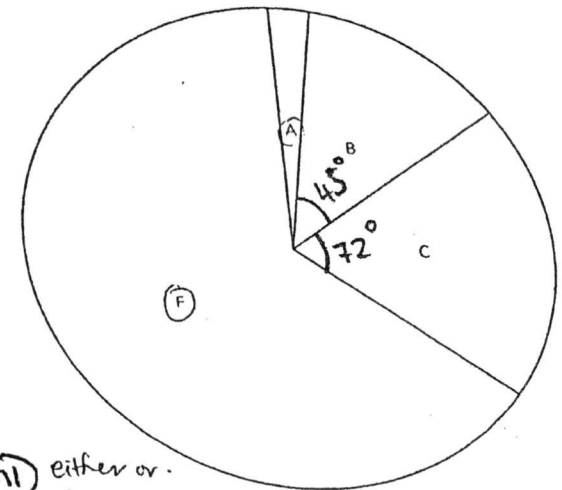
(accept 45)
 44 (B) [1]
 Answer (a)(i)..... [1]
 (ii)..... 20 (B) [1]

- (b) The following box and whisker diagram illustrates the marks obtained when the same group of students sat for a Science examination. Which subject did the students do better in? Explain your answer in 2 ways. [2]



Science - median higher -
 (B) (B)

- (C) The mathematics examination data can also be represented by a pie chart. Calculate the missing angles that represent each area. [3]



- A: 80 marks and above
 B: 60 to ~~74~~⁷⁹ marks
 C: 50 to 59 marks
 F: below 50 marks

(M) either or.
 $A = \frac{5}{200} \times 360^\circ = 9^\circ$ (A)
 $F = \frac{130}{200} \times 360^\circ = 234^\circ$ (A)

Name: _____ ()

Class : Sec _____

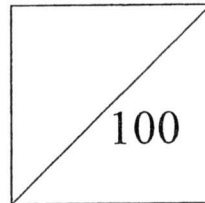


TAMPINES SECONDARY SCHOOL
MID-YEAR EXAMINATION 2017
SECONDARY FOUR EXPRESS/4N1
SECONDARY FIVE NORMAL (ACADEMIC)

MATHEMATICS
PAPER 2
4 May 2017

4048/2
2 hours 30 minutes

Additional Materials:
Answer Paper
Graph Paper (1 sheet)



READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 100.

This paper consists of 11 printed pages

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

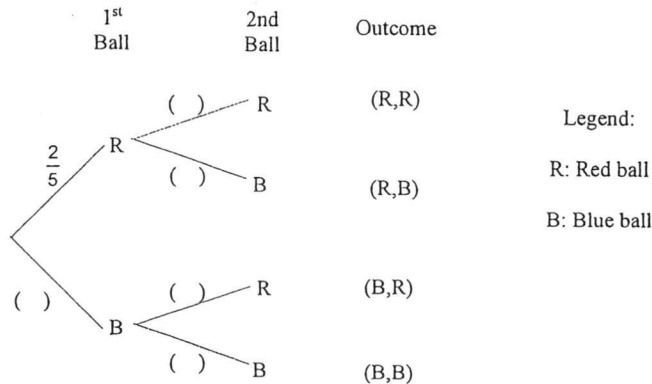
$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer all the questions.

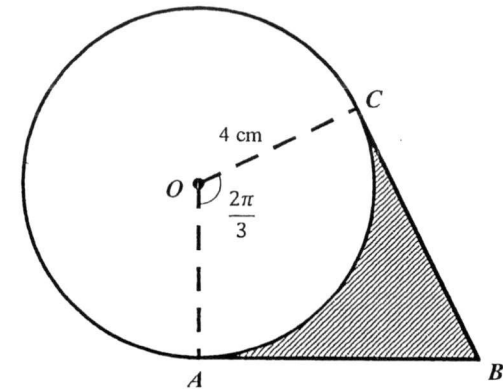
- 1 (a) Make z the subject of the formula [3]
- $$v = \frac{2uz - 3u}{5z + 2}$$
- (b) (i) Factorise $9(x - 1)^2 - 64$ completely. [3]
 (ii) Hence, find two factors of 18161, other than 1 and 18161. [2]

- 2 Sally had 4 red balls and 3 blue balls initially. She bought more balls such that the number of blue balls bought is twice the number of red balls bought. All the balls are placed in a box and one is randomly picked.
- (a) Given that the probability that she picked a red ball is $\frac{2}{5}$, find the number of red and blue balls that Sally bought. [3]
- (b) Sally replaced the ball. She then randomly picked two balls out of the box, one after another, with replacement.
- (i) Copy and complete the following probability tree diagram. [2]



- (ii) Find the probability that Sally picked two balls of the same colour. [2]

3(a)



The diagram above shows a circle with radius 4 cm. AB and BC are tangents to the circle at A and C respectively. Given that $\angle COA = \frac{2\pi}{3}$ radians, calculate

(i) the perimeter of the shaded region, [3]
 (ii) the area of the shaded region. [4]

(b)

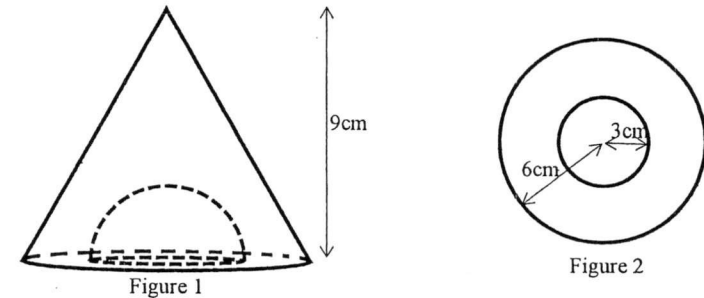
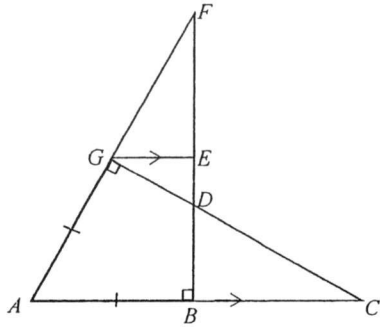


Figure 1 shows a solid ornament. Figure 2 shows the base of the ornament. The ornament is a solid cone of height 9 cm and radius 6 cm with a hemispherical hole of radius 3 cm. Find the surface area of the ornament. [3]

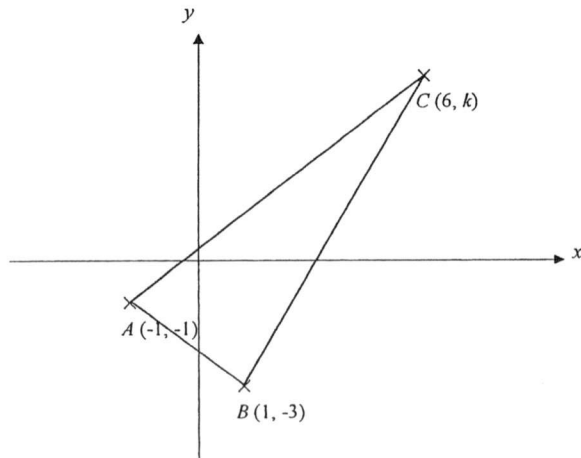
4



In the diagram, $\angle ABF = \angle AGC = 90^\circ$, GE is parallel to AC and $AG = AB$. G is the midpoint of AF .

- (a) Prove that $\triangle ABF \cong \triangle AGC$. [2]
- (b) Name two pairs of similar triangles. [2]
- (c) Given that $\angle BFA = 30^\circ$ and $AC = 26$ cm, find
 - (i) the length of BF , [2]
 - (ii) $\frac{\text{area of } \triangle GFE}{\text{area of } AGE B}$ [2]

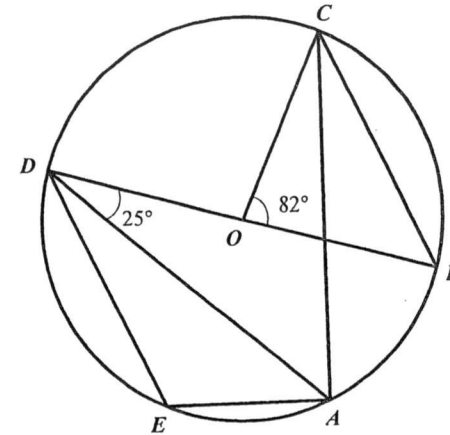
5



The diagram shows a triangle ABC with $A(-1, -1)$, $B(1, -3)$ and $C(6, k)$. The gradient of AB is $-2n$ and the length of BC is 13 units. Find

- (a) the value of n , [2]
- (b) the value of k [3]
- (c) the equation of the line AB . [2]
- (d) one possible coordinates of the point D if D is vertically above B and $ABCD$ is a trapezium. [4]

6



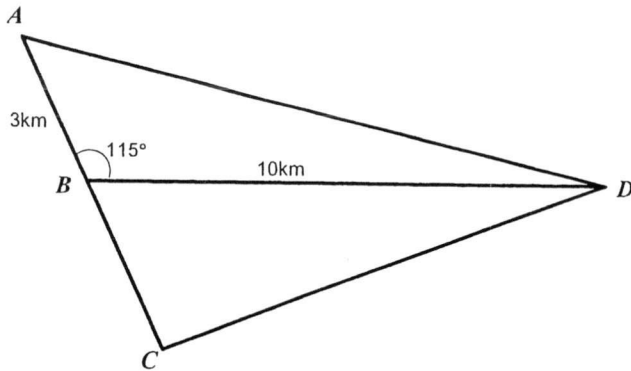
Points A, B, C, D and E lie on a circle with centre O .

BD is a diameter of the circle.

ED is parallel to BC , $\angle ADB = 25^\circ$ and $\angle COB = 82^\circ$.

- (a) Giving your reasons, calculate
 - (i) $\angle CBO$ [1]
 - (ii) $\angle CAD$ [1]
 - (iii) $\angle ADE$ [1]
 - (iv) $\angle AED$ [2]
- (b) Prove that CO produced will pass through E . [3]
- (c) Given that the radius of the circle is 3 cm, calculate the perpendicular distance between the chords DE and BC . [3]

7



The diagram shows four points A, B, C and D on a flat piece of land. ABC is a straight line.

Given that $AB = 3$ km, $BD = 10$ km and $\angle ABD = 115^\circ$,

(a) calculate the length of AD . [3]

It is also known that the area of $\triangle BCD = 15.9$ km².

(b) Find the length of BC . [2]

A man walked along BD , from B until he reached a point E , where CE is the shortest distance from C to BD .

(c) Calculate the distance he walked. [3]

The same man flies a drone vertically above E such that the angle of elevation from C to the drone is 14° .

(d) Find the angle of depression from the drone to D . [3]

8 A plane flew a distance of 1700km from Singapore to Bali at an average speed of x km/h.

(a) Write down an expression in terms of x , for the time it took in hours to complete the journey. [1]

The same plane returned via the same route with an average speed that is 100km/h faster.

(b) Write down an expression in terms of x , for the time taken in hours to complete the return journey. [1]

Given that the return journey took 22 minutes shorter than the journey from Singapore to Bali,

(c) form an equation in x and show that it reduces to $11x^2 + 1100x - 5100000 = 0$. [3]

(d) Solve the equation $11x^2 + 1100x - 5100000 = 0$, leaving your answers correct to 1 decimal place. [3]

(e) If the plane departs from Singapore to Bali at 8am and stays in Bali for 3 hours for maintenance before returning back to Singapore, find the time that the plane will arrive back in Singapore. [4]

9 Answer this question on a piece of graph paper.

The variables x and y are connected by the equation $y = \frac{x^2}{8} + \frac{4}{x} - 4$.

The table below shows some values of x and the corresponding y values corrected to 2 decimal places.

x	1	2	3	4	5	6	7
y	0.13	-1.5	-1.54	-1	p	1.17	2.70

- (a) Calculate the value of p . [1]
- (b) Using a scale of 2cm to represent 1 unit on the horizontal x -axis and 4cm to represent 1 unit on the vertical y -axis, draw the graph of $y = \frac{x^2}{8} + \frac{4}{x} - 4$ for $1 \leq x \leq 7$. [3]
- (c) By drawing another straight line, find the solution to $\frac{x^2}{8} + \frac{4}{x} - 6 = 0$. [2]
- (d) Find the gradient of the curve at $x = 4$. [2]
- (e) (i) On the same axes, draw the graph of $y = -x + 2$. [1]
(ii) Write down the x -coordinate of the point of intersection of the line and the curve. [1]
(iii) This value of x is a solution to the equation $ax^3 + bx^2 - 6x + 4 = 0$. Find the values of a and b . [2]

10 Paul wants to travel from Tampines Secondary School to Marina Barrage at a certain time of day. He has decided to hire a private car to bring him to the location. His three choices of car companies are Comfy, Ubab and Grer.

The pricing information for each of his choices and other relevant information are given in the next page.

Assuming he wants to make the trip at 4pm on a Monday,

- (a) calculate the amount he will need to pay if he chooses Grer. [1]
(b) Justify which company you would recommend to be the most economical for Paul to choose for this trip. Show your calculations clearly. [6]

If Paul wants to make the trip at 6.30pm on a Wednesday instead,

- (c) would you recommend a different company to be more economical for Paul to choose? Why or why not? [3]

Distance between Tampines Secondary School and Marina Barrage		23km
Average time to travel from Tampines Secondary School to Marina Barrage	During non-peak period	25 minutes
	During peak period	30minutes

Comfy Taxi

Base Fare (1km or less)		\$3.20
Every 400m thereafter or less, up to 10km		\$0.22
Every 350m thereafter or less, after 10km		\$0.22
Every 45 seconds of waiting time or less		\$0.22
Peak period surcharge:	Monday to Friday, 6am – 9.29am Monday to Sunday, 6pm to 11.59pm	25% of meter fare
	Monday to Sunday, midnight – 5.59am	50% of meter fare

Ubab

Base Fare	\$3
Per km	\$0.45
Per minute	\$0.20

Grer

Base Fare	\$3
Per km	\$0.80

End of Paper

1 (a)

$$V = \frac{2uz - 3u}{5z + 2}$$

$$V(5z + 2) = 2uz - 3u \quad (M1)$$

$$5Vz + 2V = 2uz - 3u$$

$$5Vz - 2uz = -2V - 3u$$

$$z(5V - 2u) = -2V - 3u \quad (M1)$$

$$z = \frac{-2V - 3u}{5V - 2u} \quad (A1) \quad \text{or} \quad \frac{2V + 3u}{2u - 5V} \quad \text{or} \quad -\frac{2V + 3u}{5V - 2u}$$

(b) (i) $9(x-1)^2 - 64$

$$= [3(x-1)]^2 - 8^2 \quad (M1)$$

$$= [3(x-1) + 8][3(x-1) - 8] \quad (M1) \quad \text{or}$$

$$= (3x+5)(3x-11) \quad (A1)$$

$$9x^2 - 18x - 55$$

$$= (3x+5)(3x-11)$$

(ii) $9(x-1)^2 - 64 = 18161$

$$(x-1)^2 = 2025$$

$$x-1 = 45$$

$$x = 46 \quad (M1)$$

Ans: $3(46) + 5$ and $3(46) - 11$

$$= 143 \quad \text{and} \quad 127 \quad (A1)$$

other acceptable answers

$$11, 1651,$$

$$13, 1397$$

2 (a) $P(\text{red ball}) = \frac{2}{5}$

let x be the no. of red balls bought.

$$\frac{4+x}{7+3x} = \frac{2}{5} \quad (M1)$$

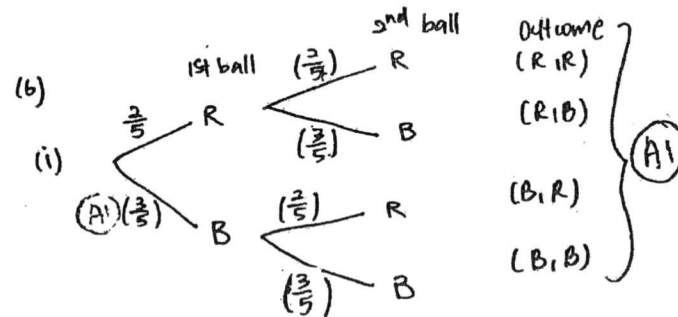
$$5(4+x) = 2(7+3x)$$

$$20+5x = 14+6x$$

$$6 = x$$

no. of red balls bought = 6 (A1)

no. of blue balls bought = 12 (A1)



total no. of red = 10

total no. of blue = 15

(ii) $P(\text{both balls same colour}) = \frac{2}{5} \times \frac{2}{5} + \frac{3}{5} \times \frac{2}{5}$ (M1)

$$= \frac{13}{25} \quad (A1)$$

3 (a)(i) length of minor arc AC = $4\left(\frac{2\pi}{3}\right)$ (M)
 $= 8.37758$

$\tan\left(\frac{\pi}{3}\right) = \frac{BC}{4}$

$BC = 4\tan\left(\frac{\pi}{3}\right)$ (M)
 $= 6.9282$

Perimeter of shaded region = $8.37758 + 2 \times 6.9282$
 $= 22.23398$
 $\approx 22.2 \text{ cm}$ (A)

(a)(ii) area of half of minor sector COA = $\frac{1}{2}(4)^2\left(\frac{\pi}{3}\right)$ (M) $\approx \frac{1}{2}(4)^2\left(\frac{2\pi}{3}\right)$
 $= \frac{8\pi}{3} \quad 8.3776$
 $= 16.755$

area of $\triangle OAB = \frac{1}{2} \times 4 \times 4\tan\left(\frac{\pi}{3}\right)$ (M) 27.713
 $= 8\tan\left(\frac{\pi}{3}\right)$

area of shaded region = $\left[\begin{matrix} 8\tan\left(\frac{\pi}{3}\right) \\ 13.856 \end{matrix} - \frac{8\pi}{3} \right] \times 2$ (M)
 $= 10.9577 - 8.3776 \times 2$
 $\approx 11.0 \text{ cm}^2$ (A)

(b) surface area of ornament = $\frac{\pi(6)(\sqrt{117})}{203.88}$ (M) + either one (M) $[\pi(6)^2 - \pi(3)^2] + 2\pi(3)^2$
 $= 345.26$ 84.83 56.549
 $\approx 345 \text{ cm}^2$ (A)

4 (a) $AG = AB$ (given) } (M)
 $\angle ABF = \angle AGC = 90^\circ$ (given) }
 $\angle GAC = \angle BAF$ (common) }
 $\therefore \triangle ABF \equiv \triangle AGC$ (AAS) (M)

(b) $\triangle EGD$ and $\triangle BCD$ (B) $\triangle GDE$ and $\triangle CDB$
 $\triangle FGE$ and $\triangle FAB$ (B) $\triangle FGE$ and $\triangle GDE$
 etc.

(c) (i) $BF = GC$.
 $\sin 30^\circ = \frac{GC}{26}$ (M)
 $GC = 26 \sin 30^\circ$
 $BF = 13$ (A)

(ii) $\frac{\text{area of } \triangle GFE}{\text{area of } \triangle AFG} = \left(\frac{1}{2}\right)^2$ (M)
 $= \frac{1}{4}$

$\frac{\text{area of } \triangle GFE}{\text{area of } \triangle GEB} = \frac{1}{3}$ (A)

Q4(c) (i) * Triangle should not exist — it does not follow trig ratio.
 Question should have been $\angle BFA = 30^\circ$, not $\angle BAF$.
 Because of this error, the following answers are all acceptable.

① Using Pyth Thm:

$$BF = \sqrt{26^2 - 13^2}$$

$$= 22.5166605$$

$$\approx 22.5 \quad (M1)$$

$$(M1)$$

$$\downarrow$$

$$BF^2 = 26^2 - 13^2$$

② Using TOA CAH SOH

$$\tan 30^\circ = \frac{BF}{13} \quad (M1)$$

$$BF = 13 \tan 30^\circ$$

$$\approx 7.51 \quad (A1)$$

③ Using sine rule

$$\frac{BF}{\sin 30^\circ} = \frac{26}{\sin 90^\circ} \quad (M1)$$

$$BF = 26 \sin 30^\circ$$

$$= 13 \quad (A1)$$

④ Using cosine rule:

$$BF^2 = 26^2 + 13^2 - 2(26)(13) \cos 30^\circ \quad (M1)$$

$$BF^2 = 259.566827$$

$$BF \approx 16.1 \quad (A1)$$

5(a) gradient of AB = $\frac{-3+1}{1-(-1)} \quad (M1)$

$$= \frac{-2}{2}$$

$$= -1$$

$$n = \frac{1}{2} \quad (A1)$$

(b) length of BC = 13

$$\sqrt{(6-1)^2 + (k+3)^2} = 13 \quad (M1)$$

$$\sqrt{25 + (k+3)^2} = 13$$

$$25 + (k+3)^2 = 169$$

$$(k+3)^2 = 144$$

$$k+3 = 12 \quad \text{or} \quad -12$$

$$k = 9 \quad \text{or} \quad -15 \quad (\text{rejected})$$

(A1)

or

$$k^2 + 6k - 135 = 0$$

$$(k-9)(k+15) = 0 \quad (M1)$$

(c) equation of AB: $y = mx + c$

$$y = -x + c$$

Sub $(-1, -1)$:

$$-1 = -(-1) + c$$

$$c = -2$$

$$\therefore \text{eqn of AB: } y = -x - 2$$

(d) $D(1, y) \quad (M1)$

grad of CD = grad of AB

$$\frac{y-9}{1-6} = -1 \quad (M1)$$

$$y-9 = 5$$

$$y = 14 \quad (A1)$$

$$D(1, 14) \quad (A1)$$

or grad of AD = grad of BC

$$\frac{y+1}{1+1} = \frac{9+3}{6-1}$$

$$\frac{y+1}{2} = \frac{12}{5}$$

$$y = 3.8$$

$$D(1, 3.8)$$

(a) (i) $\angle CBD = \frac{180^\circ - 82^\circ}{2}$ (base \angle of isos Δ)
 $= 49^\circ$ (B1)

(ii) $\angle CAD = 49^\circ$ (\angle in same seg.) (B1)

(iii) $\angle ADE = 49^\circ - 25^\circ$ (\angle H \angle , $ED \parallel BC$)
 $= 24^\circ$ (B1)

(iv) $\angle BED = 90^\circ$ (\angle in semicircle) } (M1)
 $\angle BEA = 25^\circ$ (\angle in same seg.) }
 $\angle AED = 90^\circ + 25^\circ$
 $= 115^\circ$ (A1)

(b) $\angle DAE = 180^\circ - 115^\circ - 24^\circ$ (\angle sum of Δ) (M1)
 $= 41^\circ$

$\angle CAE = 41^\circ + 49^\circ$ (M1)
 $= 90^\circ$

since $\angle CAE = 90^\circ$, CE is diameter (\angle in semicircle)
 \rightarrow CO produced will pass through E.

(c) Let x be the \perp distance from BC to O.

$\sin 49^\circ = \frac{x}{3}$ (M1)

$x = 3 \sin 49^\circ$

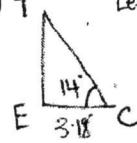
perpendicular dist between DE and BC = $2 \times 3 \sin 49^\circ$ (M1)
 $= 4.53 \text{ cm}$ (to 3sf) (A1)

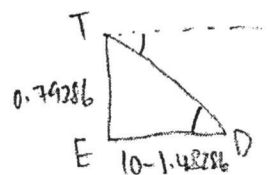
(a) $AD^2 = 3^2 + 10^2 - 2(3)(10) \cos 115^\circ$ (M1) (M1) sqrt
 $AD = 11.5913$
 $\approx 11.6 \text{ km}$ (A1)

(b) $\frac{1}{2} \times BC \times 10 \sin 65^\circ = 15.9$ (M1)
 $BC = 3.50874$
 $\approx 3.51 \text{ km}$ (A1)

(c) $\frac{1}{2} \times 10 \times CE = 15.9$
 $CE = 3.18$ (M1)
 $\tan 65^\circ = \frac{3.18}{BE}$ (M1)
 $BE = \frac{3.18}{\tan 65^\circ}$
 $= 1.48286$
 $\approx 1.48 \text{ km}$
 He walked 1.48 km (A1)

(d) T Let T be the position of the drone.

 $\tan 14^\circ = \frac{TE}{3.18}$ (M1)
 $TE = 3.18 \tan 14^\circ$
 $= 0.79286$

 $ED = 10 - 1.48286$ (M1)
 $= 8.51714$
 $\tan \angle TDE = \frac{0.79286}{8.51714}$
 $\angle TDE = 5.3183$
 $\approx 5.3^\circ$

\angle of depression from drone to D = 5.3° (A1)

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(a) $\frac{1700}{x}$ h (B1)

(b) $\frac{1700}{x+100}$ h (B1)

(c) $\frac{1700}{x} - \frac{1700}{x+100} = \frac{22}{60}$ (M1)

$$\frac{1700(x+100) - 1700x}{x^2 + 100x} = \frac{22}{60} \quad (M1)$$

$$170000(60) = 22x^2 + 2200x$$

$$22x^2 + 2200x - 10200000 = 0 \quad (M1)$$

$$11x^2 + 1100x - 5100000 = 0 \text{ (shown)}$$

(d) $11x^2 + 1100x - 5100000 = 0$

$$x = \frac{-1100 \pm \sqrt{1100^2 - 4(11)(-5100000)}}{2(11)}$$

$$= 632.7 \text{ or } -732.7$$

(A1)

(A1)

(e) SG to Bali $\cdot \frac{1700}{632.7} = 2\text{h } 41\text{min}$ (M1)

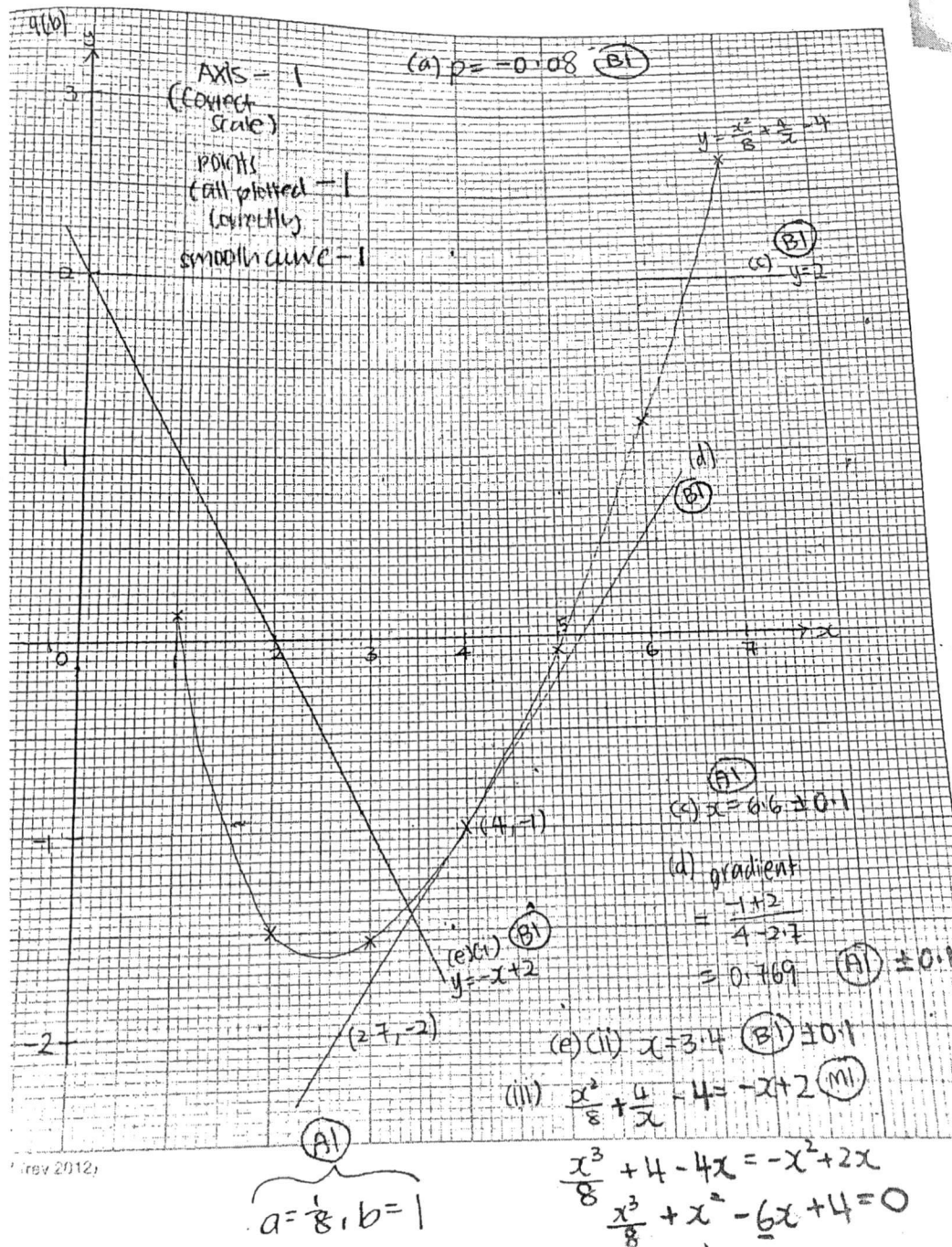
stop over = 3h

Depart Bali : 8am + 2h 41min + 3h = 1.41 pm (M1)

Bali to SG $\cdot \frac{1700}{732.7} = 2\text{h } 19\text{min}$ (M1)

Arrival Time: 1.41 pm + 2h 19min = 4 pm. (A1)

(M1) show correct subⁿ of values



Q9 e iii) question paper has a typo.

the equation is supposed to be $ax^3 + bx^2 - 6x + 4 = 0$
not $ax^3 + bx^2 - 8x + 4 = 0$.

(M) was given to equate eqⁿ of curve to eqⁿ of line.

remaining (A) would be given if student had tried to manipulate the equation and reduce it as much as possible.

a) amount to pay = $\$3 + \$0.80(23)$
 $= \underline{\$21.40}$. (B)

b) Ubab

amount to pay = $\$3 + \$0.45(23) + \$0.20(25)$
 $= \underline{\$18.35}$. (B)

Comfy

base fare = $\$3.20$

first 10km : $\frac{10000}{400} = 25$ } (M)
 $25 \times \$0.22 = \underline{\$5.50}$

Next 13km : $\frac{13000}{350} = 37.14$ } (M)
 $38 \times \$0.22 = \underline{\$8.36}$

Waiting time : $\frac{25 \times 60}{45} = 33.33$

$\$0.22 \times 34 = \underline{\$7.48}$.

either (M)
for calculating
base fare
or
waiting time

total cost = $\$3.20 + \$5.50 + \$8.36 + \7.48
 $= \underline{\$24.54}$. (M)

Conclusion : Ubab (A)

1016)
12) Comfy.

$$\begin{aligned} \text{new waiting} & \\ \text{time} & \quad \frac{30 \times 60}{45} \\ & = 40. \end{aligned}$$

$$\begin{aligned} \text{new waiting} & \\ \text{time} & = 40 \times \$0.22 \\ \text{charge} & = \$8.80. \end{aligned}$$

$$\begin{aligned} \text{meter fare} & = \$3.20 + \$5.50 + \$8.36 + \$8.80 \\ & = \$25.86. \end{aligned}$$

$$\begin{aligned} \text{price after} & \\ \text{surcharge} & = \$25.86 \times 125\% \\ & = \$32.33. \end{aligned}$$

MI for showing
waiting to
calculate
surcharge

Ubab.

$$\begin{aligned} \text{amount to} & = \$3 + \$0.45(23) + \$0.20(30) \\ \text{pay} & = \$19.35 \end{aligned}$$

(MI)

Conclusion: Stay with Ubab
because still the
cheapest.

(A1)